Numerical Optimization

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Lecture - 25
Second Order KKT Conditions

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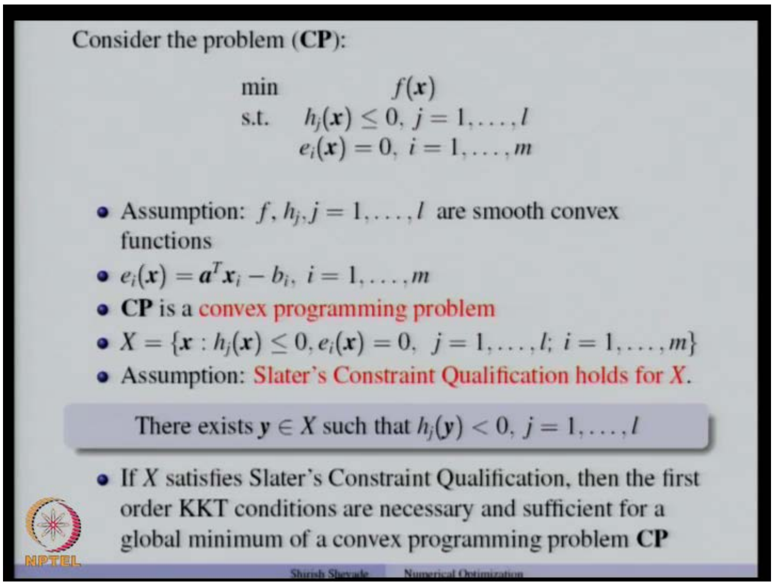

Hello, welcome back. In the last class, we discussed about convex programming problem and in particular, we considered the problem on this type where we want to minimize f of x subject to some inequality constraints and equality constraints. Now, they are essentials that we need what of that f and h j x are smooth convex functions and e is the equality constraints. Related functions are assigned functions of the type a i transpose x i minus b i. Then, under this condition, these problems become convex programming problem and remember that for the convex programming problem, we minimize a convex function which is subject to constraints which is a convex set.

So, this set x is a convex set which is an intersection of any convex sets and we have earlier shown that Slater's set is a convex set and we assume that Slater's constraint qualification holds for x. So, that is to exist at least one point which in the interior of the set or in other words, the set x has a non-interior.

Only to say that Slater's constraint qualification holds that exists some y in the set x, such that y is less than 0 and as I mentioned last time that when y belongs to x. So, that means that clear e i y equals to 0 or all i going from 1 to m, so that condition is always satisfied. So, it is just to make sure that h j of y is less than 0 for all z n, that is there exists upon which is in the interior of the set and you can say Slater's constraint qualification holds. So, if Slater's constraint qualification holds, the first order KKT conditions are necessary and sufficient for a global minimum of a convex programming problem, and we saw the proof of this which was a step forward extension of the earlier proofs where we considered the convex programming problem for minimizing f x subject to h j are equal to j i.

Now, what do the meaning of Lagrange multipliers associated with a non-linear programming problem? So, let us try to see that interpretation.

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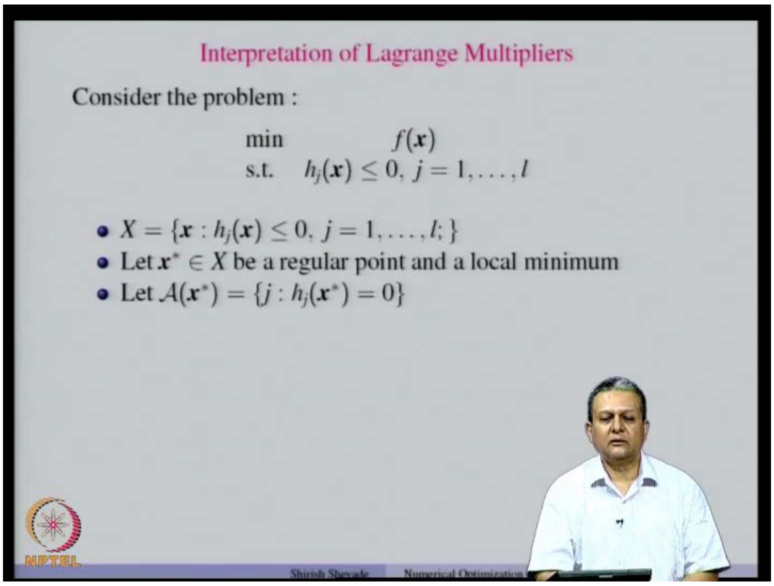

For the time being, let us consider this problem where to minimize f of x subject to the constraints h j of x less than or equal to 0 and the ideas that we are going to study can be easily extended to a general non-linear programming problem, where we also have equality constraint. So, let us denote the constraints set as usual by the single capital X and let us assume that x star belongs to x in a regular point and is also local minimum. So, let us consider the active set at x star, the group sets of all j's such that h j x star equal to 0. Now, when you want to study the interpretation of Lagrange multipliers, so what you want to know is that if any of these constraints update, then what is the effect of that on the objective function, optimal objective function?

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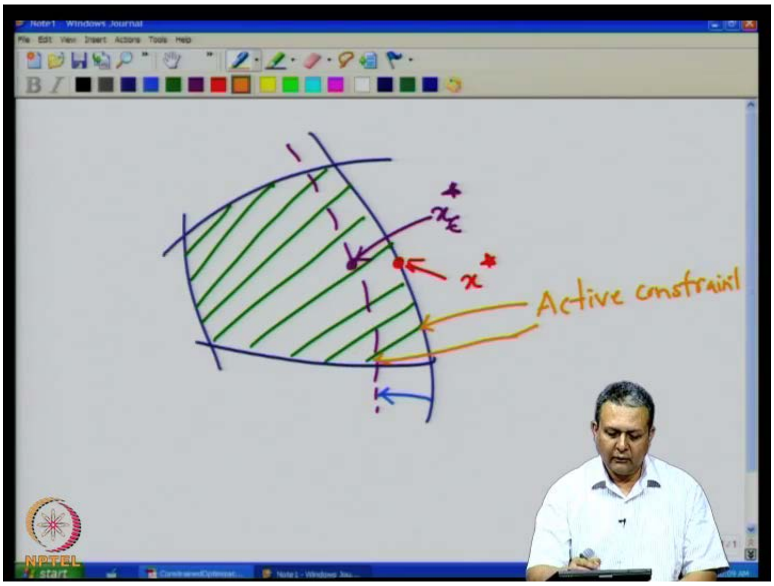

In other words, suppose we have a constraint set. So, our constraint set is the points on the boundary as well as the interior of the set. Now, suppose let us take a point which is p square. So, suppose for some objective function, this point is our solution. Now, at this point, only this constraint is active. So, the only constraint which is active at x star is this constraint.

Now, other constraints are inactive. That means the other constraint, this constraint, this constraint and this constraint, they follow h j x star less than 0, while this constraint follows h j x star 0 or it is satisfied with the equality. Now, suppose we perturb this constraint update. So, suppose we perturb this constraint to something like this and we get a new solution which is somewhere, suppose this point is at the new solution. Now, remember that this is the new solution. The same constraint is active, this constraint which was here which is not perturbed to this constraint.

So, let us assume that at the new solution, the same constraint is active. In other words, to perturb the constraint by a small amount, the state of active constraints does not change at this new term. So, suppose we perturb this constraint by some amount epsilon, then let us call this point as x star epsilon and remember that the same constraint. So, this was the active constraints x star and now, this is the same constraint which is active at X star epsilon. Now, one crude way to check the change in the objective function is to resolve the problem, but without resolving the problem, can we get a rough idea about how much will be the change in the objective function if we perturb this constraint by epsilon and that is what we want to study.

