Numerical Optimization

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Lecture - 1
Introduction

Hello, and welcome to this series of lectures. My name is Shirish Shevade, I am associated with computer science and automation department, at Indian institute of science, and I will give a course on numerical optimization. So, this course is about studying optimization algorithms, and their applications in different fields. The field of optimization is very important, and it has applications in lots of disciplines. See in our day-to-day life, we optimize things; for example, if you want to move from one point to another, we find out the path which is shortest or suppose if you want to invest money, we find out those instruments, where the risk is minimum, and gains are high. So, the process of optimization, can be thought of as making best use of resources under given constructs.

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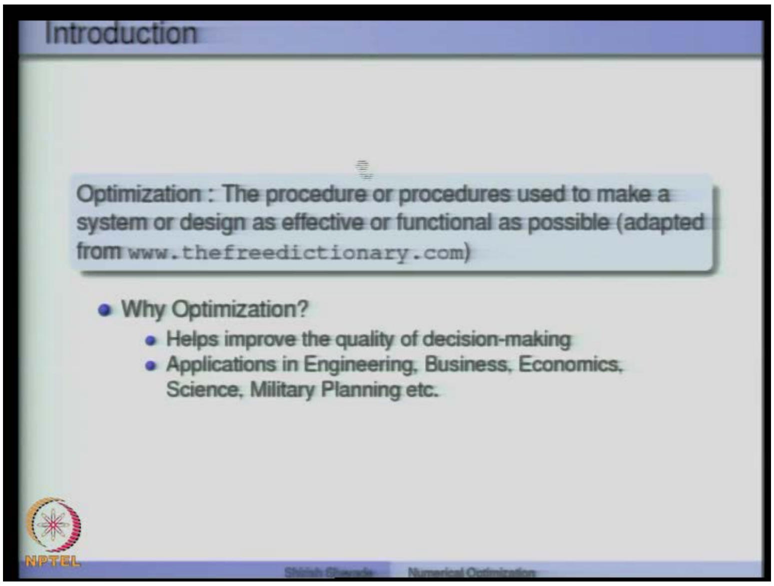

So, one definition of optimization is the procedure or procedures used to make a system or design as effective or as functional as possible. Not only human being, even the nature has also a way of optimizing things; for example, the light rays follow certain path so as to reduce the travel time. So, why should one optimize different tasks? So, the optimization helps us, to improve the quality of decision making. So, use of optimization tools, help the policy maker to make better decisions, and the decision maker gains by doing so. As I said earlier that it has applications in engineering, business, science, economics, military planning, etcetera. So, optimization is very important field, and it would not be a exaggeration if I say that, in everything that we do, we try to optimize. So, in this today's lecture, what I will do is, I will introduce you to some of the preliminary ideas, that we want to study in this course. So, I have tried to avoid the mathematical details as much as possible, and tried to make it very introductory lecture, but the details will follow in the later course.

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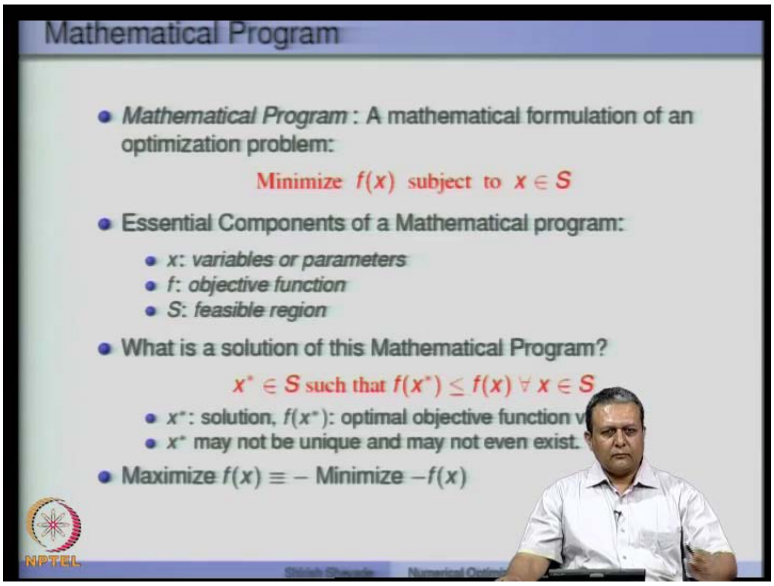

A mathematical program is basically a mathematical formulation of an optimization problem. So, a typical mathematical program, would look like this; minimize f of x subject to x belongs to S, where x is a set of optimization variables, or they are also called optimization parameters, these are typically unknown. The f is a objective function which we want to minimize, and S is called a visible region, which is also called a constraints set. So, we want to choose x, within this set S, such that some objective is minimized. Now, what is the solution of a mathematical program. So, we are looking for some x star, which belongs to the set S such that the value of the function at x star, is less than or equal to the value of the function, at any other point in the set S. So, such a x star is called, a solution of this problem, and f of x star, is called optimal objective function value. Now, remember that, x star may not be unique; for example, there could be some problems, where you will have multiple solutions, and there could be some problems, where the solution even may not exist.

So, it is very important, to check the existence of an optimization solution, optimization problem solution, then we solve a optimization problem. So, in this course, we will be working with a problem of this type. Now, in many situations you have to maximize, objective function. For example, as I mentioned, that the when we want to invest money, we want to maximize the returns, but the maximization problem, can also be equivalently written as, minimization of minus f of x, and once we solve that problem, you change the sign of the objective function, and you will get the solution of this problem. So, many times in this course we will, restrict ourselves to minimization of a objective function, but truly it is not a limitation, because any maximization problem can be written as a minimization of, modified objective function or objective function.

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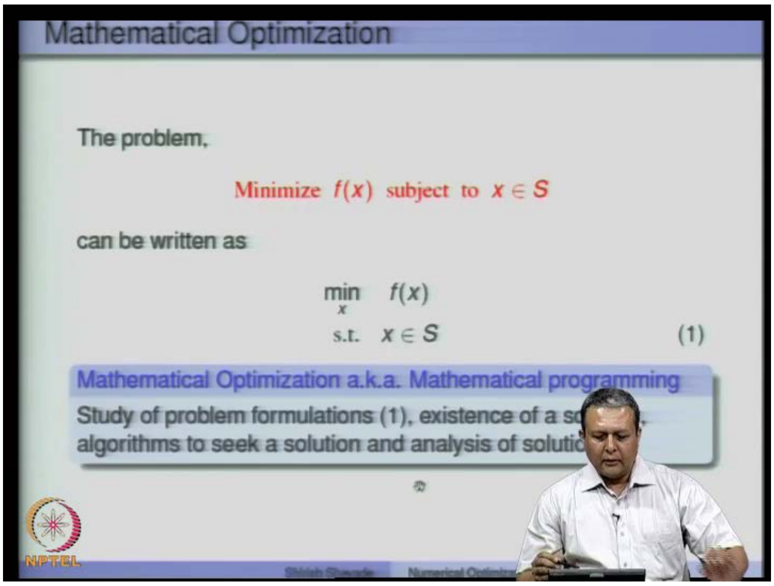

Now this, the optimization problem, the minimize f of x subject to x belongs to S, it can be compactly written in this form. So, this is the objective function, this is the x written below the minimization symbol, is the optimization variable, and this is the constraint sets. So, hereafter I will write the optimization problem, as minimize f of x subject to x belongs to x in this form. Now, mathematical optimization, or it is also known as mathematical programming, it is basically study of, this problem formulations of this type. It is also study of existence of a solution, and then use of algorithms, to seek one such solution, and then the analysis of such solutions. So, this is called mathematical optimization, or mathematical programming.

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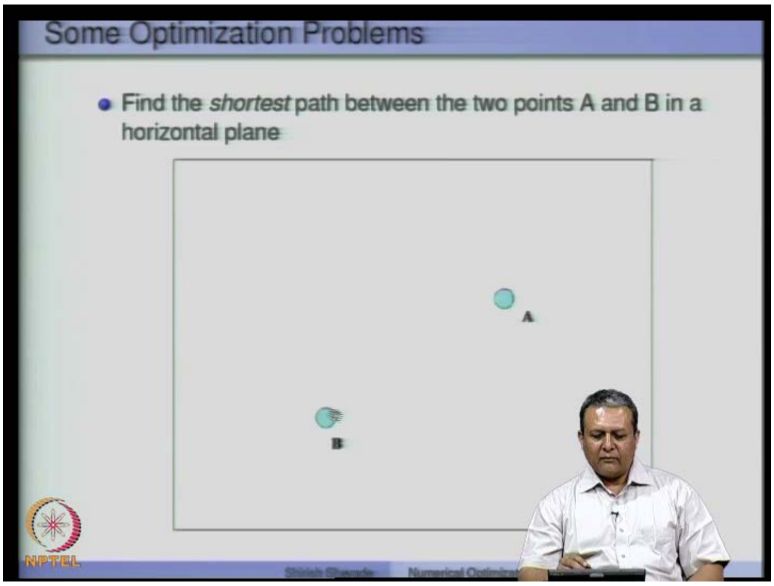

Now, let us look at some interesting optimization problems, that we come across in our day-to-day life; for example, if you are given two points A and B, and our aim is to find the shortest path, between the two points A and B, in a horizontal plane. Now you will see that there exists, plenty of, or infinitely may curves, which can connect point A to point B. So, this is the problem, where we have to search over, the infinite dimensional space, to look for a solution, of find, of this problem where we want to find out the shortest path between A and B, but suppose if you restrict ourselves to, the Euclidean distance. Then you know that the straight line path joining a and b, will be the shortest path. So, a lot depends on the kind of distance measure that we want to use. Now, let us vary this problem a little bit, let us assume that A and B are two cities, and they are connected by a set of roads. So, think of a situation, where we have a network of cities, different cities and A and B are two of those cities, and those cities are connected by a road, then the problem becomes little difficult. We have to look for the shortest path, because we want to travel on the road, rather than finding out the straight line distance between the 2 points.

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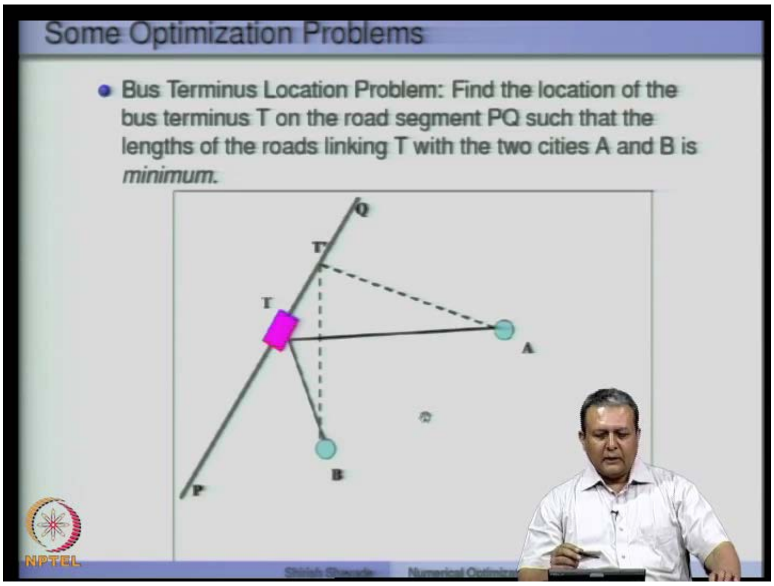

Now, let us consider another problem, this problem also, which called bus terminus location problem. Assume that P Q is a segment, rectilinear segment of a highway, and A and B are two cities, which are on one side of this highway, and the problem is, to construct the bus terminus, on this section of highway; say bus terminus T, which is conveniently located to the residence of, this two cities. So, which means that, their travel time to the bus terminus, should be as small as possible. Now, suppose if you decide to use Euclidean distance, then you know one has to look out for this path length, and then there are again infinitely many possibilities, for the bus terminus T, so one such possibility is shown here. So, the path length here is, B to T dash and T dash to A. So, the problem is, to find out the best position of the bus terminus, so that it is convenient, to the residence of these cities A and B. Now you might have seen, similar problem in geometry earlier, and also would have solved it using some geometry ideas, but then this problem also can be posed as an optimization problem. Now the problem also can be extended to, say three cities or four cities.

