Combinatorics
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Lecture - 35
Partition Number - Part (3)

Welcome to the thirty fifth lecture of Combinatorics. In the last class, we were discussing about partitions of integers.

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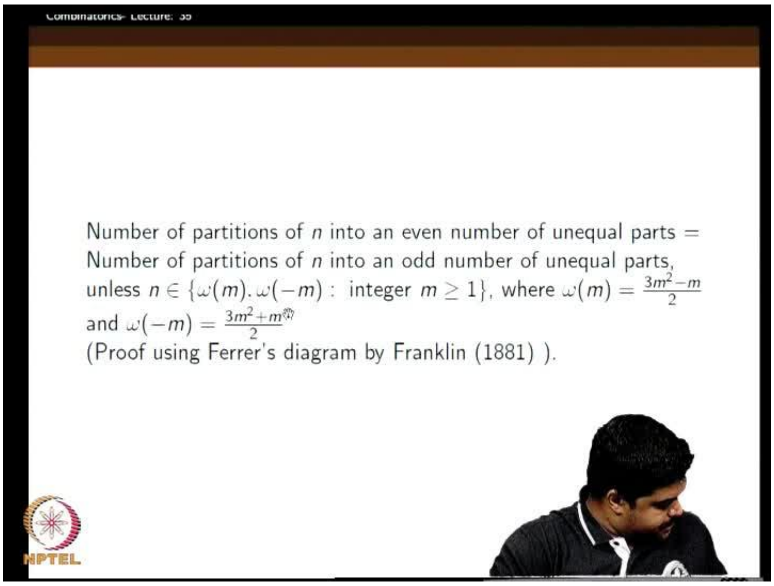

We were considering special kind of partitions, namely the partitions of n where each part is different, there is no repetition. Each summand is a different number, unequal parts and among these say that the distinct parts that are what we called before. So, earlier we had seen that these number of such partitions is equal to the number of partitions of n where each partition is odd number. Now, we are looking at this kind of things along the partitions, where we have distinct summands and unequal parts. Now these partitions obviously, we can group it into two categories; namely those of this kind of partitions, which have an even number of parts and those of this kind of partitions which have an odd number of parts.

We are saying that both these groups have the same cardinality they are equal I mean, even number of parts and odd number of parts. There are equal number of partitions of that is what once we concentrate only on partitions of n, where the parts are all unequal summands, but this is true for most n. There are also some values of n for which that is not true, where for such n the difference between the cardinalities of the two groups will be at most one that is what we are trying to prove.

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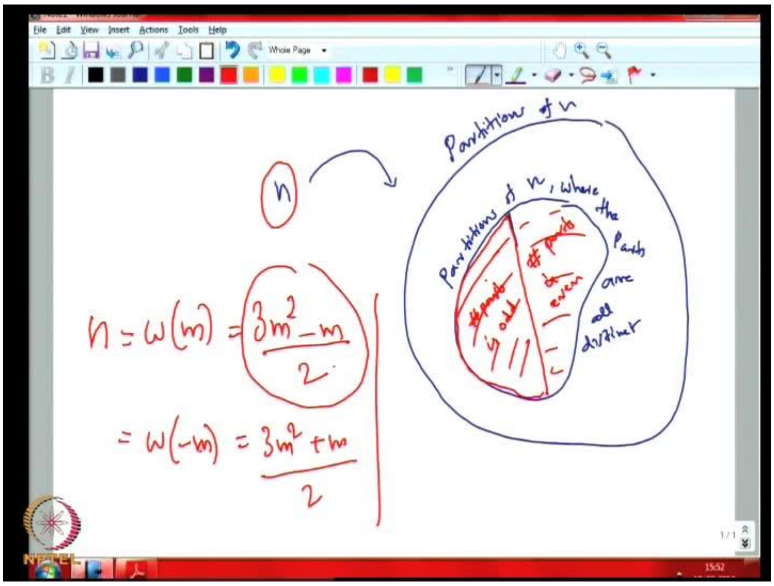

I will just draw a diagram here, this is for a given n, this corresponds to the partitions of n. Now, within this we are identifying certain type of partitions, this is partitions of n, where the parts are all distinct means there is no equal parts, the parts are all unequal. Now, within this we can definitely group into two category say like this. Let us say we can group them into two; this is within this partition number of parts is odd, this is number of parts is even. We are trying to say that depending on n for most of the n, this cardinality of this portion will be equal to the cardinality of this portion.

This is what we are saying, but there are certain n's where the cardinality of this or this may be bigger, but even if it is bigger, it will be big by just one more. What kind of numbers that is true now; if n is of the form omega of m, then it can happen omega of m means it is omega of m is this 3 m square minus 3 m square minus m by 2 if or n is of the form omega of minus m that is 3 m square plus m. We are just putting a instead of m in this first formula we are putting minus m, will get 3 m square will not be affected because minus will go away there when you put minus m here it will become plus m by 2.

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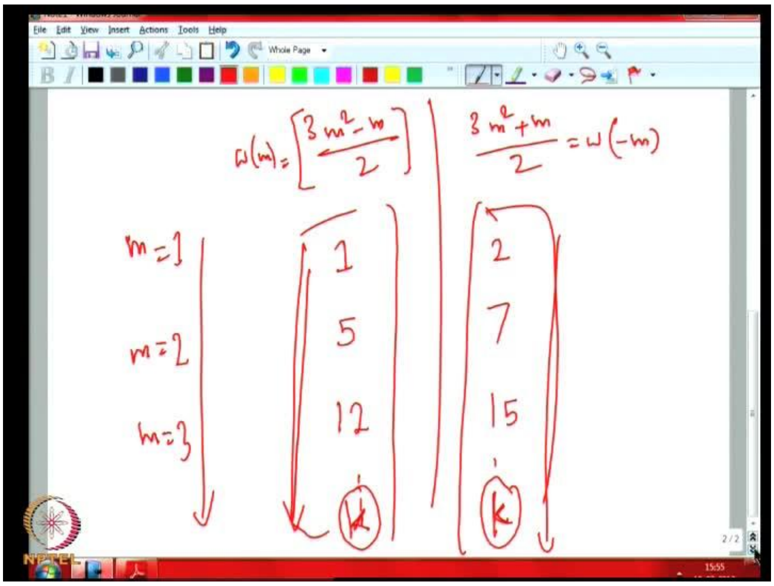

So, we will wonder what kind of numbers are this so, we can just write down 3 m square m by 2 and 3 m square minus, let us say this is minus m by 2 and plus m by 2. This is omega of m this is omega of minus m see for m equal to 1 what will happen. So, m equal to 0 what will happen m equal to 0 this is just 0, we are not interested in that, this is also 0 we are not interested in that. So, let us start with m equal to 1, 1 onwards only we are considering this so, 3 m square minus m is 3minus 1 by 2 that is 1, this is 3 m square plus m that is 3 plus 1 by 2, 4 by 2 is 2.

When m equal to 2 this will be 12 minus 2 that is 10 by 2 is 5 and this is 12 plus 2, 14 by 2 is 7. When m equal to 3 this will be 27 minus 3 is 24 by 2 , 12 and this is when I put m equal to 3 here 27 plus 33 by 2 15 and so on this will go. So, if you want to if you are wondering whether this can ever become a fraction, it will not because you know suppose m was an even number, you can see that this 3 m square is an even number, m is an even number an even number minus an even number or plus an even number is always even, so that divided by 2 is going to be an integer.

On the other hand if m was an odd number this 3 m square is an odd number and this m is an odd number the difference or sum is going to be an even number so that divided by 2 is an integer all the time. So, you can see there are numbers which are not of this form, either 3 m square minus m or 3 m square plus 2. So, for instance 1, 2, 3 is missing here and 5, 7, 6 is missing here 8, 9, 10 are missing here all this things.

You can see that, when we give bigger values this one only increase, bigger values of m so, as m increases this will increase. Similarly, this will also increase so if something is missing here it is missing right. Now, one question which may come to your mind, this is possible that we may find some k here which also appears here, is it possible that k will come here, k will come here also. We are telling that it will never happen so whatever we see here in this column will be different from what we see here in this column.

