Numerical Optimization
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Lecture - 24
Convex Programming Problem

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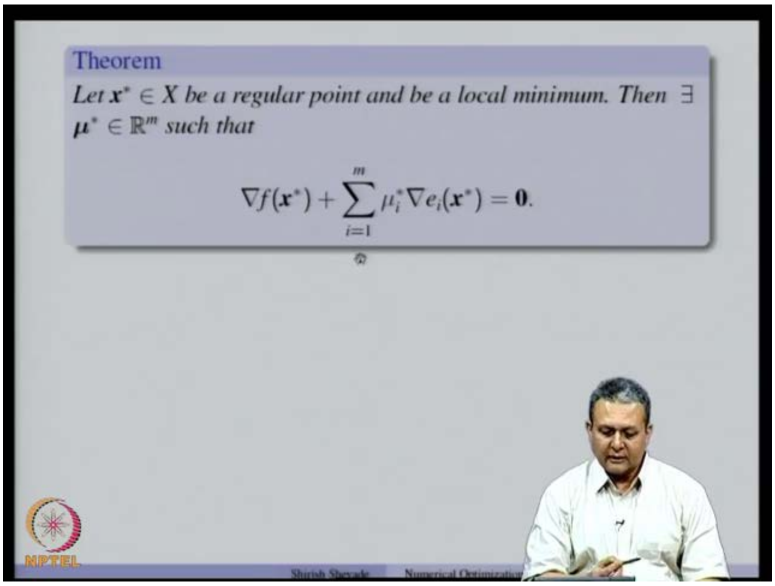

Hello welcome back in the last lecture, we started discussing about equality constrained optimization problems; and we looked at these theorem we says that. Let x star belong to X be a regular point and also be a local minimum, then the necessary condition for that local minimum is that there exists. Some mu star in m-dimensional space, note that m is the number of equality constraints that we have and the equality constraints are of the type e i x equal to 0.

So, there exist mu star in m dimensional space such that the gradient of f of x star is a linear combination of gradient of e i x star or all i is going from 1 to m. Now, remember that at x star all the equality constraints are active. So, that is why unlike the inequality constrained problems, we have consider all the equality constraints. Because, they are activate in a x star belong to X and by regular point what, we mean is that the gradient e i x star for all i is going from 1 to m are linearly independent. So, the only way that the there will be a trivial combination, which will make this component 0 and that is all mu i star 0. So, we wanted to avoid that so, we use the regular regularity assertion, now what this result means is that. So, let us consider set of for objective objective function contours.

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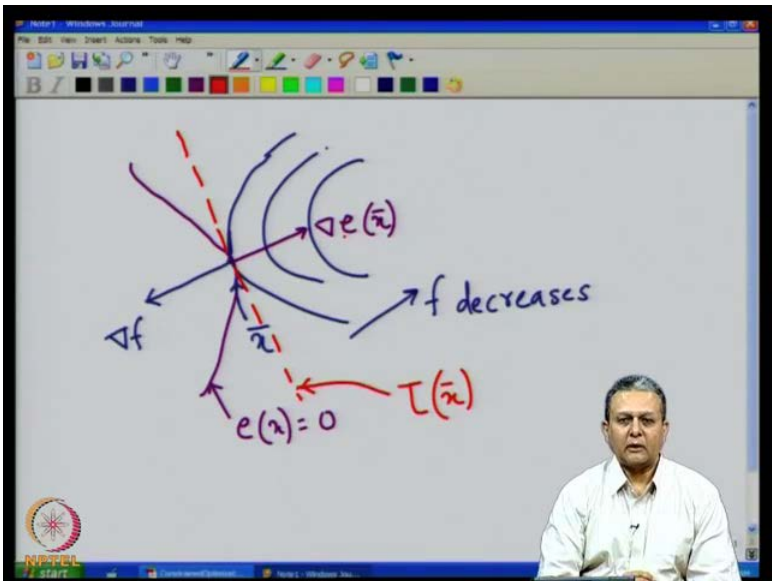

So, objective functions or contours are like this, so let us consider a point and let us call this as x bar, now the objective function value. So, f decreases in this direction, so the gradient at this point assuming that the function is differentiable. So, the gradient of f will be in this direction. Now, let us look at a contour of a equality constrained problems. So, that contour is suppose, like this. So, this is the surface e x equal to 0, now if you look at the the gradient gradient e x bar is pointing in this direction. So, we will see that the if, we draw the tangent plane to this equality constraint at at the point x bar, it will be like this. So, this is t x bar and we will see that at this point the gradient of f can be written as a some multiple of gradient e x bar.

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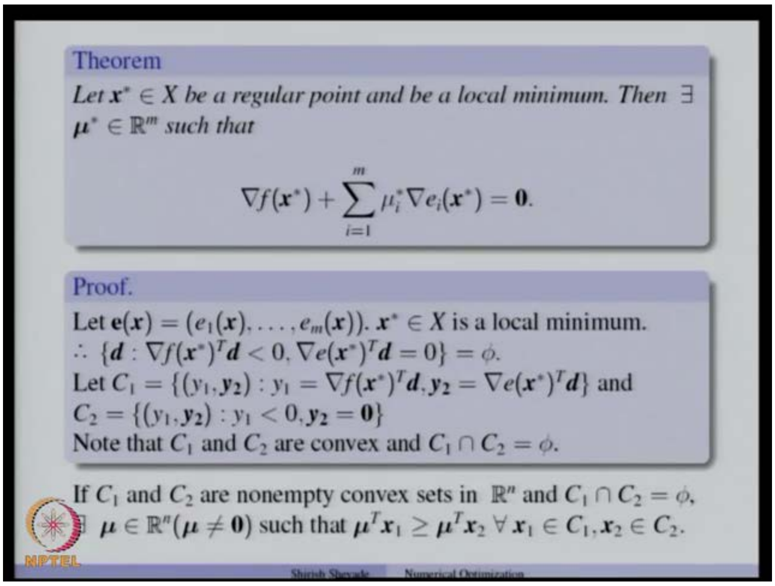

So, in general when, we have more number of equality constraints then at a local minimum the gradient of the objective function is written as a linear combination of the gradients of all the equality constraints. So, let us look at the proof of this so, let us arrange all the equality constraints in e x and then since x star, which is a feasible point is also a local minimum. What, we have is that the set of all directions d, which make an obtuse angle with gradient f x star and which are orthogonal to gradient e x star.

So, orthogonal to gradient e x star means, they are orthogonal to each and every constraint in the constraint equality constraint set. So, they e 1 to e m x, so this set is a null set because, x star is a local minimum, we have seen this earlier. So, let us define 2 sets C 1 and C 2. So, C 1 is a set in m plus 1 dimensional space, where which consists of 2 components y 1 and y 2. Y 1 is a scalar and y 2 is a vector and y 1 is written as gradient f x star transpose d and y 2 is written as gradient e x star transpose d and C 2 be that set of y 1 y 2, such that y 1 less than 0 and y 2 equal to 0.

Now, clearly since x star is a local minimum the intersection of C 1 and C 2 is a null set and more over C 1 and C 2 both are convex sets. Now at this point, we would like to recall some the results that, we studied when m we, discussed about convex sets and convex functions and one of the results is related to 2 non empty, non intersect secting convex sets.

So, if we have 2 non-empty, non-intersecting convex sets then there exists a separating hyperplane, we separates the 2 sets. So, let us recall that result that, if we have C 1 and C 2 is are nonempty convex sets and they have empty intersection. So, if the 2 sets have empty intersection then, there exist mu which, is non zero, such that mu transpose x 1 is greater than or equal to mu transpose x 2 for all x 1 in C 1 and x 2 in C 2 or in other words.

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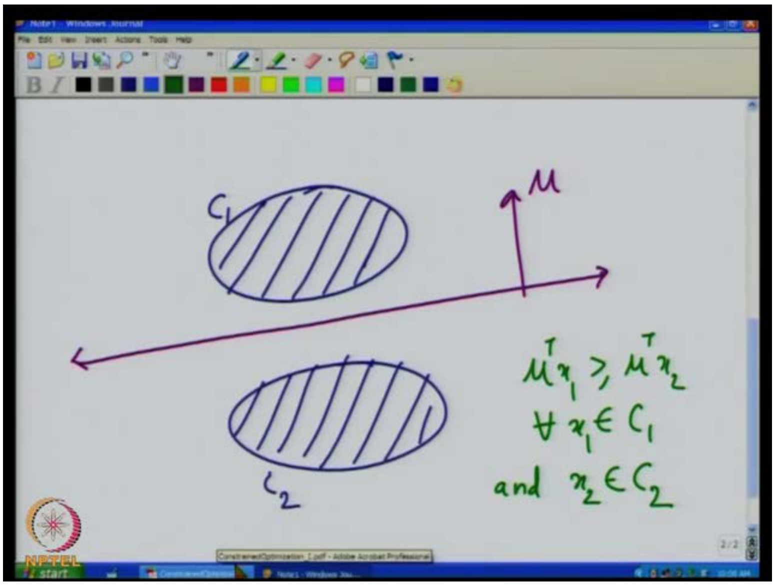

So, if we have 2 sets which, are convex and which, are not intersecting which, have empty intersection. So, this is our set C 1 and this is our set C 2 then there exists a separating hyperplane, that mu be the normal to this hyperplane. So, mu transpose x 1 where, x 1 belongs to C 1 will be always greater than or equal to mu transpose x 2. So, what we have is mu transpose x 1 is always greater than or equal to mu transpose x 2 for all x 1 belongs to C 1 and x 2 belong to C 2. So, this is the result that, we saw last time when, we talked about the convex sets.