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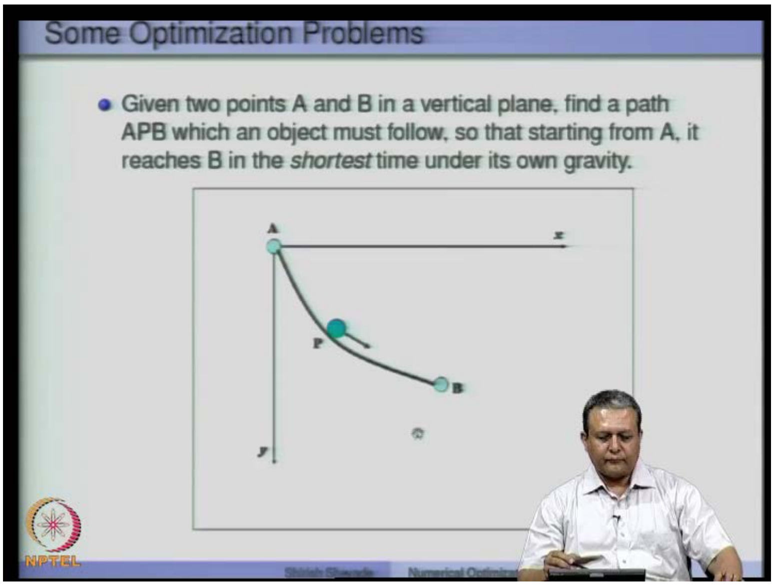

Now, here is another problem, the points A and B are on a vertical plane, and there is a small ball, which we want to roll, from point A to point B, and m is to find the path, or to find the curve; such that if the balls follows that path A P B, it will reach B in min shortest time under its own gravity, so there is no other force acting on this ball. And we are interested in finding out, the curve which, the ball should trace so as to reach, B in the shortest possible time.

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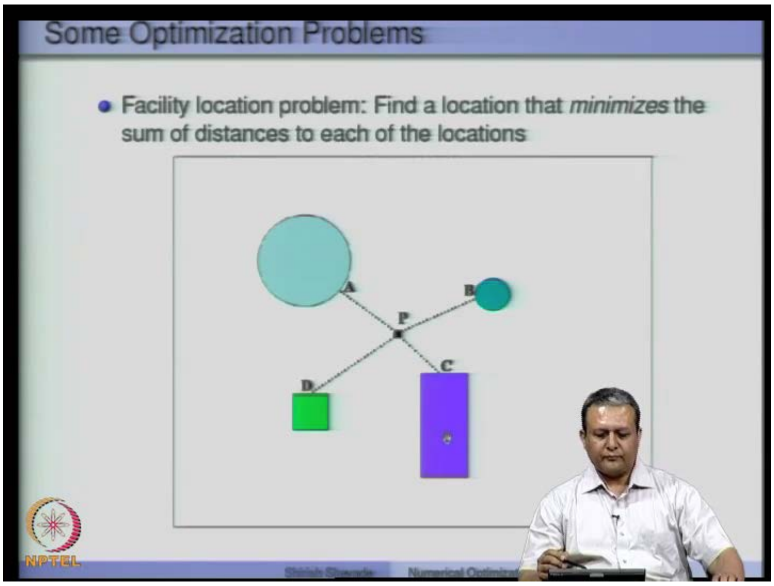

There is a another important problem in optimization, which is called the facility location problem. Suppose that A B C and D, are four population centers, in a city. And it has been decided to construct, a facility, which is located at some point, convenient to all the population centers. For example, we want to you know construct a convenience store, at certain place, such that all the residence of this population centers, can make best use of that. Now suppose again we want to use the Euclidean distance, so we basically want to minimize this, the path link A P plus P B plus P C plus P D. The problem becomes more interesting, when suppose there are sets of roads, on which you know the traffic is allowed, so that puts some constraints on this optimization problem. So, one has to find a proper location, so that the road distance traveled by each of the, each of the I mean by residence from each of this population centers, is minimized. Now, the similar ideas also can be extended to a different kind of problem, suppose we want to have a garbage dump area, for this population centers, and that should be as far away as possible, from this population centers, but within the particular limits of that township. So, we want to maximize, the distance of that garbage dump area, from each of this population centers. So, this problem again can be posed as an optimization problem.

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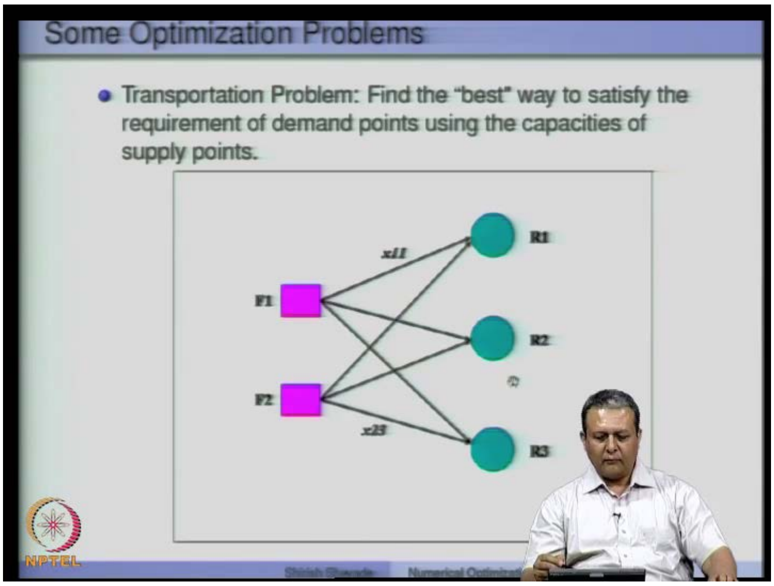

So, there is another interesting problem, which typically is reported in the optimization literature, and that is called the transportation problem. So, here is a case of a company, which is dealing with say agricultural related products. So, suppose the company has two plants, fertilizer plants, which are called f 1 and f 2, and the company all has retail outlets, so let us call them as R 1 R 2 R 3. Now, each of these plants has some, weekly capacity of producing, a particular fertilizer, and each of the retail outlets, has a weekly demand of, the fertilizer that it should get from these plants. So, the transportation problem, is to find out the best way, to satisfy the requirement of the demand points, using the capacities of supply points.

So, these are the supply points, and these are the demand points, and what is the best way to satisfy the requirements of these demand points, using the capacities of this supply points. The problem becomes again more interesting, if suppose, let us call f one as a fertilizer plant, and f two as plant which packages seeds, and this seeds need to be supplied, to this outlets, instead of the fertilizer. Now the fertilizer produced, can be a any real number, while the number of packets produced, is an integer variable. So, the problem becomes bit difficult, because on one hand we have the real variables, and on one hand on the other hand we have the integer variables. So, this fall under the category of mixed integer programming, we will see later more about this, in the later part of this lecture.

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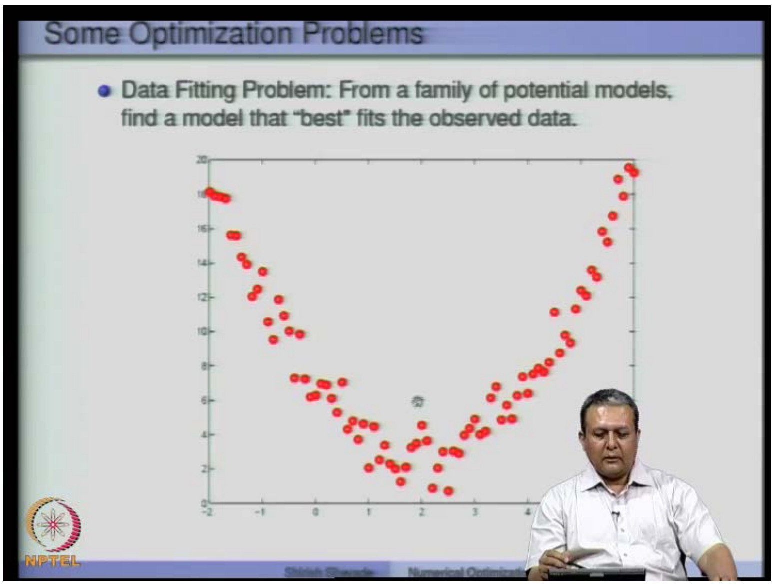

There is another interesting problem which, is typically addressed by statisticians; that is called the data fitting problem. So, from a experiment, a data set is generated, which relates the dependent variable, and the independent variable. So, suppose we plot the independent variable on the x-axis, on the horizontal axis, and dependent variable on the vertical axis. So, these are the data points, which are obtained from some experiment. Now, they are assume to have, come from a particular function, and the idea is that, from a family of all potential function models, find that function which bests, fits the observed data. So, this problem is very popular in statistics, also called regression problem.

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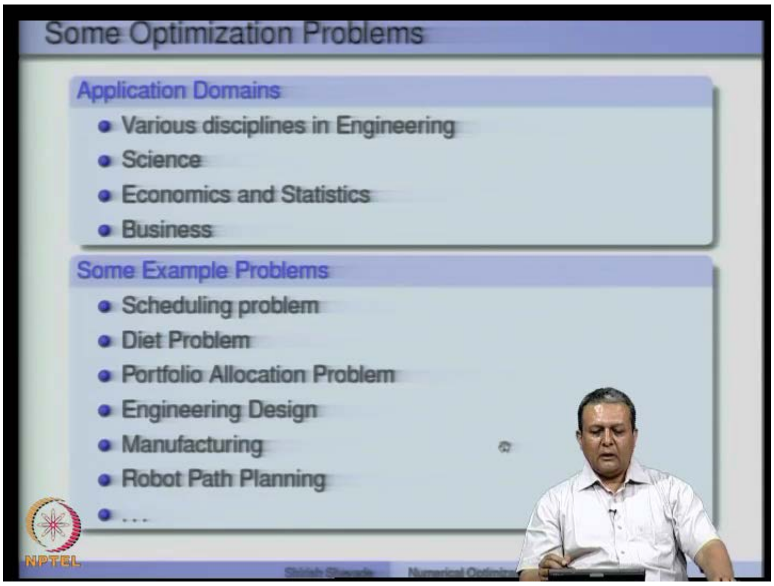

So, you might have notice that, optimization problems are applied in variety of domains right from engineering, to science, to economics, statistics, and business. So, some examples of these optimization problems are given here. So, one of the important problems is the scheduling problem; for example, airline companies, would like to schedule their crews, and their craft, to minimize the cost. Diet problem is, one of the oldest problems, in this literature so every day we consume some food items, and they have some nutritional contents in it. So, what is the optimal quantity of those food items, that we should consume, to satisfy the daily nutritional requirements, and also minimize the cost. So, this is a well-known diet problem in optimization literature.

