Numerical Optimization
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Lecture - 22
First Order KKT Conditions

Hello, welcome back to this series of lectures on Numerical Optimization. In the last class, we started looking at constrained optimization.

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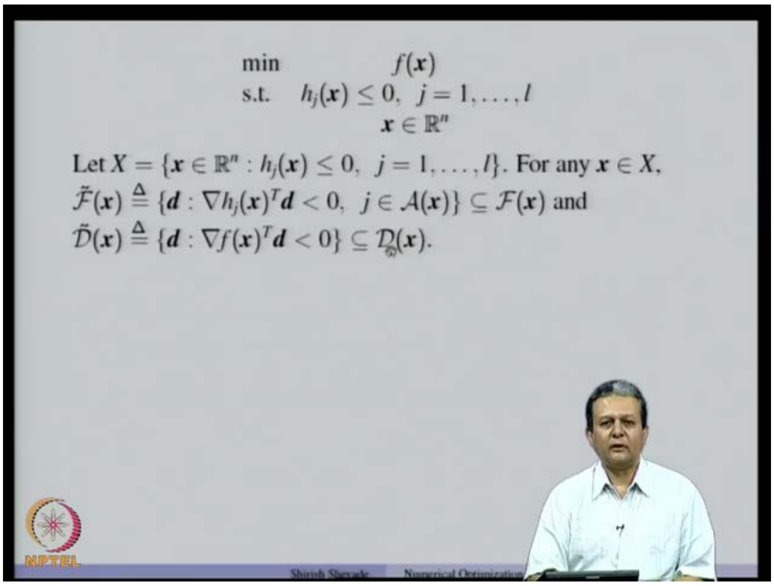

In the problem that we are interested in solving is a minimize f of x subject to the constrained that h j x less than or equal to 0; there are l such inequality constraints. For the time being, we are not considering equality constraints, but we will consider those constraints as well sometime later. And we define the constrained set x to be the set of all vectors in R n such that h j x less than or equal to 0 for all j is going from 1 to l.

And then, we defines a two sets, one set is the f tilde x set and that is corresponding to the set of active constraints, at the current point x and the set of active constraints is the set of all inequality constraints, which are binding at a given point x or in other words those are the constraints which are satisfied with equality at a given point x. So, we collect all those active constraints at a given point x and call that set as script A x. And the set F tilde x defined as the set of all directions d the non zero directions such that they make an obtuse angle with the gradients of the active constraints.

And we also showed in the last class that F tilde x is a subset of f x, f x is a set of all feasible directions the non zero feasible directions. And we also define the set D tilde x to be the set where the set of all directions d such that they make an obtuse angle with the gradient of f at given x; and we have already seen that this is also a subset of D x.

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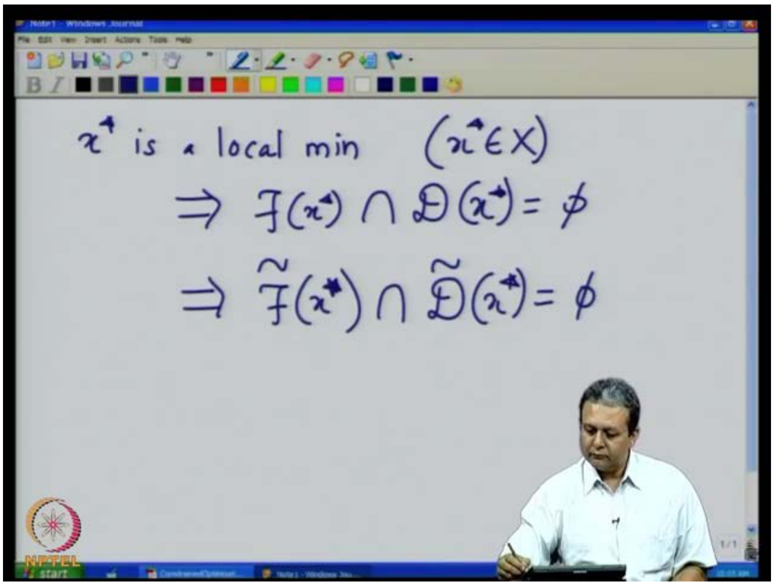

Now, we have seen that x star is a local min, that implies that f x star remember that x star belongs to the set of feasible points. So, f x star intersection D x star is a null set and we saw that f tilde x star is a subset of f x star and D tilde x star is a sub set of d x star and therefore, we can write this as f tilde x star intersection D tilde x star is a null set. The reason for doing or the reason for using f tilde and D tilde is that they can be express using the gradients of the objective function and the active constraints. And that can be use to derive algebraic conditions, which can be used as necessary conditions for extra to be a local min.

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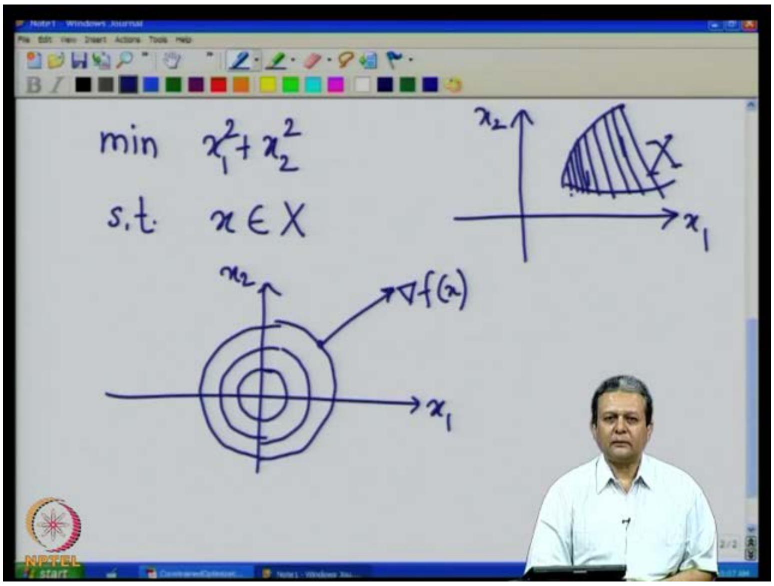

So, so, let us consider a simple problem suppose that we want to minimize x 1 square plus x 2 square subject to x belongs to x, remember that this x is a vector having two component x 1 and x 2 and let us define our constraints set as. So, this is our constraint set X now, if we look at the contours of the objective functions they are the circular contours. So, in other words the objective function contours would be something like this. So, you have two coordinates x 1 and x 2 and then this contours are circles of different radii.

So, these are the objective function contours and as we see that at a given point x since, the function is differentiable we can take the gradient. So, gradient f x will be in this direction and as we move in the direction towards the negative of the gradient be decrease the objective function. So, suppose we want to solve this constraint problem.

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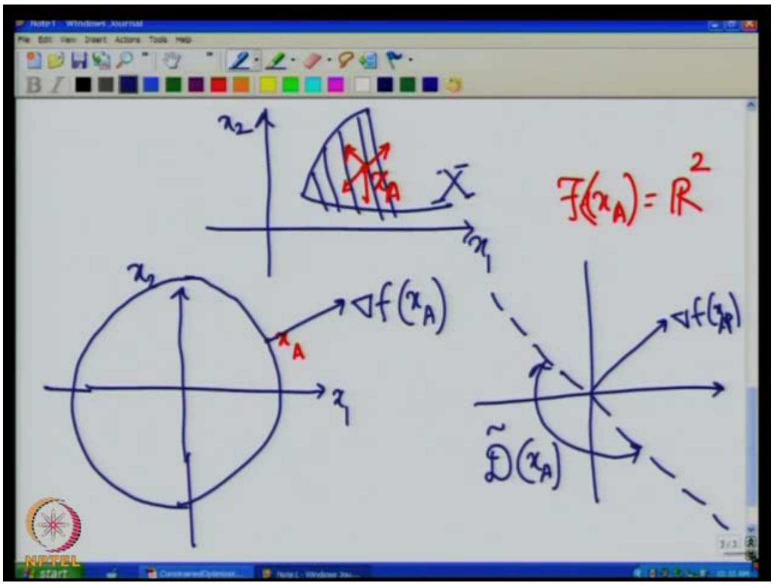

Now, let us look at the let us take the constraint set but, we have seen earlier and let us take a feasible point. So, let us choose some point it is call this as x A now, this x A lies in the interior of the set and none of the constraints are active. So, if you look at the feasible directions, the feasible directions at x A is R 2.

