Graph Theory

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Module No. # 04

Lecture No. # 26

Second Proof of WPGT, Some Non-Perfect Graph Classes

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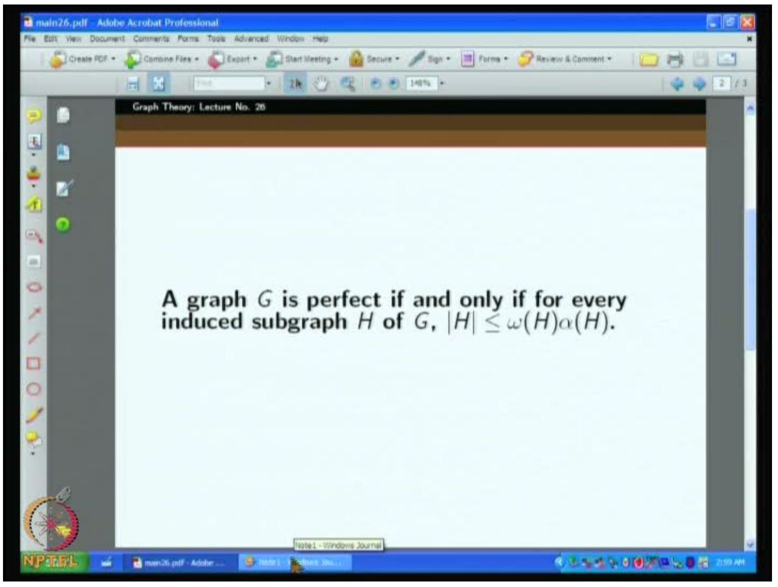

Welcome to the twenty-sixth lecture of graph theory. In the last class, we were discussing about perfect graphs, and we did a proof of Lovasz of weak perfect graph theorem, which states that if G is a perfect graph, then its complement is also a perfect graph. This was the theorem.

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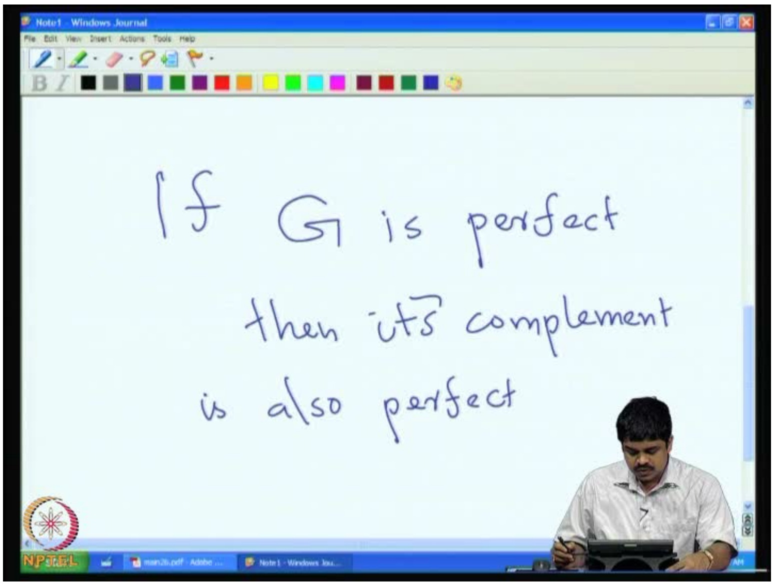

If G is perfect, then its complement is also perfect. The theorem is obvious from strong perfect graph theorem, which was proved much later, but then again, we decided to do a proof of weak perfect graph theorem because it gives good idea about the perfect graphs and good insides. So, in this class, we will consider another proof of weak perfect graph theorem by (( )) which uses some linear algebra in the proof. In fact, instead of proving directly the statement that if G is perfect then its complement is also perfect, what we are going to do is to prove a different statement, which was also proved by Lovasz earlier.

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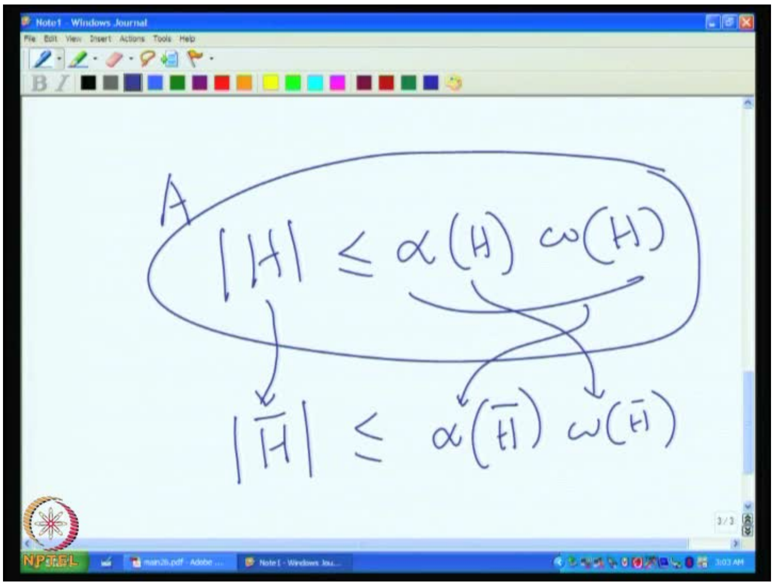

This is the statement: a graph is perfect if and only if for every induced sub graph H of G, the cardinality of H is less than equal to omega into alpha. If you multiply the clique number and the independence number of any induced sub graph, then there should be greater than the cardinality of the number of vertices in the induced sub graph we are considering.

If it happens for a induced sub graph, it is a perfect graph and for a perfect graph, it will happen. So, it is if and only if statement. Why are we proving this in order to prove the weak perfect graph theorem, which states that if G is perfect, then G complement is also perfect, because it is obvious that if this theorem is true, that means, any graph which satisfies the condition, that every induced sub graph H satisfies inequality cardinality number of vertices H is less than equal to alpha and omega, should satisfy that statement also, that if G is perfect, G complement is also perfect.

It says, because this statement is symmetric with respect to the complement on the graph, because any induced sub graph if you take in the complement that the corresponding induced sub graph will again have the same number of vertices that will be less than this product because this omega will become alpha of H bar and this will become omega of H bar because in the complement the independence number of the original will become the clique number and the clique number of the original will become the independence number.

This product is not going to change. Just the meaning of this alpha and omega changes. The numbers remain same. The total product remains same and this left side L H Salso remain same. Therefore, inequality will be true for every induced sub graph for the complement also.

Therefore, the complement will be perfect because any graph it satisfies this inequality for every induced sub graph is perfect. So, the complement is also perfect. It is enough to show that this inequality characterizes the perfect graphs. It is an if and only if statement for the perfect graph. Then, we have already proved the weak perfect graph theorem.

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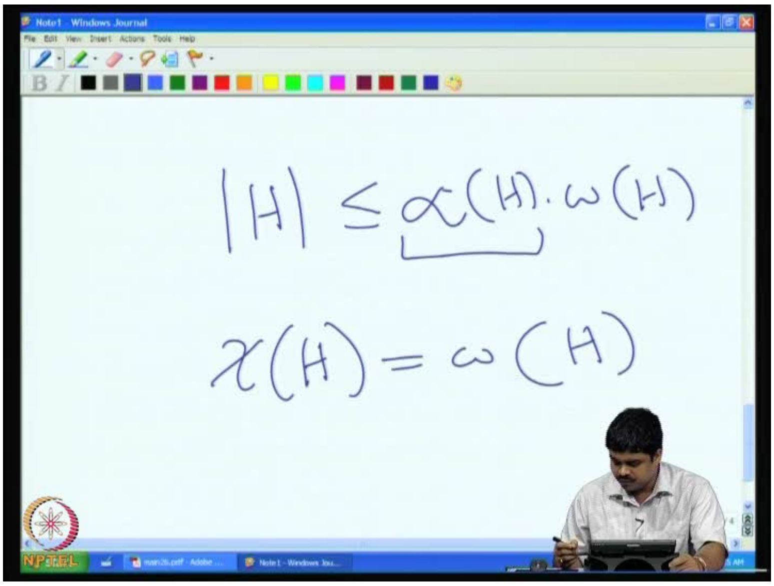

Therefore, we will try to prove this statement. One side of this statement is very easy, that means if it is perfect graph, then for every induced sub graph, this inequality should be true because you taken induced sub graph H and its cardinality has to be less that equal to its omega into its clique number alpha into omega of it because it is a perfect graph, this induced sub graph is also a perfect graph and then the chromatic number of this H is equal to the clique number. That means we have a coloring of H using omega colors but each color class is an independent set. So, it can contain only at most omega of H vertices because omega of H is the biggest cardinality of an independent set in alpha of H is the biggest cardinality of an independent set in H.alpha of H is the maximum possible number of vertices in any color class and we can cover all the vertices of H using omega colors. So, omega into alpha will be an upper bound for the cardinality of H. It is very straight forward for a perfect graph.

