Combinatorics
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Lecture - 41
Summary

So, I have finished all the 40 lectures required for this course combinatorics. Now, I would like to makes some concluding remarks. First the contents of this course; it is an elementary combinatorics course, in the sense that if somebody wants to teach combinatorics alone as a topic and it is the first level course, the kind of material that will be included typically in such course is what is available here. I would also mention that I have not included any material from graph theory, because I have already done a course in a graph theory. I know that usually people include a little bit of graph theory in a combinatorics course, but I thought it is not necessary because the full course is available for graph theory in the NPTEL collections.

And there is no number theory or no probability related topics like probabilistic methods or some discreet probability it is not included, because anyway they are separate topics. And I stress this for combinatorics and the usual very usual elementary first level topics which are usually thought in the course. So, the kind of topics I have selected are first I started with pigeonhole principle, but that is the simplest technique, but I have taken several examples to illustrate the kind of problem which we see in combinatorics and taken examples from graphs, though I told I do not have not thought graph theory. I have taken some example problems from graph theory, the required definitions, etcetera, I myself has given, but it would be nice if the student was aware of some notions about graphs but nothing much these are all for specific examples that solve, right. And many examples regarding numbers are taken, but no deep, I mean no detailed knowledge of number theory is required at all.

We just have the twelve standard level knowledge of number theory numbers and elementary initial concepts of graph theory; that is all required even to understand those examples; otherwise, you can skip those examples or you can just quickly browse the net to see what is the meaning of those words, that is enough. And the pigeonhole principle we have not included several problems on that to introduce all kinds of things in

geometry where some combinatorial question in geometry, some combinatorial questions involving numbers, some problems from graph theory and things like that.

And then I have introduced all the elementary notions which is required for a combinatorial course, but even there I have assumed a twelve standard level knowledge; that means you already know what permutations are, what combinations are, and what functions are and the relations. This much knowledge you should be having if you have done the topics you have studied well the mathematics up to twelve standard only that much is assumed. So, after that I discussed elementary counting principles, then many of the elementary problems which come in combinatorics like selections from a multiset and of our combination of a multiset, R permutations of a multiset, such topics which are elementary, the balls and bins problems, such topics are discussed in detail.

And then after that again consider some more strategies like double counting strategy, then the inclusion-exclusion principle such things, but I gave several examples and this is so covered in several classes all these things. And I also discussed some of the bounds which are commonly used in combinatorics like the Sterling's approximation for n factorial and using that what kind of approximations you can get for a n choose r and some of the usual upper bounds, lower bounds, etcetera discussed and then the generalization; see the concept of n choose r how do you generalize is to when r is also negative, such questions are discussed. So, that we can have a generalization of the binominal theorem is discussed which is required later in the discussion of generating function, and all but again I thought it is important to have a glimpse of this generalization.

So, because moving little away from the counting problems and knowing how to go little more general, so that is why that topics are introduced and after that again recurrence relations and generating functions are discussed in detail. Several examples are taken and solved and then exponential generating functions are discussed and after that I have considered some special number sequences which appear in combinatorics mainly the Catalan numbers, the partition numbers, how many ways you can partition a given antigen n and the Sterling numbers of two kinds, the first kind and second kind; all these things are discussed.

So, in the last part the Sterling numbers of the first kind that comes in the fortieth lecture, so it was not discussed in such detail but again you get an idea what this is. These are the kind of topics which are discussed. So, I have taken time today to discuss all that; it is not that I have quickly discussed each of these topics. I have given several examples, I have explained them in detail; lot of examples are taken to give how to use it, right, to give an idea of how to use it. Then the kind of audience I expect for this course, again I stress this is a very elementary course and this is not for the extremely intelligent student who I believe should read the books and learn. So, their thinks will be more precisely written and he can learn faster because the kind of time I have spent on examples in those, such a student need not spend, probably, he can see couple of examples and go further, he can do things quickly.