So, now let us look at the, set of descent directions that R 2. So, so if you look at the objective function. So, if you look at the, suppose this is the point x A now, we draw the objective function which passes through this point. So, this is the objective function which passes through this point and we will see that the gradient at this point is pointing in this direction. And therefore, if you want to find out the set of descent directions or in other words we are interested in finding f tilde I am sorry D tilde x A.

And that D tilde x A will be something like this that we have the two directions and then, we take a normal to this gradient f of x A. So, this is the tangent plane at x A, so we have shifted x A to the origin and then, we are looking at those directions which make an obtuse angle with gradient of f of x A. So, so this is the set which will be D tilde x A, note that the gradient f of x A is pointing in this direction and the set of all directions which make an obtuse angle with gradient of f of x A.

That that set is D tilde x A and we will see that, this set is non-empty so; that means, that if we are at x A it is possible to decrease the objective function further. So, that we we can minimize the objective function. Now, let us take the same example.

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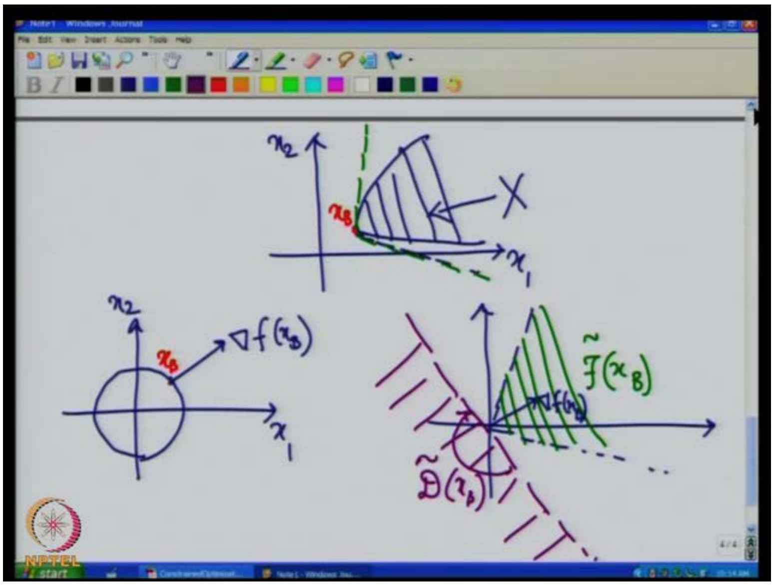

So, let us take a point x B. Now, if you take this point x B let us look at the the, set of active constraints; so this constraint as well as this constraint, this active here. So, let us take the gradients of this constraints and they will be and the gradient with respect to this; so if we take this gradients and then use them.

So, this region is now, the region f tilde x B remember that this was our set constraint set X and what we did was we considered or the active constraints at x B, which are these two constraints and took tangent planes. And then took their the intersection of the respective sets and that terms out to be f tilde x B. Now, if we consider the objective function at x 1 x at x B. So, this is going to be the point x B then the objective function here, would be this and this is going to be the gradient of f at x B.

So, let us take the gradient of f at x B and then, if we take a tangent plane at x B. So, the tangent plane will be, and then this will be D tilde x B. So, you will see that, in this case f tilde x B and D tilde x B does not have any intersection because, D tilde x B is this cone, open cone and f tilde x B is another open cone and they do not have any intersection. So, in this case it turns out that x B is indeed the, global minimum of this problem on the other hand if we look at the previous case.

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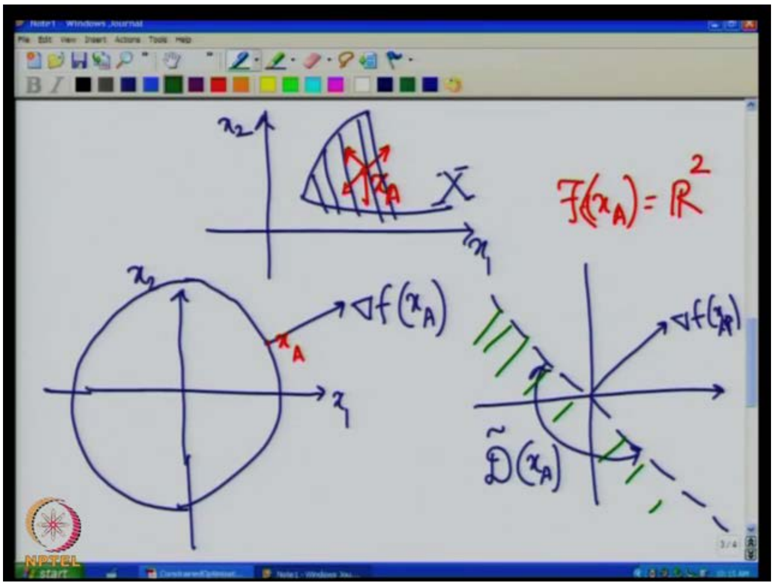

We will see that the D tilde x r x A. Since, f x A is R 2 the entire R 2 plane D tilde x A intersection f x A is a non-empty set so; that means, are there exist are descent direction at x A. So, that we can if you move along the descent direction, we can always decrease the objective function.

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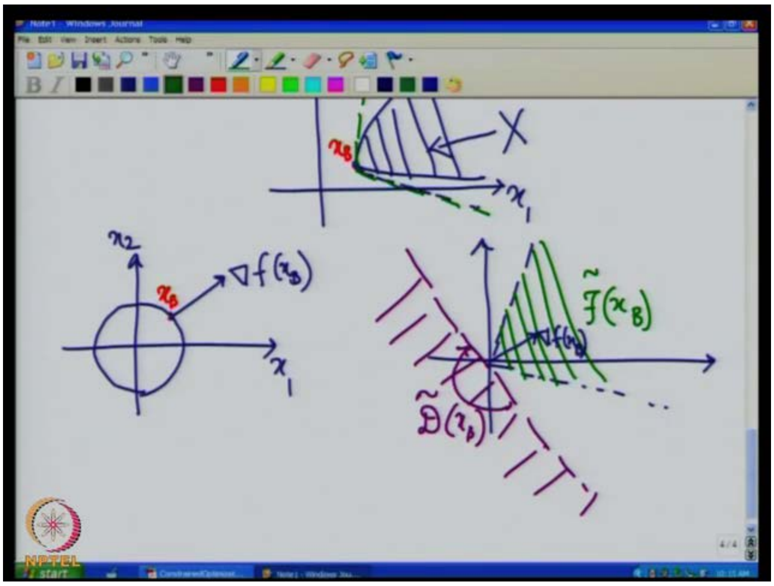

And that is not possible, if we look at x B at x B the D tilde x B and f tilde x B have a non-empty intersection a have empty intersection and therefore, x B turns out to be, in this case global minimum of the problem.

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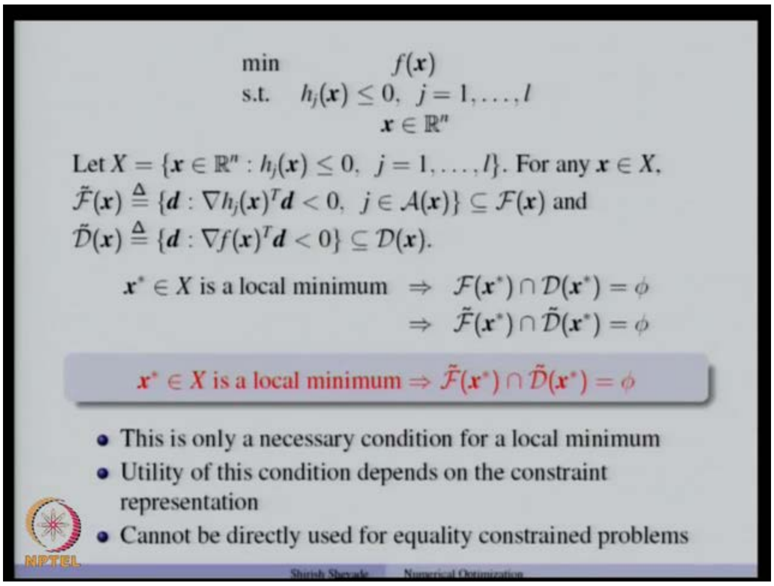

So, so if x star belong to X is a local minimum then that implies that f x star intersection D x star is null set and since, this sets are cannot be written algebraically we wanted to use some sets, which can be represented algebraically. And therefore, we defined f tilde and D tilde x and therefore, the intersection of, at a local minimum x star the intersection of f tilde x star and D tilde x star is a null set. So, we have this important result and note that this is only a necessary condition for a local minimum.