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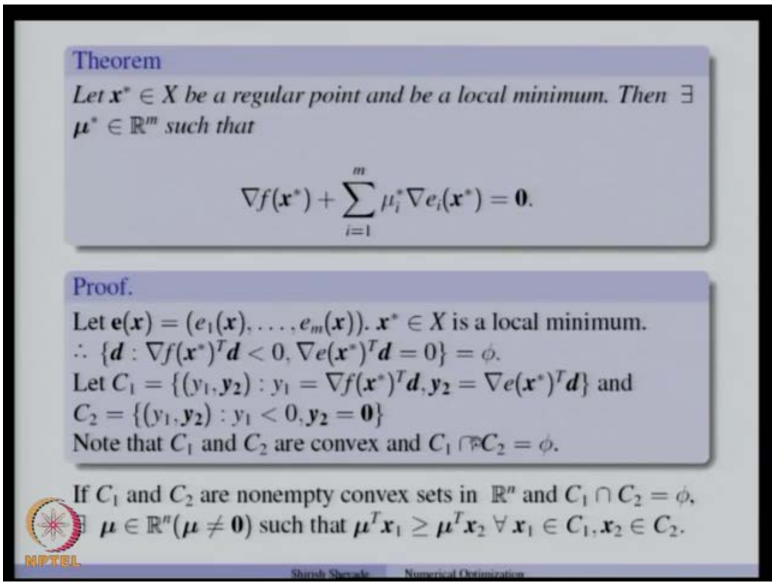

So, let us use this theorem in this case. So, now we have this 2 sets C 1 and C 2 and there intersection is empty, they are convex.

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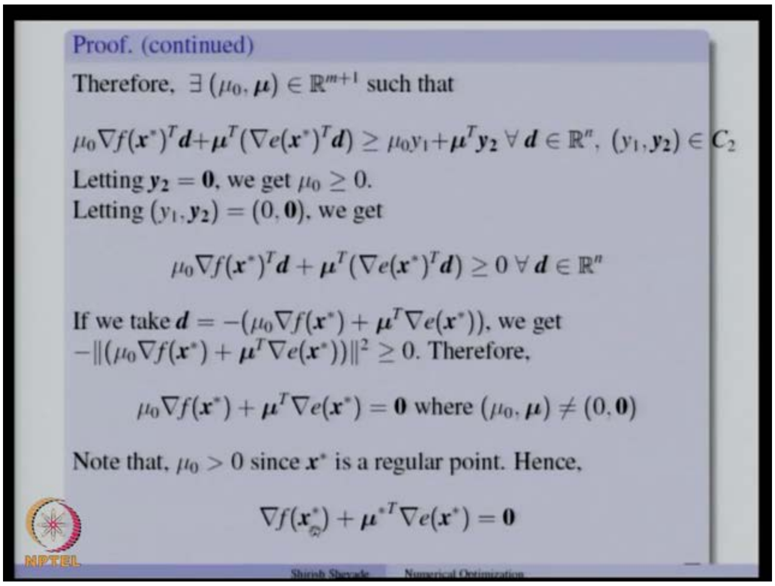

So, therefore, there exist vector in m plus 1 dimensional space which, has 2 components mu 0 and mu note that, this mu 0 will be associated with gradient f x star and mu will be associated with gradient e x star. So, there exists non zero vector, I repeat that this is a non zero vector not all components are 0.

Such that mu 0 into gradient f x star transpose d plus mu transpose into gradient, e x star transpose d is greater than or equal to mu 0 y 1 plus mu transpose y 2 for all d, so for all d in r n and y 1 y 2 in C 2. So, for all ds in r n, we are able to get this result, because of the separability of the two convex sets.

Now, if we let y 2 equal to 0 here then, what happens is that y 1 can be made arbitrarily large negative quantity and because of which, this number if mu 0 is less than 0. This number will be very large and we are saying that the quantity on the left is greater than this large number. And mu 0 could be infinity minus infinity and y 1 is very large negative number. So, this quantity will be close to infinity and not the that means, the left quantity is greater than or equal to infinity, which is not possible. So, this by letting y 2 equal to 0 and noting that y 1 can be made arbitrarily large negative number, we have to have mu 0 greater than or equal to 0.

So, we note that this mu 0 has to be greater than or equal to 0, now if, you let y 1 y 2 both close towards 0 then what, we get is that the left hand side quantity should be greater than or equal to 0, for all d belong to r n. And since this quantity is greater than or equal to 0, if we take d to be so, this holds for all d. So, we are free to choose any d in r n. So, if we chose d to be minus mu 0 gradient f x star plus mu transpose gradient e x star then, what we get is that a minus norm of mu 0 gradient f x star plus mu transpose gradient e x star whole square equal is greater than or equal to 0.

Now, we have already seen that the norm any vector is a non negative quantity. So, this quantity is non negative and the negative of that has to be greater than or equal to 0, now the only way this inequality satisfied is when this quantity is 0. So, that means, we have mu 0 gradient f x star plus mu transpose gradient e x star equal to 0, where mu 0 and mu are not they do not constitute 0 vector.

Because, we are constitute 0 vector then this relationship is trivially satisfied and why we, are not interested in this relationship in such trivial relationship. Now, remember that we, have assume that x star is a regular point and earlier, we showed that if x star is a regular point this mu 0 has to be greater than 0. Because otherwise again we, will end up in a contradiction.

So, mu 0 has to be greater than 0, since x x star is a regular point and therefore, we we can divide the equation by mu 0 and what, we get is gradient of x star plus mu star transpose gradient e x star equal to 0 and this is what we, wanted to prove. So, by ensuring that because, of the regularity of x star mu 0 is greater than 0, we are able to say that at if x star is a local min then gradient f x star plus mu star transpose gradient e x star has to be 0 of gradient f x star is written as a linear combination of gradient e i x star, i going from 1 to m. Now unlike the inequality constraints note that there are no sign restrictions on mu. Mu is a vector in m dimensional space and therefore, for equality constraint problems, we can have any linear combination of equality constraints and that is equal to a gradient f x gradient f x star in this case.

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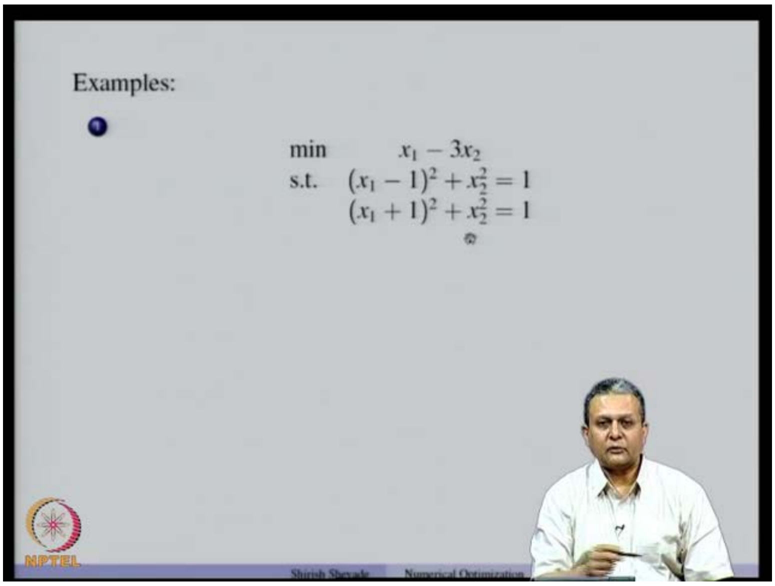

Now, let us take some examples. So, here we have 1 example where, we want to minimize x 1 minus 3 x 2 subject to x 1 minus 1 square plus x 2 square equal to 1 and x 1 plus 1 square plus x 2 square equal to 1.