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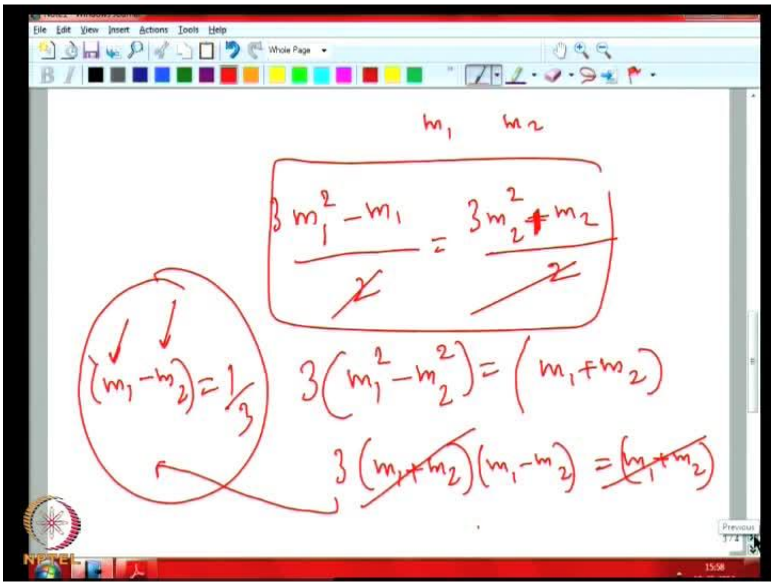

Suppose, it is not so that means there should be some m 1 and m 2 such that 3 m 1 square minus m 1 by 2 is equal to 3 m 2 square minus m 2 by 2. So, we cancel 2 first, then we rearrange we get 3 into m 1 square minus m 2 square is equal to m 1 minus m 2. So, we can cancel this from this side, because this m 1 square minus m 2 square is m 1 plus m 2 into m 1 minus m 2 so, this is just 3 into m 1 plus m 2 is equal to 1. So, which means that sorry I meant this is plus so, we wanted to consider whether there is an m 1 such that 3 m

1 square minus m 1 by 2 is equal to 3 m 2 square plus m 2 by 2; that is what we are considering therefore by rearranging, we will get m 1 plus m 2.

Now, see this is m 1 plus m 2 into m 1 minus m 2 so, this is equal to m 1 plus m 2, of course this is not 0. Therefore, I can cancel this of so we get from this thing how do we get 3 into m 1 minus m 2 is equal to 1. So, these three, I can take to this side this will become 1 by 3. So the difference between m 1 and m 2 is equal to 1 by 3, which is not possible, because all this m 1 and m 2 all are in integers so, there difference cannot be 1 by 3. So, this are contradiction so, this will never happen.

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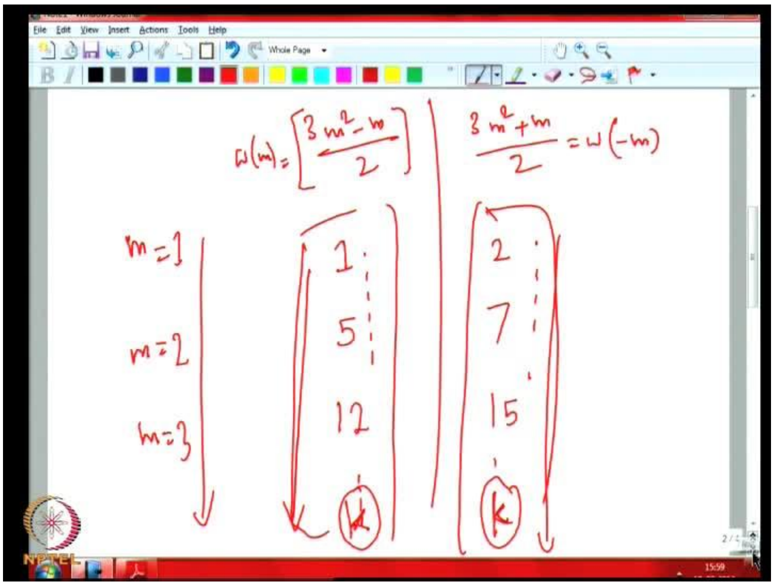

So, what I have proved now is that what to see in this first column are all different from what you see in this second column, right?

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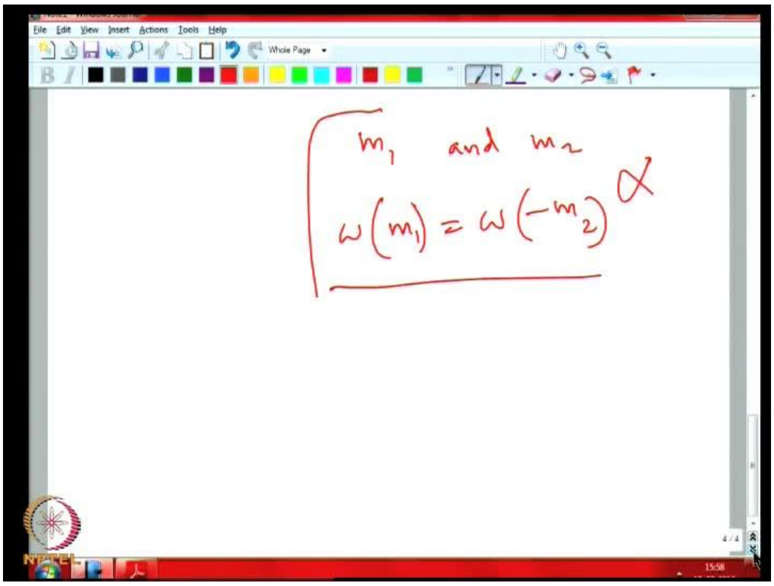

So, it is not possible to have some m 1 and m 2 such that omega of m 1 is equal to omega of minus m 2 this is not possible so, this will never happen, this is not possible.

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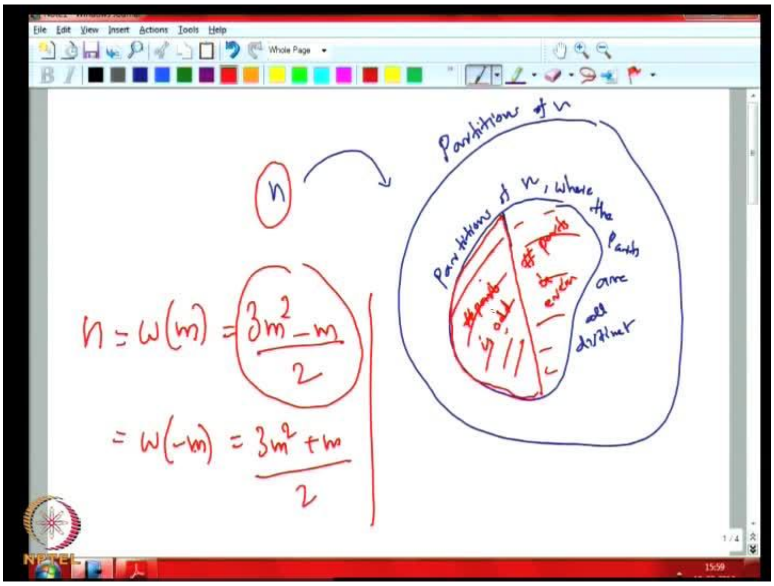

Now, so what we were discussing is about the special numbers where the cardinality of this group is one less or more compare to this so, we will tell when is this bigger and when this zone is bigger of which m which is bigger. So, it is possible that for some m this is bigger for some other m this is bigger. We will actually show that if an m is odd this part is bigger, when m is even this part is bigger, right? So, that means there number of even partitions will be more when m is even and when m is odd the number of odd partitions will be bigger.

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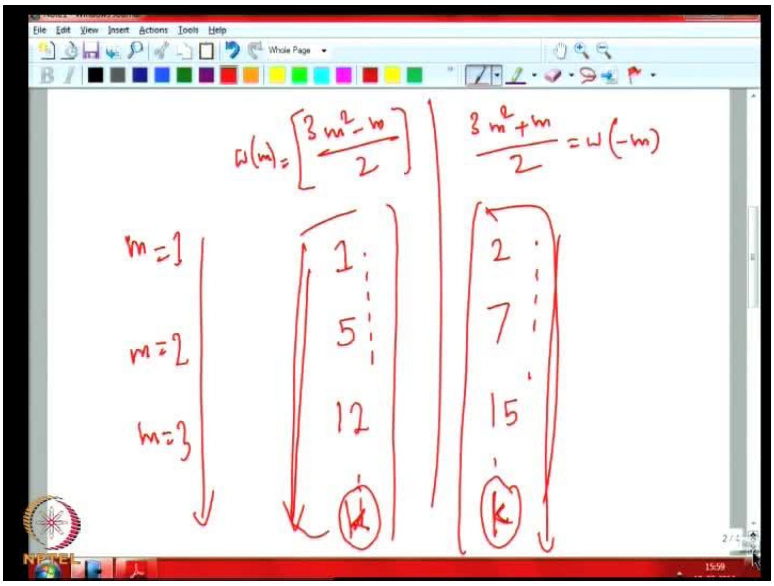

This is what we will show, bigger by 1 just 1 , if at all it is bigger it can only be the bigger by 1 it will be bigger only for these two cases this kind of values only and for this kind of values it will be bigger. If m is odd number the partitions with the group the collection of partitions with odd number of partition will be more by 1 and if m is even the collection of partitions with the even number of parts will be bigger this is what we want to prove otherwise both groups will be of equal cardinality.