For example, if at a particular point the gradient of a function is 0 then; that means, that at that particular x star gradient f x star is 0. And then therefore, this set becomes a null set and this condition is automatically satisfied, but that does not mean that x star is a local minimum for a constraint problem. We also saw that the utility of this condition depends of how do we represent the constraints set. So, in the last class we considered one example, where the same constraint set if it is represented using two different waves.

Then in one case, we got f tilde x star to be a null set and in the other case we got f tilde x star not to be a null set. So, once any of this sets become empty, the intersection become automatically empty sets. So, we have to avoid such cases and we also saw that, this condition cannot be directly used for the equality constraint problems.

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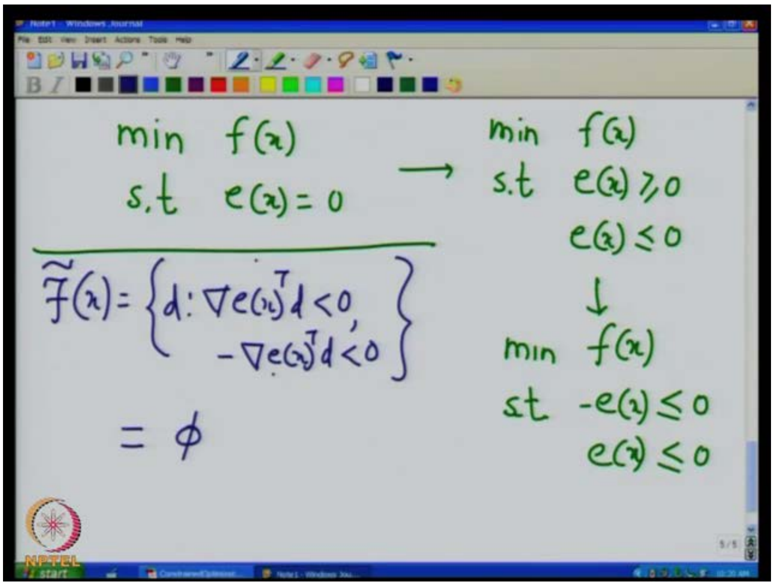

So, if we have a problem minimize f x subject to e x 0 this is a equality constraint problem, let us assume that we have only one equality constraint. So, we can write this problem as minimize f x subject to e x greater than or equal to 0 and e x less than or equal to 0; or in other words, we can write this in our usual form minimize f x subject to minus e x less than or equal to 0 and e x less than or equal to 0. Now, if you look at this constraint and then write f tilde x to be note that this is the equality constraint.

So, both this constraint should be active at a any given point x. So, set of all these such that gradient e x transpose d less than 0 and minus gradient e x transpose d less than 0. So, this corresponds to the second constraint and then the this condition minus gradient x transpose d less than 0 corresponds to the first constraint. So, you will see that, this set is always a null set because, we cannot find out direction d where which is making obtuse angle with gradient e x as well as negative of the gradient of e x.

So, f tilde it always becomes a null set and therefore, this condition, the condition that we derived earlier that x star is a local minimum implies that f tilde x star intersection b tilde x star is null set, that condition is trivially satisfied for the equality constraint problem.

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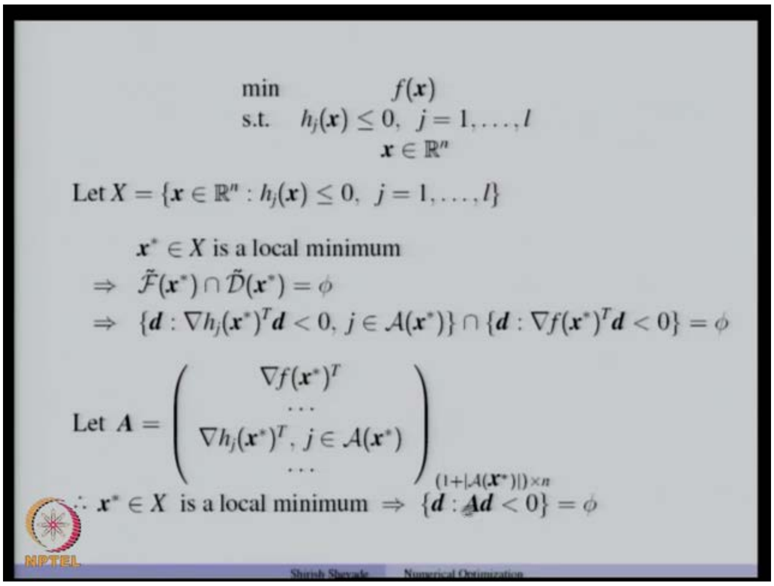

So, now let us look at this problem and again we define constraint set x in this way and x star is a local minimum which implies that this holds. And let us rewrite, those conditions in the form of the gradients of the constraints and the objective functions, note that we are always working with the active constraints at any point of time. So, at x star we collect all the active constraints and find this set gradient h j x start transpose d less than 0 and intersection of that with the set, where set of all directions, which make an obtuse angle with gradient f x star and x star is a local min implies that this intersection is empty.

Now, can we write this conditions in a more compact form. So, for that purpose, let us define a matrix A, whose one row is the transpose of the gradient of f at x star and the remaining rows are the the gradient vectors, transposed and put in the form of different rows of the matrix A. Remember that, this matrix A depends on a x star, but just for notational convenience, we have drop the dependents on x star we have not written that here. Now, this matrix if you look at the number of rows.

So, there is a row corresponding to the gradient of the objective function and there are, rows corresponding to the set of active constraints. So, the number of rows will be 1 plus cardinality of the set A x star and since, these are gradient vectors in n-dimensional space the number of columns will be n. So, if we define this matrix A like this, then we can say that x star is a local min implies that the set of all directions d says that A d less than 0 is a null set. I again repeat that the A is always depends on x star, but to avoid notational later, we have not indicated it here. So, this is the compact way of representing this constraints and slowly, we will move to the algebraic conditions which are necessary, for x star to be a local minimum.

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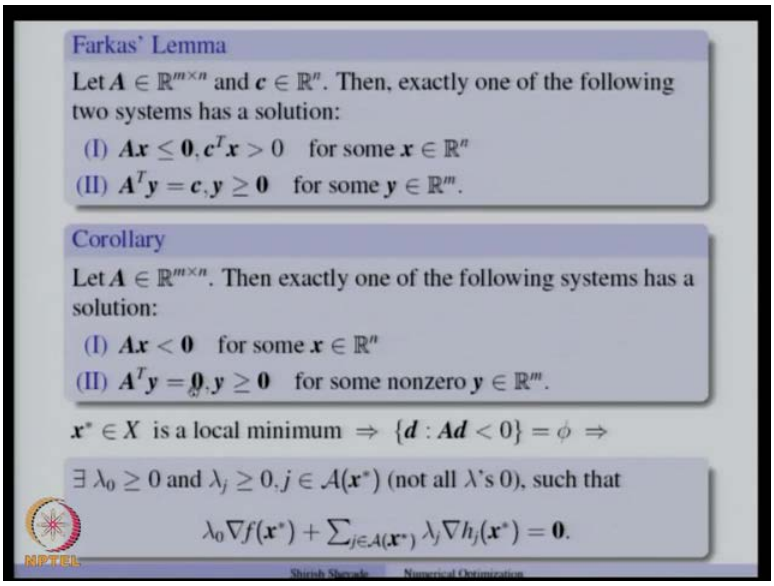

Now, to write those are algebraic conditions, we will need Farkas' lemma which we studied when we, discussed about convex sets and convex functions. So, let us recall Farkas' lemma. So, if we have matrix A which is m by n matrix of real numbers and a vectors see which is a n-dimensional vector, then exactly one of the following systems has a solution.