Then there is also a portfolio allocation problem, where the idea is to invest the capital dynamically, in the portfolio of instruments, so as to have maximum returns. Another important application of optimization problems is engineering design; for example, optimization problems can also be used, to design aircrafts, design aircraft engines, design VLSI chip designs VLSI chips and so on. Optimization also has applications in manufacturing, and robot path planning, and there are many more applications. So, you will see that, optimization problems are useful in variety of domains, and they are useful for the practitioner, as well as a theoretician, and it is important to make use of some of the optimization ideas, to take quality decisions, and for the benefit of the business or whatever.

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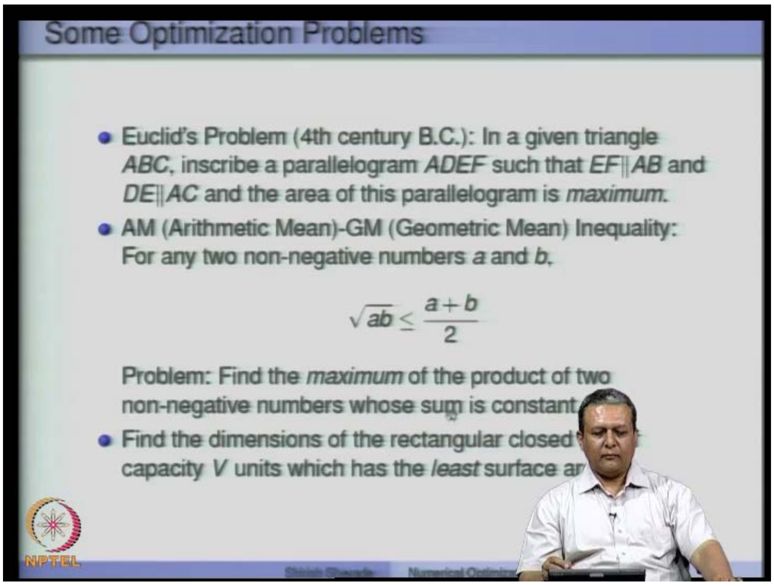

Interestingly, the optimization problems, have applications in geometry, algebra, and many other fields. So, this was the problem posed by Euclid in fourth century b c. So, in a given triangle A B C, inscribe a parallelogram A D E F; such that the side E F is parallel to A B, and side D E is parallel to A C, and area of this parallelogram is maximum. Now you might have solve this problem using geometry ideas, but this problem can be posed as an optimization problem, and where we are looking for inscribing a parallelogram, which has a maximum area. You also might have seen the arithmetic and geometric mean inequality. As you know that for any two non-negative numbers A and B, square root of A B is less than or equal to A plus B by 2. Now this problem can be posed in optimization problem.

So, from this inequality, you will see that the geometric, inequality does not exceed the, geometric mean does not exceed the arithmetic mean. So, the problem can be posed as, find the maximum of the product of two non-negative numbers, whose sum is constant. There is a another interesting application in geometry, where optimization problems can be used, is that, suppose there is a company which manufactures the boxes, for packaging, and the box the box is of rectangular size, and it has a capacity of v units. So, the idea is to find the dimensions of such a box, which has the capacity of v units, such that the surface area of that box is least. So, you might have seen that, optimization is very useful. You might have solved these problems using different ideas, but they can also be solved, by posing them as a optimization problem.

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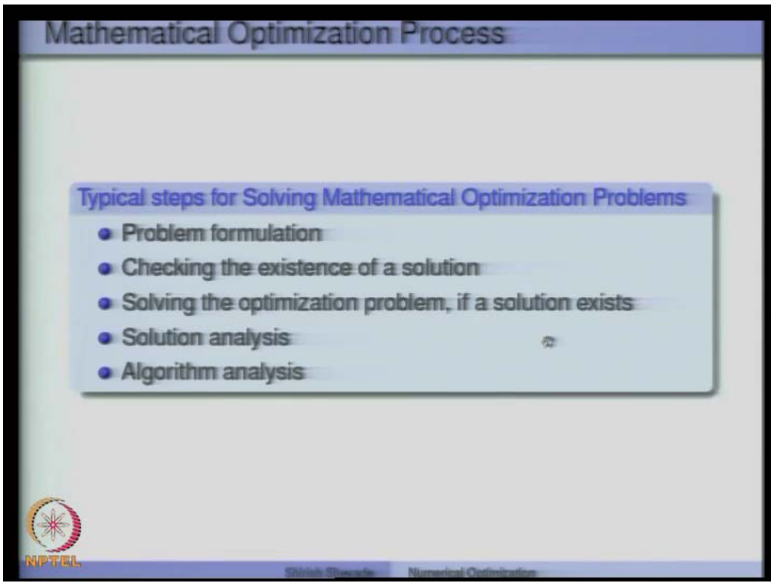

Now, typical steps involved in solving a mathematical optimization problem are; the first step is mathematic, of the problem formulation. Then to check whether there is the solution exist, to such a problem or not, then solving the optimization problem, using some algorithm, if a solution exists. Then analysis of solution, analysis of algorithm, and then one way have to go back, to the problem formulation if there is a need, or one may have to go, to solve the problem using different algorithm, if the algorithm that we have previously considered is too complex, for the problem. So, this is the iterative process, till we find out a reasonable solution, and such solutions will help the decision maker, for his own benefit.

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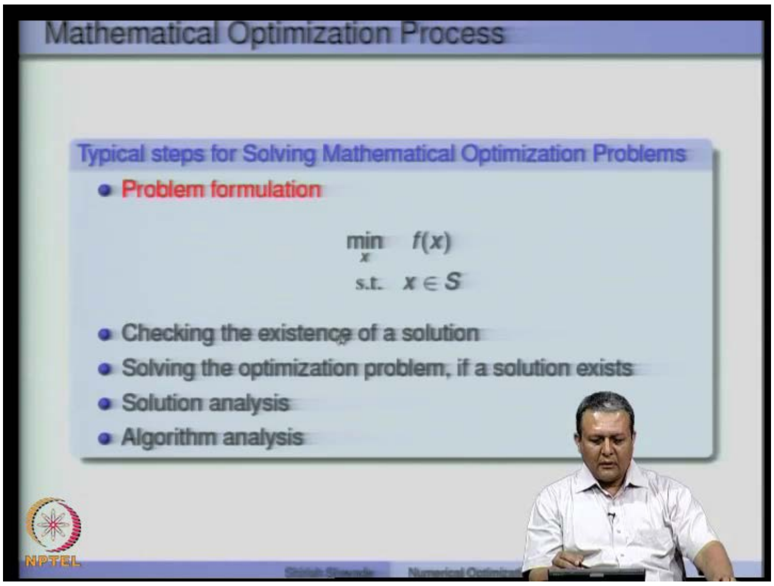

Now, the first and important step of any optimization problem is, to minimize the objective function, subject to certain constraints. Now, we will look at some of the problem formulations now.

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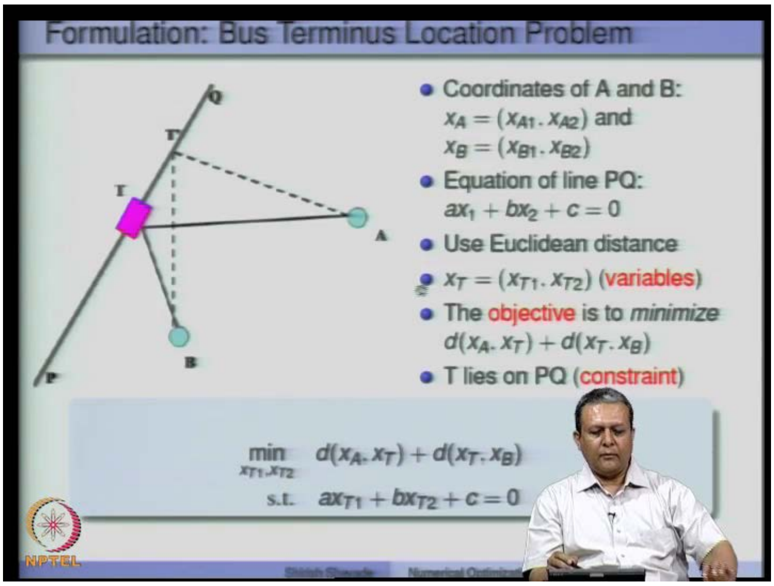

So, let us look at the bus terminus location problem, where the idea was to find a bus terminus on the segment p q, such that the lengths of the paths from a, lengths of the roads going from A and B to T are as minimum as possible. So, let us take it as a two dimensional problem, and let x a 1 and x a 2 denote the coordinate of x a, and similarly for x b, and then this being a straight line, let us assume that its equation is a x 1 plus b x 2 plus c is equal to 0, and assume, we assume that we use Euclidean distance. Now our aim is to find the location of the bus terminus. So, it will have two coordinates; x T 1 and x T 2, these are called the variables. The objective is to minimize the distance from A to T plus distance from B to T. So, x A and x T are the coordinates of the points A and T respectively. So, distance between x A and x T plus distance between x T and x B, so this is going to be our objective, to minimize this quantity.

And the constraint is that, the bus terminus should lie on the line segment p q, so this is going to be our constraint, T lies on p q. So, we can write this as minimize, the sum of the distances, subject to the constraint that, the point T lies on, the segment p q. So, the coordinates of this point, should satisfy the equation a x T 1, plus b x T 2, plus c equal to 0, and x T 1 and x T 2 are the optimization variables here, which are determined by solve, of the optimal value of them, this are determined by solving this problem, and this will give us, the exact location, of the bus terminus, which is convenient to the residence of the two cities A and B. Now, the problem can be varied a little bit, suppose that, instead of a straight line segment, the bus terminus should lie, the segment P Q is a curvilinear segment, and the bus terminus should lie on the, that curvilinear segment P Q. So, if we know the equation of that curvilinear segment P Q, then we can use that equation instead of, this equation and reformulate the optimization problem.