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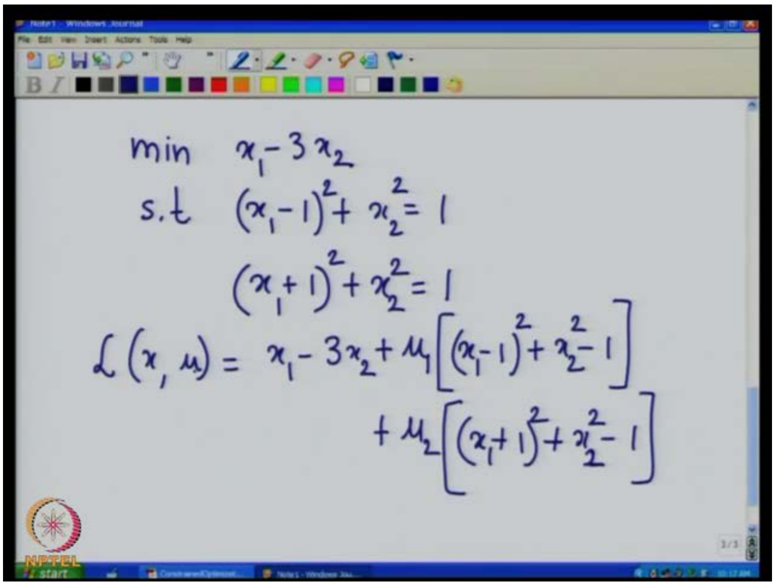

So, minimize x 1 minus 3 x 2 subject to x 1 minus 1 square plus x 2 square equal to 1 and x 2 x 1 plus 1 square plus x 2 square is equal to 1. So, let us look at this problem, now we have 2 equality constraints. So, both are active at a given point and so, let us first write the lagrangian, so lagrangian of x mu to be x 1 minus 3 x 2 plus mu 1 into x 1 minus 1 square plus x 2 square minus 1.

So, remember that this constraint the first constraint is written in the form e 1 x equal to 0 and that is why, that e 1 x is x 1 minus 1 square plus x 2 square minus 1 that is multiplied by the lagrangian multiplier for that constraint plus mu 2 into x 1 plus 1 square plus x 2 square minus 1. So, this is our lagrangian function, which is a function of both x and mu remember that, this x is a vector and mu is a vector x contains x 1 and x 2 as it is components mu contains mu 1 and mu 2 as it is components.

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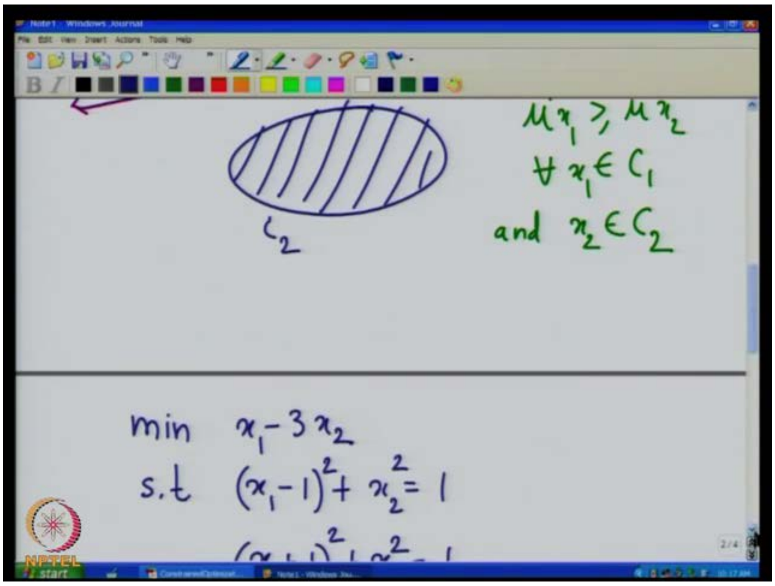

Now so, let us write down the gradient of the objective function and the gradients of the constraints. So, we write this function in this problem in the form.

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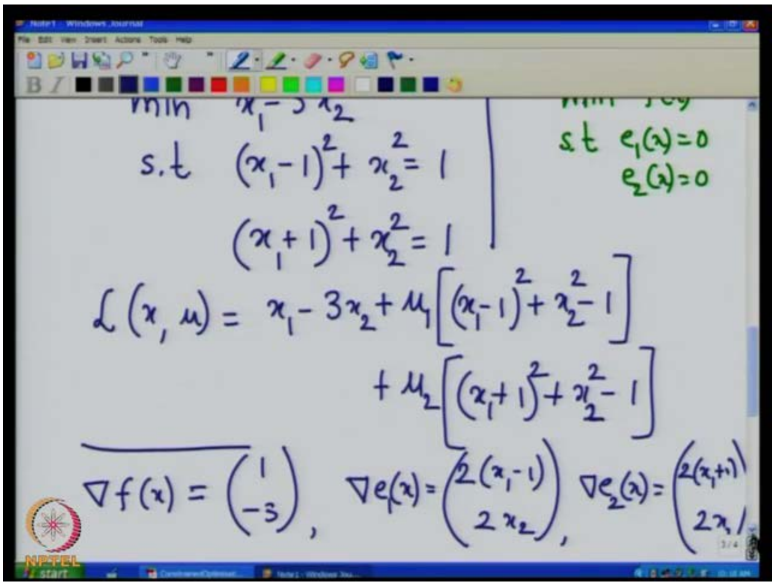

Minimize f x subject to e 1 x equal to 0 and e 2 x equal to 0. So, so let us write down the gradient of f x gradient of f x is equal to 1 minus 3 then gradient of e 1 x is equal to 2 into x 1 minus 1 and 2 x 2 and gradient of e 2 x is equal to 2 x 1 plus 1 2 x 2.

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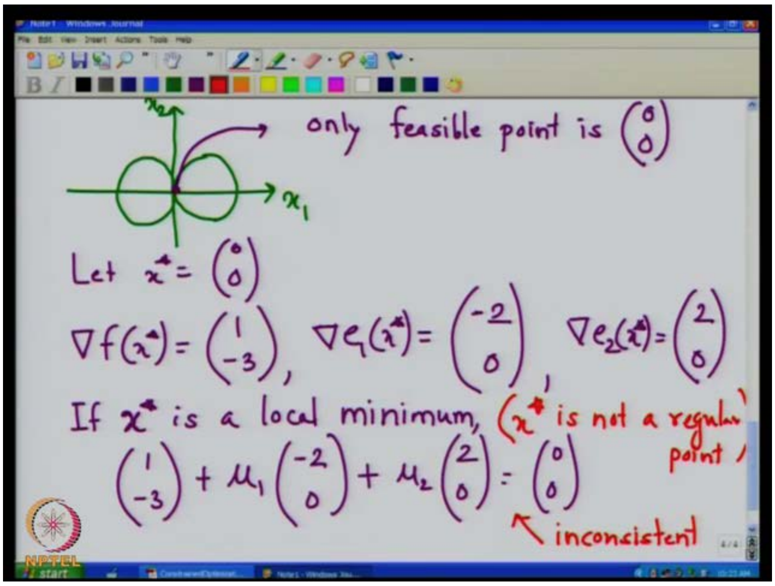

So, now, let us draw the feasible region. So, we have x 1 and x 2. So, x 1 minus 1 plus x 2 square equal to x 1 minus 1 square plus x 2 square equal to 1 is a circle with centre 1 comma 0 and radius 1. So, it is a circle with this centre and then the second constraint uses another circle, now if you intersect them. So, we get only feasible point is. So, let us consider this x star, so let x star be 0 0.

So, gradient of x star since it is a linear function, this gradient is not going to change then gradient e 1 x star is equal to minus 2 and 0 and gradient e 2 x star will be 2 0, now if x star is a local minimum then, what we want is that. So, if x star is a local minimum what, we want is that gradient f x star. So, 1 minus 3 plus mu 1 into gradient e 1 x star plus mu 2 into gradient e 2 x star is equal to 0 now.

So, you will see that it is not possible to construct or to find mu 1 and mu 2 which, satisfy this. Because, the first equation gives 1 minus 2 mu 1 plus 2 mu 2 equal to 0, while the second equation gives minus 3 plus 0 plus 0 equal to 0 and which, is not possible. Because, of the left side, we have minus 3 and the right side of the equation, we have 0 they cannot be same. So, the problem here, that we cannot use the K K T condition is that, x star is not a regular point because, this is the only feasible point is not a regular point and therefore. So, this system is inconsistence. So, therefore, the regularity is very important when, we want to solve any optimization problem to get a

KKT point and then check whether that point is a local minimum or not. So, although x x star is a solution of this problem, we know that x star is not a K K T point in this case.