So, either we have x less than or equal to 0 and c transpose x greater than 0; that means, that one can find x, which makes an obtuse angle with, which makes an acute angle with c and obtuse angle with I am not an not an acute angle with the rows of the matrix A or there exist some y, which is which satisfied this condition that a transpose y is equal to c and y greater than or equal to 0. So, either of this systems, as a solution and what is important for us is a corollary of Farka's lemma that we discussed earlier and also proved.

So, let us look at this corollary, so we have a m by n matrix A and then the corollary says that exactly one of the following systems, as a solution. So, either we have A x less than 0 or there exist some non-zero y such that such that A transpose y is 0 and y greater than or equal to 0. So that means, there exist, some non-negative vector y, which is not 0 vector; that means, not all the components y or 0 at a any kind of time and A transpose y is 0.

So, either of this two systems has a solution. Now, you will see that, if the rows of the matrix A are linearly independent, then A transpose y equal to 0 will happen only in the trivial case where y equal to 0 and therefore. So, the system to may not have a solution, but then, that will be some x which is in the n-dimensional space such that A x is less than 0. So, now let use this corollary, to write the algebraic condition for a local minimum.

So, we have X star is a local minimum implies that the set of all directions d such that, A d less than 0 is a null set and we use this corollary. So, the A d less than 0 is similar to this system one and if this is a null set then system two has a solution. So that means, that there exist some lambda 0, which is non-negative and lambda j, which is also non-negative, where j belongs to the set of active constraints at x star and remember that, there is a non-zero y. So, we how these lambdas, which are all non-zero, not all lambdas are 0.

So, such that this condition is satisfied So, this condition is same as the system two which is written here where y's are replace by lambdas and A is the matrix, which is written using or which is obtain using gradient f x star and gradient h j star, where j belongs to A x star. So, the system one has no solution, means that system two has a solution and therefore, there exist some lambdas not all lambdas are 0. And remember that, there is a lambda 0 associated with gradient of f x star and lambda j associated with the set of active constraints h j x.

So, so in this case we are able to write the necessary condition for local minimum in the form of and algebraic condition which uses, gradient vectors of the objective function and gradient vectors of the of the active constraints.

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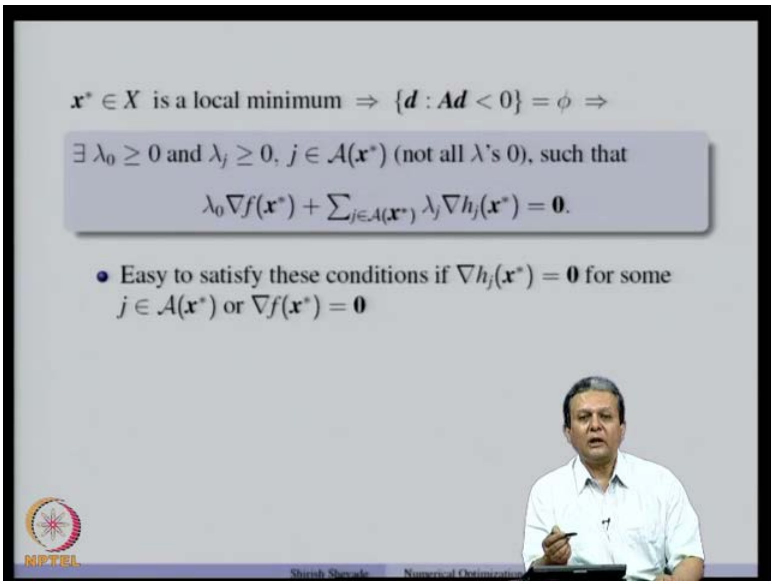

Now, if you look at this condition, it is easy to satisfy this condition, when suppose gradient f x star is 0. So, in that case what we can do is that, we can set lambda 0 to any positive value and set the remaining lambdas to 0 and this condition get satisfied. Also if any of the gradient h j x star is 0 then, we can set all the remaining lambdas to 0 accept that particular lambda which can be set to a positive value. So, this conditions becomes trivially satisfied under those special conditions.

Now, as I mentioned earlier that x star is local minimum implies that f tilde I x star intersection D tilde x star is a null set. Now, the D tilde x star is a null set, that condition suppose if it is satisfied for given set of active constraints, then what happens is that f tilde x star intersection D tilde x star is automatically a null set. And therefore, we would not be considering the objective function at a point x star there, because object regardless of what the objective function value is that condition is always satisfied because, D tilde x star is always is if it a null set.

And somehow to we have avoid D tilde x star to be a null set. So, one way of ensuring the D tilde x star is not a null set is that let us assume that this gradient h j x star, where h j belongs to the set of all active constraints, they are linearly independent. So, this quantity can become 0 only in the trivial case, and in that case the lambda 0 becomes greater than 0 because, what we want is that not all lambdas are 0.

So, if we ensure that the gradient h j x star there j belongs to script x star, there are linearly independent then the only way this combination can become 0 is web setting all lambda this to be 0. And in that case lambda 0 cannot remain 0 and therefore, we can ensure that, the set D tilde a I am sorry the set f tilde x star, which is corresponds to the constraint set, will not be a empty set because, if we look at this.

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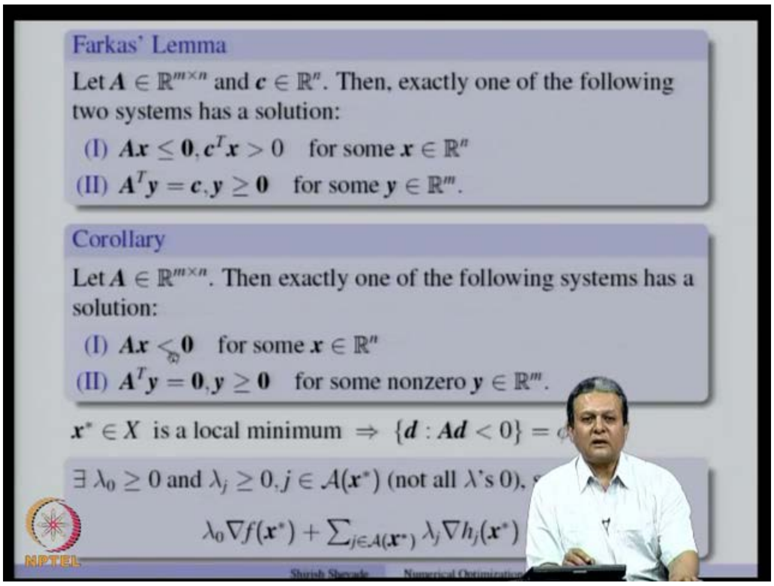

So, if there exist only a combination where all y's are 0 then this system does not have a solution; that means, that A x is less than 0 as a solution. So, if you just consider the rows of the matrix A corresponding to the active constraints and apply this corollary you will see that, if the gradients of the active constraints are linearly dependent, then the set f tilde x star will not be an empty set and therefore, we can avoid the trivial cases.