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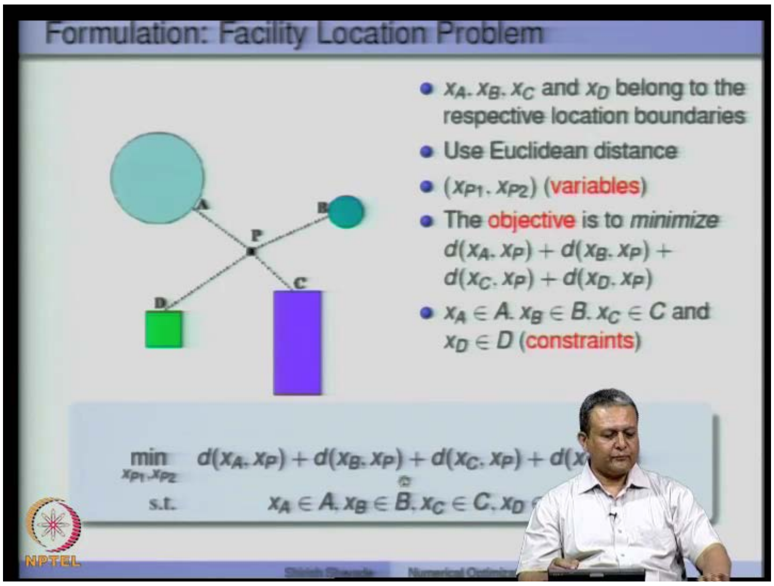

So, if you look at the facility location problem, so if we say that X A X B X C and X D, are the points belonging to the boundaries. Remember that I am you know looking at the boundaries of this population centers, and again we want to use Euclidean distance. So, the problem is to find out the best location of p, so that this location is convenient to the residence of this population centers. So, the objective is to minimize, the distance between x A X P plus X B X P plus X C X P and X D X P, but what are the constraints. The constraints are that the points X A X B X C and X D should lie on the boundaries of, the respective population centers. So, the problem can be written in this form, so you will see that this is the objective that we want to minimize, these are the constraints that the point should belong to their boundaries, the respective boundaries, and the optimization variable is X P 1 X P 2. Now, when it comes to the, the other problem of finding out the location for garbage dump area, we may have to change this maximization minimization problem to a maximization problem.

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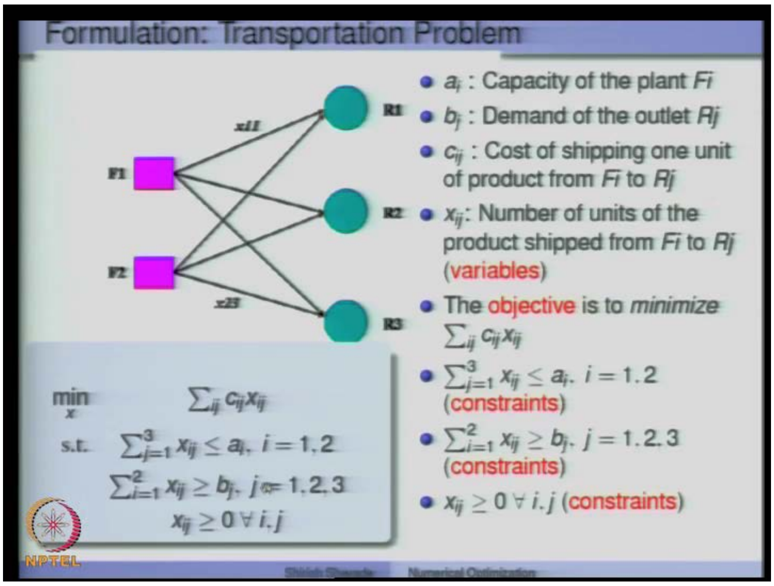

The transportation problem also can be easily formulated, so suppose. Let us first consider the, case of two fertilizer plants; a F 1 and F 2, having capacity A 1 and, weekly capacity of producing A 1 A 2 units of the fertilizer, and the retail outlets R 1 to R 3, they have a weekly demand of B 1 B 2 and B 3 units respectively. And then to ship the fertilizer from the plant, to the outlet, there is a cost inward, and let c i j denote the cost of shipping one unit of the fertilizer, from the factory F i, to the outlet R j, and then x i j, let x i j denote the number of units of product that needs to be shipped from the factory F i to the retail outlet R j. So, the x i j's are our now our variables, and the objective is to minimize the cost. So, if x i j is a unit, and c i j is the cost of shipping one unit. So, the cost of shipping, the x i j units from factory i to retail outlet j, is c i j x x i j, and then we have to summit over all possible combinations of factories, and retail outlets. Now, what are the constraints that every fertilizer plant, cannot supply anything which is greater than its capacity.

So, this constraint is shown here, and what is the retail, what is the constraint on the retail outlets, that they should at least get their weekly demand every time. So, their weekly demand for the jth outlet is b j. So, they should get at least that much quantity of, that much units of fertilizers. And also we need to ensure that, all the quantities involved are, all the units produced, all the quantities shipped, are non-negative. So, the optimization problem can be written as, minimize the cost, subject to the capacity of, the capacity constraint for the ith factory, or ith plant, the demand constraint for each of the outlets, and then the non-negativity constraints. Now as I said earlier that if we change this problem a little bit, that the company also wants to sale the packaged seeds, and those packets packages are integers. So, this constraint has to be modified a little bit, for this particular plant. We have to ensure that, for this particular plant, the variables, related to this plant are integers, and they are non-negative integers. So, that modification has to be made in this problem formulation.

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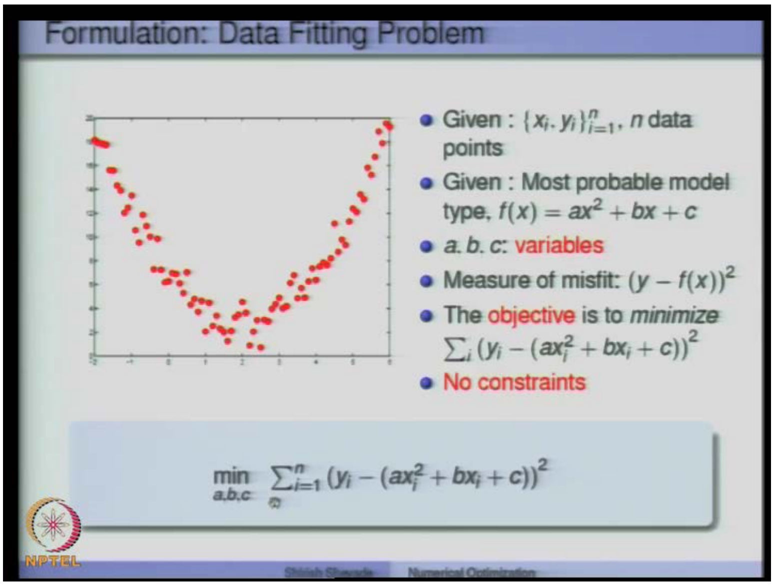

Now, when we look at the data fitting problem, so we are given the x i and y i i going from one to n, n data points, which are shown here, and it is also given to us, that the probable model is a bit quadratic type, which takes the form a x square b x plus c. So, the idea is to fit a quadratic model, given this data. So, a b c are unknown parameters, so they are also called variables, and the major of misfit is, the distance between the difference between the y and f of x. So, we want to fit a quadratic objective, or quadratic function to this data, such that this objective is, the misfit, for every point is minimized. So, remember that there are no constraints given to us. So, there could have been some constraints on a b and c, but nothing is given to us. So, we can write the optimization problem as, minimize the misfit for every point, and minimization is done over a b and c, so these are our optimization variables. So, these are some ways of formulating the optimization problems. Formulating any optimization problem is an art, and care should be taken, to formulate the problem properly, so that the lot of time is saved.

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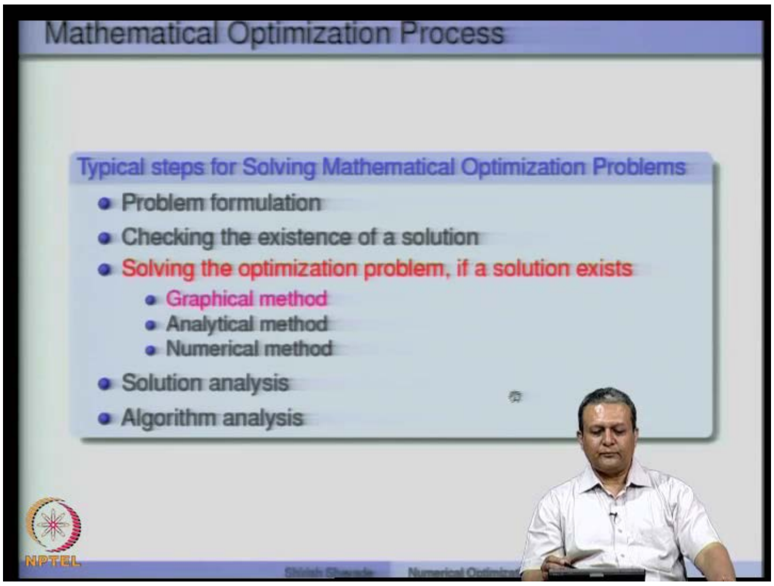

So, the next step in the optimization process is, checking the existence of a solution, but we will study the step later in this course, so in today's lecture I will not talk about this. So, in today's lecture, I will mainly concentrate on, some of the preliminary ideas for solving a optimization problem, if a solution exist. So, one of the simplest method to solve an optimization problem, in one variable or two variables, is a graphical method. So, we are now going to see this graphical method.

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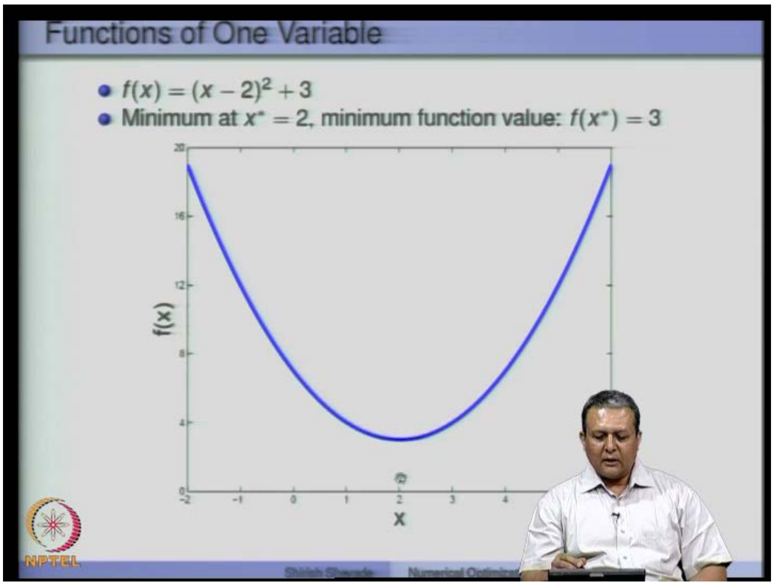

Now, let us consider a function, in one variable, which is of this from f of x is x minus 2 square plus 3, and the function, the plot of the function is given here, and you will see that the, minimum value of this function, is somewhere here, and the corresponding value of x is 2. So, the minimum, of this function, occurs at 2, and the minimum function value or the optimum function value is 3.