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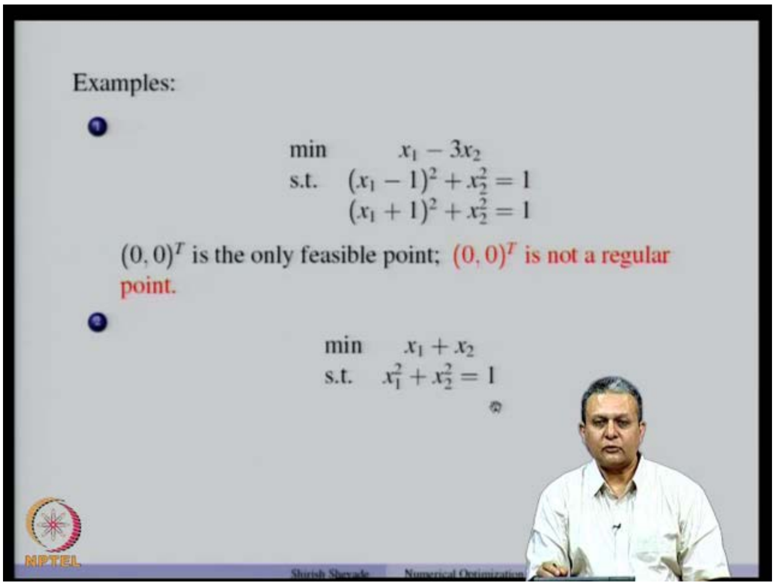

Now So, the only feasible point is the origin and is not a regular point and therefore, although it is a solution, t is not a KKT point. Now, let us look at another example minimize x 1 plus x 2 subject to x 1 square plus x 2 square equal to 1.

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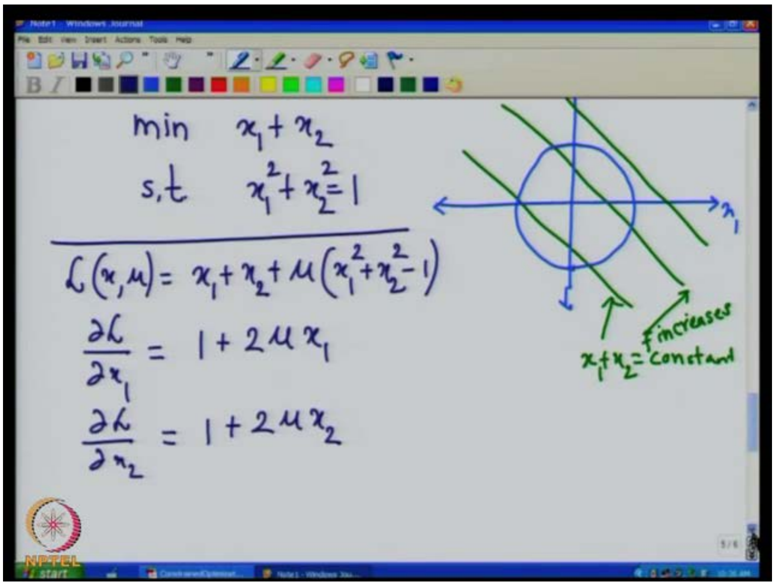

So, minimize x 1 plus x 2 subject to the constraint x 1 square plus x 2 square equal to 1, now if we plot the constraint. So, this is a constraint, that we have and we look at the x 1 plus x 2. So, x 1 plus x 2 equal to constant is are the. So, these are the lines x 1 plus x 2 equal to some constant. So, we will see that as, we move in this direction. So, f increases such, we move in this direction f increases.

Now So, let us write the lagrangian of this problem. So, the lagrangian x mu will be the objective function plus mu time the constraints, note let us take the partial derivative of l with respect to x 1 and that will be 1 plus 2 mu x 1 partial of l with respect to x 2 will be 1 plus 2 mu x 2. Now, if we look at the KKT conditions, what the KKT conditions demand is that.

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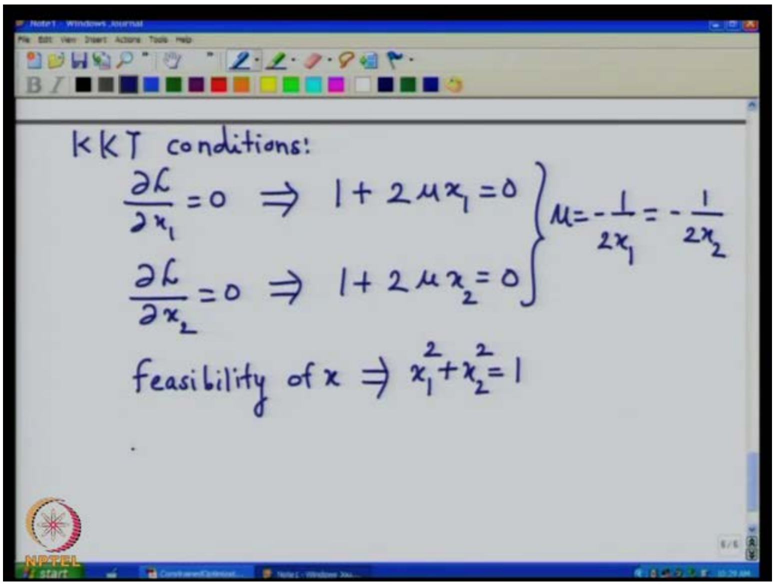

So, we have partial of l with respect to x 1 equal to 0 and that implies 1 plus 2 mu x 1 is equal to 0, then partial of l with respect to x 2 is equal to 0, which means 1 plus 2 mu x 2 is equal to 0 and then we also want to satisfy the feasibility of the points x 1 and x 2. So, the feasibility constraints x 1 square plus x 2 square equal to 1, so we have 3 equations and the unknowns are x 1 x 2 and mu.

Now, these 2 equations together, they give 1 and equal to minus 2 mu x 1. So, mu is equal to minus 1 by x 2 x 1. So, mu is equal to minus 1 by 2 x 1 and that is equal to minus 1 by 2 x 2. So, this means that x 1 and x 2 is equal to x 1 is equal to x 2 and if we substitute x 1 equal to x 2 here, what we get is 2 x 1 square equal to 1 and therefore, what we get is.

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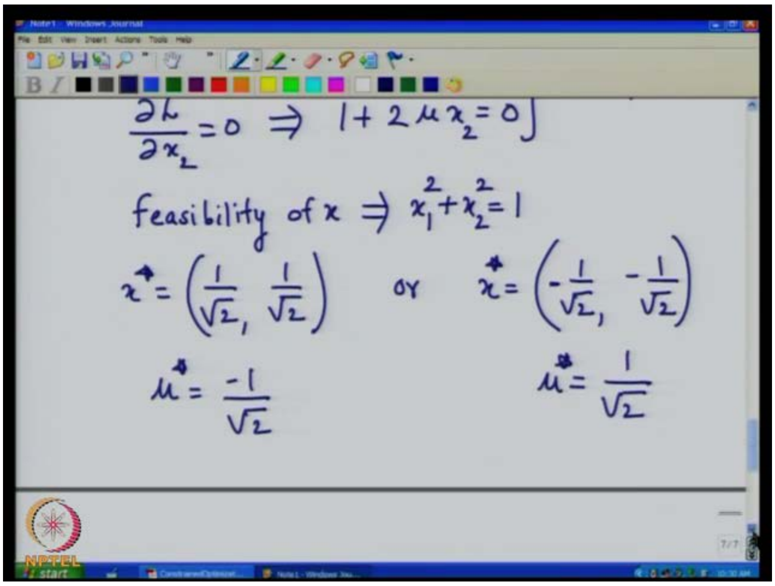

X star equal to 1 by root 2 1 by root 2 or x star equal to minus 1 by root 2, minus 1 by root 2, so these are the possible solutions and the correspondingly mu star. So, if x star is 1 by root 2 then in this case mu star will be minus 1 by root 2 and in this case mu star will be plus 1 by root 2. So, we get a KKT point, we in fact, we get 2 KKT points 1 is x star is 1 by root 2 1 by root 2 and mu star is minus 1 by root 2 and other 1 is x star is minus 1 by root 2 minus by root 2 and mu star is 1 by root 2.

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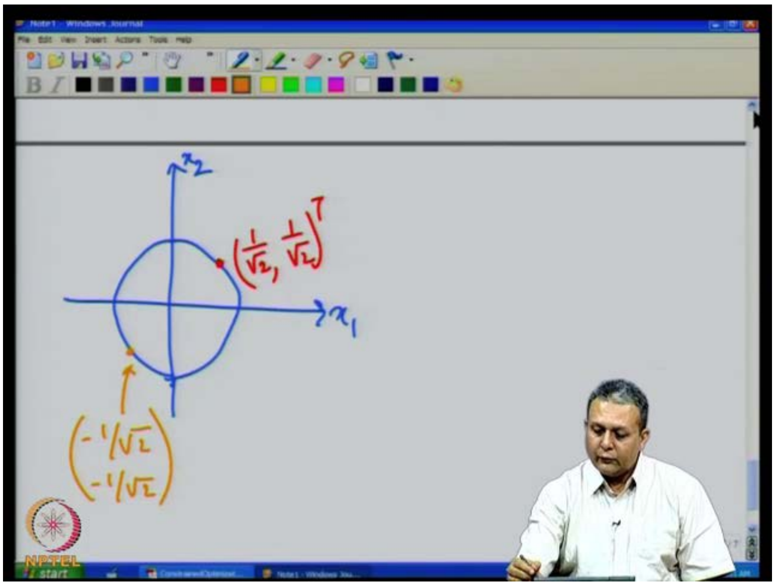

Now, let us again re plot the constraint set x 1 and x 2, now here is a point, which is 1 by root 2 and 1 by root 2 and here is a point, which is minus 1 by root 2 and minus 1 by root 2.