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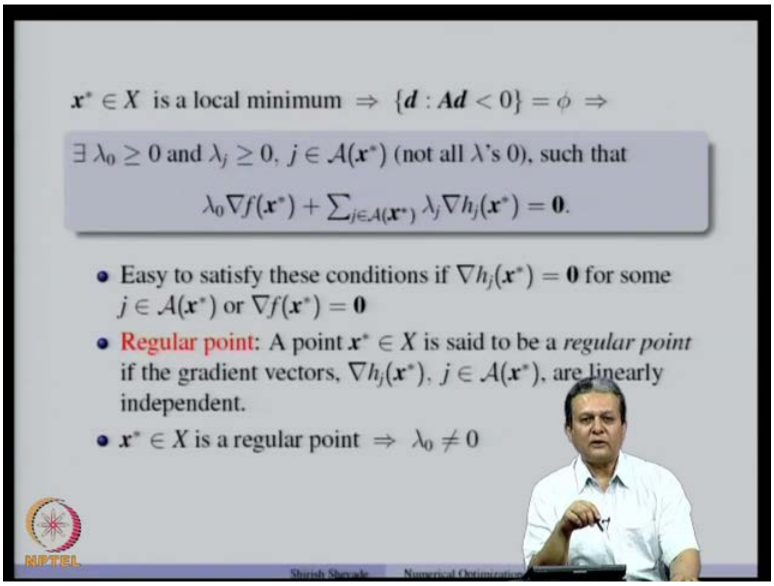

And that is done by, using this assumption that x star is a regular point. So, x star is the regular point, if the if the gradient vectors of the active constraints are linearly independent. So, this will guarantee that lambda 0 is greater than 0 and therefore, we do depend on the objective function. Because otherwise if you do not ensure that then what will happen is that D tilde I am sorry f tilde x star will become a null a null set and f tilde x star intersection D tilde x star also will be automatically a null set.

And; that means, that we are not giving importance to the objective function and that should not be the case. So, we are looking for those conditions under which, if you look at certain points, which are regular and satisfy certain conditions those are a possible candidates for a local minimum. So, as I mentioned earlier that, if it is a regular point then lambda 0 is certainly greater than 0 it cannot be equal to 0, because since this constraints are independent. The only way this quantity can become 0 is lambda j equal to 0 and under those are circumstances lambda 0 cannot be equal to 0, because according to the theorem this one, not all lambdas are 0.

So, what we do is that now, we can write this condition only with respect to the active constraints. Now, what happens to the constraints, which are inactive now, we can safely assume that the lambdas corresponding to those constraints are 0.

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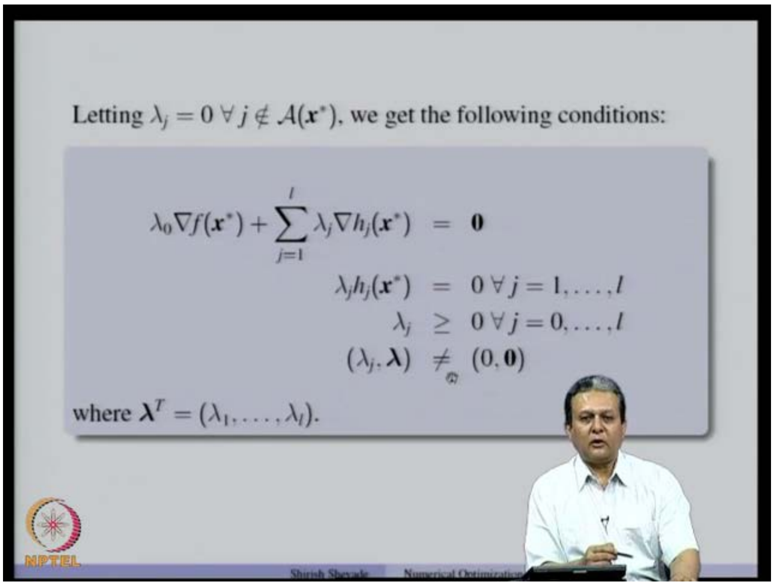

So, let us would those lambdas to 0 and what we get is this condition lambda 0 gradient f x star plus sigma lambda j gradient j x star equal to 0. Now, remember that we have introduce this. So, all those constraints which are inactive for them h j x star will be less than 0 and if you multiply lambda j by j x star then that becomes 0. So, for active constraint h j x star is always 0. So this conditions is satisfied, for inactive constraints h j x is less than 0, but by setting lambda j corresponding lambda j is to 0 this condition is satisfied.

And lambda j's are greater than or equal to 0 that that is as per the theorem or corollary of farka's lemma that we saw earlier and not all lambdas are 0. So, the lambda 0 and lambda, this should be lambda 0 and lambda which is a vector containing lambda 1 to lambda l. So, these are all non-zero.

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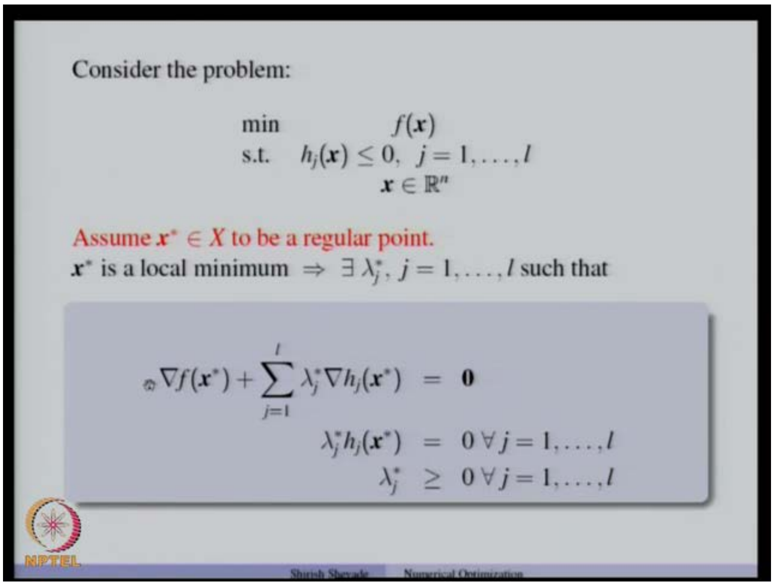

Now, let us consider the problem, where we want to minimize the same objective function f x, subject to this constraints and now, one assumption that we make is that x star is a regular point. That means, that if we consider the set of all active constraints that x star, then the gradients of those active constraints are linearly independent. So, if x star is a local minimum then there exist lambda j star, such that gradient f x star plus sigma j lambda j star gradient h j x star equal to 0.

Now, since we have assumed x star to be a regular point, we have already seen that lambda 0 is greater than 0. So, ideally there should have been a lambda 0 here, but since we have regular point and lambda 0 is greater than 0 we can divide the those conditions by lambda 0 and by dividing that, the coefficient of this is made one and then we have the corresponding lambda j star. And then the lambda j star j x star equal to 0 is as discussed earlier and all the lambdas are lambda j stars are non-negative.

So, this set of conditions are derived from the conditions that we saw earlier, by ensuring that lambda 0 is strictly positive and that is ensured by assuming that x star is a regular point.

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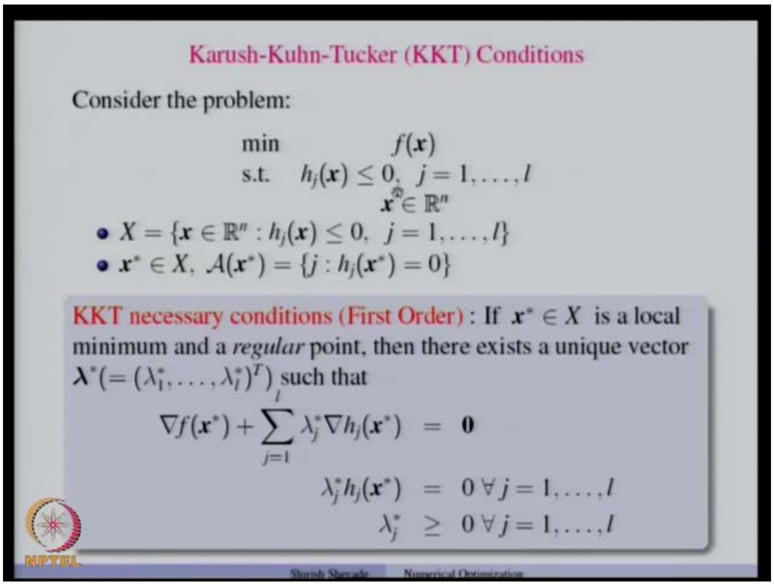

Now, this set of conditions are called Karush-Kuhn-Tucker or in short KKT conditions and, they form a set of conditions which need to be ensured at a local minimum. Now, the conditions were derived, by Karush some time in 1939 and they were independently derive by Kuhn and tucker in 1951. So, the credit goes to, all the three persons who derived those conditions and that is why these conditions are called KKT conditions. So, in this course, whenever I refer to KKT conditions. So, I mean the conditions derived by Karush and Kuhn and Tucker.