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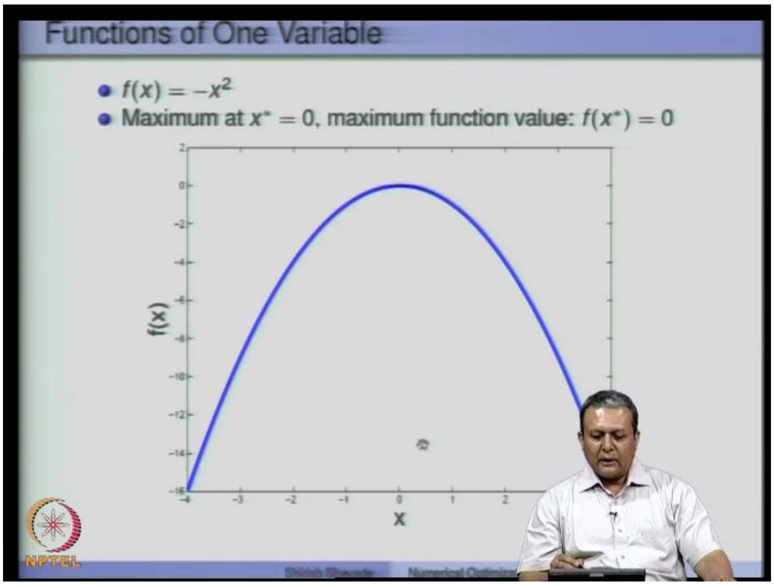

Let us look at another function; f of x is minus x square which is plotted here. So, you will see that the function goes to minus infinity, when x goes to plus infinity, or x goes to minus infinity. So, the function does not have a minimum, but this function has a maximum, and the maximum function value is attained here, and the corresponding value of x is 0. So, you can see that the corresponding value of x is 0, and the optimal function value, the maximum function value is 0.

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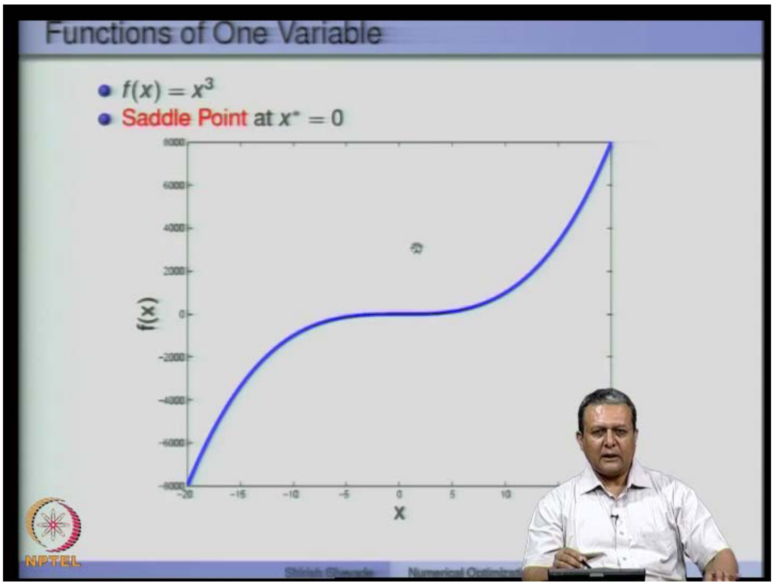

Let us look at another example; f of x is x cube. So, you will see that this function, when x goes to infinity, the function goes to plus infinity.

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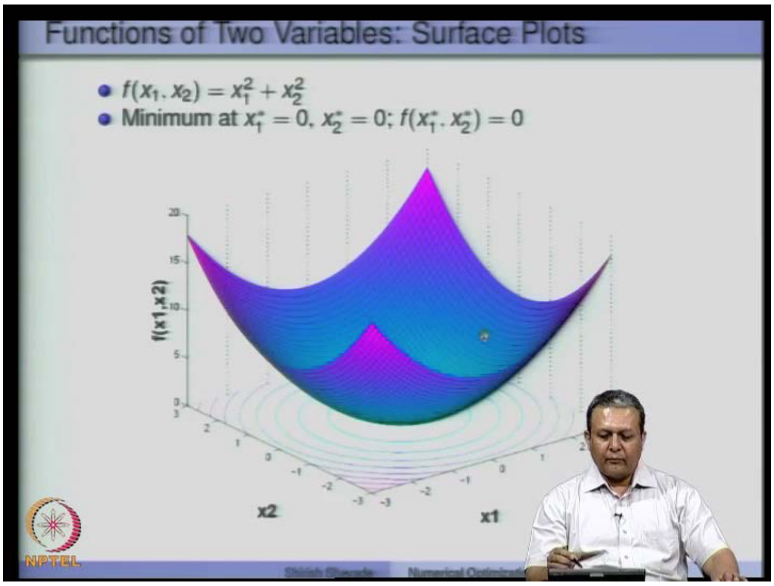

And when x goes to minus infinity, the function goes to minus infinity. So, this function does not have a minimum, or a maximum, because the function just goes to plus infinity or minus infinity, in either direction, but at this point. So, if you see that if you move in this direction, on the right side, the function value increases, and if you move in the opposite direction, or the other direction, the function value decreases. So, such points are called saddle points. And for solving any optimization problem, it is very important to locate such points.

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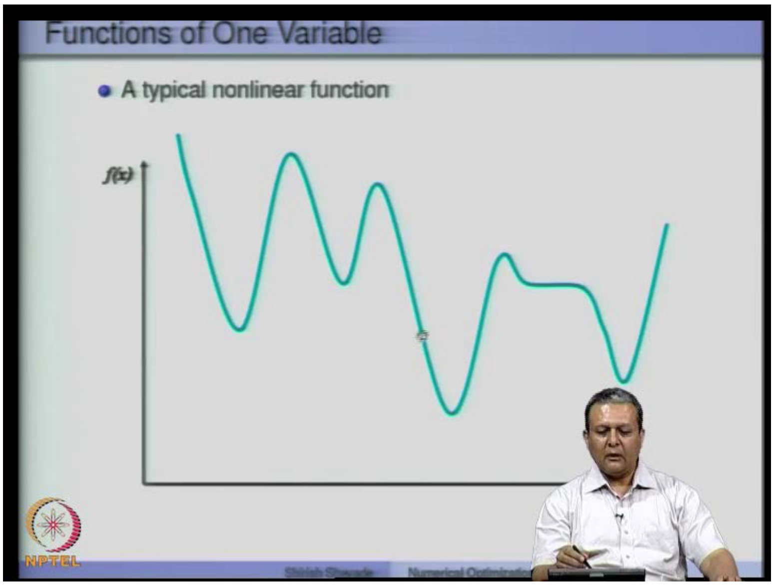

Now, a typical non-linear function, would have graph of this from, and if we are looking for the minimum of this function. Let us assume that the function goes to infinity in both the directions, so the minimum value of the function occurs here. But in many optimization problems, it is very difficult to locate such minimum points, instead if you look at this points you will see that, in the vicinity of these points, the function value is increasing right. So, many optimization algorithms that we are going to study, we will try to find such minimum points. So, these are also called local minima, and we will formally define them later in this course. And such a point, where the function value is the least, it is called a global minimum, and such minima are, it is difficult to characterize such minima, and also verify such minima. So, in this course, we will mainly study about the solutions, which are typically local minima. Now, when it comes to two dimensional space; two dimensional functions, so one function is given here.

So, it is function of two variables x 1 and x 2, and function is f of x 1 x 2 is x 1 square plus x 2 square. So, the surface plot of the function is given here. So, now if we cut this function, the surface plots horizontally, what will get is a circle, and those circles are plotted beneath this surface. So, this is the x 1 coordinate, x 2 coordinate, and on the vertical axis is, the function value. So, if we cut the surface horizontally; that means, that we are fixing a particular function value, and that would result in what is called a contour plot. So, contour plot, for the two dimensional function, is basically a set of point, where the function value is constant. Now if you cut this surface at different horizontal levels, you will get different contour plots, and all and some of them are plotted, beneath this surface. Now, you will see that the function has a minimum somewhere here, which is a origin and the function value is also zero.

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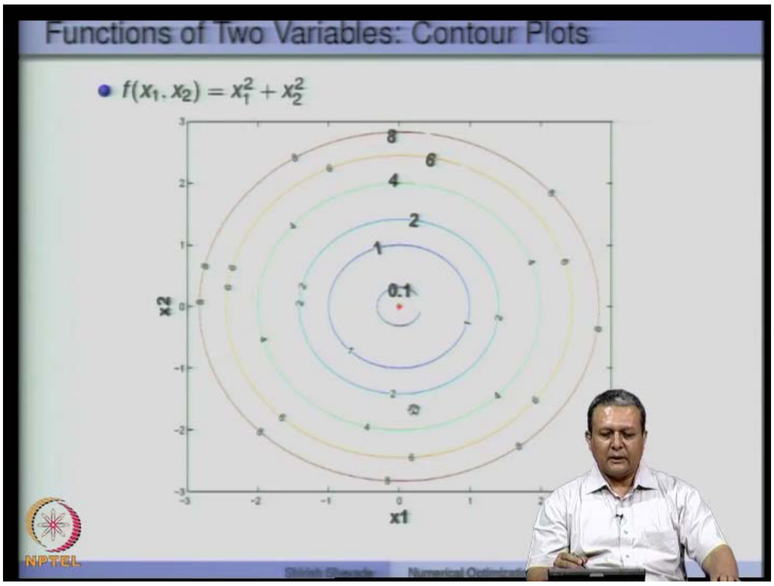

Now, let us just look at the contour plots, and how do they look like. So, the contour plots for the previous function are given here. So, remember that, this is a quadratic function, so the outer plot which is given here the function value is 8, and then the inside the function value is 6, and then so on, and the minimum of the function occurs here, which is a origin. So, you will see that the function value, in this case has been decreasing.

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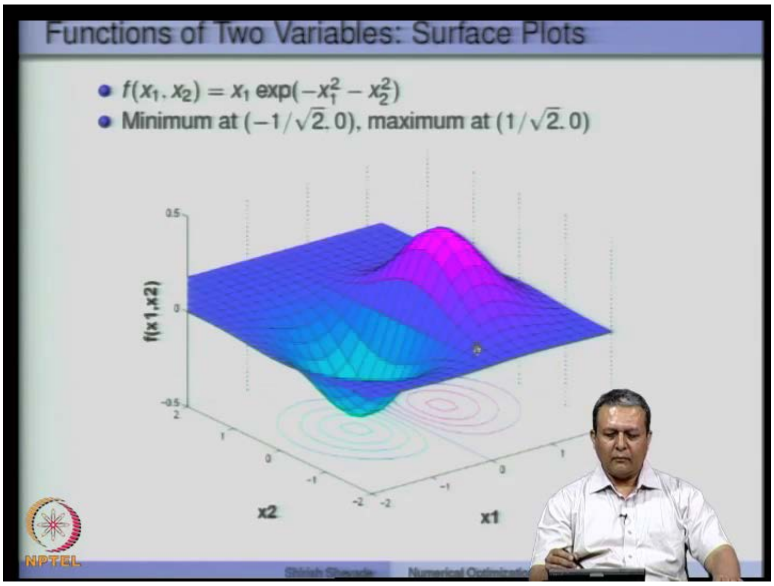

Now, there is another function, which is not a quadratic. You will see that this function has a peek, and a valley alright. So, this valley, will give you the minimum of this function, and this peek will give you the maximum of this function. So, the maximum occurs that 1 by root 2 0, and the minimum occurs at minus 1 by root 2 0.