The more non trivial part is to show the other way, that means if a graph satisfies this inequality for all of its induced sub graph, then it has to be perfect. How will we prove this thing?

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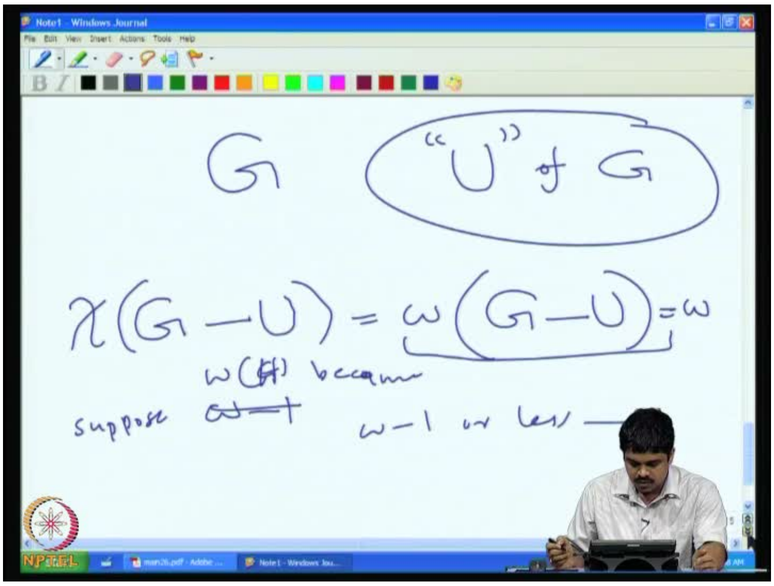

The first observation is to see that, suppose, you have taken an independent set. Suppose, G is the given graph satisfying the property and you identified any independent set U in it U of G.

If you take the independent set U of G and remove U from G, let say G minus U is the graph which we get, what can I tell about the chromatic number of G minus U? What can you tell about the chromatic number of G minus U? You have just removed an independent set. You know G is a perfect graph. So, G minus U is also a perfect graph because you have just removed some vertices from that. An induced sub graph of G is going to be a perfect graph also.

So, chi of G minus U has to be equal to omega of G minus U. Can you tell anything more than this? Can we tell what is the actual value of this thing? We can see that it has to be omega. So, is it trivial?

Because, you started from G, you removed some vertices. The clique number can only reduce. It can remain the same. But, we are saying that it has reduced. Suppose, it has reduced. It has become omega minus 1 or less. Suppose, omega of H became, so if G minus H omega minus 1 or less, then what will be happen?

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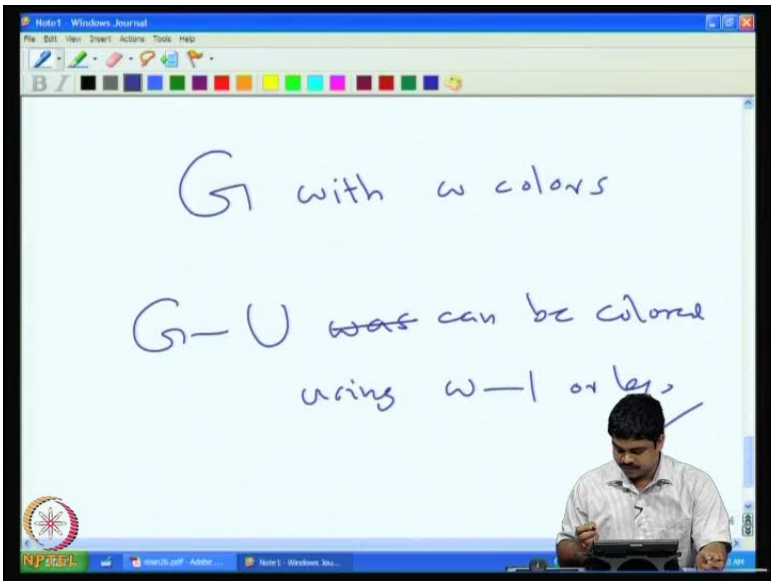

Then, we can put back U to the graph. U is an independent set. We need only one more color to color the entire graph. If this omega of G was equal to omega G minus U was omega minus 1 or less we have colored using omega minus 1 colors and one more color for U, then we will be able to color G with omega colors because omega minus 1 was enough for G minus U. G minus U can be colored using omega minus 1 or less colors.

G can be colored with omega, because U is an independent set. we can get all the vertices when you color and then G is colored with omega colors. So, G becomes a perfect graph. To repeat, we are trying to prove that if G satisfies the inequality omega and alpha. For every induced sub graph H, the cardinality of H less than equal to omega of H into alpha of H, then we want to show that G is perfect.

We will do it by induction. For small graphs, it is true, for one known graph, two known graph, it is very easy to check. We are assuming that for all the graphs with smaller number of vertices, it is true.

When we took an independent set and removed from G, G minus U will definitely satisfy this inequality for all the sub graphs. So, by induction, G minus U was perfect. That is why, we could say that chromatic number and clique number for G minus U is same. Not only that, this clique number and chromatic number of G minus U has to be equal to omega itself, the original clique number of G irrespective of which independent set we remove.

The reason is that suppose it reduced, then we could have used one more color to color U because U is an independent set and then G is colored with omega colors and the clique number of G is omega and then G is also perfect because every induced sub graph of G is perfect by induction assumption and then G is perfect now like this.

Therefore, we know that we cannot do that. Color chromatic number of G has to be greater than omega. Then only there is anything to prove. Otherwise, it is already proved. So, we can say that G minus U for any induced independent, whichever independent set, we try G minus U requires omega colors to color. That is why probably G may require more colors. We have to come up with contraction from that/

If G minus U requires any colors, interestingly, U can be anything. It can be one vertex, it can be the biggest independent set size itself, whatever it is G minus U requires omega colors to color.

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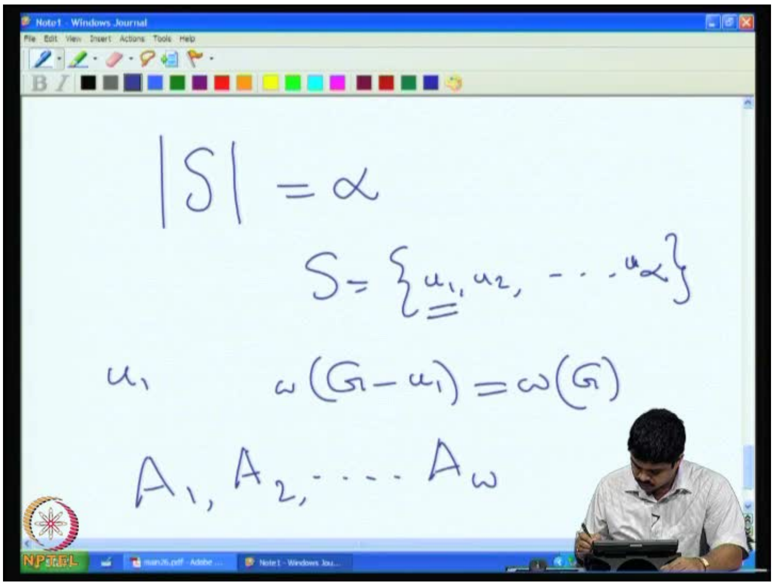

Now, you select a maximum independent set, some alpha. Let say S is a maximum independent set. The cardinality of S is equal to alpha. Now, let's say S equal to U 1, U 2, U alpha. This is the set of elements in S.

Now, what we are going to do is to consider G minus S first. When you remove that independent set S, the remaining graph as we mentioned already can be colored using omega colors. The main thing is its clique number has not reduced. it still has an omega size clique on it. That means, this independent set size cannot intersect with every clique in the graph. Otherwise, if it intersects with every maximum clique in the graph, the clique number would have reduced. That does not happen. That is why the clique number remains omega and the chromatic number is also equal to omega because it is a perfect graph by induction hypothesis smaller graph.

Now, you consider an omega coloring of this G minus S and then what will happen is it has infact omega colors and any omega size clique inside G minus S has to have a representative color in it.