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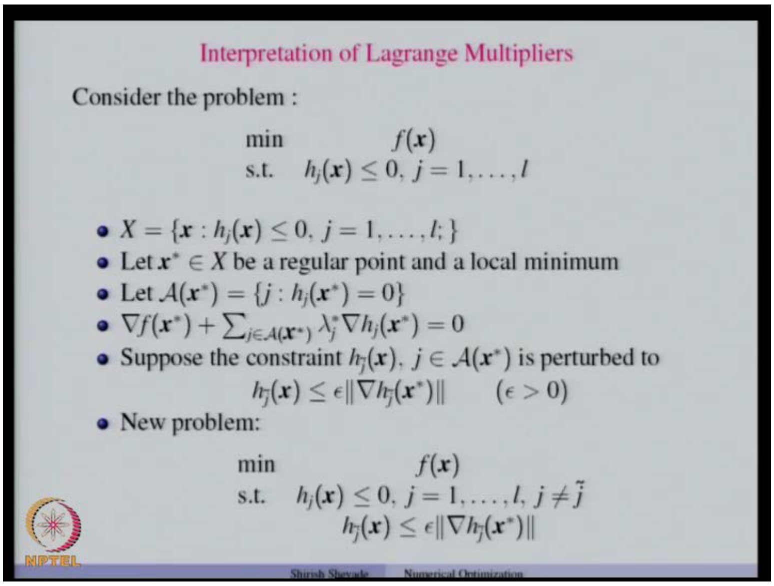

So, x star is a regular point and is a local minimum. So, the first order condition are satisfied and in other words, gradient f of x star plus sigma j belongs to the active x star lambda j gradient h j x star is 0 or in other words, x star lambda star is a (( )) required.

Now, suppose you perturb the constraint of bit and we write a new constraint as h j x less than or equal to epsilon into norm normal gradient h j x star. Now, this norm is taken just to do the normalization. So, let us assume that epsilon is greater than 0. Now, how much effect such perturbation of the constraint will have? So, here we have picked one constraint which is active (( )) till this is a constraint at that is perturbed. So, this should be j h belongs to a x star and this is going to be our new constraint. So, the problem that now we are going to be looking at is to minimize f x subject to the same constraint that we had except the constraint associated with j theta. So, the same constraint holds for this and for this h j (( )) constraint h j theta x is less than or equal to epsilon into norm of h j theta x star. So, only constraint got changed as showed in the diagram that only one active constraint was perturbed. Now, how much will be changed in the objective function that is what we want to see.

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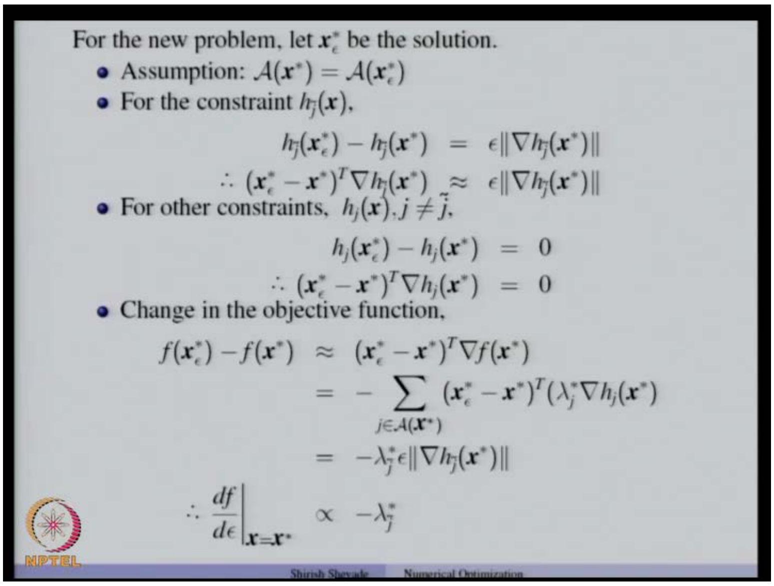

So, let us assume that x star epsilon is the solution of the new problem because the procedure is one of the constraints, one of the active constraints with perturbed, suppose that means in the x star epsilon. Now, one important assumption we make is that the state of active constraint at x star at the state of active constraints at x star epsilon are the same. So, the perturbation is not large, but it is small enough, so that the state of active constraints remains the same.

Now, if you look at the constraint h j till the x which is perturbed. We know that h j till x star was 0 because that was the active constraint and h j till the epsilon x star epsilon is nothing, but epsilon norm of h j till the x star. So, h j till the x star epsilon minus h j x star is quantity by which the constraint got perturbed. Now, if you use the first order Taylor series approximation, then the quantity on the left side is intact. If we write exactly the h j till the x star epsilon as an approximate as assign approximation of h j till at x star. So, what we get is as till the x star epsilon equal to h j till the x star plus x star epsilon minus x star transpose gradient h j x star. So, this quantity on the left side is nothing, but epsilon minus x star transpose gradient h j till the x star and that is approximately equal to the quantity on the right side. Note that we are talking about first order approximation and that is why approximation sign is there.

Now, for the other constraints which were active at x star, they also were active at x star epsilon because we changed only one constraint. For those constraints, this quantity is the difference between these two is 0 or in other words, if we again do the first order Taylor series approximation, so this is approximately equal to 0. So, now let us look at the change in the objective function. So, we can write f of x star epsilon as an affine approximation around x star and that will be f of x star plus x star epsilon minus x star transpose gradient of x star.

Now, here we use the (( )) condition. Now, we know that x star is a (( )) x star (( )) point. So, gradient f x star can be written as gradiative of sigma gradient lambda x star gradient h j star, where z over the set of active constraints. So, this quantity is written as this minus sigma j belong to x star minus x star epsilon minus x star transpose lambda based star gradient of h j and we have seen that except the j theta constraint, this quantity x star epsilon minus x star transpose gradient h j x star is 0 except constraint for j theta constraint. This will be lambda x star epsilon into gradient exactly the x star. Now, epsilon is the small positive quantity. Now, we can take epsilon in the left side and then, take the limit as epsilon tends to 0 and therefore, what we get is d f by d epsilon at x equal to x star is proportional to negative of the Lagrange multiplier for j theta constraint.

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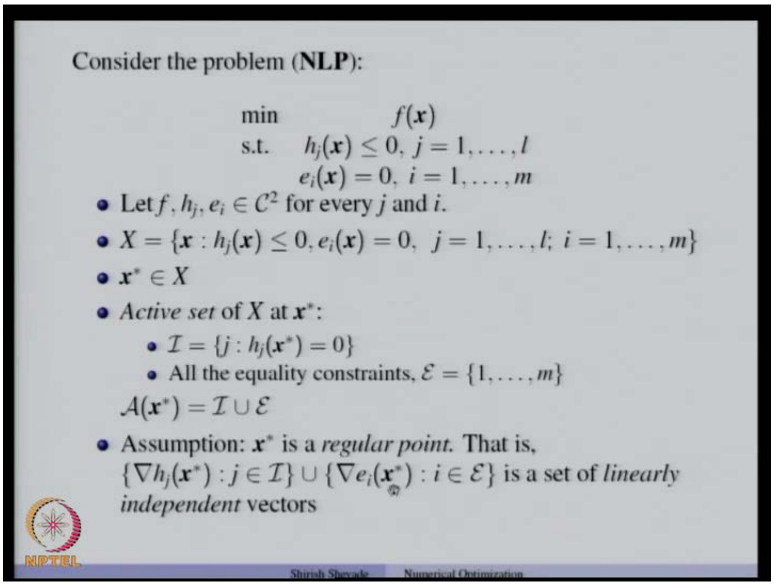

So, what it means is that it would perturb one particular constraint. The change in the small amount, the change in the objective function is proportional to the negative of the Lagrange multiplier of that constraint. So, this gives some idea about how much will be the change in the objective function when a particular constraint is perturbed, but not that all this analysis was done based on the assumption that active set remains the same at the new solution. If the active set changes, then this analysis does not hold, but the important point to note is that this Lagrange multiplier gives some idea about the change in the objective function if a particular constraint is perturb. So, if the Lagrange multiplier for a particular constraint is small, the change in the objective function could be small. If that constraint is perturbed, this is also called economic interpretation of Lagrange multiplier or sensitivity analysis.

Now, let us go back to then on liner programming problem that we considered, a general non-linear programming problem where we remain f x subject to the constraint h j x could be 0 and e i x could be 0, and we saw the first order KKT conditions for the scrolling and now, we will see the second order conditions for this problem. These conditions will see without leaving any details of the proof. So, let us assume that f h j and e i, they belong to class of twice continuously differentiable functions and let us denote the constraints by the symbol capital X, and let us take a point x star which is feasible. So, as we saw last time, the active set of x at x star is the set of all the equality constraints of the type h j x star equals to 0, and all the equality constraints we have, n equality constraints we have m equality constraints. So, all those constraints constitute the set of active equality constraints. So, this put together we get the active set x star to be union of the two sets i and e.

So, we assume that x star is a regular point. So, that is the gradient h j x star j belongs to i and gradient e i x star i belongs to e is a set of linearly independent vectors.