So this is essentially for a good student at the B-Tech level, say, and who is interested in the topic. But on the other hand see may not be extremely fast and also the kind of students who would like a visual media; so in another words who would like to listen and learn, find a class rather than reading from the book. It depends on the person, some people like to read the book and they do not lose interest when you keep on reading the books, but some other people can concentrate, focus in a class and listen. So, such people can to take this thing, and I claim that if you really good student even the twelve standard level is enough to go through this course, because as I told you before the expected level of knowledge is only the knowledge of what functions are, what relations are, may be what partial orders are, and some elementary facts about numbers, so what is mod operation and things like that, reminders and things like that, nothing more than that.

And just a notion of what is NCR, what is NPR, such things and nothing else. If you are B-Tech student but ideally if you are B-Tech student and if you have usual discreet math course, then you know all these things which is required for this course, even the kind of notions we need from the graph theory are covered in such a course; just little bit of graph theory is covered in such a course, little bit of combinatorics is covered, little bit of probability is covered. So, everything is what required for this course will be covered in such a course. After taking a discreet math course if you do these thing, it will be perfect for you; that is kind of audience I except.

Again I should stress that somebody is advanced listener, for instance who is a PhD student and doing well may be in a different area but is familiar with many of these

topics, and I will not advise such a student to go through this course also because though probably they might not have done a formal combinatorics course even at the elementary level, but I am sure by the time they have reached there, they already know many of the topics I discuss and if they go through this elaborate presentation of each of the elementary topics very simple topics, they may get bored. So, I do not advise that an advanced listener even if he has not done an elementary course in combinatorics before should go through this thing; he can just quickly look at topics he wants and maybe he start reading more advanced topics.

So, then there are of course the nature the way I have presented things, so again to discourage the advanced listener; listeners who have already gone through the academic setup, the theoretical computer science or mathematics if they have done sufficient time. They have spent sufficient time in this setup, then they already know many of these topics is what I believe, right, even without sitting formally in a course like this, right. And in this course the method I have adopted is I take a topic may be very elementary topic like a pigeonhole principle, but I will spend several hours may be four hours, five hours taking several examples of different different nature and explaining them.

And also when I consider inclusion-exclusion principle, I have worked out the details for every problems, most of the problems I have worked out, which is really not required if you are good enough because I just give you the way you can start off, the rest you can easily figured out. But I thought if you are an average for whom am I recording this video lectures? It is for the average student and the kind of student who just want to lazily sit down and listen, right. So, he wants to sit in front of the computer and without working out he wants to figure out. So, I have worked out everything for him. So, I think even he does not do much homework he will get to know how he does, he should do things because all the details I have worked out and it is done again and again, right.

So, I am aware that it can make things very dull for the more brilliant ones. They can either skip those portions or they can go and read from the book and also there is another issue. Because this is I am recording, I am sitting and talking to an empty room and there is no body to listen to me; I am writing there is no body to correct me when I am doing this but if I was teaching in a class, so if I make a small mistake. For instance if I write a superscript instead of subscript or if I make mistake in the notation, then somebody will tell sir, it is not like that, right.

So, it is automatically corrected there but here sometimes I make a mistake; I realized only much later, so I come back and correct it. So, it may create some trouble for the students who really want everything perfect. I would suggest that such student should read from the book because in the book the writers go through it again and again, and they make several round of correction, so everything will be perfect. So therefore, such students who want everything precise should go through a book but if you are happy in a class, so you can probably go through these lectures, but I should tell that the satisfaction level would be slightly less than what you will get in a class, because in a class you can always shout and say that see there is mistake there and the teacher corrects. Here definitely you cannot shout, and then while recording there was nobody here in front of me to shout and say that sir, you are making a mistake.

So therefore, there is a possibility that you may get a little annoyed because of some mistakes here and there but if you can keep that in mind and be okay with such drawbacks then this is for you, right. So, again that means the students who want to see things written perfectly without any mistakes and if they have that patience to read from the text book it is always advisable to study from the text book; you do not have to listen to this course. And I should also mention that there is nothing special about this course. There is nothing more what you can get from a book, because these are only standard materials. There are no special insights I could add to the proofs or the methods or anything; I have taken the usual materials and presented.

So therefore, it is always advisable to read from well-written books but if you think that you want to listen and learn then this is a good option, right, you can. So, I apologize for all the mistakes and probably for all; I mean whatever drawbacks the courses have I apologize for all of them. So, apart from this thing I do not have anything to tell. So, I wish you good luck and happy listening.