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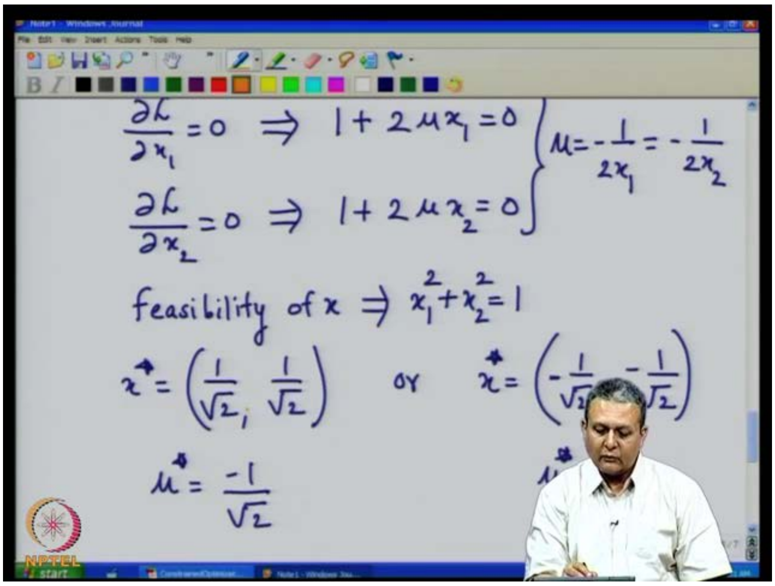

So, if you look at our KKT points 1 was 1 by root 2 1 by root 2 and the corresponding mu star was minus 1 by root 2. So, this point is a shown here and the other point is shown here.

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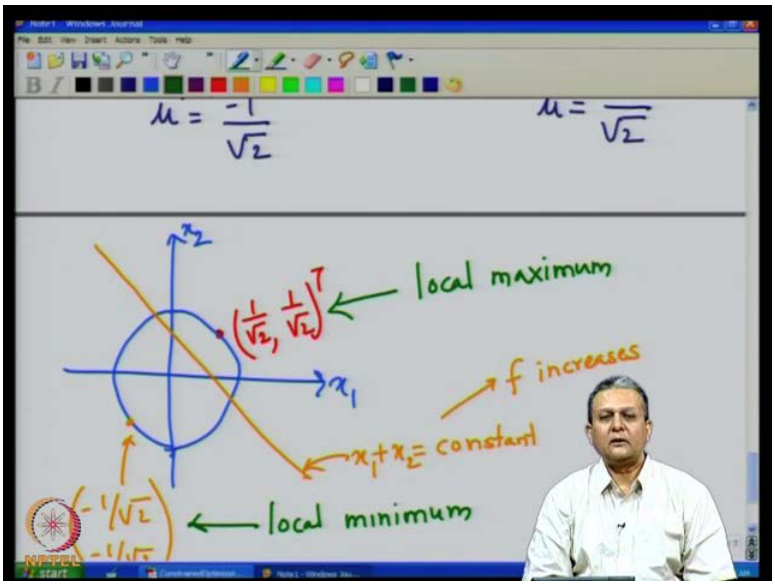

And our function is x 1 plus x 2 is equal to constant is of this form and it increases in this direction f increases in this direction. So, you will see that, we got 2 K K T points and as a function increases in this direction, this point turns out to be a local maximum and this points turns out to be as the function decreases in this direction. This points turns out to be local minimum because, of we cannot go beyond this point because then, we will violent the constraint. So, we got 2 KKT points and one of them turn turned out to be a local maximum and the other 1 turned out to be a local minimum and the main reason for this is that the first order con KKT conditions are just necessary conditions.

If you recall our discussion on unconstrained optimization, we saw that if x star is a local minimum then gradient of x star equal to 0, for differentiable function f. But, gradient f x star equal to 0 does mean that, x star is a local minimum unless, we are talking about some special functions like convex functions. So, the gradient of the lagrangian equal to 0 gives you a K K T point, which can be either a local minimum or local maximum. And so, we need to look for some second order conditions, which make use of the curvature of the lagrangian function and then only, we can conclude whether a given point is indeed a local minimum or not.

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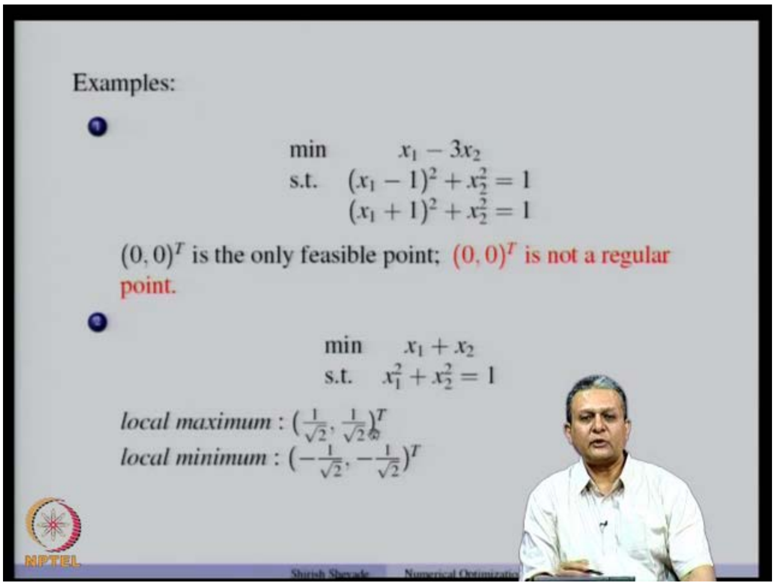

So, we have a local maximum, which is at 1 by root 2, 1 by root 2 and a local minimum at minus 1 by root 2 minus 1 by root 2. Incidentally, both this points along with the respective mu star where, the KKT points of this problem.

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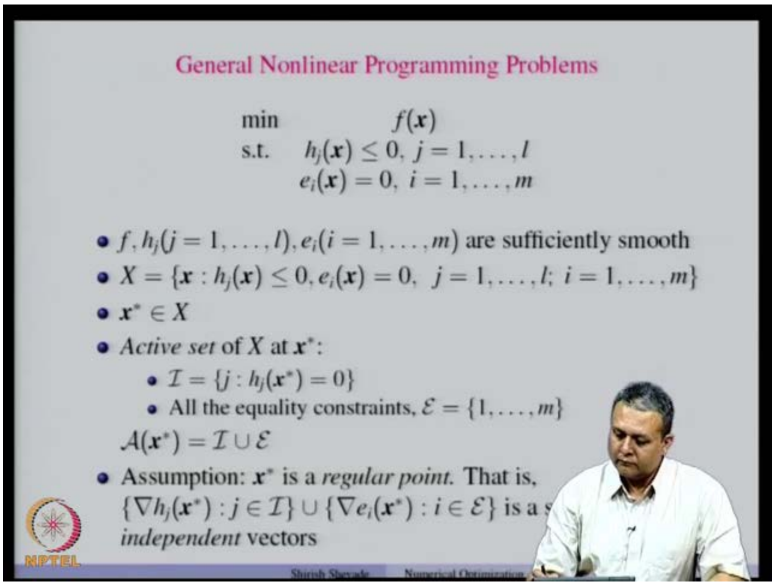

Now, we have considered so, far the problems of the type minimize affects subject to the inequality constraints and problems of the type minimize affects subject to the equality constraints and for both the cases. We derived the KKT conditions KKT conditions necessary for the local minimum.

Now, let us combine the equality as well as the inequality constraints and write a general non linear programming problem. And that problem is of the type minimize f x subject to the l inequality, constraints of the type h j x less than or equal to 0 and m equality constraints of the type e i x equal to 0. So, this is our general non linear programming problem.

Now, again we assume that f in all h j’s and e i’s are sufficiently smooth. So, for first order conditions, we just need this f h and e to be continuously differentiable and for a second order conditions, we need all this function f h and e to be twice continuously differentiable. So, depending upon the condition, we will require the sufficient smoothness of this functions. So, let us denote the feasible set by capital X. So, set of all points, which satisfy all h j x greater than or equal to 0 and all e i x equal to 0.

Now, in the context of inequality constraint problem, we discuss about the regular point and we also discuss about the linear independence constrained qualification. So, if we take a feasible point and if we take all the inequality constrained, which are active then we said that the linear independence constraint qualification holds. If the gradients of the active inequality, constraints are linearly independent.

Now, similarly we assume the linear independence of the gradient of e i x star at a some x star belong into x for regularity. Now we, have to combine those ideas to define the regular point for this problem. So, let us consider a feasible x star and let us see, which sets are active at x star. So, this let us denote by this script I the set of all inequality constraints, which are active. So, that is h j x star equal to 0 or this constraint is satisfied with equality.