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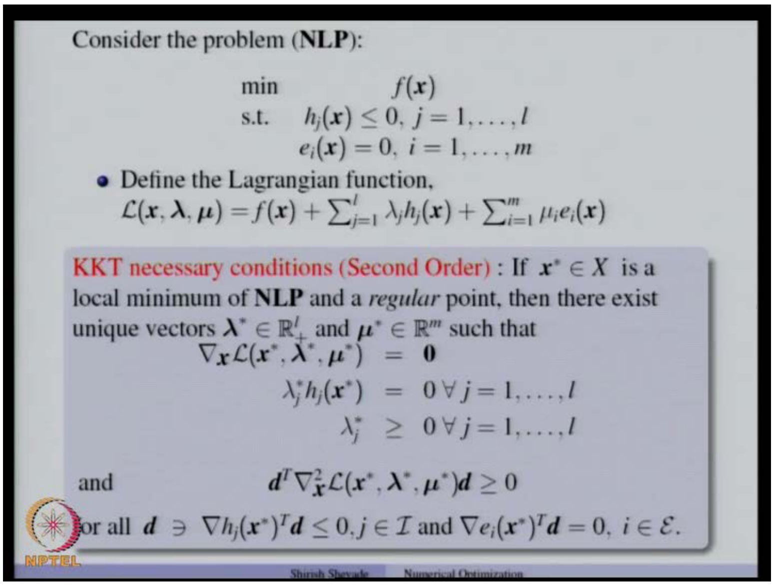

Now, we define the lagrangian function as a function of x lambda and mu and the function is defined as objective function plus linear combinations of the inequality constraints plus the linear combination of the equality constraints. We remember that whenever we define the lagrangian function and constraints are written in the form, the inequality constraints are written in the form h j x less than or equal to 0. Then, in this lagrangian, this lambda says which are the lagrangian multipliers corresponding to the inequality constraints are non-negative under this rep functional representations and if you write the lagrangian like this, this lambda says the non-negative and note also that there is no sign restriction on the mu's which are associated with the equality constraints. So, there are line quality constraints. So, there are l lambdas which are non negative and there are n equality constraints. So, there are n mu's which are not restricted in signs.

Now, let us look at the second order necessary conditions for this non-linear programming problem. If x star is a feasible point is a local minimum of this n l p and is also a regular point, then there exist unique vectors lambda star mu star, such that the gradient of the lagrangian vanish with respect to x vanishes at star lambda star and star and in addition to that, the complementary condition hold that is lambda j star h j x star is 0 for all inequality constraint and lambda j star are non negative.

So, I have already mentioned here that lambda star belongs to r l plus. So, what it means is that it belongs to the set of non negative real numbers, but this condition is again explicitly stated here. Now, as I mentioned earlier many times, you will see that the feasibility conditions of x star are also written here that h j x star less than or equal to 0 and e i x star equal to 0, but we have already assumed that x star belongs to x. That means, those conditions are always we are considering x star such that those conditions are satisfied. So, those conditions are not explicitly mentioned here.

So, these were the part of the first order condition. Now, if you recall when you studied unconstrained optimization, the first order necessary condition was that if x star is a local minimum and then, gradient f x star equals to 0. So, here instead of gradient f x star, we have gradient of m with respect to x evaluated at x star lambda star mu star is 0. So, this algebraic condition is easy to verify.

Now, the second order conditions have some additional details and those details are that if you consider the hessian matrix of the lagrangian evaluated at x star lambda star mu star. Then, that hessian matrix is positive semi-definite along the direction d, such that gradient h j x star transpose d less than or equal to 0 where j belongs to i and gradient e i x star transpose d is equal to 0 for i in the set of equality constraints. So, if we take the active inequality constraints, then for those constraints if we write down the set of directions d, such that gradient h j x star transpose d less than or equal to 0 and for all the active (( )), for all the equality constraints if we consider those d satisfy gradient e i x star transpose d equal to 0. Now, this d put together for most at and we chose any d form that set and the hessian of the Lagrange should be positive semi-definite allow any of those directions. So, these are further second order KKT necessary conditions.

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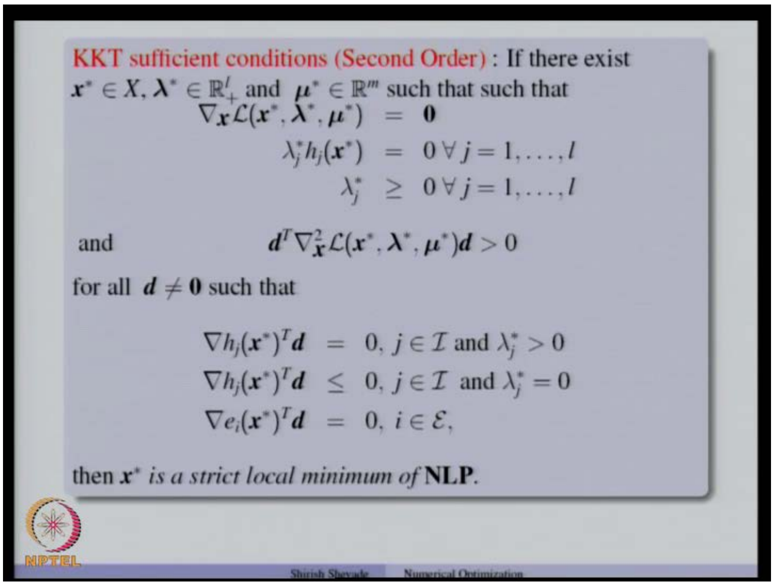

Now, let us look at the second order sufficient conditions for langrage program. Now, there exist x star which is feasible lambda star which is vector of non negative real numbers and we star in r n because there are m equality constraint. So, u star in r n such that this condition whole gradient of the langrage zero complementary (( )) four and all the lambda is thus on negative. Now, in addition to that if we consider all non zero's d's, such that if you take the active inequality constraints which have positive Lagrange multipliers for the gradient x star should be 0. We take all the active in inequality constrains for which the lambda star x 0, then gradient variant at x star transpose to less than equal 0 and if we take all the equality constraint for them, gradient x star transpose d is equal to 0.

If we consider this set of this non zero d satisfies this and then, take any zero that can say the hessian of the Lagrange should be positive definite along those function. If that condition is satisfied, then we say that x star is local minimum of (( )). So, let us again look at the unconstraint optimization problems and we saw that e transpose for the unconstraint problem, we saw that hessian matrix is positive. Now, since talking about unconstraint column, there was no restriction on d at that point. Now, we are talking about the concern problems. So, d transpose hessian of the Lagrange into d will be greater than 0 or in other words, the hessian of the Lagrange in at extra lambda star new star should be positive definite for all the directions d which all the non-zero directions will satisfy these conditions.

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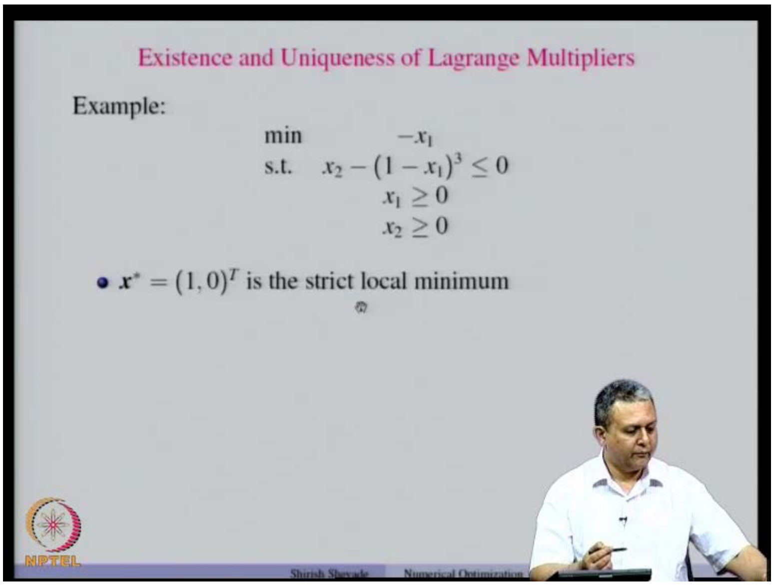

Now, the proof of these conditions stand out reference books, but in today's class, what we will do is will consider several examples and see how to make sure that the KKT conditions are used to solve to get a solution of a non-linear programming problem. So, the first example that we are going to look at is about the existence and uniqueness of Lagrange multipliers under what conditions to the Lagrange multipliers have under, what conditions do the Lagrange multipliers exist and under what condition are they unique. So, let us consider problem where we want to minimize minus x 1 subject to x 2 minus 1 minus x 1 cube less than or equal to 0 x 1 greater than or equal to 0 and x 2 greater than or equal to 0. Now, we have already seen this problem and we also saw that 1, 0 is a strict local minimum of this problem.