But, when we consider this coloring, we would rather do see like this. You take U 1 and consider G minus U 1. This can be because U 1 is an independent set by itself. You know the clique number would not reduce. So, the omega of G minus U 1 is equal to omega only. The clique number will remain same.Now, because by induction hypothesis, if this G minus U 1 also can be colored with omega colors, consider that omega coloring and this omega coloring correspond to omega color classes. we can call it A 1, A 2, A omega. These are the color classes.

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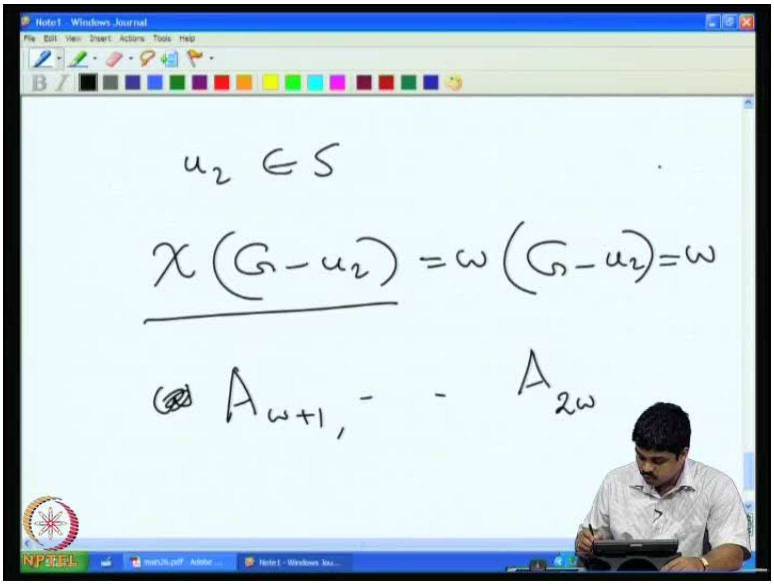

Now, what will happen if I consider U 2 from S and remove U 2 from G, G minus U 2. Again, the same thing applies for G minus U 2 because its chromatic number is equal to its clique number because it is a perfect graph induction and also it is equal to omega. As you have seen that its clique number cannot reduce by removing this thing, because if it reduce, then U can get new color and G also becomes perfect as we want.

We can consider an omega coloring of G minus U 2 and let say omega A omega plus 1, 2 A omega 2 omega are the color classes.

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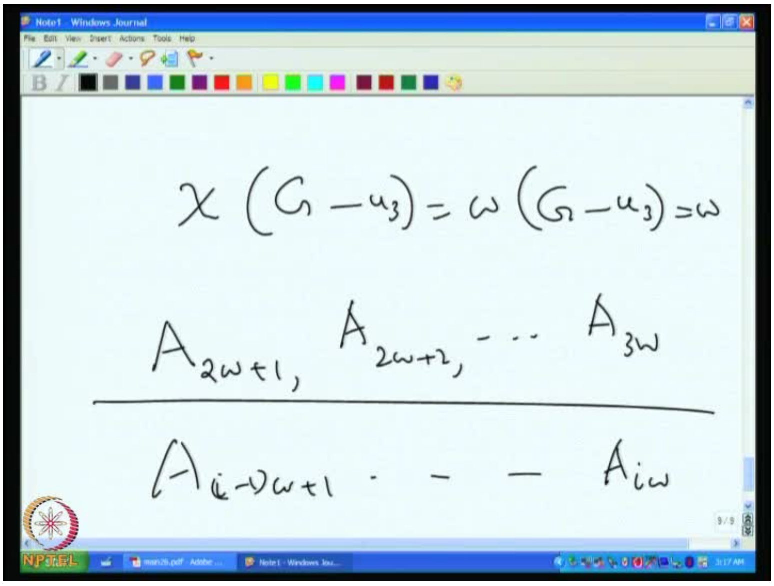

Now U 3 also we can do the same thing. For instance, because chi of G minus U 3 will be again equal to omega of G minus U 3 and that will also be equal to omega as we know and then omega coloring of this G minus U 3, if we consider, we can come up with another set of omega color classes. These are all independent sets. Each color classes in independent set that will be A 2 omega plus 1, A 2 omega plus 2 into A 3 omega like. that for when you remove U I, you will get, i minus 1 omega plus 1 2 A i omega.

So, total how many independent sets do you collect like this, each color class from what we are doing is, you remove a vertex of S because S is U 1, U 2, U 3, U alpha.

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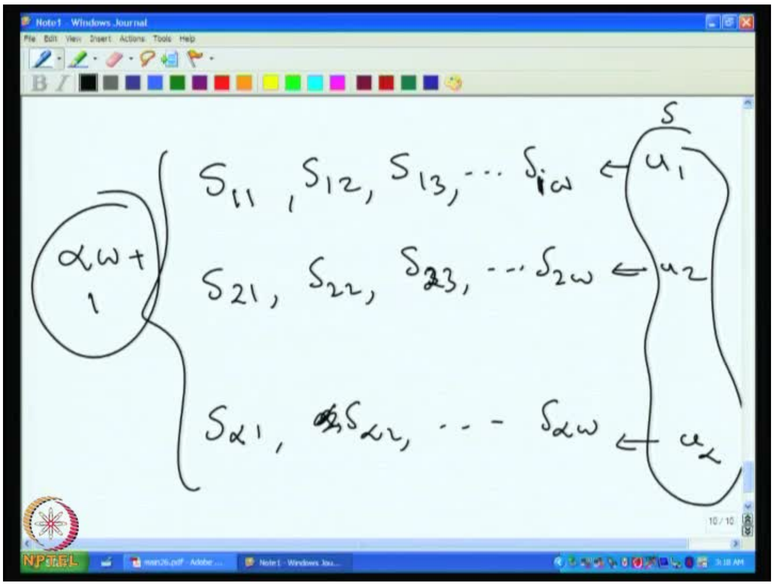

A vertex U i is removed from the graph and the remaining graph has a coloring using omega colors, not less and we collect all the color classes and these color classes are listed. So, for it may be even more convenient to see this color classes S when U 1 is removed, the color classes we got can be written as S 11, S 12, S 13, S 1 omega and then when U 2 is removed, the color classes obtained can be seen S 21, S 22, S 23 and S 2 omega and then S when U alpha is removed, you got U alpha minus U alpha one comma S alpha 2 S alpha omega.

So how many sets are produced because you are there omega of them, another omega total of alpha omega sets are produced and add to this the original set, S itself because this itself was a independent set. This is S. So, this also is added.

Together, this entire collection is alpha omega plus 1 independent set and this alpha omega plus 1 independent set have some interesting properties.

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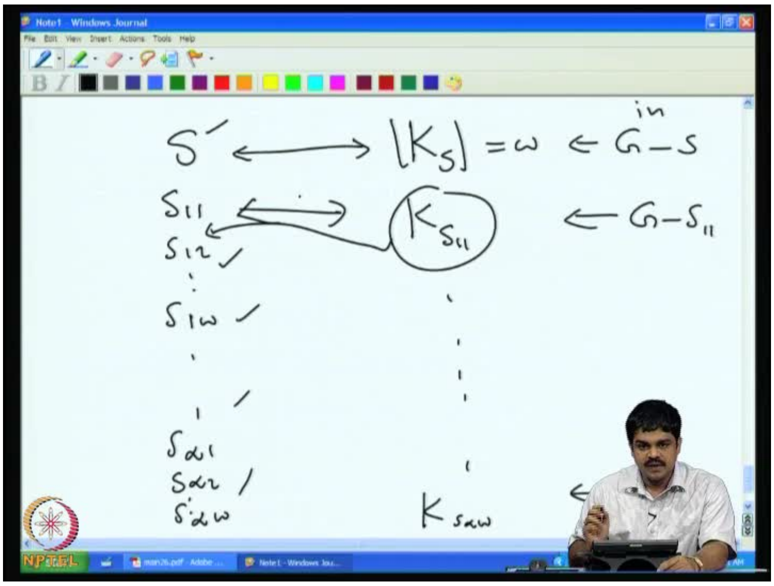

The most interesting properties, if you take any clique of the graph so one thing is for instance if I had listed this alpha omega this S is the first independent set, second is a S 11 to S 12 up to S 1 omega and then finally end of with S alpha 1, S alpha 2 S.Corresponding to each of this independent set, we know that even if you remove S from that, there is a clique of cardinality omega in G because the clique number cannot reduce by removing an independent set. If you remove an independent set and if the clique number reduces, then the remaining graph has a coloring with less number of colors than omega of the original. Then, you can add one more color to the independent set and then we will get omega coloring of G which would make it perfect.