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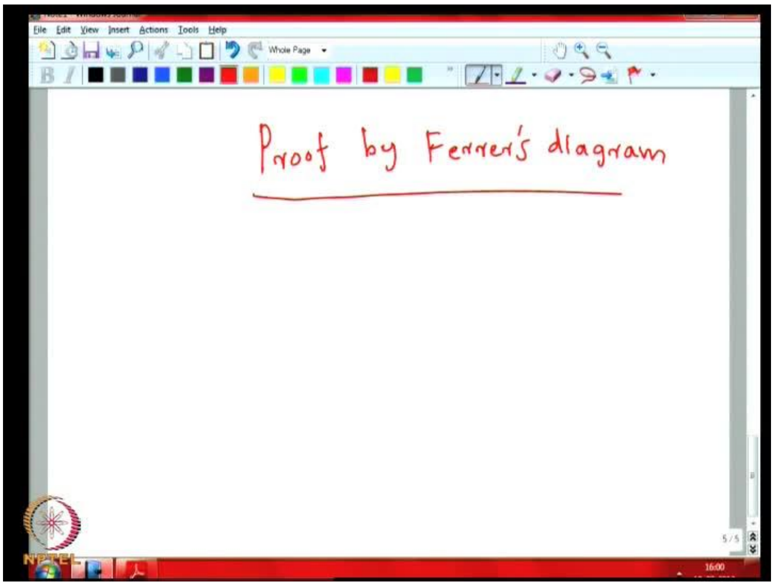

Now, to prove this thing we use Ferrer's diagram with the proof by Ferrer's diagram. This proof was done by Franklin in 1881. Now, let us look at the Ferrer's diagram for a partition from say this part.

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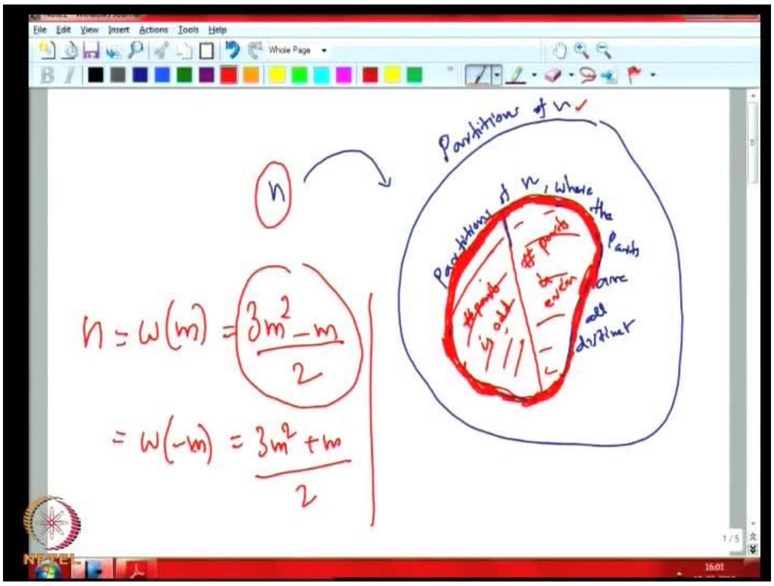

So, as we are considering partitions of m, any partition has a Ferrer's diagram, but we are bothered about only partitions inside this part only partition inside this part. We are not interested in partitions outside this so, what are this kind of partitions is a partitions of n where each part is distinct that means unequal values.

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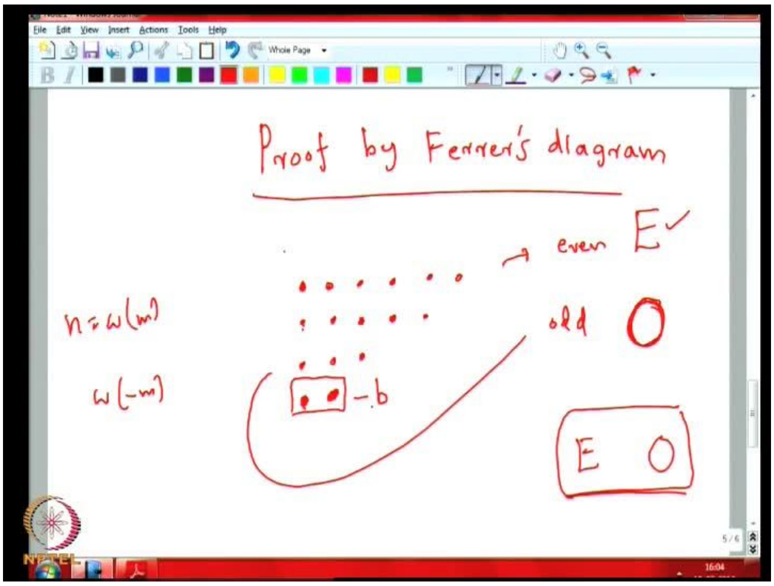

Now, if you come back to the kind of partitions, it will look like this the first row may look like this, the second row may look like this, but the point is because they are unequal values so it can come up to here, it will never be like this. It can only come up to here, you can have up to here or it can you can be like this, not like this and the third value will be even smaller, may be like this ,fourth value can be like this, but not like this. It will keep on reducing so, if you go like this, it will keep reducing the length. This is the property of such partitions in unequals, when you look at the Ferrer's diagram this is what you will see.

We have two kinds of Ferrer's diagram one as see, count the number of rows that means 1, 2, 3, 4. How many dots are there in the first column, this is the number of rows, this can be even or if this is odd then that is a different kind of partitions. So, let us say this set is called E and this set is called O, if the number of parts are even then we will see with red belongs to E, if this number of parts are odd then it belongs to O, here, this two sets we defined.

Now, what we plan to do is to set up a bijection between E and O; in most cases will set up a bijection. In the exception cases where n equal to omega of m or omega of minus m, we will set up a bijection almost a bijection with one exception that means we will be able to remove one element from one of these sets and then we will be able to set up a bijection, that is what our plan is. So, how will we do this? We define an operation first, the operation is like this.

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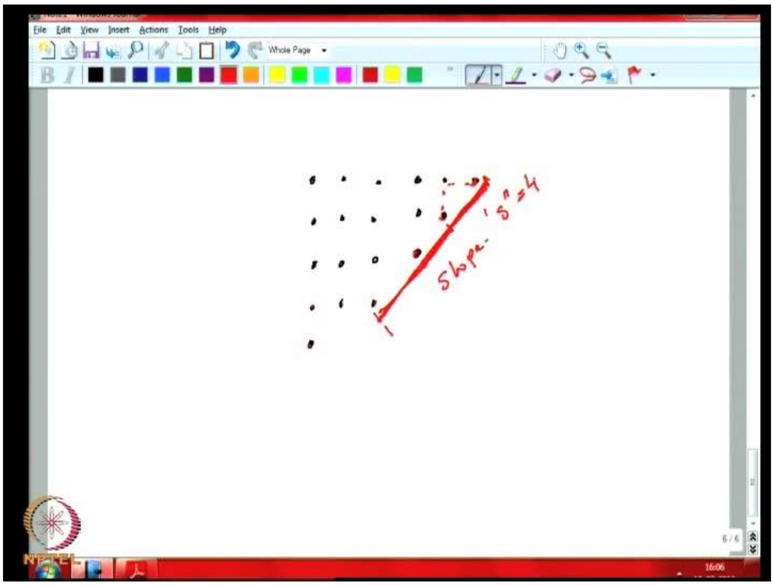

Look at this partition, if this is the last row, suppose this is the last row two elements then this will be called the base, in this we leaves this number of dots. In the last dot as b, that means the value of the smallest part in the partition, this will be called b and another parameter for instance, we can take another partition here may be a long partition. Suppose this one, here the base is just 1 that means b equal to 1, if partition was up to here then b equal to 3 so, this is our partition. Now, we will define another thing look at this last one so, this is next class. This is the slope, it can go like this so for instance suddenly- and ofcase the next is like this we can jump here, but the next you should have seen here, but it is not here that means you should stop here.