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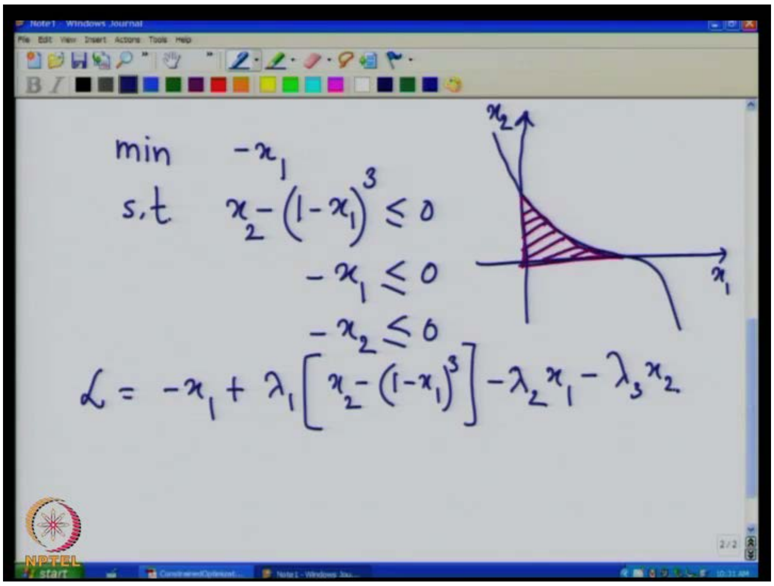

So, let us consider the problem, minimize minus x 1 subject to, so let us write the problem in our required form and that is 1 minus x 1 cube less than or equal to 0 and then, we have the constraint x 1 greater than or equal to 0, x 2 greater than or equal to 0. So, we will write those constraints as minus x 1 less than or equal to 0 and minus x 2 less than or equal to 0.

Now, let us look at the constraints set and the constraint set is, we have x 1 and x 2 as the two x axis. So, this is the constraint x 2 equal to 1 minus x 1 cube and we are talking about the points which are less than or equal to this. So, our constraints set is going to be the region shown here which are by shaded line. Now, let us look at the Lagrange of this problem. So, the Lagrange of this problem, remember that Lagrange is a function of x as well as the lambdas and mu's as is the case, but will drop that representation for time being we just mentioned one will assume that l is function of x as well as lambda and mu. So, l is nothing, but the objective function plus we have the first constraint which is x 2 minus 1 minus x 1 cube. The second constraint which is minus x 1, this gamma is equal to 0. So, we have minus lambda 2 x 1 minus lambda 3 x 2.

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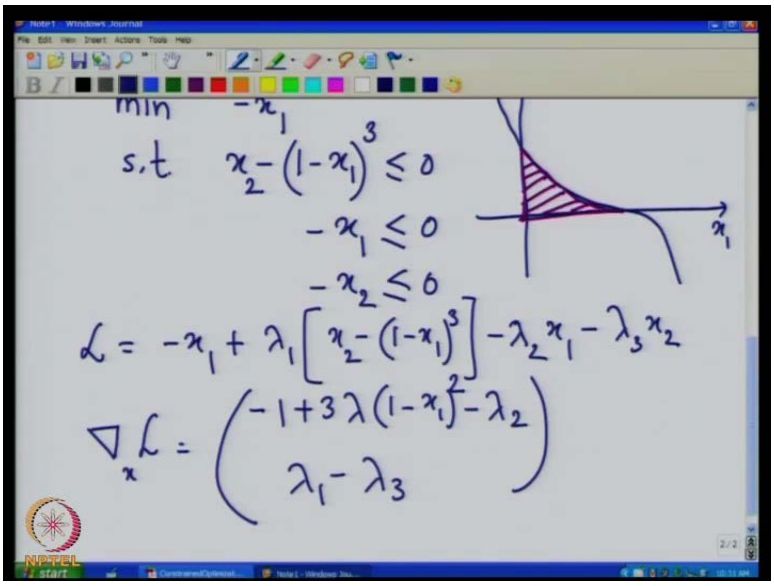

So, the gradient of the Lagrange with respect to x will write it as minus 1 plus 3 lambda 1 minus x 1 square minus lambda 2 and with respect to x 2, this lambda 1 minus lambda 3. This is the gradient of the Lagrange.

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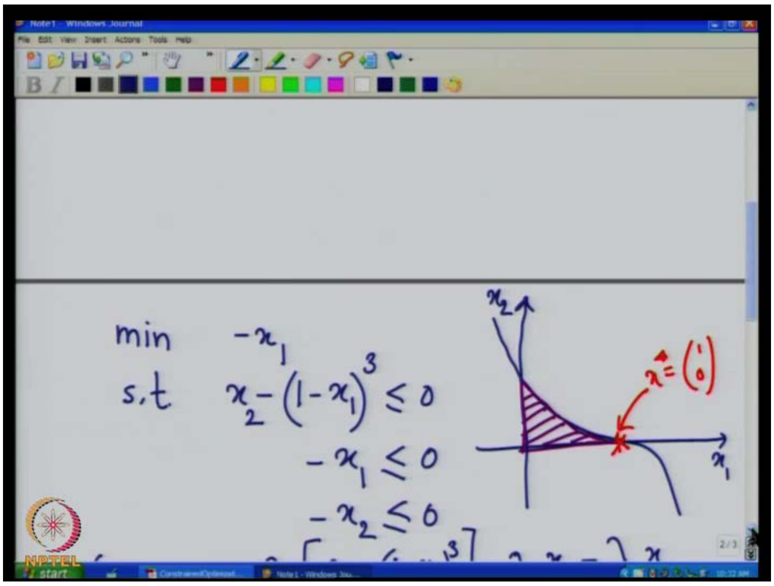

Ok, so if you recall that we had seen earlier that this point is x star which is 1, 0 that is local minimum of this problem. We want to minimize minus x 1 means, maximize x 1 and that with respect to this feasible region. The minimum occurs at this point or the maximum x 1 occurs at this point. Minimum was minus x 1 occurs at this point or maximum of x 1 occurs at this point. So, let us see what happens to the KKT conditions at this point.

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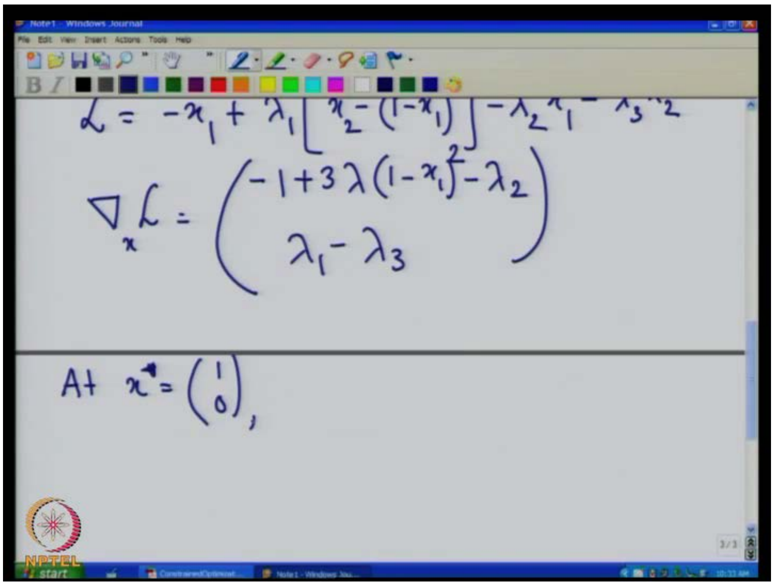

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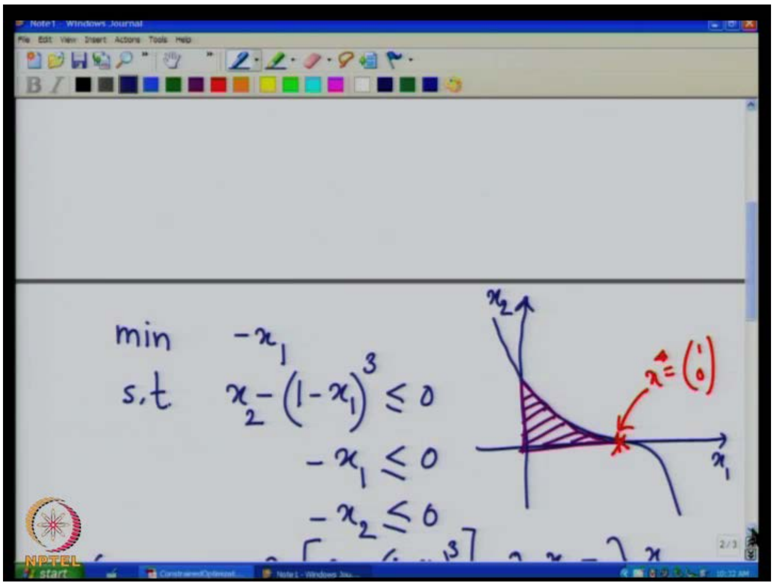

So, let us consider the case where we have x star to be 1, 0. Now, at this point, the constraint x 1 greater than or equal to 0 is inactive. Only two constraints are active, the first constraint and the third constraint. So, the second constraint is inactive.