So, let us look at this problem and the constraint set x and let us assume that x star is a feasible point and let us, collect all the possible set of active constraints at x star and denote them by script A x star. So, we have the first order KKT necessary conditions for, the local minimum of this problem. Note that, as I mentioned earlier that this functions and the the function f as well as the h j x are differentiable. In fact I also mention that, they belong to c two, but for this conditions it is enough that they belong to c one.

So, if x star which is the feasible point is a local minimum and it is a regular point, then there exist a unique vector lambda star, such that gradient f x star plus sigma j lambda j star gradient h j x star is 0 lambda j star h j x star is 0 and lambda j is non-negative. So, this conditions are called, first order necessary KKT conditions for the program which is given here. Now, the KKT necessary conditions are more general in the sense that they all, so use the equality constraints, but. So, for we have not at talked about the equality constraint problem.

So, we will call this conditions as KKT necessary conditions for this program, but later on we will bring in the quality constraints programs also.

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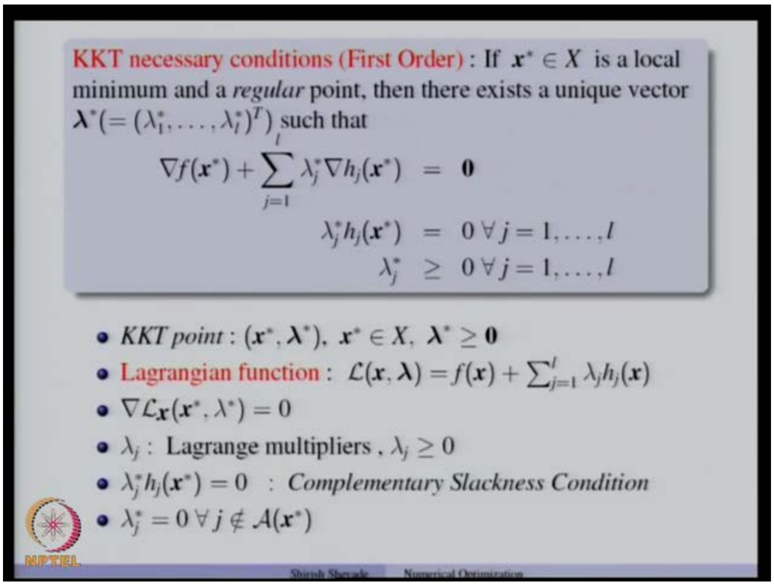

So, these are the KKT necessary conditions. Now, the point x star lambda star, is called KKT point and many a times as part of the KKT conditions people also write x star belongs to the feasible set, but here we have mentioned earlier mentioned this. So, it is not part of the KKT conditions, but sometimes there is the practice to you write x star belong to f x part of the KKT condition. So, this x star and lambda star together, is called a KKT point were x star is a feasible point and lambda star is a vector of non-negative numbers.

Now, if we define a function which is called a Lagrangian function, that function is defined as f x plus sigma lambda j h j x and remember that, we are assuming that f and h j is are differentiable. So, the first condition of this result, says that the gradient of the Lagrangian function with respect to x vanishes. Now, recall that when we discussed about un constrained optimization problem, the condition that we got was that if you want to minimize f of x. Then x star is a local minimum of f implies that, gradient f x star is 0 that was for a unconstraint optimization problem.

Now, here we have constraint optimization problem and the condition that now, we get is that x star is a local minimum and if it is a regular point, then the gradient of the Lagrangian at x star lambda star, well with respect to the variable x vanishes. So, gradient of the Lagrangian with respect to x evaluated at x star and lambda star the vanishes. Now, this lambda j's are called Lagrangian multipliers and they have to be non-negative. Also, we have this condition lambda j star h j x star equal to 0 for all j this condition is called complementary slackness condition.

And note also that lambda j star are 0 for all j not in x star or in other words for all constraints which are inactive the Lagrangian multipliers at the solution are 0. So, so these conditions are called the KKT conditions. So, which say that, the gradient of the Lagrangian at x star lambda star, evaluated with respect to x vanishes, the complementary slackness conditions hold and all the Lagrangian multiplies corresponding to the inequality constraints are negative. Remember that, there is a Lagrangian multiplier associated with every constraint.

So, if your l constraints, we have l Lagrangian multipliers and at solution at the solution, the Lagrangian multipliers corresponding to all the inactive constraints are 0 and a since, at the since for the active constraints h j x star is 0. So, lambda j star h j x star is 0 note that, lambda j star could be 0 for active constraints.

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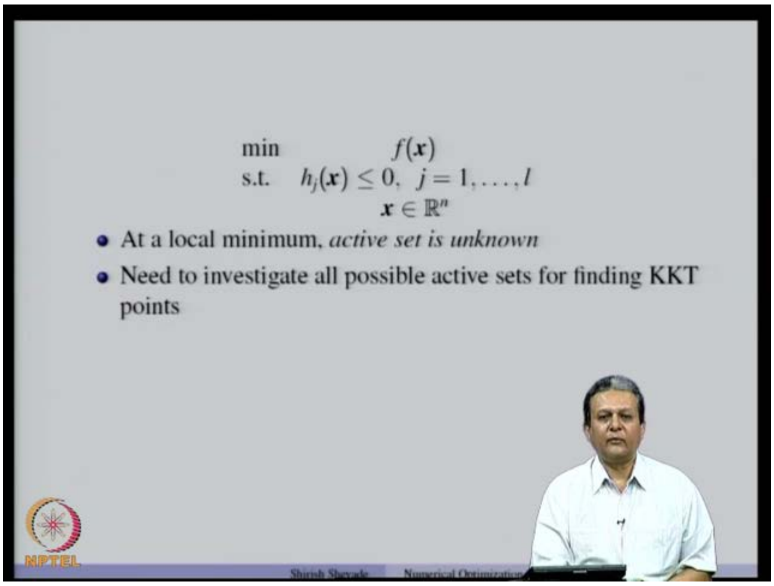

Now, typically when we want to solve this problem, at a local minimum we do not what are the set of active constraints. So, what we have to do that we have look at all possibilities or all possible sub set of the active sets and see finally, at the solution which set of constraints are active. Suppose, we had known the the active set at the solution earlier, then this problem could have been converted to un equality constraint problem and we would a solved that, but that is typically not the case. So, one needs to investigate, all possible active sets for finding KKT points.

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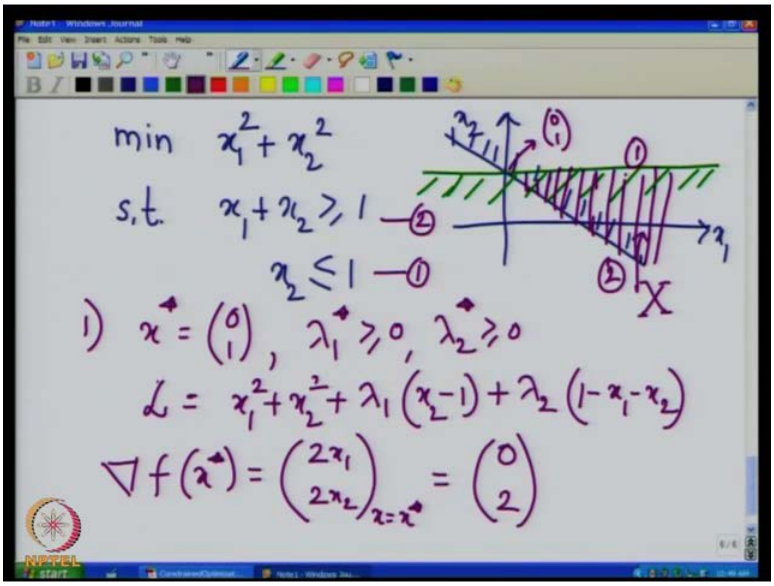

So, let us take a example suppose, we want to minimize x 1 square plus x 2 square subject to x 1 plus x 2 greater than or equal to 1 and x 2 less than or equal to 1. So, we look at the constraint set. So, x 1 plus x 2 greater than or equal to 1. So, this is x 1 and this is x 2. So, it is this set x 1 plus x 2 greater than or equal to 1 and x 2 less than or equal to 1 x 2 less than or equal to 1 is set and therefore, what we have is the constraint set which is shown it. So, this is going to be the constraint set X and, we want to minimize this objective function.