Therefore, at least one clique corresponding to, I will say K of cardinality omega remains in G minus S. Similarly, we will get a K S 11 in G minus S 11 and so on. This is also an omega size clique. Similarly, here also. K S alpha omega in G minus S alpha omega. This is what will happen.

Now, not only that, if you take any of this clique, for instance, you can take some cliquehere, we know that this clique does not intersect with this because this outside in G minus S 11 is an omega clique in G minus S 11.

But then, that will intersect with all of the remaining things here, means the only independent set in this collection which it is not intersecting is exactly its partner to which it is paired. All others it will intersect. Why is it so?

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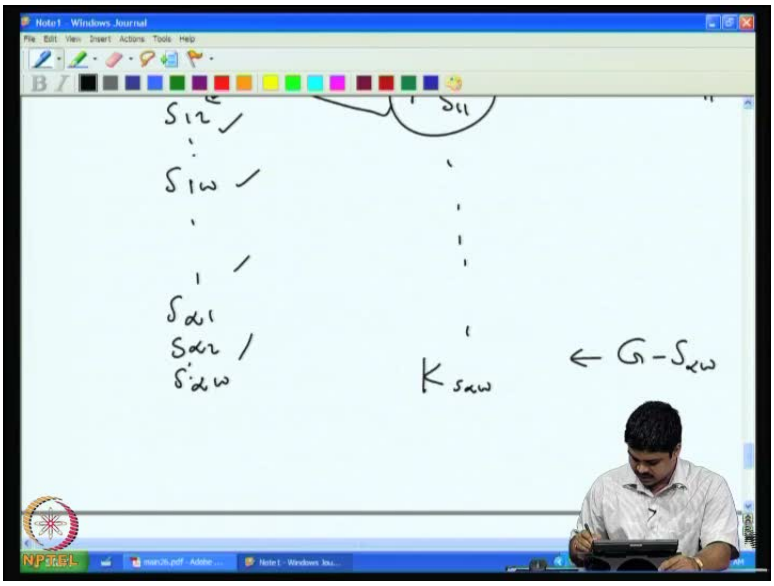

Because, you can take any clique, you can say that from this collection, any clique of omega cardinality can intersect with only one of these things and have to intersect with all of these things except one. So, we know which one for the clique listed there, which one it is not intersecting from that. Therefore, with all others, it has to intersect. For instance, you can take a clique K.

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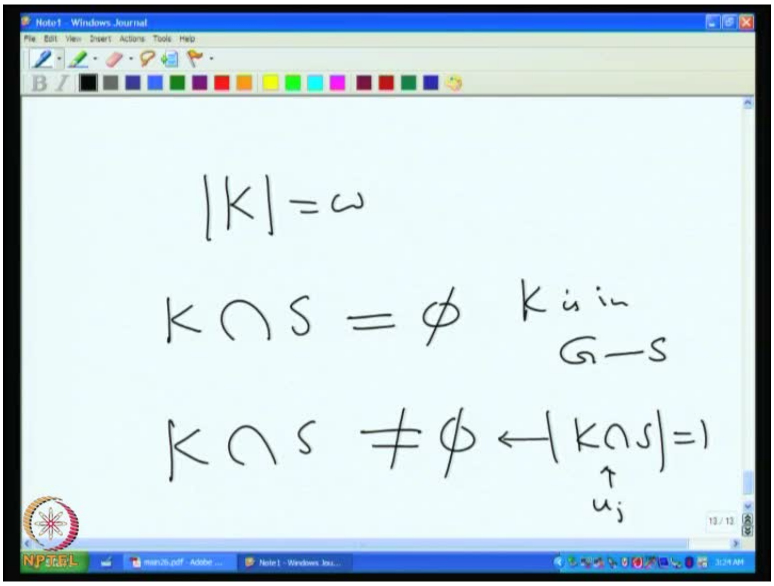

Let's say the K is of cardinality omega. Now, there are two possibilities K intersects with S, our original independent set has no intersection with that means K is in G minus S.

Then, we know that if you remove any U i from that graph because U i belong to S that will not affect this K. Any ways, K was not intersecting. We are removing vertices U 1, U 2, U 3, U alpha where from S

So, when you removed a U , that would not reduce K. That will leave K in the graph G minus U. That is a maximum clique. In the G minus U i graph, after removing U i is color with omega colors and then omega clique should all the representative colors. That means, it is part of all of S i 1, S i 2, S i omega all the sets it is available. This is true for every I, therefore, it will be intersecting with all the color classes that we generate.

That means, all of this S i j kind of sets, it will intersect. Now, on the other hand, suppose that clique was intersecting with S, but if it is intersecting with S, that is the intersection can only be of cardinality 1, because this is 1. Why is it so? K is a clique and S is an independent set, it can intersect with only one vertex in S. Intersection can be exactly 1. It cannot be 2 or more because how can the clique intersect with more than one vertices in an independent set.

Now, that is some U j, say this intersection correspond to some U j, if they are not removing, already S is intersecting and if it is intersecting with U j, then whenever any other U i is removed, this clique is not affected. It will remain in the graph. When you remove some U i other than this U j, the particular vertex to which it is intersecting in S.

Whichever other vertex you remove from the graph, this clique will be intact. This will remain as such because we are coloring the remaining graph with omega colors, each color class has to have a representative from the clique. The clique has omega vertices in it. All of them has to get different colors.

The clique has to intersect with each of this independence set except some independent set in S j 1 to S j omega. Every other color classes of this form S i 1 to S i omega S so i not equal j will have an intersection with K and among the sets S j 1 to S j omega. Only one independent set will be non intersecting with K because K even after removing this

U j, it will reduce to a clique of cardinality omega minus 1. So, it has to cut omega minus 1 color classes except one color class because only one can escape intersecting with it.

So, except one set, it has to intersect with all others. This is true for every clique. What we have told is, you take any clique of maximum cardinality in G, it has to intersect with all the independent sets that we have created that is S and S i j's, the total omega alpha plus 1 independent sets. From that all of them it has to intersect except one. Depending on the cases we have analyzed that sometimes it can be S or sometimes it can be some of the S i j's.

But, exactly one independent set we can find to which it is not intersecting and all the others in this collection, it has to intersect. This is the key point and in particular that this is true for any clique of cardinality omega. We have already seen that when you consider this each of the independent set in S and S i j's ,total alpha omega plus 1 independent set, each of them has an associated clique which is in G minus S i j or G minus that independent set. That means, there is a clique which does not intersect with it is not just that every clique has to intersect with exactly one of them, there exists one clique which for each of them which does not intersect and naturally if it does not intersect with this one, it will intersect with all others.

Because it has the possibility of not intersecting with only one set in this collection, with all others it has to intersect. We have created a collection of independent sets namely omega alpha plus 1 independent sets namely S, S 11 to S 1 omega S 21 to S 2 omega and finally S alpha 1 to S alpha omega.

And corresponding to each of this thing, we have also associated a clique which does not intersect with it and with among these sets, that is the only independent set with which it does not intersect. All others it will intersect.

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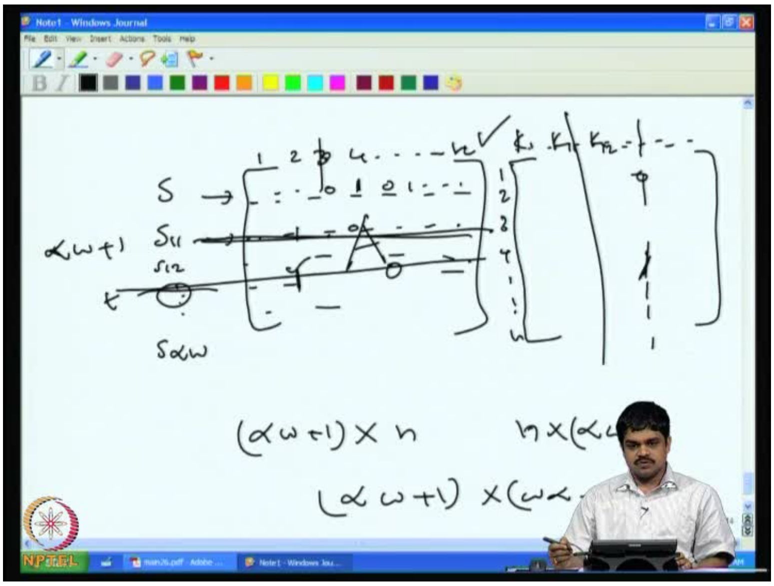

This is enough to prove our statement. We are going to construct a metrics where the rows are here. We will put it as 1 2 3 4 5. This column numbers are the vertex numbers and these rows will correspond to the independent sets, this is S, this is S 11, this is S 12 and this is S alpha omega. The number of rows will be alpha omega plus 1 and this is an alpha omega plus 1 cross n metrics. Let's say this is A and what will be in a row, we will actually take the support vector of S, means in a vertex belongs to S, then, we will put 1 there otherwise we will put a 0 there.