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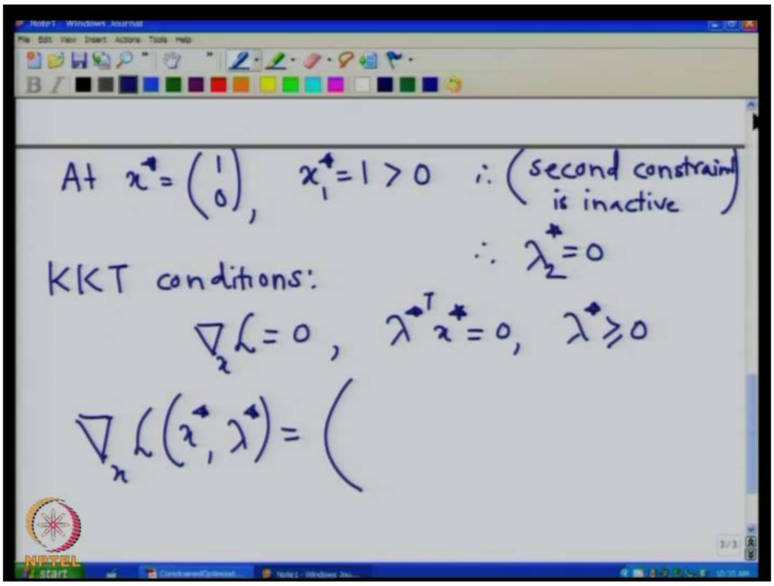

So, x 1 star equal to 1 which is greater than 0 and therefore, the second constraint x 1 star greater than or equal to 0 is inactive. Therefore, the second constraint is inactive and therefore, what we have is lambda 2 star will be equal to 0 because second constraint is inactive and we saw earlier that for the in active constraints, the corresponding Lagrange multipliers are 0.

So, now let us look at this equation. So, the KKT conditions what they say is that the gradient of the Lagrange should vanish at a KKT point. So, what we have is gradient of l with respect to x should be 0 and then, lambda star transpose x star should be 0 and lambda star greater than or equal to 0. So, let us use this KKT conditions at this point and see what happens to the Lagrange multipliers. So, gradient of l with respect to x at x star lambda star is nothing, but let us put these values of x star in the gradient of the Lagrange.

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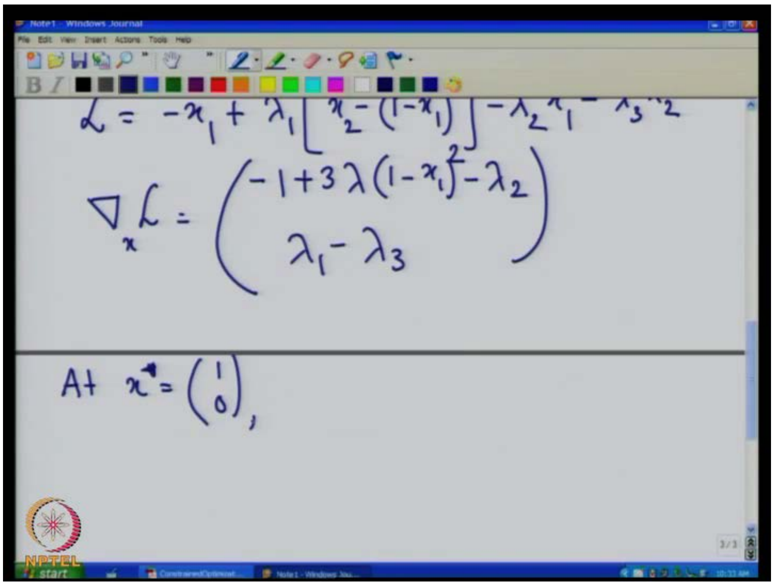

So, the gradient of the Lagrange, we have x star x 1 star to be 1 and x 2 star to be 0 and then, equate that to 0. So, what we get is minus 1 because x 1 is 1. So, this quantity vanishes and what we get is minus 1 plus 1 minus lambda 2 and since, the second constraint is inactive, lambda 2 star is 0.

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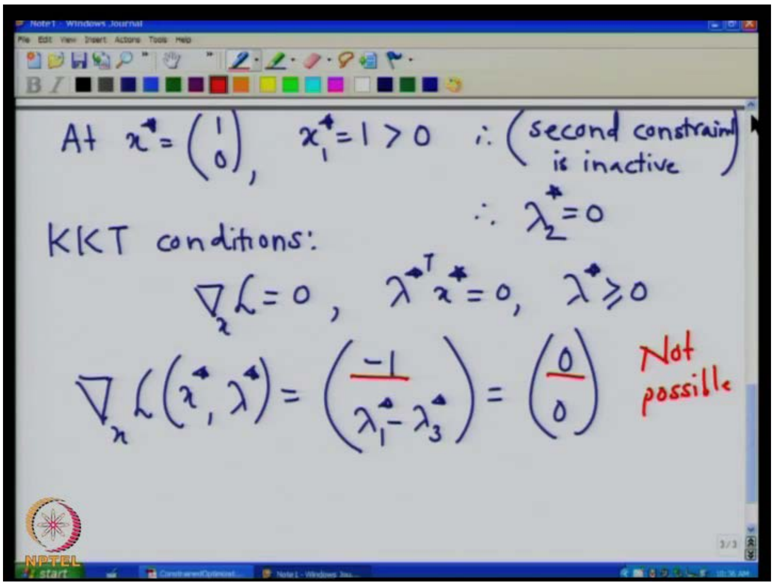

So, what we get is minus 1 and lambda 1 star. So, the second constraint lambda 1 minus lambda 3 should be 0 or in other words, lambda 1 minus star minus lambda 3 star should be equal to 0. Now, if you look at this, we have minus 1 equal to 0. So, that means that this is not possible.

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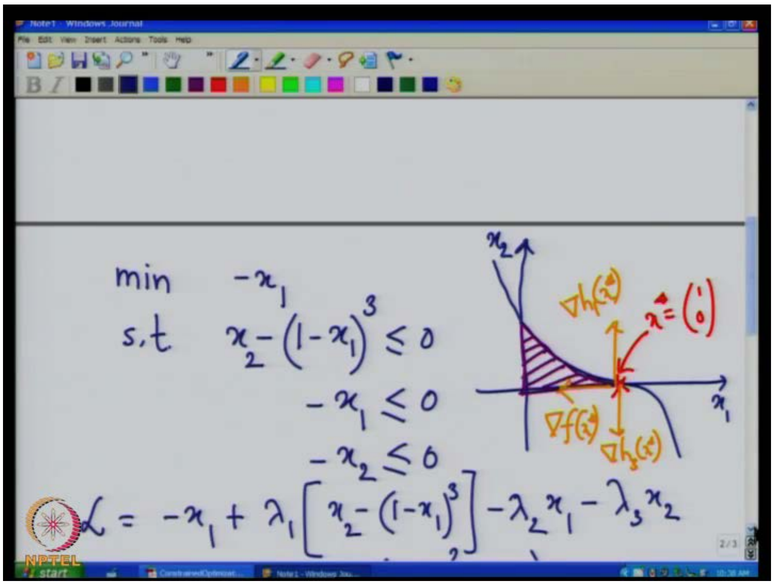

The reason why we came off with this condition is that the point is point x star is not a regular point. So, if you consider the gradient of f x star, so the gradient of f of x star will be in this direction. Then, the second constraint is inactive the third constraint, its gradient is 0 minus 1. So, the gradient h 3 x star is along this direction and the gradient of the first constraint will be along the direction 0, 1. So, this will be the gradient h 1 x star. So, we will see that the gradient h 1 x star and gradient h 3 x star which are the gradient of the active constraints at this point, they are not linearly independent and therefore, x star is not a regular point and therefore, we are not able to find a KKT point. Although, this point is local minimum, strict local minimum of the problem, we are not able to get the KKT point.