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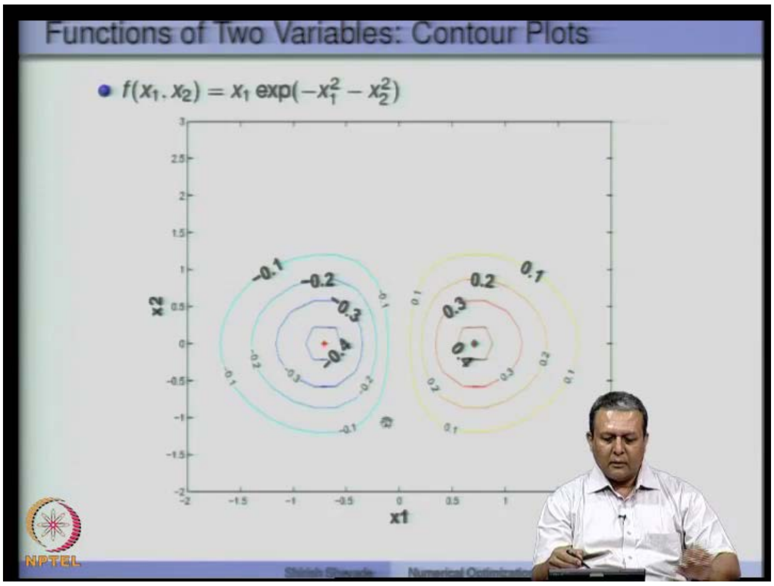

So, the same function, functions contour plots are given here. So, you will see that, if you look towards the minimum, towards the left part of the curve, the function value keeps decreasing, and then the minimum is achieved here. The right part of the curve, the function value keeps increasing, at as we go in the interior, and the maximum occurs here.

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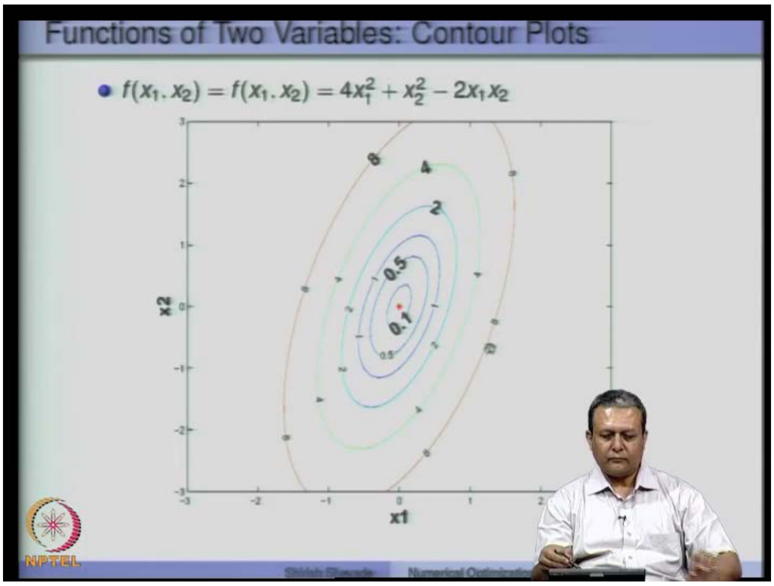

Here is another quadratic function, so notice that, unlike the previous quadratic function f of x 1 x 2 equal to x 1 square plus x 2 square. Here the contour plots are elliptical in nature. So, again the, on this plot, the function value is constant which is 8, and then as we go inside the function value has been decreasing, and the minimum occurs somewhere here, where the function value is 0.

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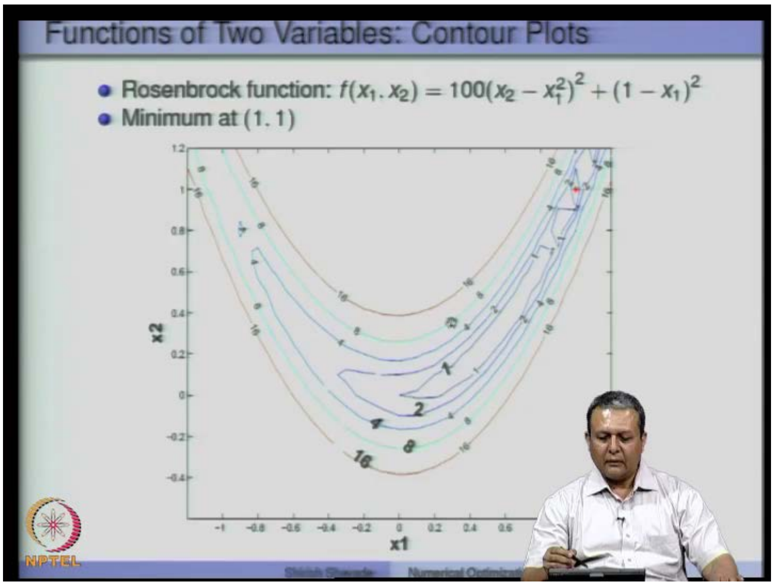

There is a commonly used test function in optimization literature, which is called Rosen Brock function, and then the contour plots of this function are given here. So, here the function value is. This is this is the contour plot for function value 16, and then as we move in the interior it goes down, and finally the minimum is at one work, where the function value is 0.

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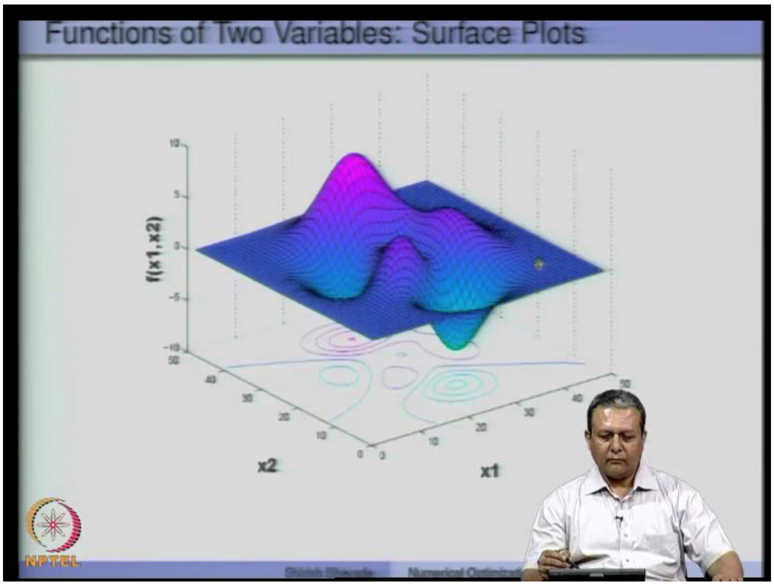

This again a surface plot of some function, now you will see that it has, multiple peeks and multiple valleys. So, well optimization problem, it can be thought of rolling a marble, along this surface, and then it will go to a point where the nearest minimum lies. So, if you roll it from this place, it will go and settled down at this place or if you roll it from this place, it will go down and settled this or settled at this place. So, the initial point is very important in solving such problems.

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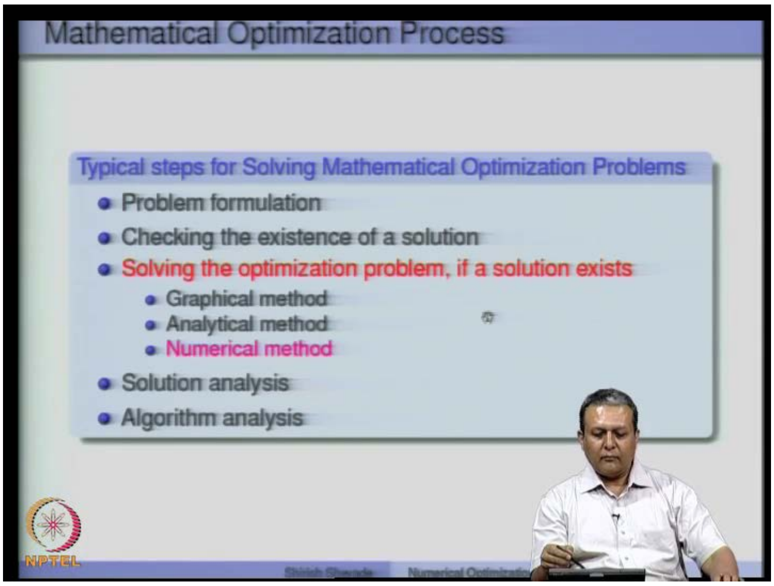

Now, beyond two dimensions, it is very difficult to use graphical methods, to solve a problem. So, one has to resolve to analytical methods, or numerical methods. Analytical methods typically give you a solution in one go, while numerical methods, will use some algorithm, which are iterative in nature, and they will reach the solution, over a period of time.

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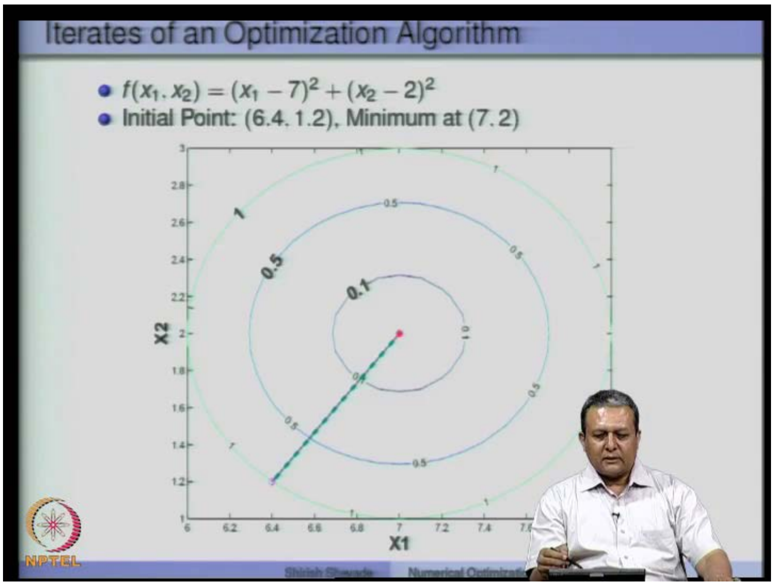

So, we will look at some of the numerical methods, I mean the solutions given by the numerical methods. So, again we take a function which is quadratic, the contours are circular in nature, and suppose we start from this point, some optimization algorithms can give you the solution in one go. So, if you start from this point, in the next step optimization algorithm will give you the solution, which is 0.72.