Now, as far as the equality constraints are concerned. If x star is feasible then all the equality constraints need to be satisfied so that means, all equality constraints are active, so I active at x star. So, let us combine all the equality constraints in dashes and put them in the set script e. Now this i and script i and script e these 2 sets together form the active set of the constraint set at x star and therefore, a x star is nothing but i union e.

So, let us assume that x star is a regular point, so that means, that gradient h j x star j belong to i and gradient e i x star i belong to e. The this put together are linearly independent vectors, so gradient h j x star where, j is coming from i and gradient e i x star i from the set e, they from a linearly independent set of vectors. So, this assumption here is very important that, x star is a regular point means that they from a linearly independent set of vectors.

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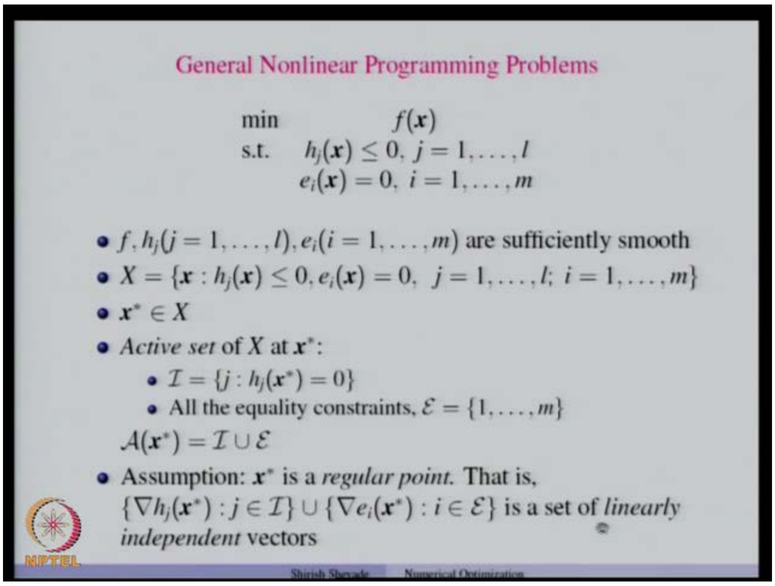

So, we assume that, they are a linearly independent set.

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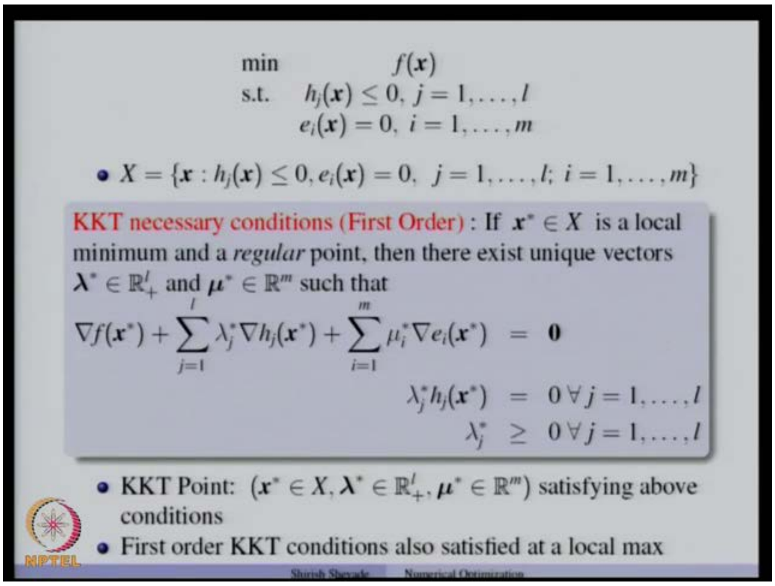

Now, let us look at this problem and we assume that X is a the feasible set, now we can write down the KKT necessary conditions. So, if x star is a feasible point remember that always start with a feasible point. So, whenever I specify the KKT necessary conditions, I do not say explicitly that h j x star is less than or equal to 0 and e i x star equal to 0. Because, it is assumed here that x star belongs to x, x star is a feasible point. Now, if x star is a feasible point and also a regular point, now if x star is a local minimum then there exist unique vectors lambda star belong to R l plus. So, this plus indicates that, this lambda stars are non negative and mu star belong to R m.

So, remember that there are l inequality constraints. So, there is 1 lagrangian multiplier lambda corresponding to every e, inequality constraints and there is 1 lagrangian multiplier mu corresponding to every equality constraints. So, the difference is that the lambda stars come from R l plus or the lambda each of the components of lambda is non-negative well, there is no such restriction on mu’s.

So, the K K T first order conditions say that, if x star is a local minimum and is a regular point and x star is feasible then the gradient of f x star plus sigma lambda j star gradient h j x star plus sigma mu i star gradient e i x star is 0. And lambda j star h j x star equal to 0 for all j’s. This is the complementary slackness condition that, we saw earlier and all the lambda’s are non-negative.

This is stated here, but again, I have repeated here just for the completeness. So, these are called the first order KKT necessary conditions and any point x star mu star lambda star where, x star is feasible lambda star is non-negative mu star is belongs to R m and we satisfies this conditions. We call it as a KKT point. So, to find a solution of this problem, just amongst to finding the KKT points and then checking, which of them give rise to a local minimum or a local maximum. And as we saw earlier that, first order K K T conditions are also satisfied at a local max.

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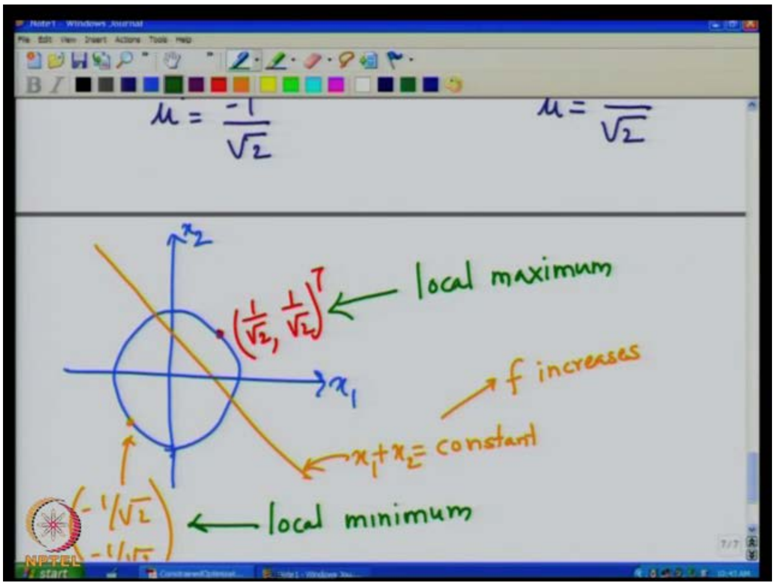

We saw that example where we, so this example we saw that both this points are KKT points, because they satisfy the first order KKT conditions. But one of them is a local maximum and the other one is a local minimum.

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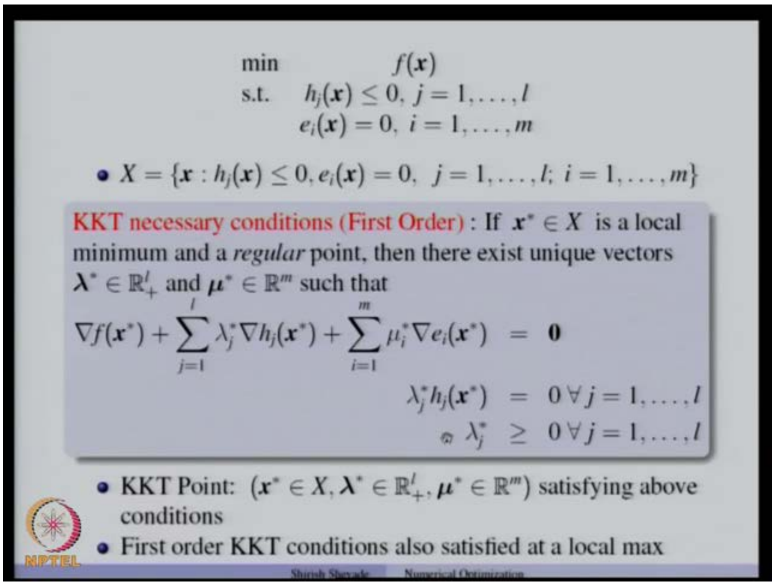

So, it is important to note that, these are just the necessary conditions, they do not guarantee that x star, if this that a KKT point is a local minimum.