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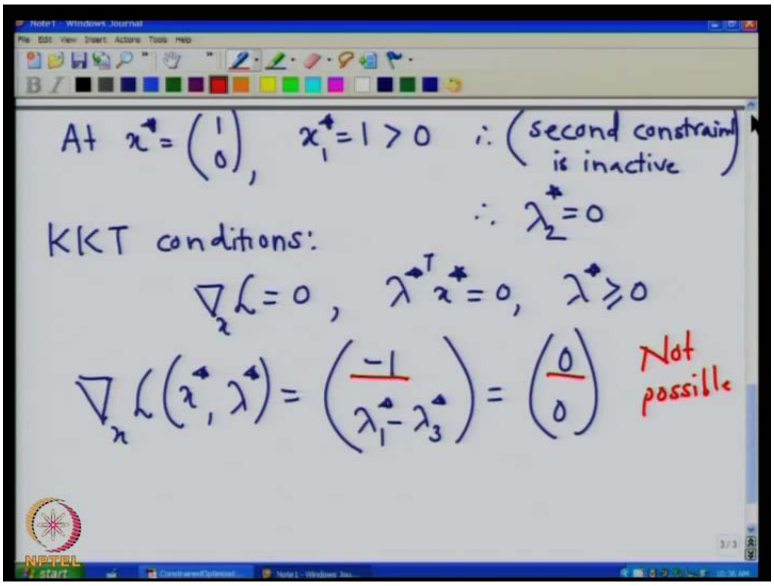

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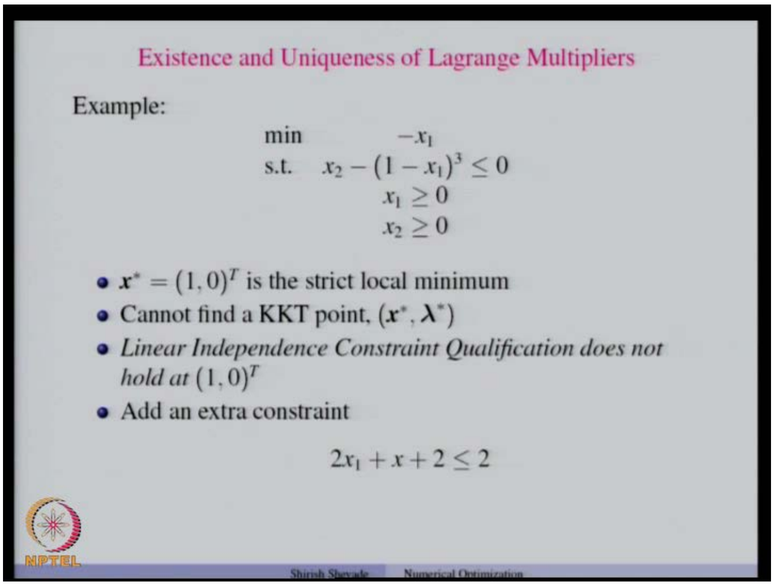

So, x 1, 0 is strict local minimum, but we cannot find the KKT point and the problem is that the linear independence constraint qualification does not hold at 1, 0. The (()) of the two constraints which are active, the first and the third constraints, they are not linearly independent at this point and therefore, the linearly independent constraint qualification does not hold and therefore, we cannot find the KKT point involving this local minimum.

Now, let us add on extra constraints to this problem. So, the extra constraints that we are going to add is this 2 x 1 plus this should be x 2 2 x 1 plus x 2 plus is less than or equal to 2. So, these constraints is 2 x 1 plus x 2 less than or equal to 2. So, let us see what happens when we add that extra constraints. So, we have the same problem and then, we have the constraints.

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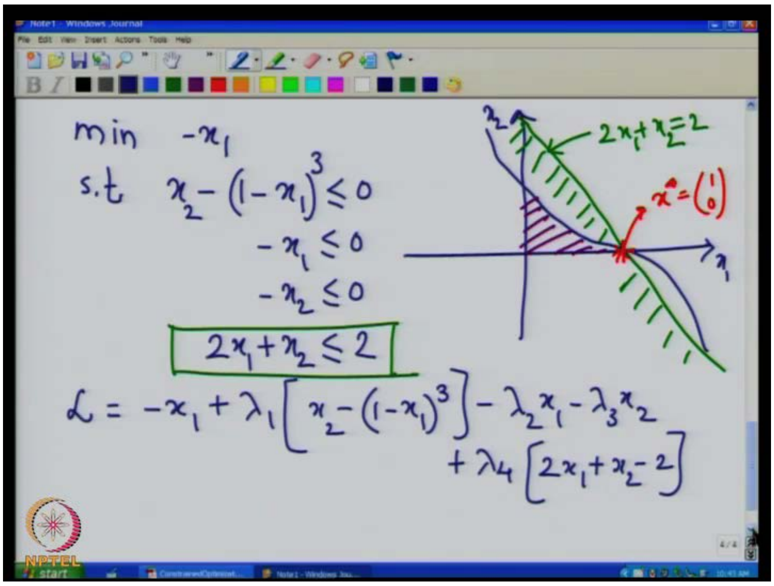

So, this was our earlier problem that x 1 greater than or equal to 0 and x 2 greater than or equal to 0 and x 2 less than or equal to 1 minus x 1 cube and to that we add a constraint. So, let us write down the constraints minimize by minus x 1 subject to x 2 minus 1 minus x 1 cube less than or equal to 0 minus x 1 less than or equal to 0 minus x 2 less than or equal to 0 and we had added one constraint which is 2 x 1 plus x 2 less than or equal to 2. So, this is the extra constraint that we have added.

Now, let us see how these constraints looks like. So, it passes through this point, the x axis and then, on the x 1 axis, the x 2 axis. So, it cuts the x 2 axis somewhere at this point. So, we can write, we can draw that constraint to be a line passing through 0, 2 and 1, 0. Now, if you look at the constraint, so what we want is that x 1 x 2 should satisfy 2 x 1 plus x 2 is less than or equal to 2 or in other words, the points below this line. So, this is the line 2 x 1 plus x 2 equal to 2 and we are interested in the region which is on this side of the line and that intersection of that region with our earlier constraints.

So, you will see that our earlier constraint which was this region, shaded region here that remains the same that has not changed. So, in other words, this point x star which was 1, 0 is still our local minimum because the constraints set is not test. Now, let us look at the Lagrange.

So, let us write the Lagrange again the same way as we wrote last time plus lambda 1 into x 2 minus 1 minus x 1 cube minus lambda 2 minus x 1 minus lambda 3 x 2 and now, we have another constraint. So, we have lambda 4 into 2 x 1 plus x 2 minus 2.

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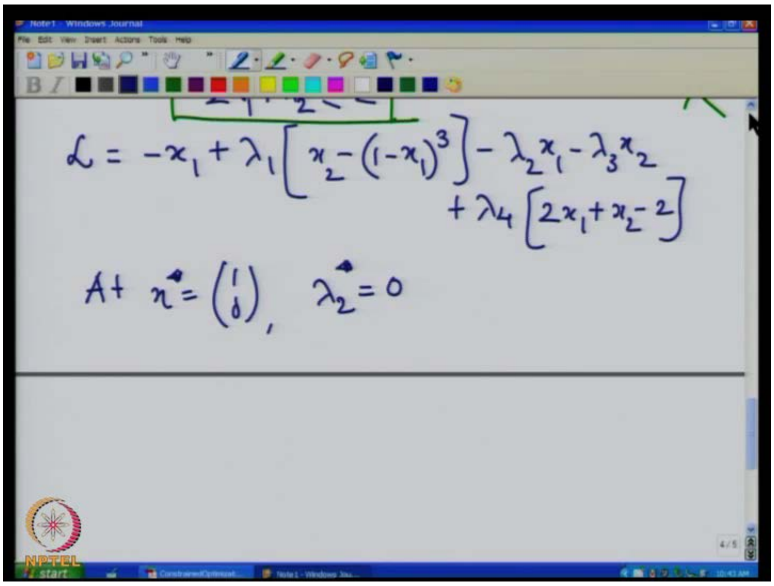