So, you are descending from here by a slope of like this, for instance just go back by one and descend by one. This is the kind of some forty five degree line we will draw in the backward direction from here. Then we will keep on seeing like this, but at some point of time here for instance you are not seeing such a dot because you know there are three dots here, there is only one dot here below, this is too less here. This is 1, this is 3, and therefore the next one is not present here. Therefore you stop here, this longest line you can draw starting from this upper most point is called s. We will call it slope that is s here, the value of s is 4.

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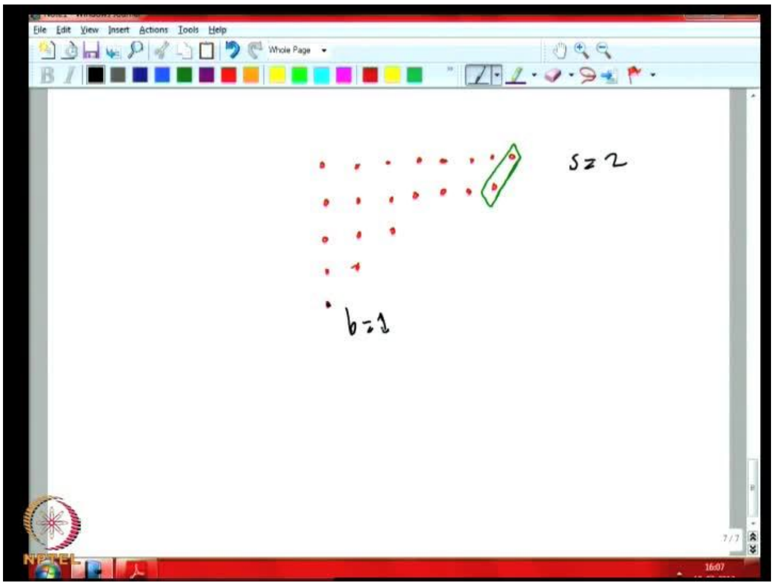

You can take another example for instance this one, for this partition this slope is this, s equal to 2 and base is 1 b equal to 1 here.

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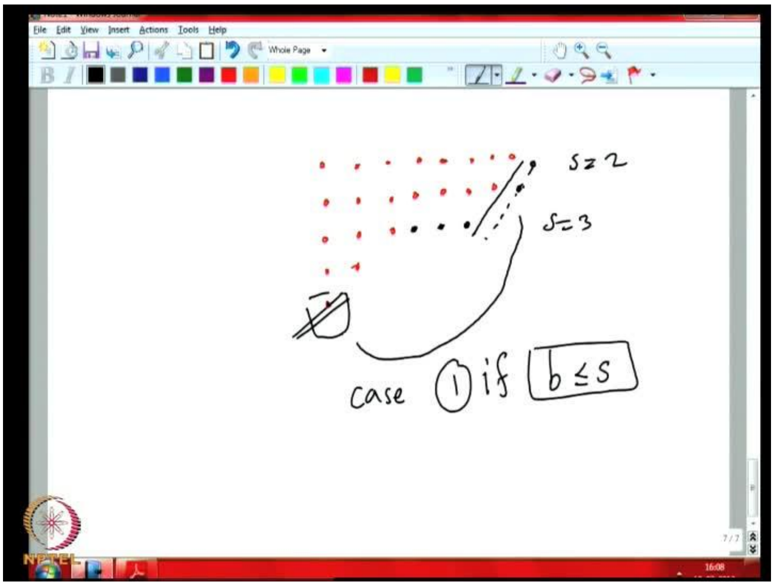

For instance I can change this partition and if I wanted the slope to be 3, I may have to add up to here so then this slope will become s equal to 3. This is the concept of slope and base now, our operation is always like this either we remove the base and take it and place it parallel to the slope like this, but this can happen only if base is less than equal to s. There are two cases one if b is less than equal to s, we will remove the base and then place it like this so, the dots there are b dots here will start from here b like this. Now, b is just 1, we will just put it here. So that, new slope after this transformation this s will become b, what will be the new base this will go away, this will become the new base this is the first operation we are defining.

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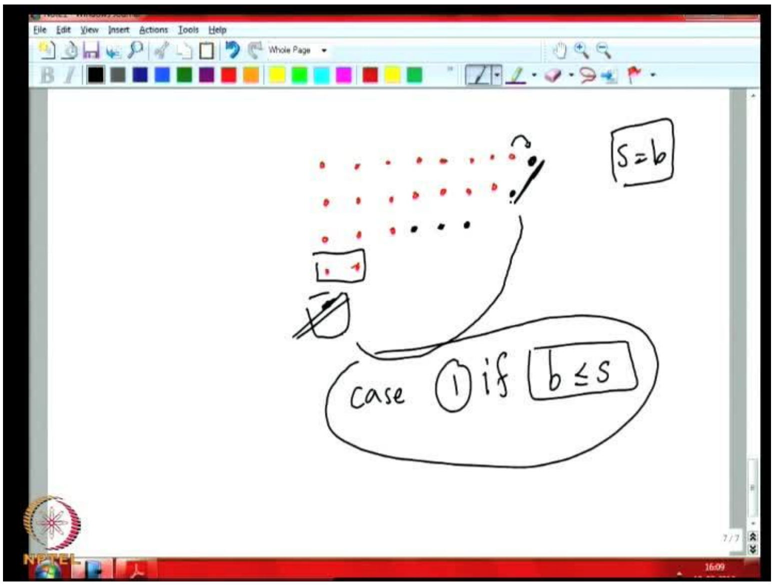

The first operation is when a partition has b less than equal to s so, you remove the dots from the last row that means you remove the base that means a last but one row will become new base. Now, what I will take? I will, I will take this to and they I will put the same dot because it should be a partition of n again. Obviously, we cannot through away the dots we will place it parallel to the slope, this in the first row will put one more the second row will put one more because b is less than equal to s. We can put it parallel to the slope without actually, in such way that this b becomes the new slope.

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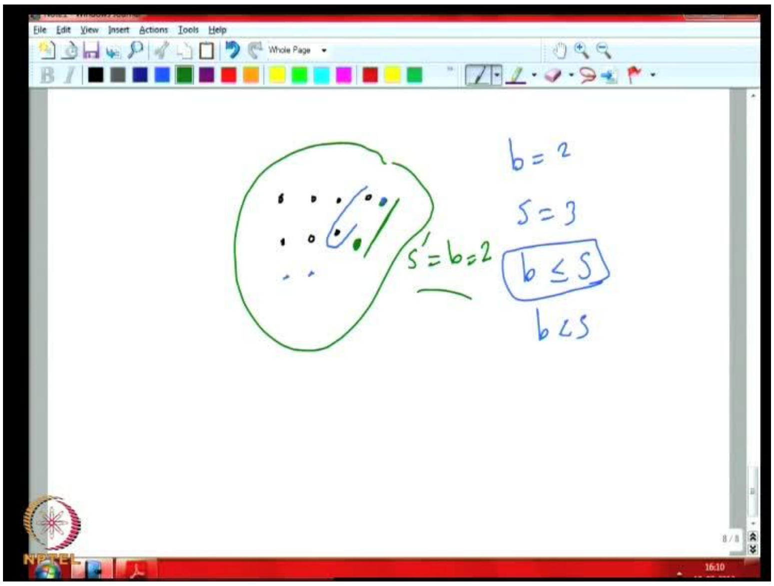

I will take few more examples for instance look at this, in this case base is this and the slope is this sorry slope is this, see base is equal to 2 slope is equal to 3, b is less than equal to s ,actually b is strictly less than s. So, this is a condition we look for and then what we do is we remove it, but two dots we remove from here and we will put it here we will put it here. Now, the new slope has become this s dash is equal to b is equal to 2, but you see that this operation is not applicable, it is not possible to do this operation in certain cases.