So, whichever vertex belongs to S, you will put 1 in the corresponding this thing, otherwise we will put 0. Similarly, here also this is what we keep doing.

So we can put 1 or 0 namely whether this particular vertex corresponding in this column belongs to that independent set.

We will also create a metrics which is an n by alpha omega plus 1 metrics namely by this. Here, the row numbers will be the vertex numbers 1 2 3 4 obtained and here this columns will correspond to the cliques K and K 11, K 12. Like that, you can say the clique corresponding to S i j will be in the corresponding column.

If it is the row, this is tth row, the corresponding clique for this independent set will be put here in the tth column.

When you multiply these two things, what will happen, you will definitely get alpha omega plus 1 cross alpha omega plus 1 metrics. Now, what will be the entries of this metrics. some thought will reveal that because what will happen, suppose, if I take ith row and ith column, if I multiply that corresponding independent set and a corresponding clique, they will not have any intersection if no vertex is common to them.

Now, you can see that, whenever there is a 1 here, there will be a 0 here, whenever there is a 1 here, there will be a 0 here corresponding because, they do not intersect.

When you multiply, you will get only zeros. But on the other hand, if you take any other rows i and j, if it is i and j, there are i not equal to j, then you say that it correspond to different independent set and clique different from the clique associated to it, a different clique which was not its clique.

Then, what will happen? There is an intersection they have to intersect and if they have to intersect and intersection is only one because, a clique andan independent set can intersect at only one place. So, when you multiply, we will get exactly 1 because all other places it will be 0 into 1 or 1 into 0 or 0 into 0. In in one place you will get 1 into 1.

Therefore, when you multiply will get 1. Through the diagonal, we will get 0 but all other places we will get 1 because ith and ith column multiplies to 0. All other places we will get 1 1 1 and we know that such a matrices rank, rank is full rank.

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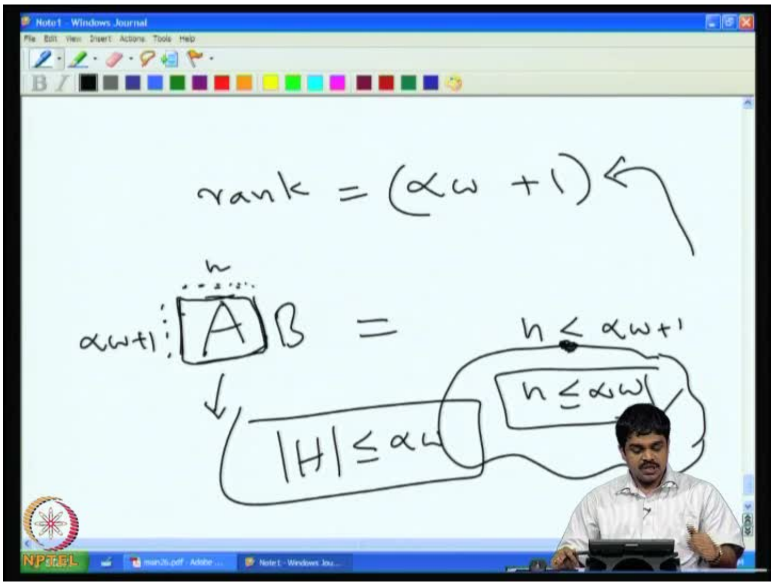

Therefore, the rank has to be alpha omega plus 1. This is alpha omega plus 1 into alpha omega plus 1 and there, if you consider the columns of that matrix, they are clearly linearly independent or rows of that matrix are clearly linearly independent.

alpha omega plus 1 is the rank of that matrix. But, in that case, we multiplied A and B and then we got a matrix of rank alpha omega plus 1. That means, the rows for instance, if this A also should have rank alpha omega plus 1, is it possible that the number of vertices in so the number of columns in A can be less than the number of columns in rows in A because, number of rows here is alpha omega plus 1 the number of columns are n.

If n was strictly less than alpha omega plus 1, that is n is less than equal to alpha omega, then what will happen is the rank of this matrix cannot be more than n because, there are only n columns less than equal to alpha omega. How can it have a rank of alpha omega plus 1?

And if the rank of A is less than alpha omega plus 1, how can the product have rank alpha omega plus 1? But we know that by assumption, this was correct because, H was less than equal to alpha omega for every induced sub graph in particular for G also.

This is clearly giving us a contradiction because, this A does not have rank alpha omega plus one and a product cannot have rank alpha omega plus 1. But then, we see that here rank has to be alpha omega plus 1. This is a contradiction.

Where is the contraction coming from? The contradiction is coming from the assumption that G was not perfect. That is why, we could select all those independent sets with that particular property, strange property.

If the G was not perfect, that is why we could assume that all and you remove all then any independent set the remaining graph can be a colored with omega colors because, the clique number does not reduce and all those assumption are based on the non perfectness of G. So, that is what is going wrong here. So, G has to be perfect.

We infer that G has to be perfect. It follows that, if this particular property for every induced sub graph H less than equal to omega of H and alpha H is true, then G has to be perfect. Then, of case complement also will satisfies this property immediately and then it is also perfect. because this statement H less than equal to alpha H into omega H is metric because alpha and omega when you go to the complement, the product remain same. alpha becomes omega, omega becomes alpha and the cardinality of does not H does not change at all.

So, we have proved it in a different way. This is the proof of Gasparian and in the next class also I plan to cover a few special classes of graphs. We have seen several special classes of graph already which are all perfect like chordal, interval graphs and then comparability graphs. Several classes we have already seen it is very important aspect of graph theory that we studied special classes of graphs with structure which comes from some applications with much neatest structure.

this is what we would like to do in this class. We will look at various very well known special classes of classes in graph theory and of course we would not be getting to very detailed analysis of all these things.

We would rather just casually look at this in some properties and then some interesting facts sometimes give a small proof. That is what we were planning because this is only to make the students aware of such classes of graphs and the interested student should read from other sources.

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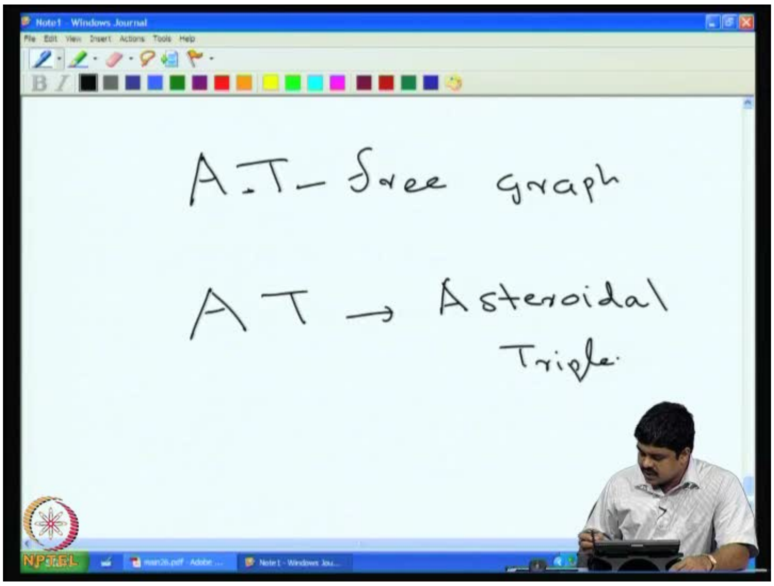

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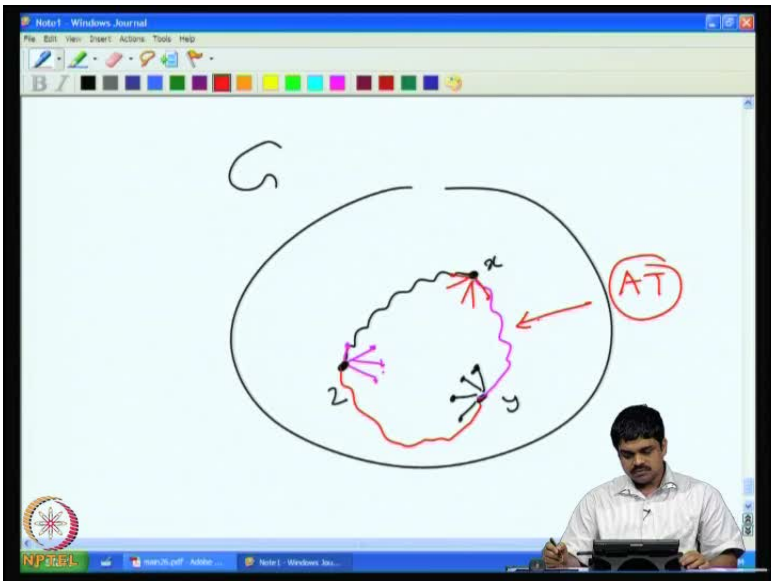

I would like to introduce the class known as AT free graphs. What is an AT free graph. T AT which is called the asteroidal triple is a independent set. In a graph G, suppose G is given, this is the graph. You can identify three vertices say x, y and z such that there is a path between x and z which does not go through any of the neighbours of y and also suppose there should be a path between x and y which avoids neighbourhood of all neighbourhood of sets that means no neighbour of set takes path in this part.