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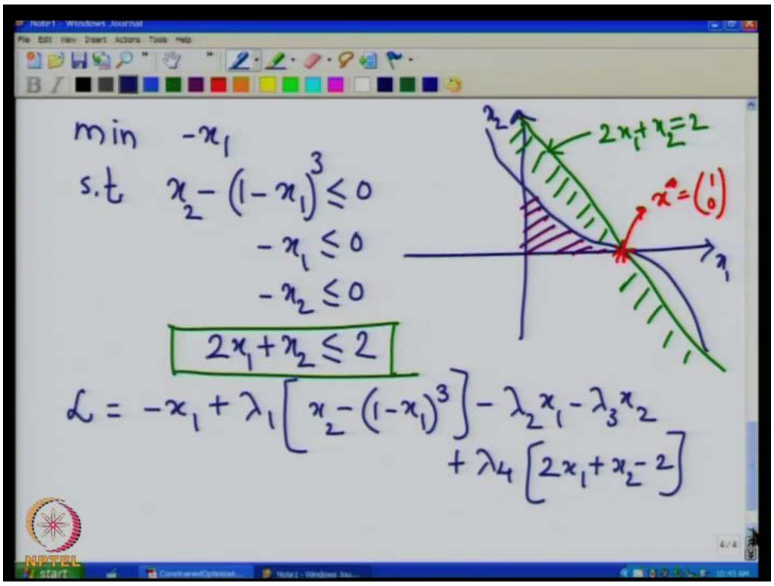

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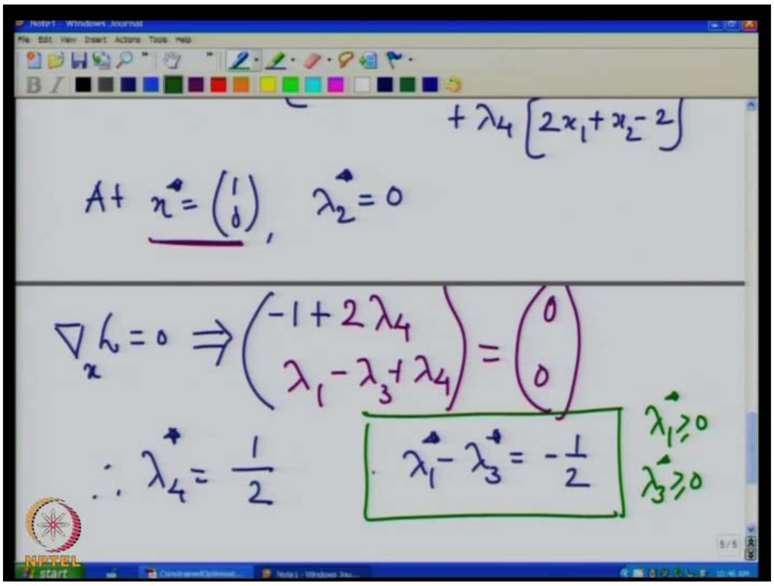

Now, if you look at this point again, so again at x star to be 1, 0, we have lambda 2 star 0 because one constraint, this constraint is inactive and the other three constraints, x 2 greater equal to 0 and then, x 2 equal to 1 minus x 1 cube and 2 x 1 plus x 2 equal to 2. So, all three in equality remaining three inequality constraints are active and therefore, if you write down the KKT conditions, so gradient of l with respect to x equal to 0, this implies what we get is minus 1 plus.

So, let us substitute at this and let us find the lagrangian at this point x star and we want to find out corresponding lambdas. So, minus 1 plus. So, this quantity vanishes lambda 2, 0. So, we have the gradient with respect to x 1. So, we have 2 lambda 4 and the gradient with respect to x 2. So, that means we have lambda 1 plus lambda 4. Sorry, minus lambda 3 plus lambda 4 is equal to 0 and therefore, lambda 4 is equal to half lambda 4 star is equal to half and lambda 4 star is equal to half and that means that lambda 1 star minus lambda 3 star is equal to minus half.

So, what does this means is that we can choose lambda 1 star and lambda 3 star to be greater than or equal to 0, such that the difference between lambda 1 star and lambda 3 star is minus half. So, lambda 2 star is 0 lambda 4 star is half and lambda 1 star and lambda 3 star can be chosen such that they are non negative and that difference is minus half. So, that means that the number of Lagrange multipliers is not unique in this case.

Now, if you look at the first order KKT necessary conditions we made a statement that if x star is regular point, then the Lagrange multipliers are unique. So, in this case in x star was not a regular point here is the situation where we do not get unique Lagrange multipliers.

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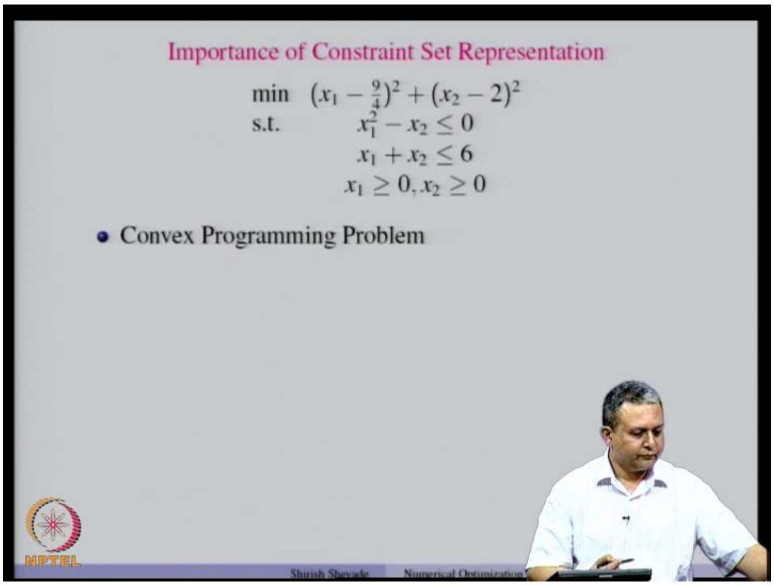

Now, let us look at other example. So, this example is about the representation of the constraints set. So, the way we represent constraint set is more important rather than the constraint set itself. A constraint set any way is important, but the way we represent, it can decide whether the particular point is KKT point or not. So, let us take an example where we want to minimize this quantity x 1 minus 9 by 4 square plus x 2 minus t square subjected to the constraint that x square minus x 2 is less than or equal to 0, x 1 plus x 2 less than or equal to 6 and x 1 and x 2 are non negative quantities.

Now, this is the convex programming problem because all these functions, the function f and j's, all are convex functions. There are no equality functions. So, this becomes the convex programming problem. So, for convex programming problem, we have to look for these constraints qualification which mean that there should at least exist one point which is in the interior of this set.

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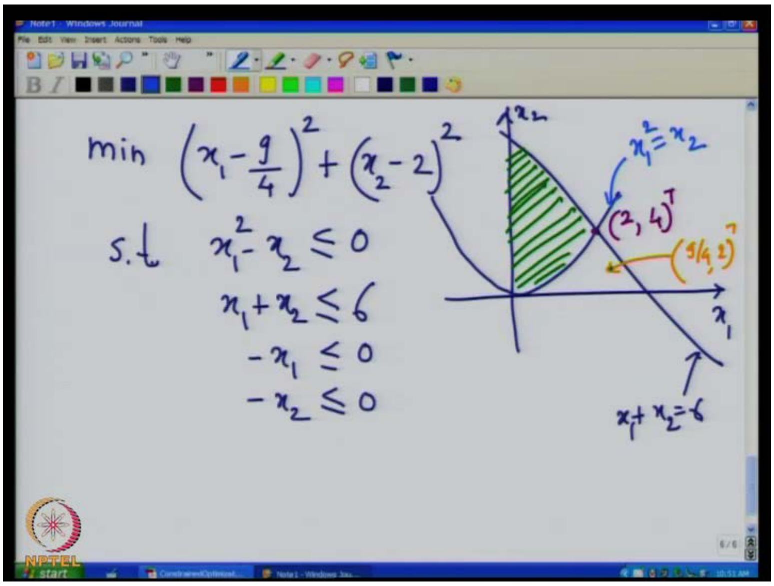

So, let us look at this problem. So, the problem that we are looking at is minimize x 1 minus 9 by 4 square plus x 2 minus 2 square subject to the constraint x 1 square minus x 2 less than or equal to 0 x 1 plus x 2 less than or equal to 6 and x 1 greater than or equal to 0 and minus x 1 less than or equal to 0 and minus x 2 less than or equal to 0. So, if we draw the feasible region, so we have x 1 x 2 as the two axes. Now, x 1 square is equal to x 2 is a parabola.