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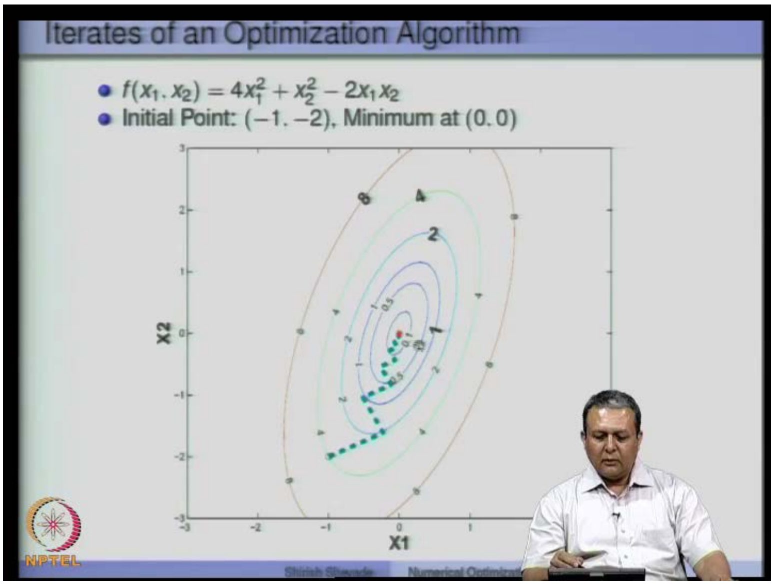

Now, if you take the, some other quadratic function whose contour plots are elliptical, some optimization problems, might give you a solution in certain number of steps. For example, if you start here, the algorithm might go to this place, and then will go to this point, and then so on, and then it will follow this zig-zag path, before reaching a solution. On the other hand some algorithms, might give to you a solution in one step. So, a lot depends on the kind of algorithm that you want to use for a given function.

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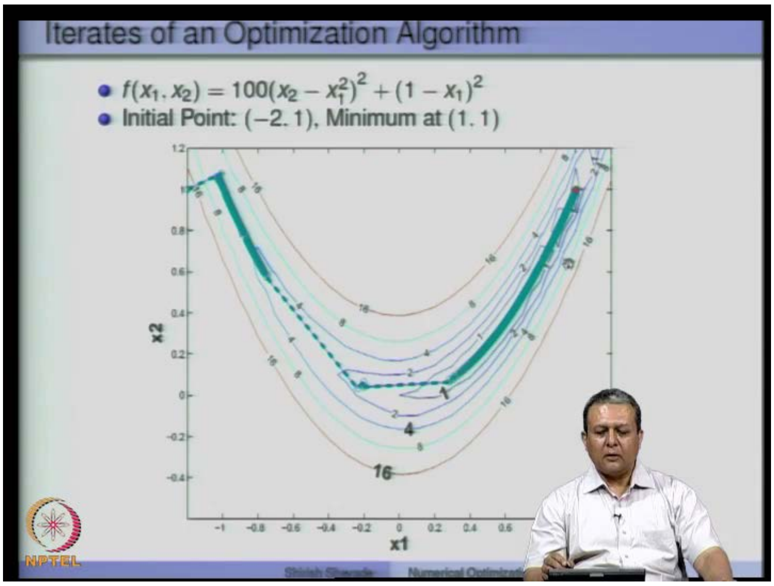

Now, if you look at Rosen Brock function. So, these are the contours of the Rosen Brock function. Now suppose the initial point, is say point 6.6, then a typical optimization algorithm, will follow this path, and then you will see the number of iterations required, to reach the solution. So, it takes lot of small steps to go towards the solution, which is 1 1. On the other hand, if you again consider the same Rosen Brock function, but start from a point, which is minus 2 and 1, then see the path which is followed by the same optimization algorithm, which was used previously. So, it starts from here, take a step to this point, and then takes lots of steps, and finally, it comes to this point, then goes to this point, again takes lots of steps to reach the solution. So, you will see that, a lot depends on the initial point of the, of initial point that you choose for solving optimization problem, using numerical methods.

As I have shown here that, this algorithm would require, the application of algorithm on this function would require less time, if you start from here. On the other hand, if you start from here, it has to take many more iteration. So, depending upon the function, one needs to decide which algorithm to use; there are no generic algorithms, to solve optimization problems. It all depends on how robust the algorithm is for a particular problem, and what is the computational complexity of the algorithm, and what is the information that the algorithm requires, to solve a given problem. So, one has to make a careful decision, about selection of an optimization algorithm, to solve a given problem, and sometimes some algorithms, may not even converge to a optimal solution.

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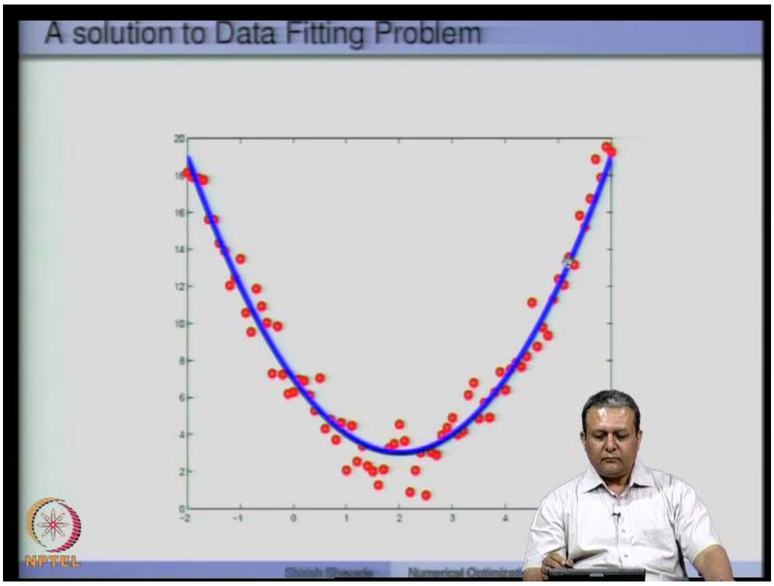

Now, this is this is was the unconstrained problem, and one solution to this unconstrained data fitting problem, is shown here. So, you will see that, we get a quadratic function, by determining the optimal A B and C, the parameters involved in the quadratic function.

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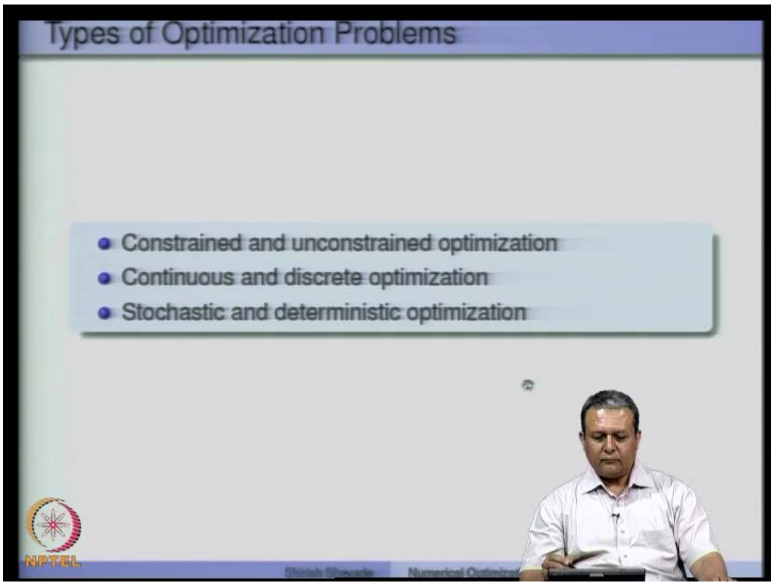

Now, there are different kinds of optimization problems, so they. There are many more types, but I have listed only few types here, because we will be concentrating only on the subset of this types. So, one type is called the constrained problem, and the other one is called unconstrained problem.

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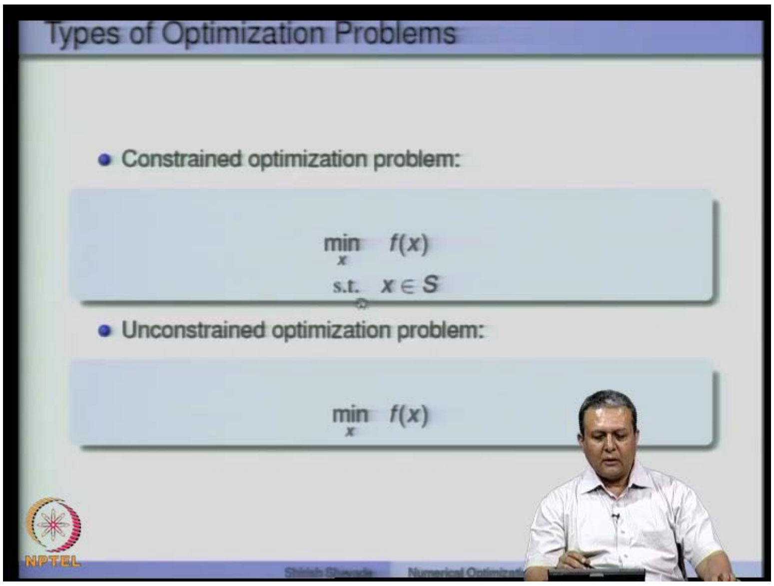

So, as has mentioned earlier that, this is the, S is a constrained set, and we are trying to minimize the objective function, subject to the constrained that, x belongs to the constrained set, so this is the constrained optimization problem. Now if there are no constraints on x, then it becomes an unconstrained optimization problem. So, you will see that, there that there is no S, the feasible region or the constrained set involved here. Many times, this constrained optimization problems, are solved by making use of the unconstrained optimization problems. So, it is important to study the unconstrained optimization problems first, before going to the constrained optimization problems.

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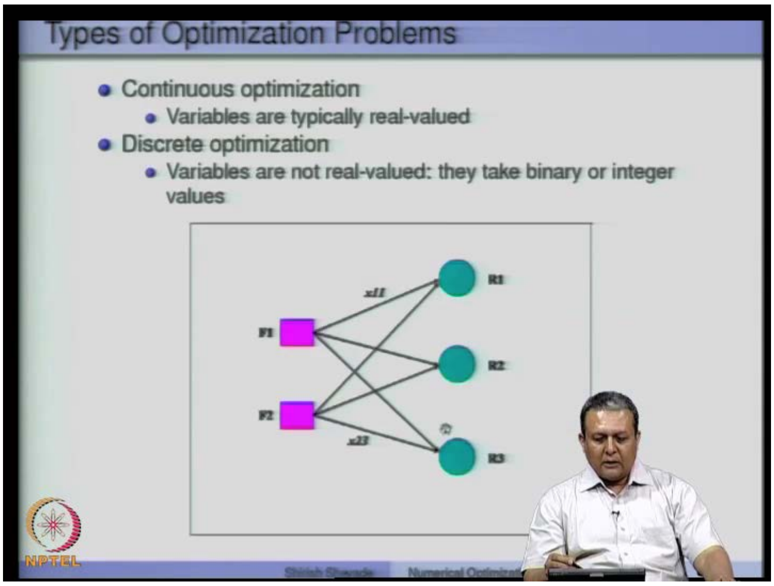

Then the other types of optimization problems are; continuous optimization problems, and discrete optimization problems. So, in continuous optimization problem, the variables are typically real-valued, while in discrete optimization problem, they are not real-valued, but they typically can be integer values, or they even can take value from the from the binary number set; that is they can either be 0 or 1. Now, if we look at fertilizer factories problem, so if both the factories are producing some fertilizers, then you can call it a continuous optimization problem. On the other hand, if one factory is producing fertilizers, and the other factory's packaging seats of a particular plant, then it becomes a mixed problem, where one set of variables is continuous, and other set of variables is discrete.