Now, at this movement we do not, where does the solution lie. So, so let us assume that this constraint is 1 this constraint is 2 and this point which is. So, we have to consider different possibilities, at the at the solution either this constraints is active or this constraint is active or both the constraints are active and both the constraint are active, at this point. So, this three possibilities need to be consider. So, let us consider a possibility that.

So, the first case that we want to consider is that, x star is equal to 0 1. Now, both the constraints are active. So, we cannot really say about, we cannot say that any of the Lagrangian multipliers corresponding to this constraints are inactive, corresponding to this constraints are 0. So, we can just say lambda 1 star is greater than or equal to 0 and lambdas 2 star is greater than or equal to 0. Now let us write the Lagrangian of this.

So, the Lagrangian will nothing, but the objective function x 1 square plus x 2 square plus lambda 1 into 1. So, we are calling this constant as 1. So, this constraint is constraint 1 and this constraint is constraint 2; so lambda 1 into x 2 minus 1 plus lambda 2 into 1 minus x 1 minus x 2. So, remember that we, have converted this 2 the form h j x less than or equal to 0. So, it will be 1 minus x 1 minus x 2 less than or equal to 0 and therefore, it is written as lambda 2 into 1 minus x 1 minus x 2.

So, now let us look at the the gradient of f at x star. So, gradient of f at x star is nothing, but 2 x 1 2 x 2 evaluated at x equal to x star and x star is 0 1. So, this gradient will be 0 comma 2.

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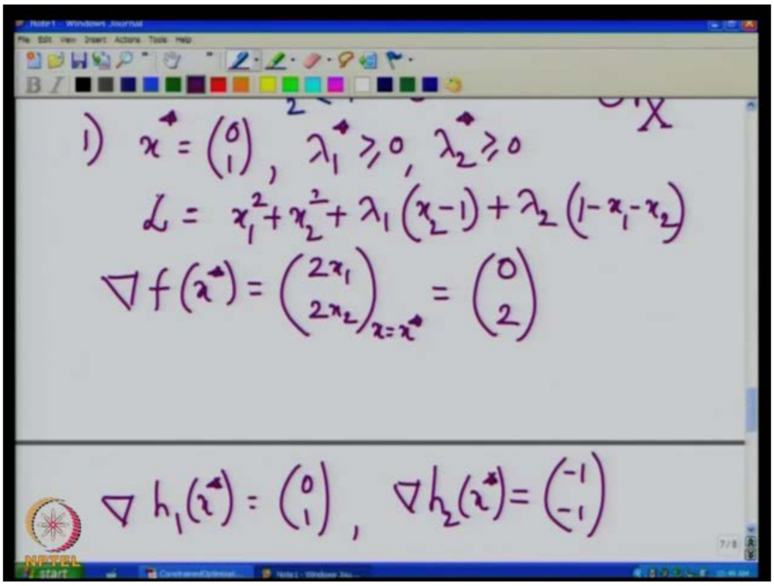

Now, let us look at gradient. So, let us take the first constraint, which is x 2 less than or equal to 1. So, gradient h 1 x star is equal to 0 1 and gradient h 2 x star is equal to minus 1 minus 1. So, what we are interested in it is.

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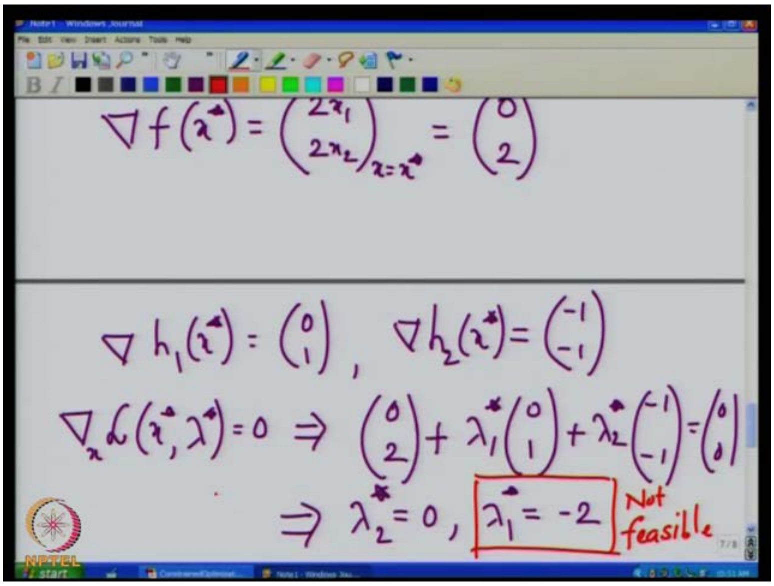

We, are interested in finding out remember that, gradient of L with respect to x evaluated at x star lambda star is 0 for KKT point. And, so that implies that 0 2 plus lambda 1 star 0 1 plus lambda 2 star minus 1 minus 1 is equal to 0 and this implies. So, if you look at this condition. So, 0 plus lambda 1 star 0 that is 0 plus lambda 2 star into minus 1 is equal to 0. So, which implies that lambda 2 star is equal to 0 and lambda 1 star. So, lambda 2 star is 0. So, which means that lambda 1 star is equal to minus 2 for this condition to be satisfied.

Now, if you look at our result, which is that all is the lambda should be non-negative. So, so this is not possible because, all lambdas have to be non-negative. So, this is this is not a feasible point, in our case.

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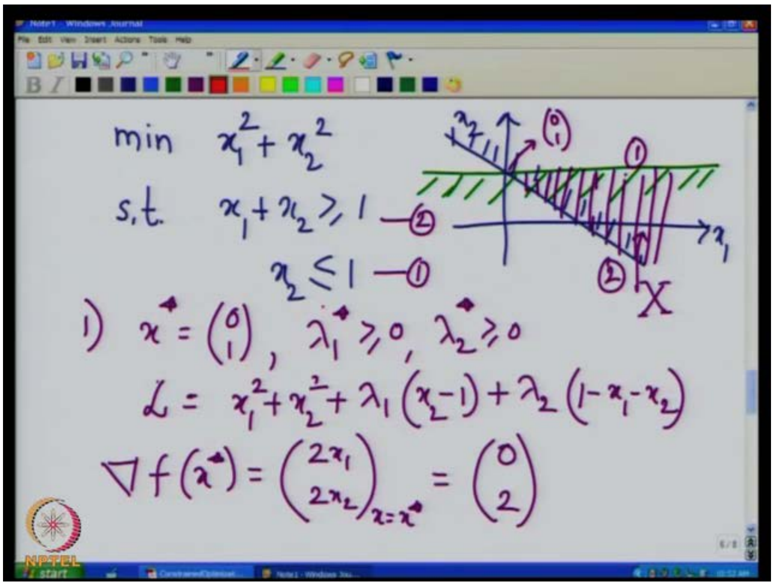

So, the solution of this problem cannot lie at the point 0 1. Now, let us assume that the constraint 2 is active. So, constraint 2 is active means x 1 plus x 2 is 1 and x 2 is strictly less than 1, so let us considered that case.