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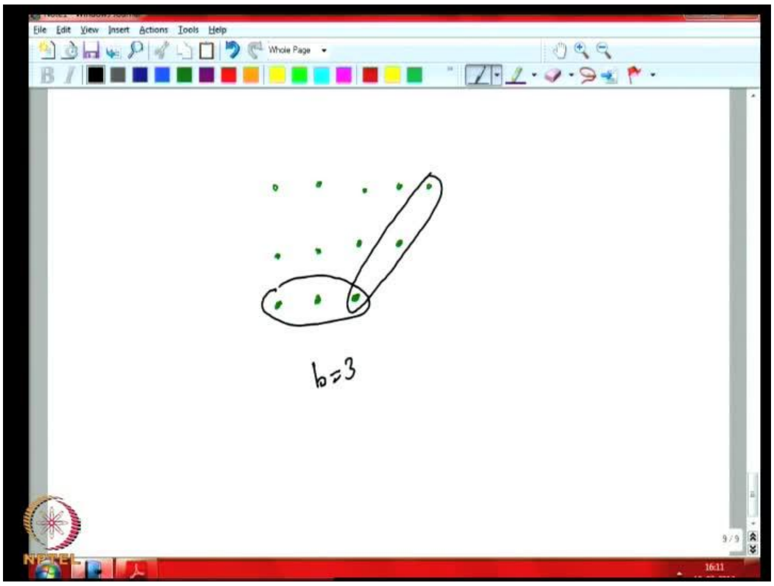

For instance what is this situation in which you cannot do this thing the only situation is suppose you can consider a situation here the base is this and that b equal to 3 and this slope is this that is s equal to 3.

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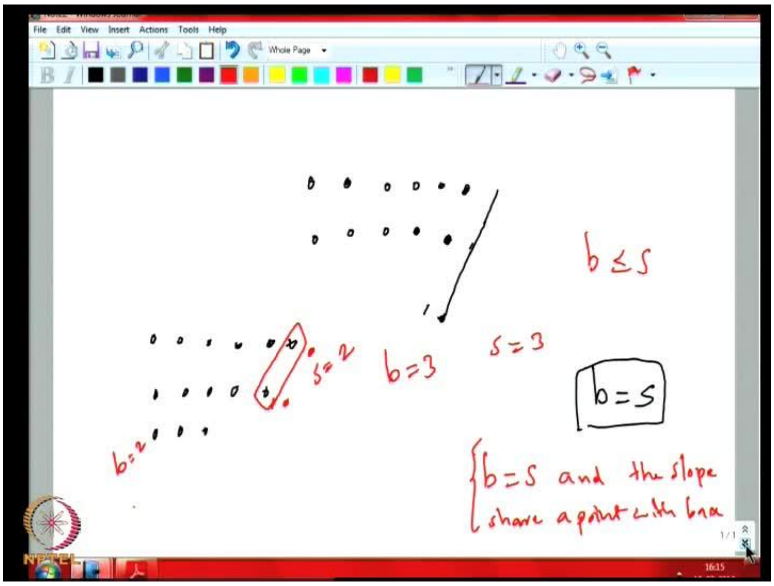

Look at this one, this part we can consider here are base is this, slope is this, here b equal to 3 s equal to 3. So, we have b less than equal to s, but if you try to apply the operation what will happen is, this will go away. So, this entire row will be removed, three of them will be taken and we will try to place it here, but then this also disturbed the slope if you place this three row here so one will be placed here, one will be placed here and one will be placed here.

So, this is not a Ferrer's diagram it will look like this, this is not correct, this is not a valid Ferrer's diagram. That means this operation cannot be applied when b is equal to s and the slope is actually sharing a point with the base means the slope is coming all the way down to the last row and actually it is taking one from the base. In that case we cannot do this thing because the base is equal therefore, when you start placing the base along this thing, it will also reach all the way to the lowest row, but then that row we have removed. So, we will see one hanging dot there therefore, that operation cannot be applied in this case.

It is easy to see that this is the only situation where we cannot apply this operation because if b was strictly less than s this will never happen because the slope can only reach all the way up to b and if b was strictly less. If I put place the base along the slope is the next dot so it will definitely reach maximum to the last but one row and that row is as such there we are not removing it. So, it would not create a problem so clearly when b equal to s only, we can have a problem even, then if b equal to s and actually the slope was ending much earlier.

For instance look at this situation, this is slope s and sorry here so this is the base right for this partition this b equal to 2 here and s equal to 2 here. So, now this slope is not actually sharing anything with the base, I mean stopping the slope is stopping above the last row if you remove it you may place it along this here, but it will not reach all the way to last row. Therefore, it is not going to create any problem therefore the only problem cases when b equal to s and the slopes shares a point with the base, this is the only problem case. In this case we will not do the operation this what we are saying, but before going further what we will do is we will study this operation, this case better so, when can this happen?

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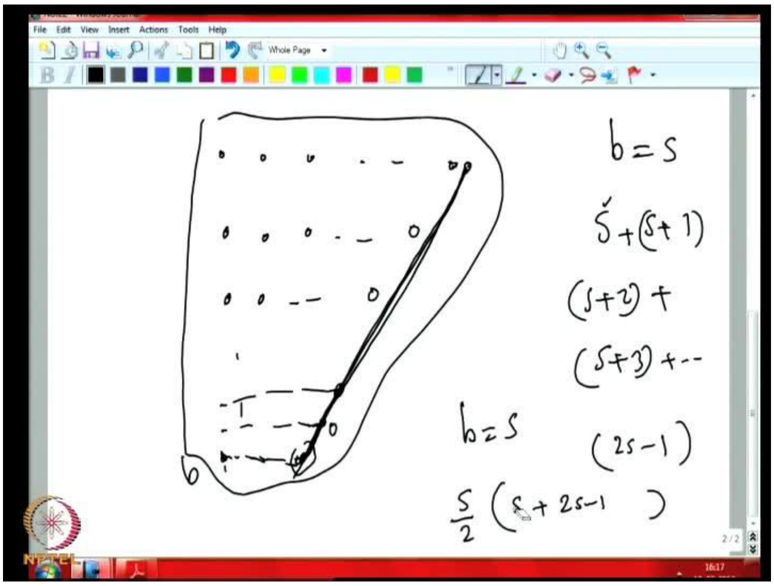

So, suppose it is like this something and then something and something like that finally, we have the base this is the base b, b dots are same and the slope is coming all the way so, let us say b dots are there. This slope is coming all the way here and is touching here and the base is actually equal to b, b is equal to s. Now, can we tell about how many dots are there? We can count from here because there are b dots here; remember b equal to s so, we can say b equal to s.

So, s dots are there here plus in this row we will have one more because the slope was like this after this thing it is coming here so that just one more here so s plus 1 in the next one. The next one will be s plus 2, in the next row we will have s plus 3 up to where we have to count all the way to here because this slope is going all the way here. There are s rows here because this slope is actually counting the number of rows in this case because slope actually touches all the rows starting from the first row to the last so, there are s such things. So, this is s, this s plus 1; so the last will be s plus s minus 1, so 2 s minus 1 total is how much. This is a sequence starting from s and reaching to s minus 1, the first term and last term can be added, s plus 2 s minus 1 and we can multiply by the total number of terms divided by 2.

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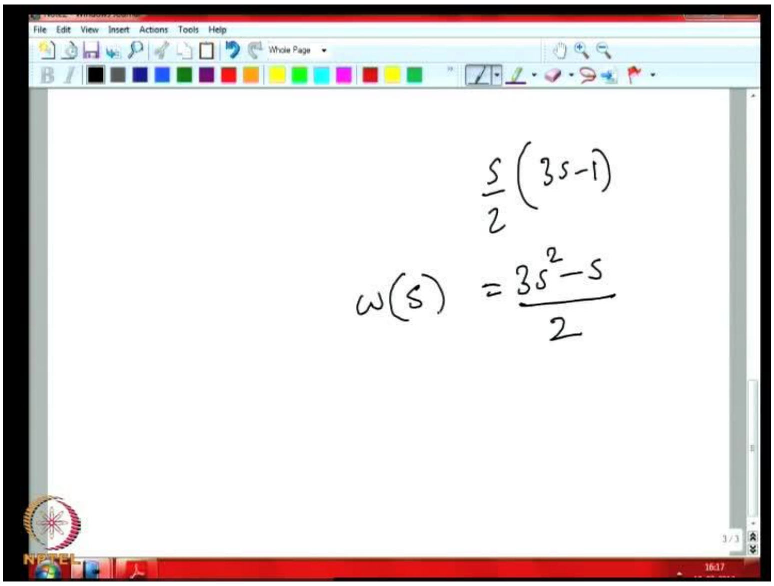

That is the sum this is what s by 2 into s plus 2 s minus 1 that is s by 2 into 3 s minus 1 that is 3 s square minus s by 2 this is actually omega of s.