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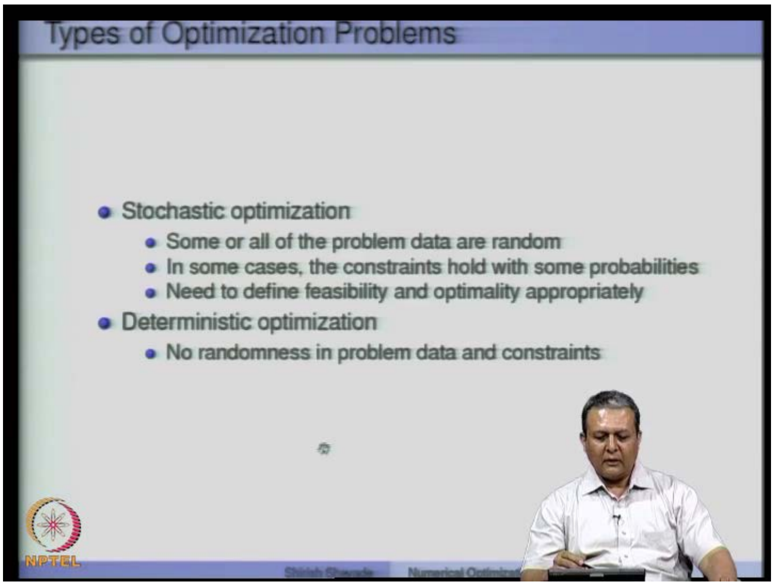

There are other types problems also, so one of the problems is called stochastic optimization problem, where some or all of the problem data are random. So, in some cases, the constraints also have, they hold with some probabilities, and in such cases there is a need to define feasibility and optimality appropriately. So, for example, if you take, this problem, so the data given to us, the weekly demands, the weekly capacities of the plants, and that weekly demands. Those could may not be fixed, those could be random variables, even the constraints also may might hold with some probabilities. So, such problems are called stochastic optimizations problems. And then when there is no randomness in problem data and constraints, those problems are called deterministic optimization problems.

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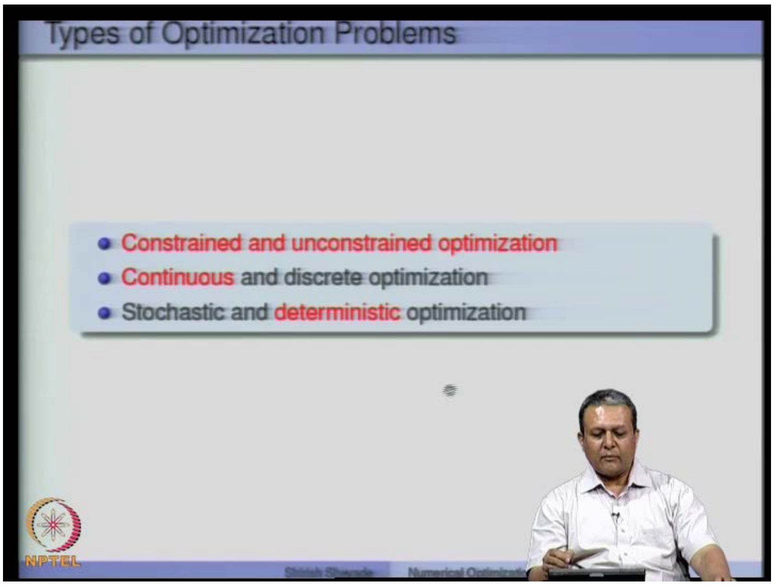

So, in this course, we will concentrate on constrained and unconstrained optimization problems, and in particular, the continuous, and deterministic optimization problems. So, the variables, that we will be dealing with, will be continuous, and the problems are deterministic, there is no randomness in the data, problem data as well as the constraint.

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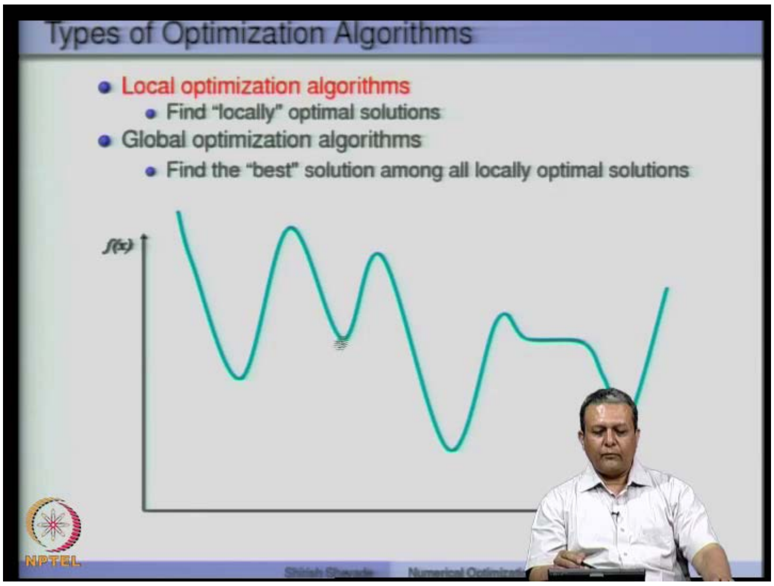

There exits, some kinds of optimization algorithms, which are called locally optimization algorithms, and globally optimization algorithms. So, as if you take general non-linear function, as I mentioned earlier, that these are the local minima, even this also can be a local minima, so many algorithms tried to find one such local minima. So, you can think of it as a now using a ball, and if you put a ball at this place, it will roll down to this point. On the other hand, if you put a ball here, it will roll down to this point, and if you put it here, if you put it here, it will roll down to this point.

So, based on your starting point, the solution that you will get or the local minimum that you will get will be decided, because all these algorithms that we planned to study in this course. Typically tried to go towards down the function, towards the nearest possible minimum, and there is no way to, there is no mechanism for the algorithm to go up, the curve. On the other hand, there are some optimization algorithms, which are called global optimization algorithms, so which will give global minimum with some probability. But in this course, our main focus will be, on local optimization algorithms, where we start from any arbitrary point, and go downhill, and go towards the nearest possible local minimum. Remember that, the local minimum and the global minimum, will be formally defined later in this course, but this is just to give you an idea about, what we planned to study in this course.

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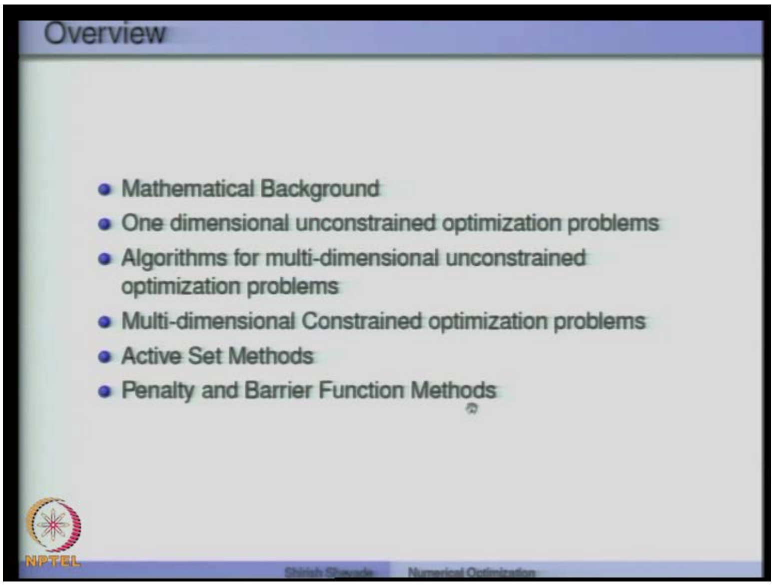

Now, here is an overview of the topics, that we planned to cover in this course. So, the subject of optimization requires some mathematical background, on differential calculus, linear algebra, and a bit of analysis, so will try to cover some of those topics, in the mathematical background. Then we study the one dimensional unconstrained optimization problems, and then move on to multi-dimensional unconstrained optimization problems. This one dimensional unconstrained optimization problems are useful in solving this, multi-dimensional problems. So, we will first study one dimensional, and then go to algorithm go to problems for multi-dimensional. Then we move on to the multi-dimensional constrained optimization problems, and some of these problems can be solved using techniques of unconstrained optimization, and then we move on to active set methods, and penalty and barrier function methods.

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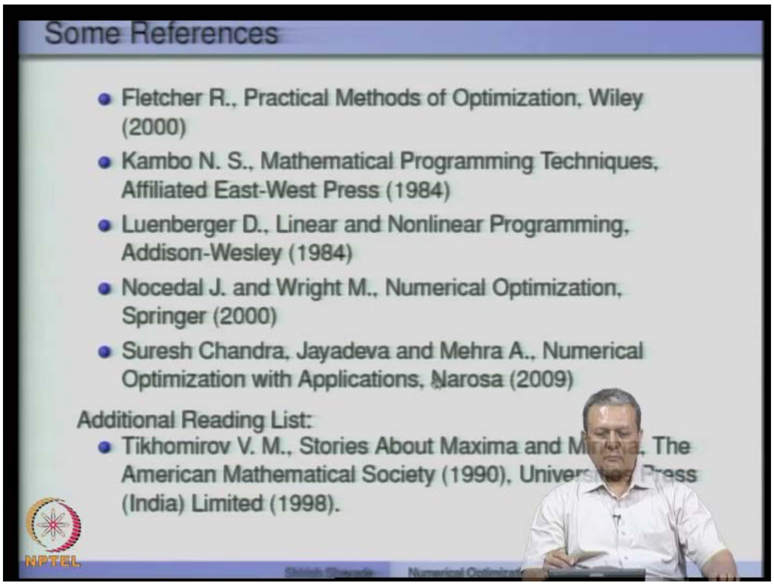

Now, there are some good references in the optimization literature. So, books by Fletcher Kambo then Luenberger, are quite popular. The book by Nocedeal and Wright also addresses this topic of numerical optimization. There was there is also a book by, Suresh Chandra Jayadeva and Mehra. So, these are some of the reference text that one can use, for studying some topics in optimization, there are many more books, but because of the space limitation, I have not included them here. There is an interesting book written by Tikhomirov, and the title of the book is stories about maxima and minima. It was initially published by American mathematical society, in 1990, and republished by universities press in 1998. So, this has some of the interesting problems, related to optimization, and belonging to different fields.

So, one can also look at some of the problems in physics, geometry, algebra, analysis, where mathematical optimization, problems can be used. So, in the next part of the course, we will start with mathematical background, we will start with the vector spaces and bit of linear algebra, and then move on to differential calculus, something about functions, and then the convergence of sequences and so on. And then we move on to the one-dimensional unconstrained optimization problem. So, in this today's talk, I just wanted to give you a big brief introduction about, what we planned to study in this course, and let us continue in the next class.

Thank you.