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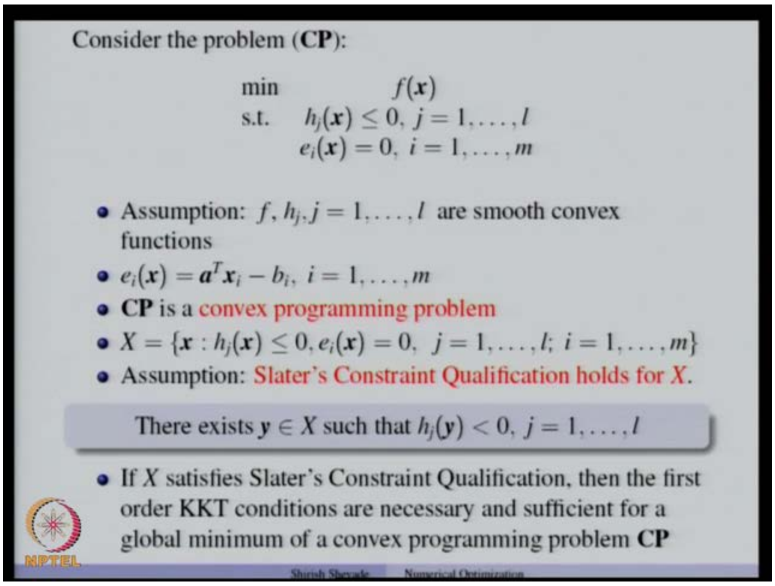

Now, there are some sets of problems for, which the first order KKT conditions are necessary and sufficient. So, consider the problem again a general non linear programming problem that we saw earlier. But, now there are some restrictions on this function. So, let us let us assume that the f and h j's are smooth convex functions. So, here smooth convex functions mean, that f and h j's are continuously differentiated.

Now, let us assume that e i x is of the type a transpose x i minus b I, for all i is going from 1 to m, So, that means, that all the constraints, equality constraints are affine constraints, now we know that, if h j x is a convex function the set h j x less than or equal to 0 is a convex set. So, the intersection of all convex sets is a convex set. So, the set of all inequality constraints under the assumptions that h j is the convex function will form a convex set.

Now, if you take a intersection of those sets with the affine sets which, are of the type e i x is equal to a transpose x i minus b i then, that intersection is also a convex set. Now here is a problem where, we want to minimize a convex function subject to a convex set. So, minimization of a convex function subject to a convex set is called a convex programming problem. And therefore, this program is also called a convex program and that is why, we are going to denote it by CP, CP stands for convex problem, programming problem or convex program.

So, C P is a convex programming problem, because we want to minimize a convex function with subject to a convex set. Now if you denote the feasible set by x then, this set x is a convex set, now let us assume that slater's constraint qualification holds for x. So, we have seen this slater's constraint qualification earlier. So, what the slater's constraint qualification says is that the constraint set x does not have empty interior, so that means, there exists at least 1 y in the constraint set, such that h j y less than 0.

So, for all j is going from 1 to l, so this is a very important thing that for all j is going from 1 to l h j y should be less than 0. Now, here we have assumed that y belongs to x. So, which automatically means, that e i y is equal to 0 for all i going from 1 to m. So, that assumption is always true when, we talk about y belongs to x. So, it is just that, we have to ensure that, there exist at least 1 point which, is in the interior of the set.

So, if x satisfies slater's constraint qualification then, the first order KKT conditions for this problem are necessary and sufficient for a global minimum of a convex programming problem C P. So, we saw a similar result for the case when, f was a convex function and h j x was h j x less than or equal to 0, where the constraint, where the inequality constraints, where h j x were convex factions. Now, we have added equality functions or equality constraints of the type a i transpose x i minus b i. So, how do we extend the result that, we proved earlier.

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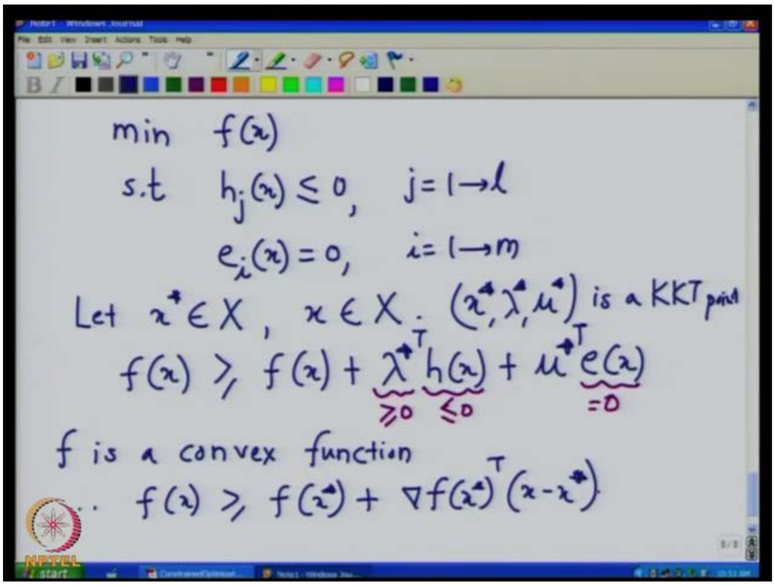

So, this is the problem that, we want to solve minimize f x subject to h j x less than or equal to 0, j going from 1 to l and e i x equal to 0, i going from 1 m now. So, let us assume that, the slater's constraint qualification holds and let x star be a unique point in the feasible set x and let us take any feasible point x other than x star. Now So, earlier case, we saw that f of x is greater than or equal to f of x plus lambda star transpose h x. Now, here h x contains all the inequality constraint h 1 x 2 h h h l x and mu star transpose e x, now the reason for this is that let us assume that x star lambda star mu star is a KKT point. Now, if you look at this quantity, since x is feasible h x is less than or equal to 0 and lambda star is greater than or equal to 0 and if, we look at e x e x equal to 0. So, this quantity is 0, this quantity is non positive quantity and therefore, we have f of x will be greater than or equal to f x. Now, here we use our result from convex functions and we says that if f f is a convex function. So, so f is convex and therefore, we can say that f of x is greater than or equal to f x star plus gradient, f x star transpose x minus x star.

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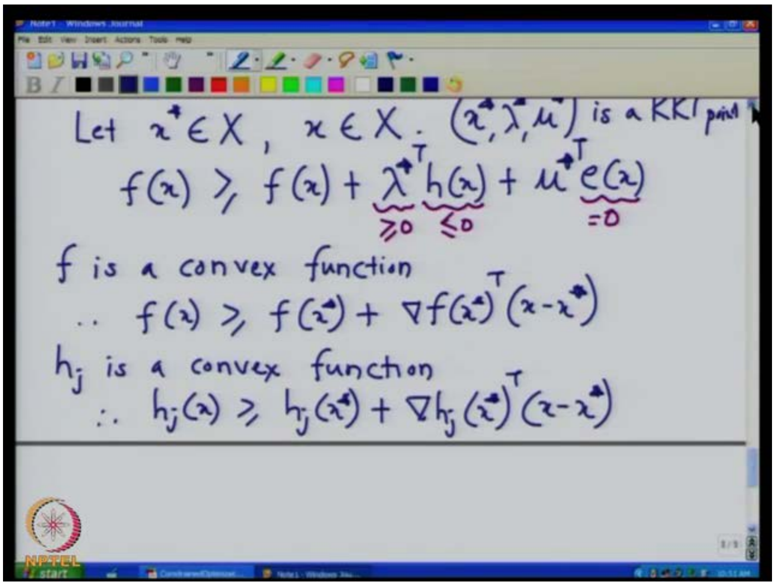

So, similarly we have h j is a convex function and therefore, h j x is greater than or equal h j x star plus gradient h j x star transpose x minus x star, now e i x are of the type.

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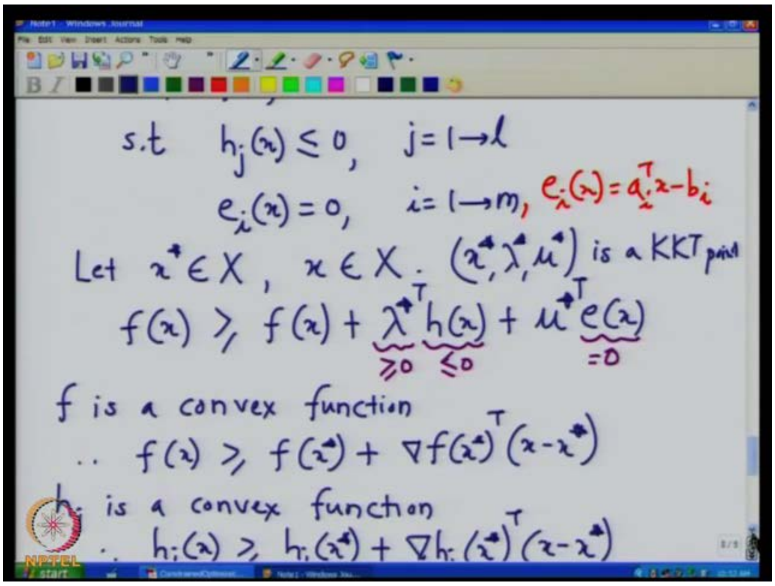

The e i x are of the type a i transpose x minus b i.