Similarly, there should be a path between z and y which avoids the neighbourhood of x. Suppose, you can identify three such vertices independent sets, that means mutually non adjacent three vertices with this property, then we say that this is an AT. A graph without an AT is called an AT free graph. If you cannot find an AT in a graph, it is called an AT free graph.

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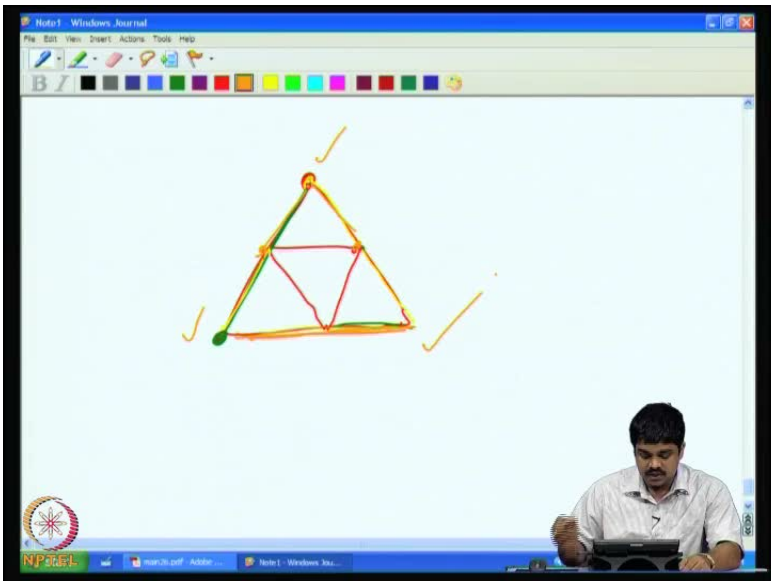

You can get some examples of AT. For instance, you look at this small graph, Can you identify any AT in this graph? If you consider these three vertices, it is an AT because between these two, we can find this path. It avoids the neighborhood. These are the two neighbours of this so and now between these two vertices, we can identify this path. This avoids the neighbourhood of this vertex because this and this are the neighbourhood of this and between this and this we can identify this path which definitely avoids neighbourhood of this vertex. This and this are the neighbors of this vertex.

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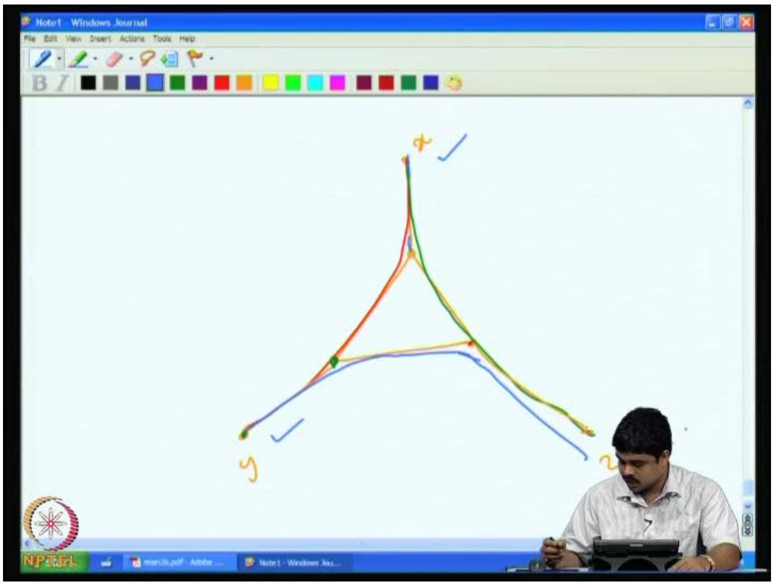

This path does not contain any of its neighbours. So, this, this and this is an AT in this graph and another AT for instance, we look at this here for instance this is x y z, between x and y we can identify this path and then see that avoid the neighborhood of set because the neighbor of set is only this. Similarly, between x and z, we can identify this path. It avoids the neighbourhood of is y because y's neighbor is here only.

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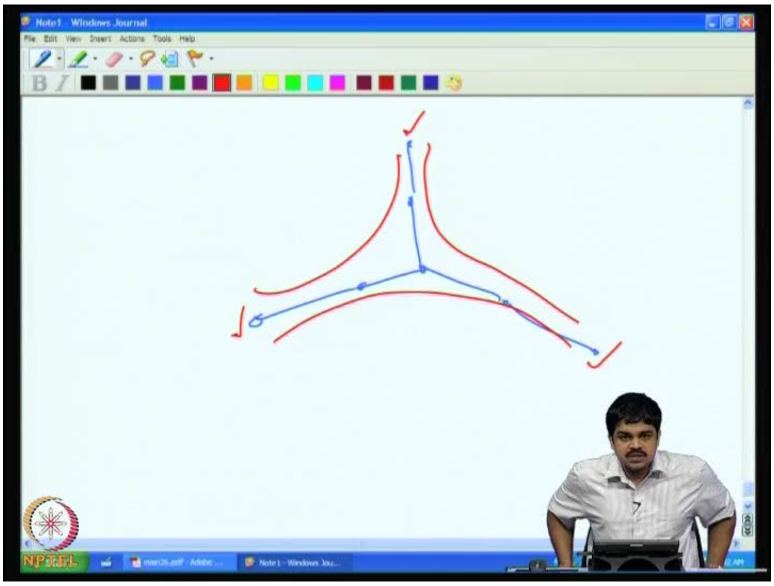

And now between y and z, instead of this path and this path avoids the neighborhood of x. So, x, y and z is an AT and so another AT here you can identify see in a tree like structure also you can identify ATs here because here is a path, here is path, here is a path. These paths avoid the neighborhood of the third vertices. If you consider these three vertices, these are ATs.

These are the kind of things ATs are. A graph without an AT is called an AT free graph. There are several such classes of graph where there are no ATs.

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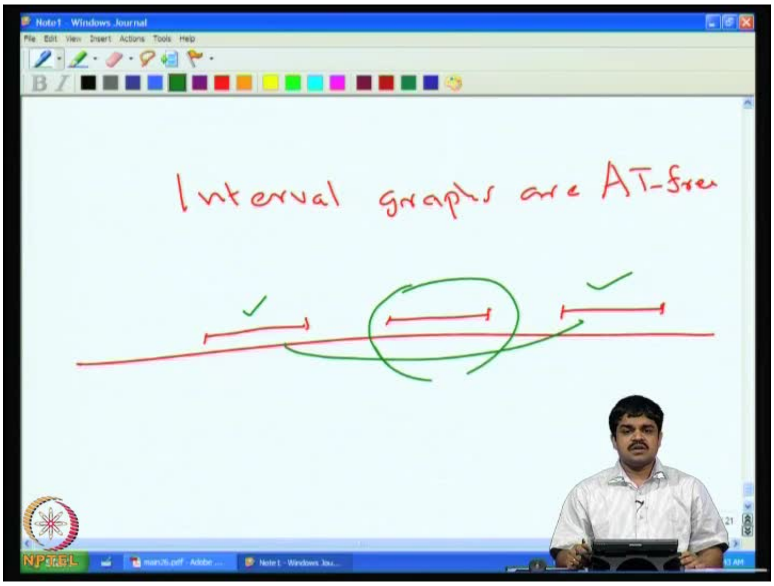

One of the very interesting class of graphs which we have already seen without an AT are the interval graphs. Interval graphs are AT free. So, interval graphs as subclasses of AT free graphs. Why do we think that there are no ATs in that? For instance, in interval graph, if there is an AT, we should be able to identify an independent set of cardinality 3. Let's say these three are the intervals corresponding to that.