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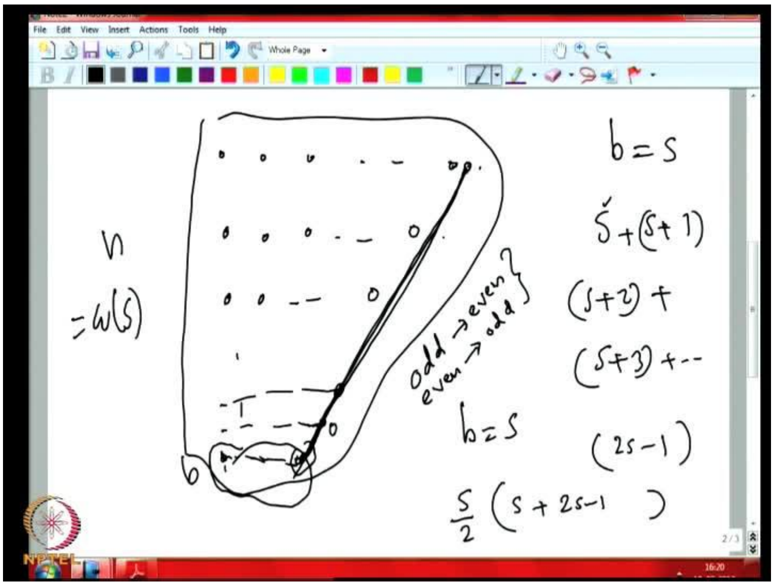

If we cannot apply this specific rule of transforming, the number of dots its actually n will be of the form omega of s that is what we are saying. In all other case if n is not of the form omega of s we can actually apply this transformation, but this what is good about applying this transformation because in this transformation we remove the base and relocate those dots to the existing rows that means the number of rows reduces by one.

So, the effect is that if the total number of rows there actually odd to begin with then that will become even, if number of rows were even to begin with then the new Ferrer's diagram will have odd numbers of rows. That means the parity of the number of parts in the corresponding partition which the Ferrer's diagram represents will change. That means if the parity of the number of parts in the partition represented by the first Ferrer's diagram was odd then after that transformation the new Ferrer's diagram will represent a partition there the number of parts is of the opposite parity so, that is even.

If initial was odd it will become even, if initial was even then it will become odd that is what. So, we say that a particular Ferrer's diagram is mapped to the Ferrer's diagram or is associated with the Ferrer's diagram which obtained after this transformation. For instance that means an odd one is map to even one and even one is map to an odd one.

So, this is a map, this actually define a map from one partition to another one of the over then number of parity are opposite.

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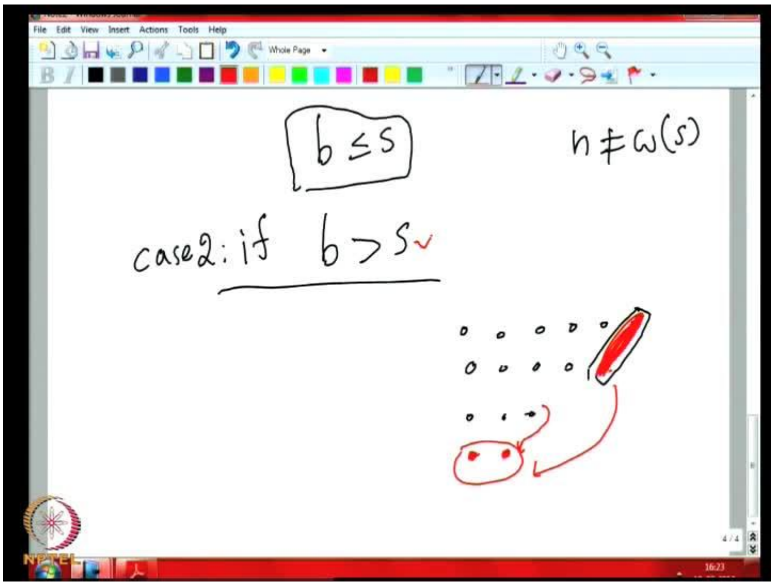

Now, this works only when b less than equal to s and the problem condition was there so, let us say if n is not equal to omega of s, we can do this thing and b has to be less than equal to s, but b can be greater than s. Suppose, the case 2 is when if b greater than s what will we do, if b greater than s this technique will not work because if I try to remove the last row and try to relocate the dots along the slope they will all should, right? It may look like the slope is like this, same this slope is like this so this is the slope. Suppose this is the partition the slope is this, but the base is this 3 is b equal to 3 greater than 2 equal to s if I try to take this and put it here it will come like this , not a valid Ferrer's diagram.

Sometimes, it may happen that if we draw something can be something up to here and suddenly there is a gap and we will see a dot, definitely that Ferrer's diagram is not valid because in the slope this one this procession is going to the empty anyway by the definition of the slope and we have more than that the number of dots and the slope to be place here so, it will our shoot. So, what we did earlier was is not feasible then b is greater than s, but we can do the other thing namely removing the slope and putting below the base.

For instance in this case what we will do is, we will just take this slope and put it here that is quite ok because 2 is less than 3. We are allow to be strictly less than 3, so this distinctness is anyway maintained. In the earlier case is also you should note that the distinctness was maintained because you are only increasing the highest rows by one in each case. So, this is distinctness is always be maintained, that I should have told before so, what that seem to see here also if you can put it here, distinctness will be maintained, remove the slope and put it just below that is because b strictly greater than s here.

New base will have now, s number of points so, the all base will be b, this s is strictly less than b because here only is removing the slope here this all contiguous again we are not creating any gap in any of the other rows. This is the second operation this case now, you see that in this when I apply the second operation you can do it in most of the case, but there is a case where we cannot do it when is that.

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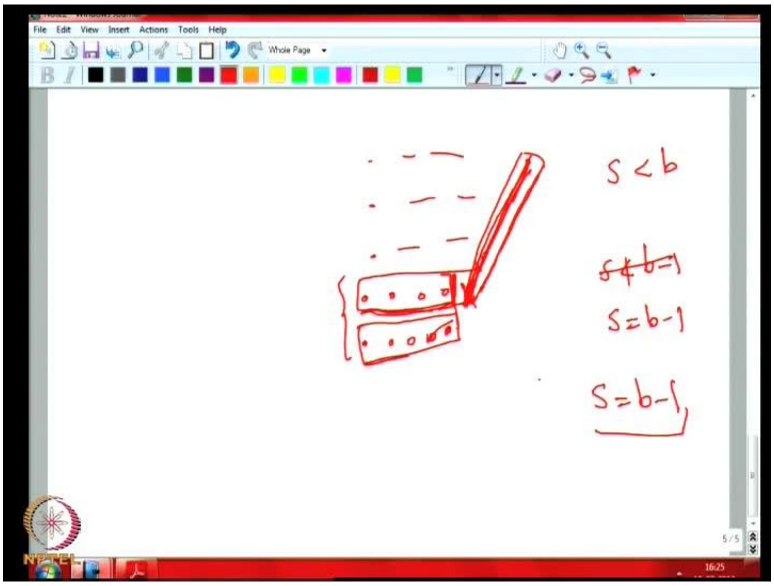

For instance we have slope definitely the smaller, but still it can come all the way down and just hit the base now, see this is way that partition look like so, this is the base and this is your slope. So, that is common point in slope and base of course we know that slope is strictly less than b. Now, if I remove it, I have also reduce the base here by one now, I try to put it here so, if it go less than this still is even after removal of losing one here still this much is bigger than s.

That means if b minus 1 itself was strictly was greater than s that fine, but if this was not true that means if s was equal to b minus 1 then what will happen is this b minus 1 thing will come and stay here our distinctness property is lost. So, there is a row here there is another row below it which is equal which is not allowed because we are only I considering partition with parts all unequal. So, that is lost we cannot map it to a partition where parts are equal.

This will happen only when s equal to b minus 1 if this was strictly less than b minus 1 we do not have a problem at all, even you if you reduce this groups still you have a unequal parts. It will without any problem will sit or stay below this last row, but if s equal to b minus 1 this distinctness property will be lost. So, in that case we cannot do it what are the cases, we cannot do and also note that if s equal to b minus 1.