So, we are interested in the points above this parabola because x 1 square should be less than or equal to x 2. So, points above the parabola and then, x 1 plus x 2 equal to 6. So, that line suppose like this, so this is the line x 1 plus x 2 equal to 6. So, the point of the intersection of these two curves is 2, 4 and so, we are interested in the points above the parabola below this line and in the first, the points which are in the first quadrants. If you look at the feasible set, the feasible set is shown here.

Now we want to find out the point on this feasible set which is close to 9 by 4, 2. So, the point 9 by 4, 2 will be somewhere here. So, this point is 9 by 4 and we want to find this feasible region which is closer to the point 9 by 4, 2. So, there are different possibilities. One two possibilities that point 2, 4 itself is a minimum point or the minimum point lies on this curve. So, this curve is nothing, but x 1 square is equal to x 2. So, either the minimum lies on this curve or minimum is here. Obviously, this point cannot be a minimum point or we get the minimum either on this line segment or this line segment.

So, let us look at these possibilities. Now, for that purpose let us write down the Lagrange.

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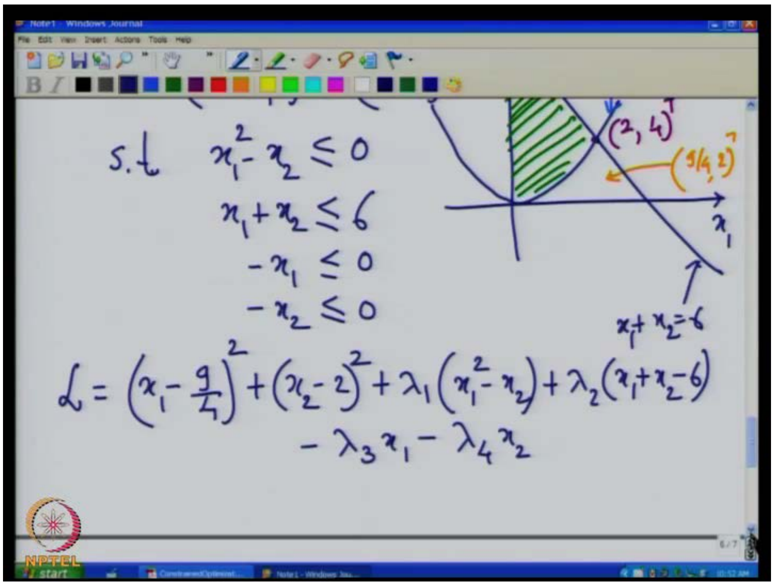

So, the Lagrange is the objective function. So, x 1 minus 9 by 4 square plus x 2 minus 2 square plus lambda 1 x 1 square minus x 2 plus lambda 2 x 1 plus x 2 minus 6. Now, still we have two constraints. So, minus lambda 3 x 1 minus 3 lambda 4 x 2. So, these are the four constraints and then, correspondingly the Lagrange will look something like this.

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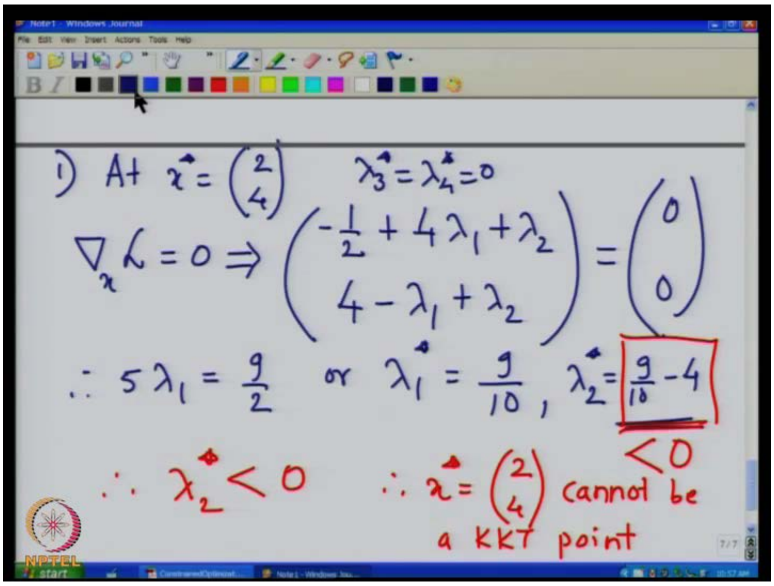

Now, let us see what happens at 2, 4. So, let us consider the case that at x star equal to 2, 4. So, what we have to do is that let us write the Lagrange and at the gradient of the Lagrange and that should vanish. So, if we calculate the Lagrange, the gradient of the Lagrange and then, calculate it at this point. So, what we get is this point. So, the derivative of this term with respect to x 1 is 2 into x 1 minus 9 by 4 and if its substitute is x 1 start to the 2, so what we get is minus half and then, next thing we get is 2 lambda x 1 and 2 lambda 1 x 1 at x 1 is equal to 2 is 4 lambda 1. So, 4 lambda 1 and plus lambda 2 and since, x 1 is greater than 0 and x 2 is greater than 0 that star lambda 2 star lambda 3 star and lambda 4 star are 0's.

Now, the derivative with respect to x 2, so 2 into x 2 minus 2. So, x 2 star is 4 minus 2 into 2 is 4 minus lambda 1 plus lambda 2 and that is equal to 0. Now, if you solve this, if you subtract the second equation from the first equation, so what we get is 5 lambda 1. Lambda 2 will get cancelled and that will be equal to 4 plus half. So, 4 plus half is 9 by 2 or lambda 1 star is equal to 9 by 10. Now, if you plug in this value of this lambda star lambda 1 star here, so what we get is lambda 2 star is equal to 9 by 10 minus 4 and this quantity is less then. Therefore, lambda 2 star is less than 0 and therefore, x star equal to 2, 4 cannot be KKT point.

The reason is that at x star, we got the value of lambda 2 star to be less than 0. So, this quantity which was less than 0 and if you recall our KKT conditions, we wanted the Lagrange multipliers to be non negative and certainly, we are not able to get the non negative Lagrange multipliers for this x star.

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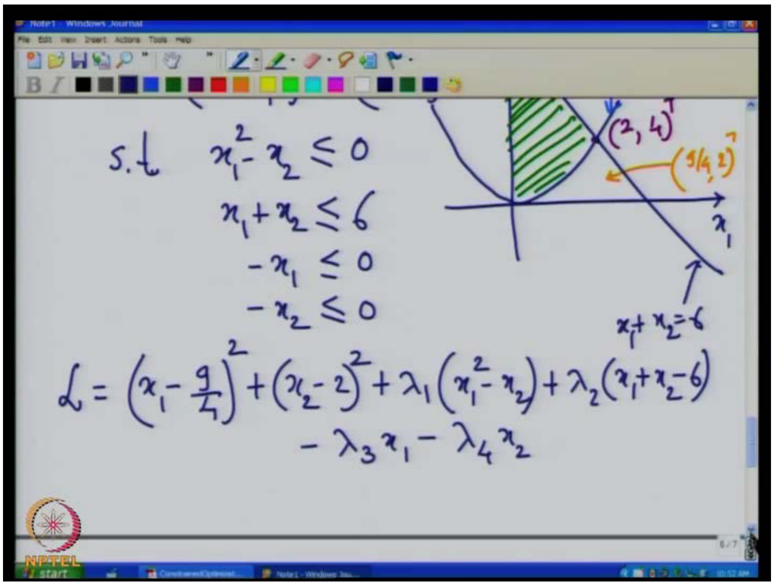

So, this point 2, 4 is not a KKT point. Now, the case that next to be checked is whether the solution lies on this curve. So, if the solution has to be applied on this curve, so only the first constraint will be active and the rest of the constraints will be inactive. Therefore, other lambda 2 star lambda 3 star and lambda 4 star will be 0. Let us assume that solution does not lie here, but it lies in these curves, curved line from this to this and 2, 4 is not a solution is not a KKT point 0, 0. Let us assume that is also not a solution. So, the idea is to get a point on this curve and this could be closest to lambda 4 and 2.

So, once we assume that the three Lagrange multipliers are 0, then it is easy to write the Lagrange and that it's gradient at a point, it is suitable point on this and we will see that in the next class.

Thank you.