Because see any three vertices should have intervals on the interval representation. Now, what will happen, for instance, if I consider this vertex and this vertex, any path between them cannot avoid the neighborhood of this vertex. It is very intuitive. Therefore you cannot have any AT in an interval graph. It is very straight forward.

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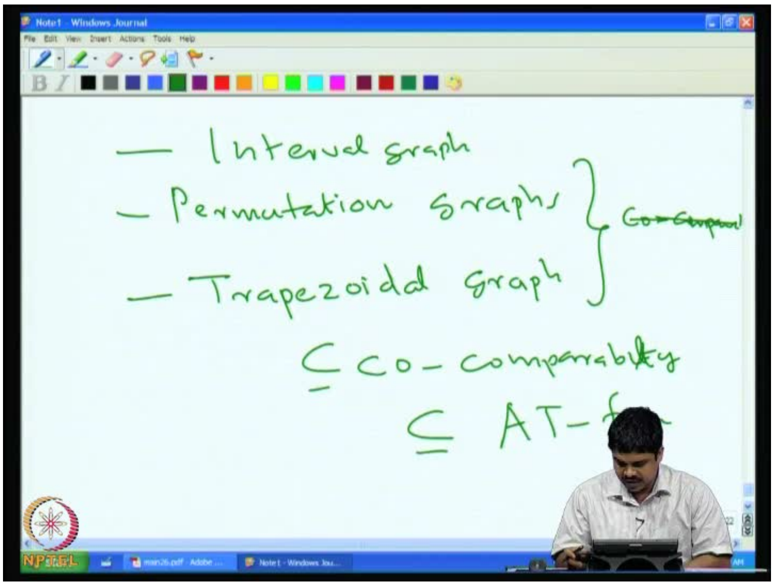

AT free graphs are very large class of graphs. It also contains many other classes like the permutation graphs which we will discuss in some detail in the next class because they are also interesting class and another class is trapezoidal graphs and a more general class which contains all these things, even interval graphs are called co-comparability graphs. Co-comparability graph means the complements of comparability graphs. These are all subsets of AT free graphs.

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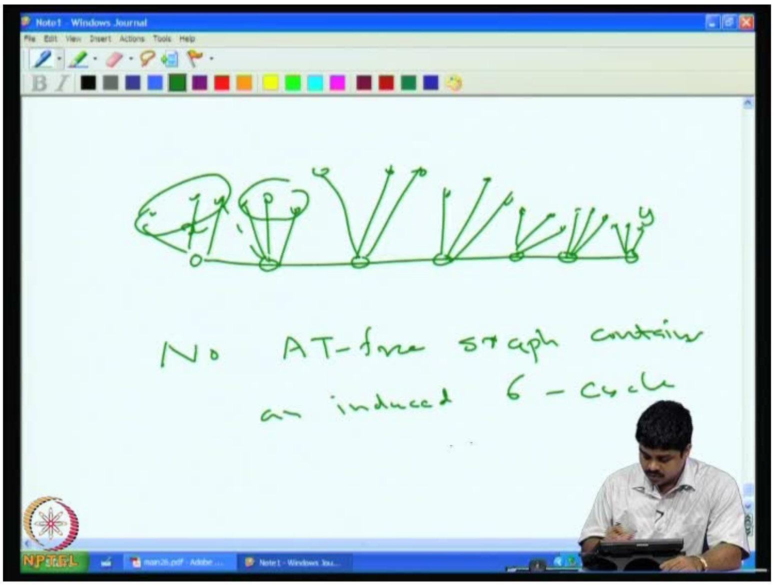

There are some interesting properties of AT. Among AT free graphs, probably the most interesting may be that AT free graphs have always got a pair of dominating vertices, in the sense that you can find two vertices such that if you consider the shortest path between them, they will be able to dominate all the vertices. It will become a dominating set. That means, every other vertex will be associated to at least one of the vertices. So, structure can be like this,

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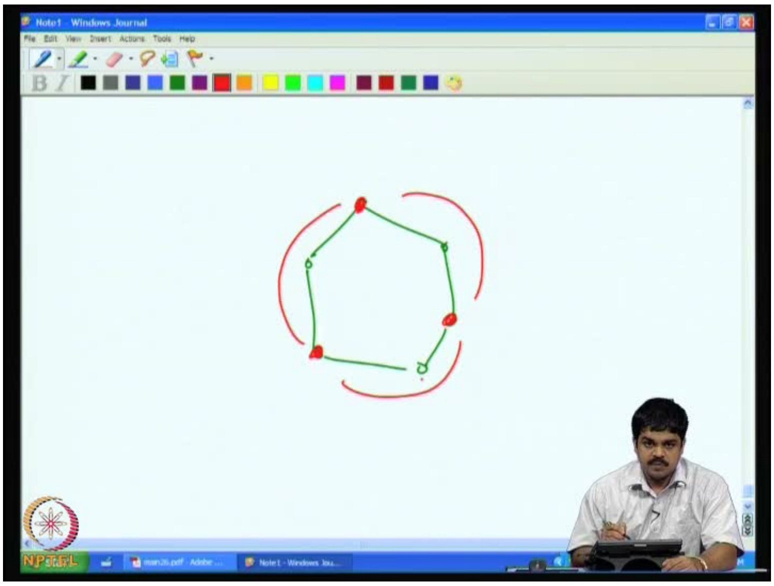

at least one of the vertices here and moreover one can tell there are connections inside this. There can be connections from here to here. We can also say that no AT free graph contains an induced 6 cycle or more 6 cycle or a greater than equal to 6 cycle. Why is it so? Suppose, you have an induces 6 cycle or bigger cycle. then you can find out an AT here very easily because for instance, here is a path, here is a path, here is a path. This is an AT because these paths between these two vertices avoid the neighborhood of this path. These two vertices avoid the neighborhood of this and so on.

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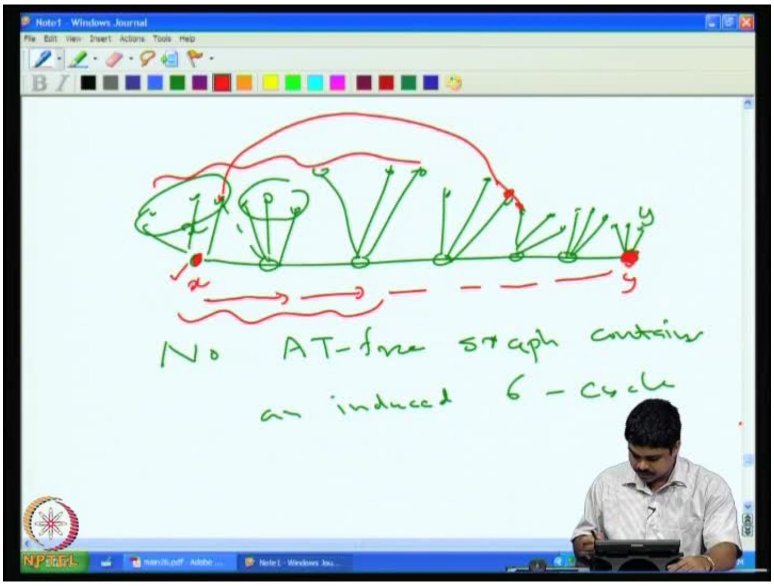

You can always say that you can select a dominating pair x y, such that the shortest path between them is the diameter of the graph. That is also known.

I do not prove all these things. I am just referring to a paper of (( )) about the AT free graphs so that this kind of structural properties are described which are very useful in studying the AT free structure. we should try to prove that there is no edge from a vertex. Here, to say a vertex to here, the edges across cannot go to for away because this is the shortest path between these two vertices. We have to try to make use of that.

We will also try to make use of the fact that there is no induced 6 cycle. You can try to see how for an edge can go and essentially we can also go out to here. I leave it as an exercise.

Interesting thing you can see is there is a certain kind of linear structure here. The graph can be seen as an expanding or progressing or growing starting from x towards y or with some little bit of growth in the as it moves toward the side. But otherwise, it is approximately linear. This a very weak statement. You should read the paper of (( )) and what AT free graphs. Our intention is not to describe lots of things is about AT free graphs alone. So, we would like to also see some of these special classes. What is the subclasses of AT graphs?