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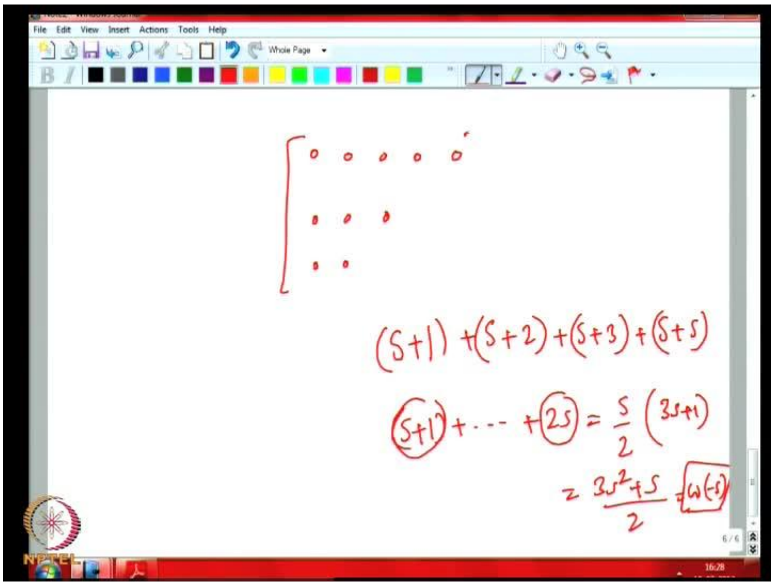

In this case what happens is, you have the last row contains s plus 1 dots and then there are how many rows there are s rows right s plus 1 plus s plus 2. So, we count the total number of points it will be s plus 1 plus s plus 2 plus s plus 3 plus so s plus s that is 2 s. s plus 1 to 2 s this is equal to s by 2 into 3 s plus 1 because is first and this last is added. So, this is 3 s square plus s by 2 which is omega of minus s.

In this case we see that omega of minus s number of dots are there omega of minus s number of dots are there this is also not allowed which are this in this case cannot do this operation. So, this can also see that here also we have increase the number of rows by one. Therefore, the parity of the number of parts and the partition has changed by that operation, but if it was the total n was equal to omega of minus s be cannot do this thing.

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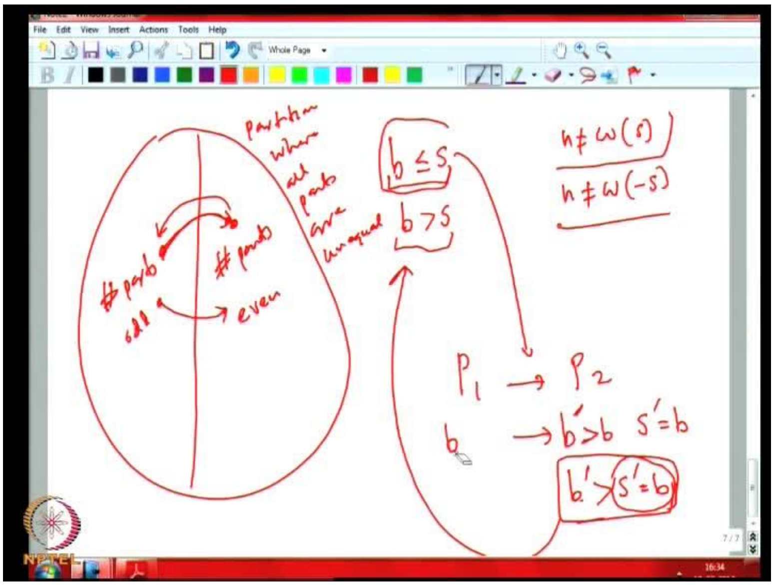

Now, what we have shown is that so, the first operation can be so, there are two cases either b is less than equal to s or b greater than equal to s. Assuming that we the total number n is not equal to omega of s and is not equal to omega of minus s then what we do is we look at this two parts. We are only considering partitions where all parts are unequal distinct now, within this thing we have two groups namely number of parts odd number of parts even.

Take a partition where the number of parts is odd. Now, look if b less than equal to s then we applied operation and we map it to something in the other side because the number of partition will become even. Suppose it was partition where the number of parts are odd that b is greater than s, apply the second operation and we will map it to something here. So, of course we from rows what suppose if you apply this operation for b less than equal to s, the resulting partition see b less than equal s so, b has become the slope.

Now, what happen is so, suppose I apply this partition p 1 became p 2 so, in p 1 b and s was the base and slope now, after this thing here this s dash is equal to b. Now, what will be the new slope it will be something so, new base b dash which can be b plus 1 or b plus 2 something more than b .I can say that b dash is greater than b why because we removed the last row the now, the last but one row will become the base that is strictly bigger than b because there all unequal parts. Now, b dash is definitely greater than s dash which is equal to b this we get.

Now, if we consider this partition in this even side and see which operation is to be applied we have to apply the other operation because s dash is greater than s dash so, what will I do there? I will pull up pull out you note that you will never get into the case where getting to a case where we cannot apply this operations because that will happen only when n equal to omega of s or n equal to omega of minus s. So, first operation cannot be applied when n is equal to omega of s the second operation cannot be applied when n is not equal to omega of minus s. Therefore, we will not get into any problem here is we can apply the operation was now what happens is s dash that we will come back to the original partition because we will be applying this thing.

You will remove that slope which was b and then put it back here below this thing. Now, the original slope will reappear there so, you will be back to the same partition so, this is inverse operation that is what we are saying. Similarly, if I had applied the second operation what would happen? So, that means the slope will be remove and put below the last row, but the slope was strictly smaller than the current base.

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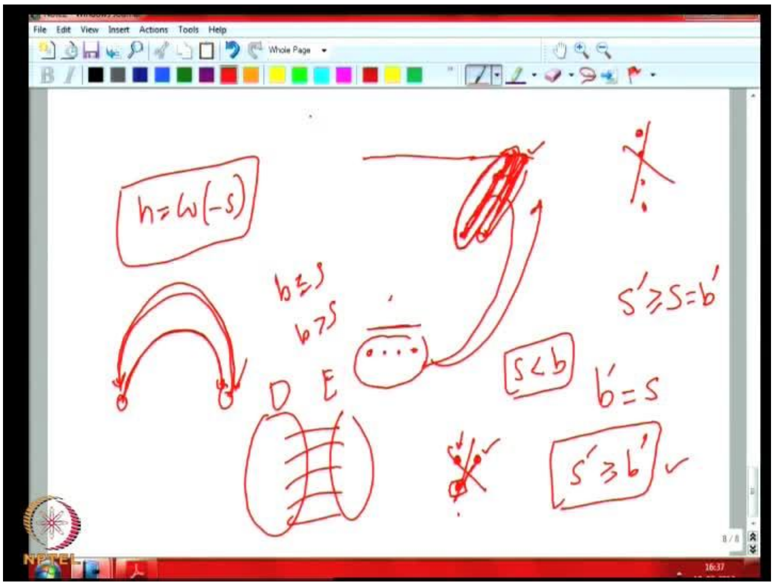

Now, what will happen is you can this so talk about this new slope, for instance this was this slope which I removed. So, the new slope clearly will be what I see so, I have taken it and put it here new slope because of all this things because this one thing, one thing here this things are also available here. So, the new slope is going to be at least as much as this at least as much as this so, therefore s the new slope s dash being at least greater than equal to s and our base new base is equal to s. So, this is s dash is greater than equal to b dash so, we are in the first case.

In newly generated partition we will have to apply the first operation and we know that the first operation will allow us to go back to the original case. So, one thing you may want to notice is when I say that I remove the slope the remaining partitions so, remove the remaining. So, if I take this slope and put it here, the resulting partition have a slope which is greater than assuming there is something behind this, something behind it something behind it.

So, because there is definitely something behind it because these are all distinct partition this cannot be just one element here. If it just one element that is just one, that is all because not possible to have a partition like this. We are not considering like this kind of partitions at all. Therefore, if the row has one thing if I remove when there is one more row definitely it means that there is something behind it. Similarly, if it stops here then there is nothing below here because if the single turn and this itself is the base.

Now, this was a case where s was bigger than base, new one is base and base is bigger than equal to s, but we assume that s is less than equal to b here. Therefore, this will not happen therefore we can assume that there is in behind. We have these dots when you remove and therefore that forms the new slope. This new slope can be even bigger that is all, but we know that this slope will not come and hit the new base if it hit the new base it means that so, it is n is equal to omega of minus s we have already calculated so, this will not happen.

So, we can indeed apply this operation back and forth so, if you apply the operation of one partition and then on the other partition we will have to apply the other operation because that is only option because either b less than equal to s or b greater than s. So, when I do this thing will go to the other case and then will have to apply the same observation you will come back so, it is a bijection. So, therefore between the odd and E we have now defined the function which forms a bijection.