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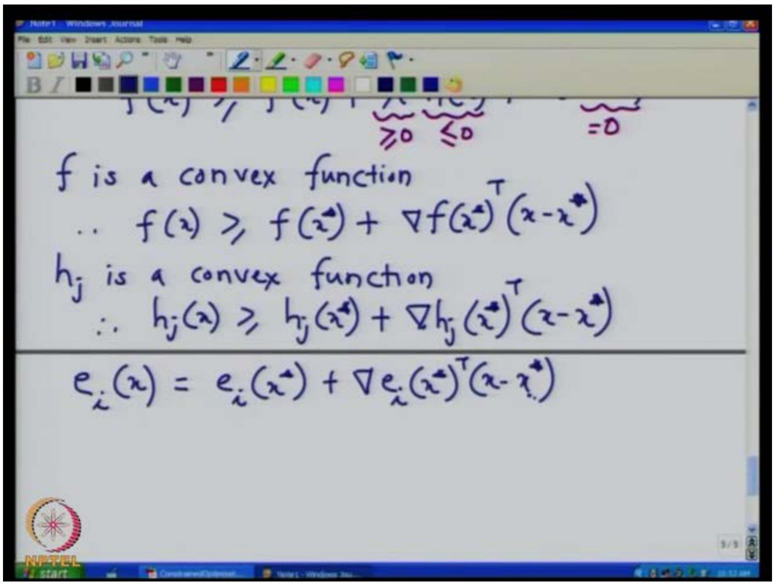

And therefore since, they are of the type e i transpose x minus b I, we have e i x is equal to e i x star plus gradient e i x star transpose x minus x star.

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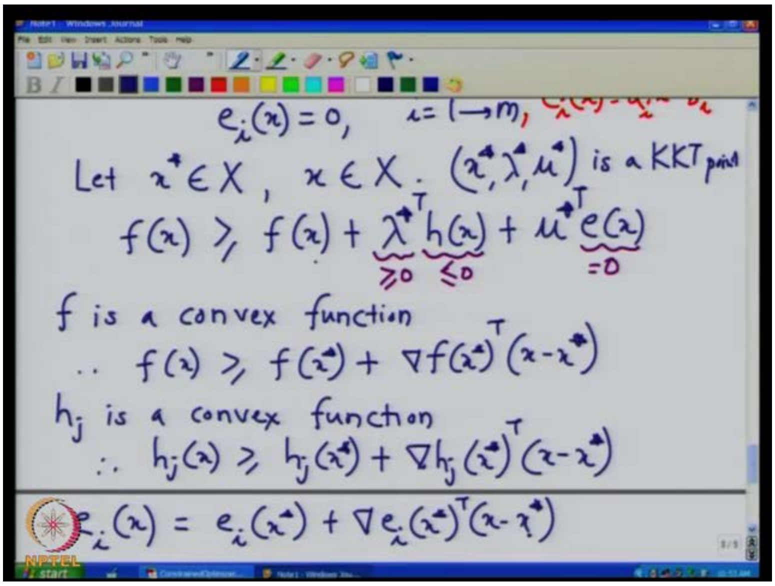

So, now, if we combine all this, if we combine all this, then this function can be written as f of x greater than or equal to f of x star plus sigma lambda j star h j x star plus sigma mu i star e i x star plus, the gradient of the lagrangian transpose x minus x star. So, if we combine this what we get is.

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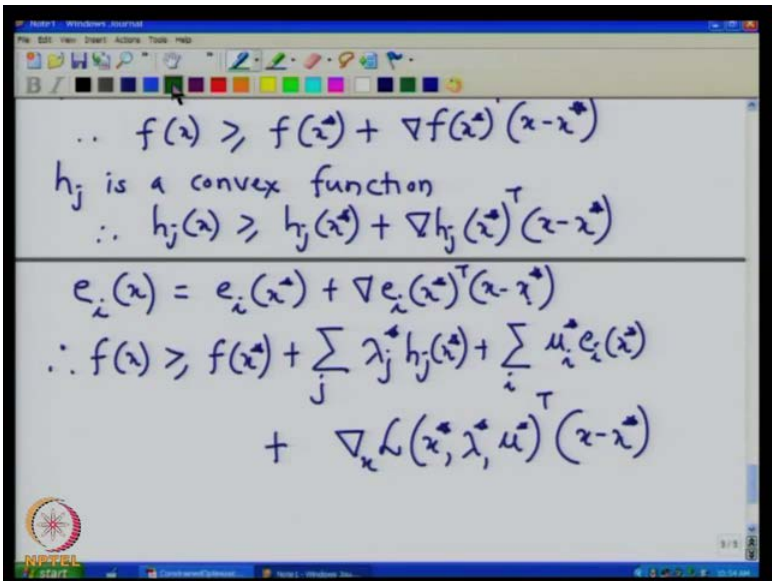

So, therefore, f of x is greater than or equal to f of x star plus sigma j lambda j star h j x star plus sigma i mu i star e i x star plus gradient of the lagrangian eval with respect to x, evaluated at x star lambda star mu star transpose x minus x star. Now what, we do is that, we use the K K T conditions. So, since x star lambda star mu star is a K K T point.

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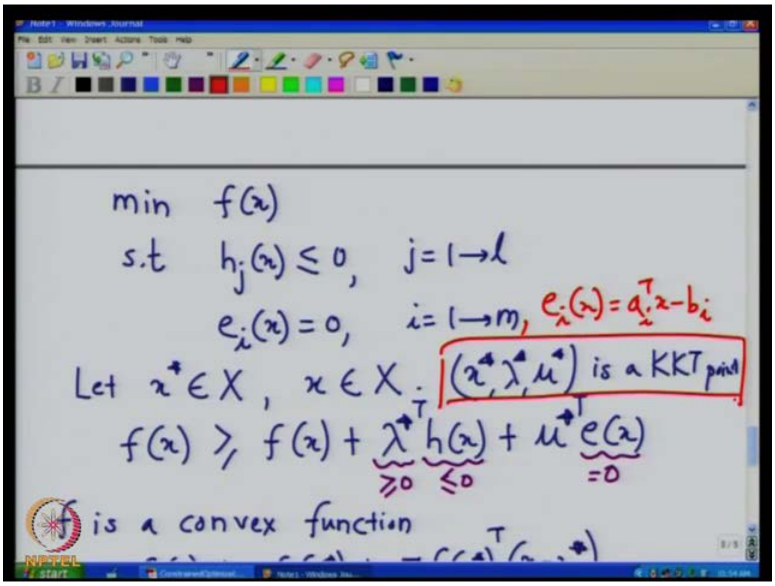

We made this assumption earlier that, x star lambda star mu star is a KKT point. So, we use this assumption along with the KKT conditions to see what happens to this expression.

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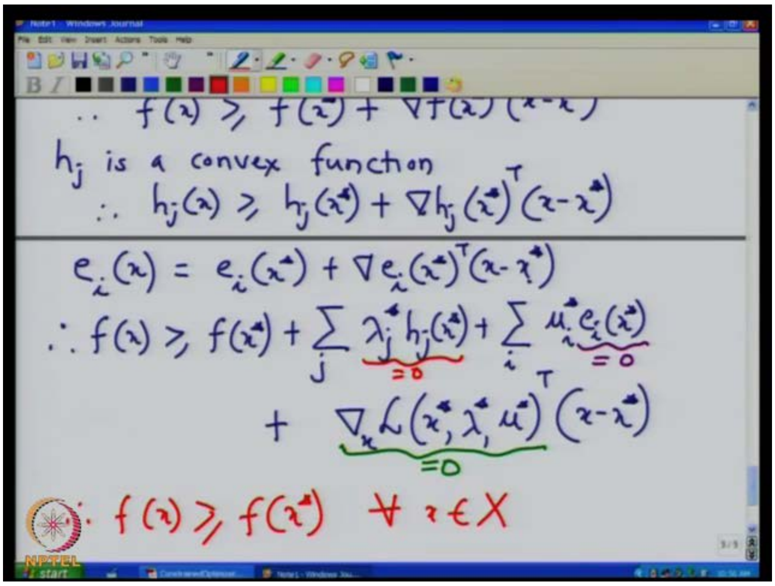

So, since it is KKT point, this quantity is 0 then since x star mu star lambda is a KKT point, because of the complimentary slackness condition. This quantity is also equal to 0. So, we have this quantity equal to 0, this quantity equal to 0 and x star is the feasible point and therefore, this quantity is also equal to 0 and therefore, what we get is. So, therefore, f of x is greater than or equal to f of x star for all x belongs to x and therefore, we get that x star is a global minimum of C P the convex program. So, under the slater's constraint qualification x, the first order KKT conditions are necessary and sufficient for a convex programming problem. Now, we will see some results related to second order conditions in the next class.

Thank you.