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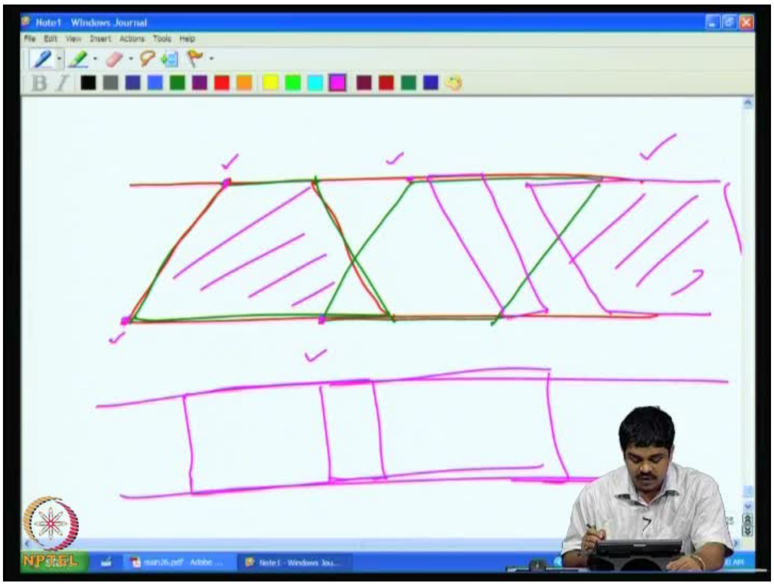

What is a trapezoidal graph? You consider two parallel lines. If you can associate with each vertex of the graph, a trapezoid can be like this such that two end points are the trapezoids. We can draw like this .Two end points of the trapezoids are here and say it can be.

TEach trapezoid correspond to a vertex and whenever the trapezoids intersect, then there is an edge between them. If we can get a trapezoid model of a given graph, then it is a trapezoid graph.

Most important thing is that you are not allowed to use arbitrary trapezoid. You have to make sure that out of the four end points, two are on these upper line and then two are on the (( )).

And it is very easy to see that if you do not allow arbitrary trapezoid, you just allow this kind of straight lines. Then it corresponds to interval graph because this will become intervals. Though we are drawing rectangles, they can be drawn one line itself interval graphs.

I will leave it to the student to verify that is trapezoidal graphs are AT free graphs.It is very easy. Just I consider an independent set here, there would not be touching three non intersecting trapezoids and try to argue that there cannot be an AT in it or you have to also try to prove that may be the trapezoidal graphs are co comparability graphs. So (( )) this interval graphs are considered. They are also co comparability graphs.

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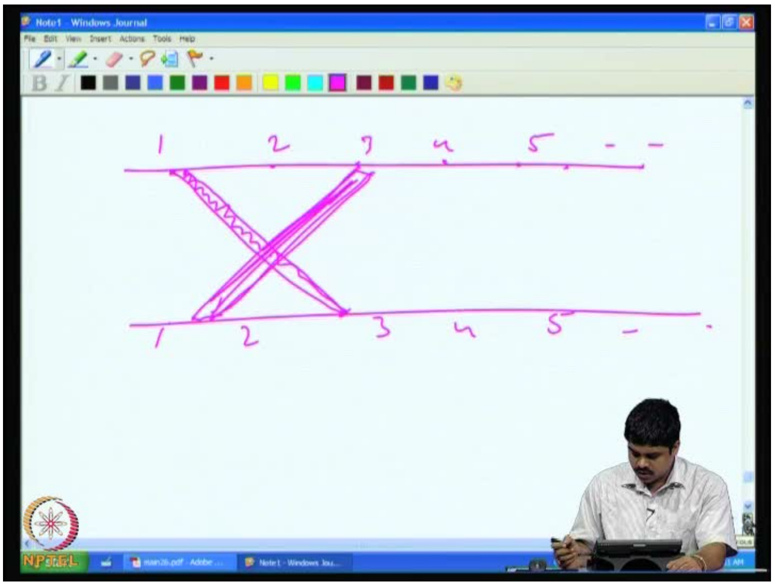

The permutation graphs which I will describe in much more detail in the next class can also be seen as a special kind of trapezoidal graphs. What we do is, 1 2 3 4 Suppose, you only allow a line for instance, this can considered as very special trapezoid.

So then, it is called permutation graph. If it is a line, you are just connecting a point here to a point here. 1 2 3 4 5 it will correspond to a permutation. I will describe in the next class and these are the some of the special classes of AT free graph. We can see this as a class which generalizes interval graph because interval graph probably the most popular classes both because of its usefulness and theoretically, it is interesting.

We have also seen that chordal graphs are generalization of interval graphs because it would not have cordless for cycles interval graphs are also chordal graphs. T Chordal graphs and all are perfect graphs but AT free graphs are not perfect because they can have 5 cycles induced 5 cycles in them 5 holds are allowed but of case 6 are more is not allowed 6 7 as we have seen why 5 cycles are allowed.

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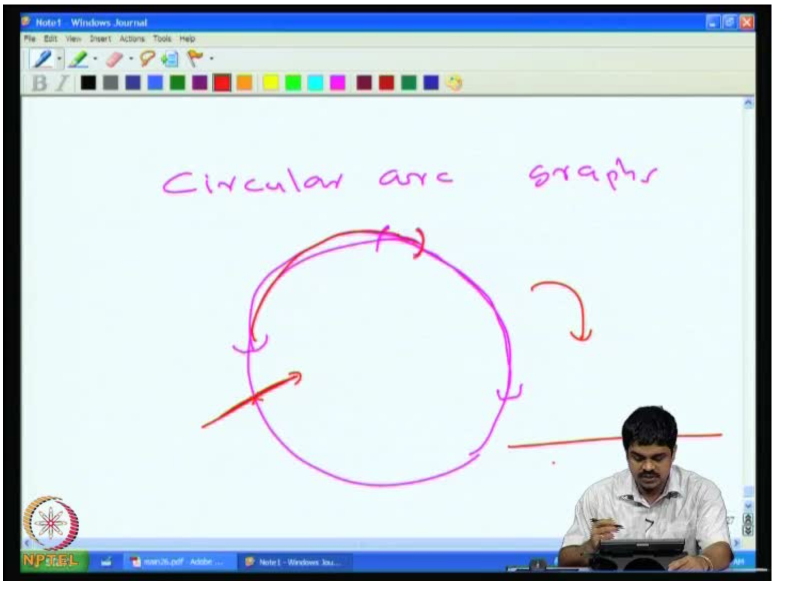

AT free graphs need not be perfect. So, that is about AT free graphs. Another class of graphs I would like to introduce, this again a generalization of interval graphs is the circular arc graphs.

What are the circular arc graphs? In interval graph, we consider intervals on the real line. In a circular arc graph, we consider intervals or arcs on the circumference of a circle.

The intersection graphs of such arcs is called a circular graph. In other words, a graph is a circular arc graph if I can associate with each vertex an arc on the circumference of circle in such a way that whenever two vertices are adjacent, the arcs intersect, otherwise, they do not intersect. The same definition intersection graph of arcs on this circumferences of a circle is called circle arc graphswhy are there generalization of the interval graphs? If in this circular arc model, if there is a point which is not covered by any arc, we can break the circlethere and then it will turn out to be a interval graph because we can draw it on the real line also. It is the generalization also. But when no point is unexposed. When exposed, that means every points has at least one arc going. Then, it need not be an interval graph. Circular arc graph can be very different from interval graph. Are they perfect graphs? They are not because you can take any cycle and draw a circular arc presentation for that. It is very easy to see that any cycle can be. Therefore, odd cycles are not perfect.

So the circular arc graphs need not be perfect. To some extent they are similar. But they are actually structurally very different. Like we define for interval graphs, if no interval is completely contained in another interval, then the corresponding order model is a proper interval model and if a graph interval graph has a proper interval model, then is an proper interval graph.

Similarly, we can define the proper circular arc graph means if a graph has a circular arc graph model, no arc is completely contained inside another arc, then it is proper circular arc graph model.

Similarly, we can define unit circular arc graphs where each arc has to be of same length. In interval graph case it is same.

But, there are some differences between the unit interval graphs and proper interval graphs are same but proper unit circular, proper circular graph and unit circular graph need not be same. We do not have much time to spend on special graph classes. Interested reader should read it from (( )) text book algorithmic graph theory or from papers. In the next class, we will consider some other permutation graphs and some other intersection graph classes. Thank you!