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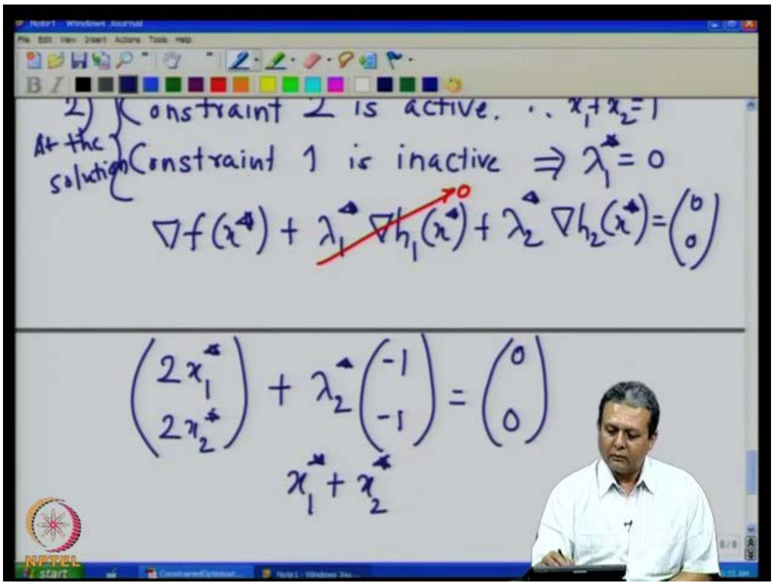

So, let us consider the case 2 where constraint 2 is active. So, which means that therefore, x 1 plus x 2 is equal to 1 because, that is our second constraint and the first constraint is the x 2 less than or equal to 1 and constraint 1 is inactive. So, this 2 are true at the at the solution. So, constraint 1 is inactive, this implies that lambda 1 star is equal to 0 and constraint 2 is active means that at a solution x 1 star plus x 2 star is equal to 1. But we do not know what are those x 1 star and x 2 star and also we need to find out lambda 2 star.

So, so we have x 1 star plus x 2 star to be 1 and the other condition is gradient f x star, f x star plus lambda 1 star gradient h j x star h 1 x star plus lambda 2 star gradient h 2 x star, we at 0 vector; and we are interested in finding x 1 x 1 star and x 2 star and lambda 2 star. So, since lambda 1 star is 0 this quantity is 0 and the gradient f x star is 2 x 1 star 2 x 2 star plus lambda 2 star gradient h 2 x star. So, gradient h 2 x star is minus 1 minus 1 is equal to 0. And along with this, we also need to satisfy x 1 star plus x 2 star is equal to 1.

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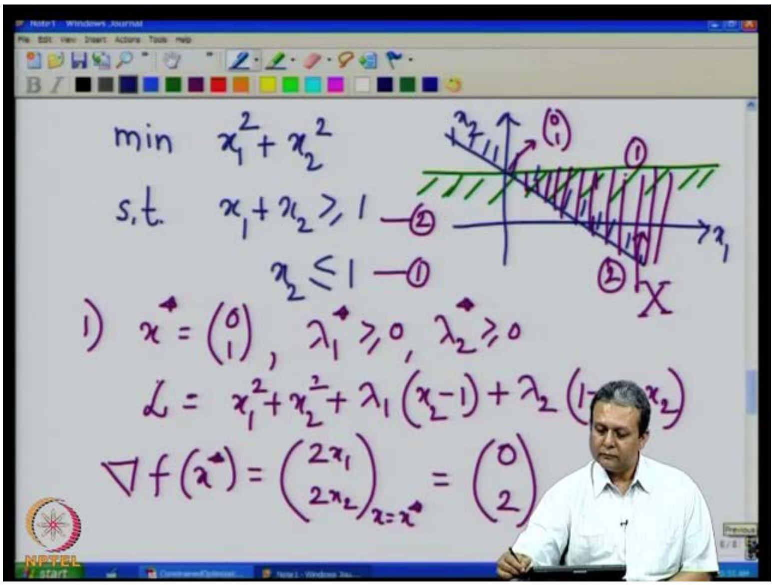

It is this condition that we need to satisfy x 1 star plus x 2 star is equal to 1.

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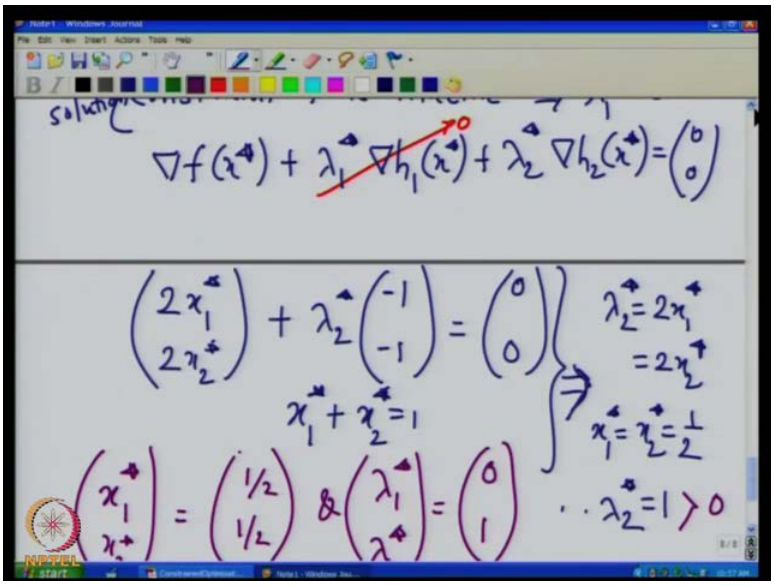

Now, from this first condition, what we get this implies that lambda 2 star is equal to 2 x 1 star and that is also equal to 2 x 2 star. So, x 1 star is nothing, but x 2 star and x 1 star equal to x 2 star equal to 1 this implies that x 1 star is equal to x 2 star is equal to half and therefore, lambda 2 star is equal to 1 and this quantity is greater than 0. So, so we have x 1 star x 2 star is equal to half comma half and lambda 1 star lambda 2 star will be 0 and 1. So, all the lambdas are non-negative and x 1 x 1 star x 2 star is a feasible point and satisfies the KKT conditions and therefore.

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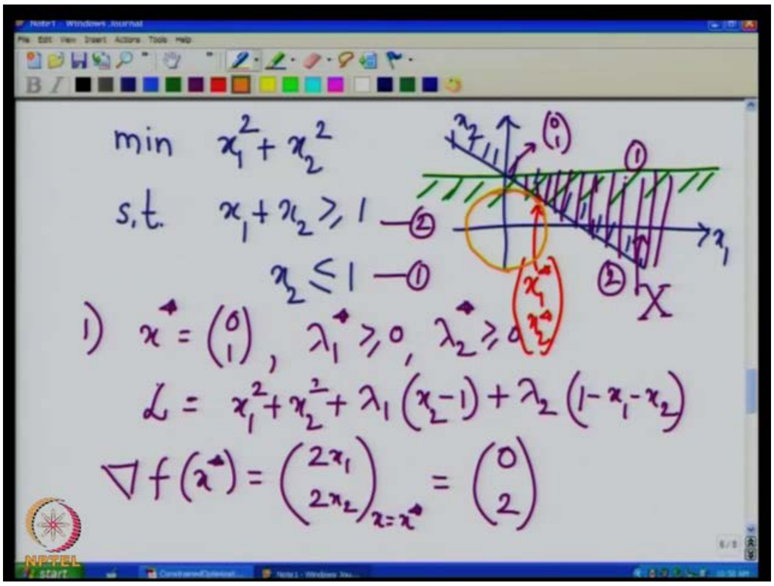

So, the the point here, half and half that is going to be our solution. So, this will be x 1 star x 2 star the actual solution. So, if you recall we started with different possibilities. So, with initially considered this as a solution 0 1, but then we came off with the condition that one of the Lagrangian multiplier become say negative. So, this cannot be a solution and the then we decided to consider this, as a active constraint at the solution and we indeed found a point on this set or on this active set such that The Lagrangian multipliers are non-negative and this indeed is a solution of this problem.

So, will see that this is the circle of smallest radius which touches this constraint set x. So, so this is a circle of radius, which smallest radius because, x 1 square plus x 2 square is nothing, but radius square of a circle center is that the origin. So, this is the smallest circle which touches this constraint set. So, we will see more about this KKT conditions in the next class.

Thank you.