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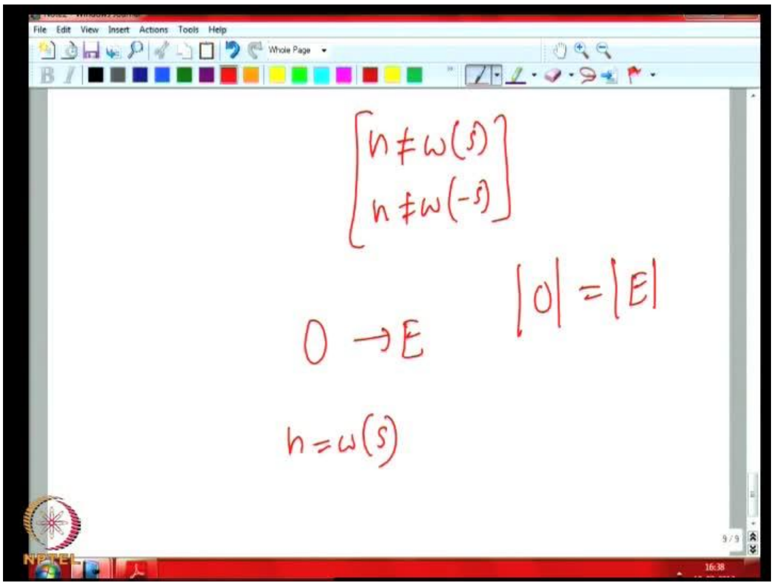

We have clearly now, proved that when n is not equal to omega of s and n is not equal to omega of minus s, we have a bijection between O and E. Therefore, cardinality of O is equal to cardinality of E. On the other hand suppose n was equal to omega of s now, we know that there is a problem case here name where the operation one cannot be applied.

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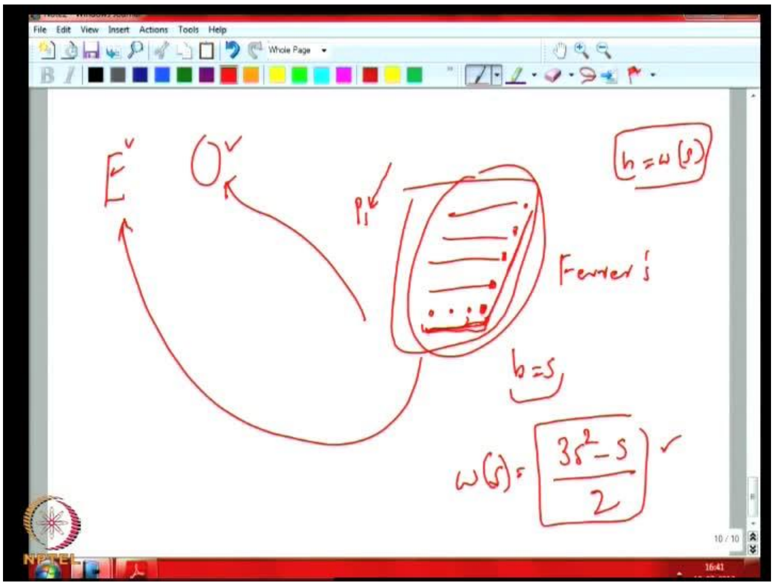

There is one partition possible namely where b equal to s and the slope actually touches the base something like this, there is a common thing between the slope and the base, but we know that there is only one such partition . Suppose this such a partition is 3 so, this total number of dots has to be equal to 3 into s square minus s by 2 that is omega of s and there is only one s for other for a given n like that we have seen that for different s's we get different n if n is equal to this thing.

If I put n equal to 3s square minus s by 2, we can solve it for two different things. So, we can just verify it by arguing from here but, we have seen the table that it increasing 1 by 1. It does not repeat because we are so, that is what and then we say that therefore, that has to be the s here and that has to be the b equal to s and then that is only one partition. If we start with another base we will be getting a different value of n, but n is given to be omega of s.

Therefore there is only one unique partition of that sort so, unique Ferrer's diagram of that sort there which not partition Ferrer's diagram of that sort partition because at of each partition. We have a Ferrer's diagram where the last row is has exactly equal to s dots and the number of rows is equal to s and so, on they share one point. Now, what we do is let us call this partition as p 1 and we will remove that partition from this, where will this partition be so, of course will it be E or O that is a question?

Now, definitely if s is odd because the number of parts is actually s, number of parts is s. So, if s is odd then this partition is its seen odd sorry if s is odd this partition is seen in O if s is even s partition is in E because number of partitions actually s depending on s parity. It will be either in even or odd if s is odd it should be in this set otherwise it will be in even.

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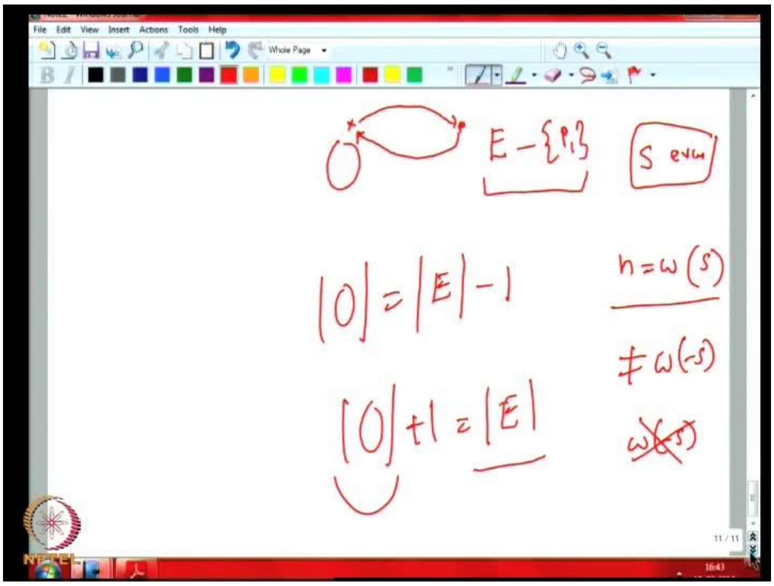

So, what I say is that we will remove that from whichever without loss of generality. Let us say it was even it was in even s so, we will remove that E minus p 1, this will be new set of partitions just remove that problem partition for every other thing we can actually apply the transformations. We are assuming that n is of the form omega of s, not that see if n is the form omega of s it is not of the form omega of minus s. Therefore we do not have the other problem case at all because the other problem case comes when n is of the form omega of minus s. So, the other problem, we do not have the other problem case

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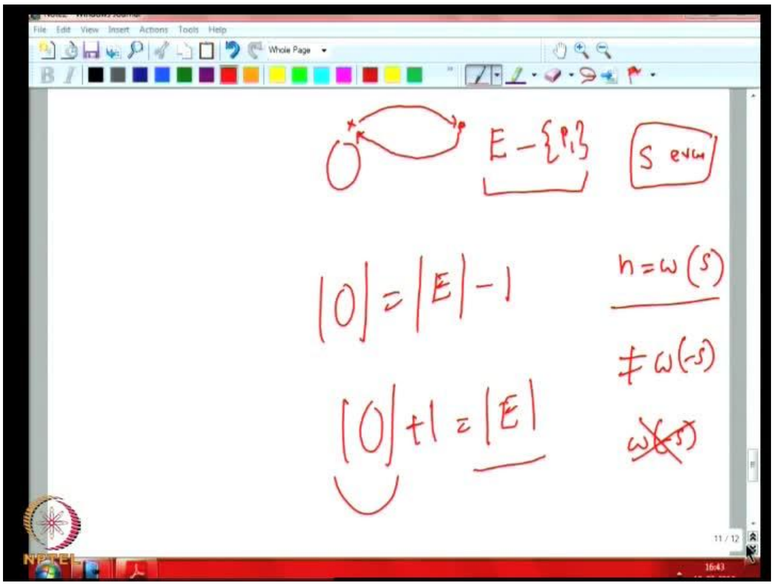

So, therefore we just have to remove one problem partition and we remove that and then O as such. Now, we can show we can see that for every partition we can apply operation one and reach here. In this partition we will reach the other case and when we can apply the other operation and reach back. So, between this and this, there is a bijection, what means is that O will be equal to E minus 1, right but, not that E. So, that means O plus 1 is equal to E. As I told E is bigger, the cardinality of E is bigger by 1 compared to the cardinality of O and this is when p 1 belongs to E.

Why p 1 belongs to E because s was an even number, s is in the number of parts slope which is equal to the number of parts also for the problem partition. So, that is even that is why it is. So, when s is equal to even number s is an even number then a cardinality of E is 1 more than this. In the other case so for instance you can consider similarly, the s is an odd number, then we will get the other situation namely cardinality of O will be equal to E plus 1.

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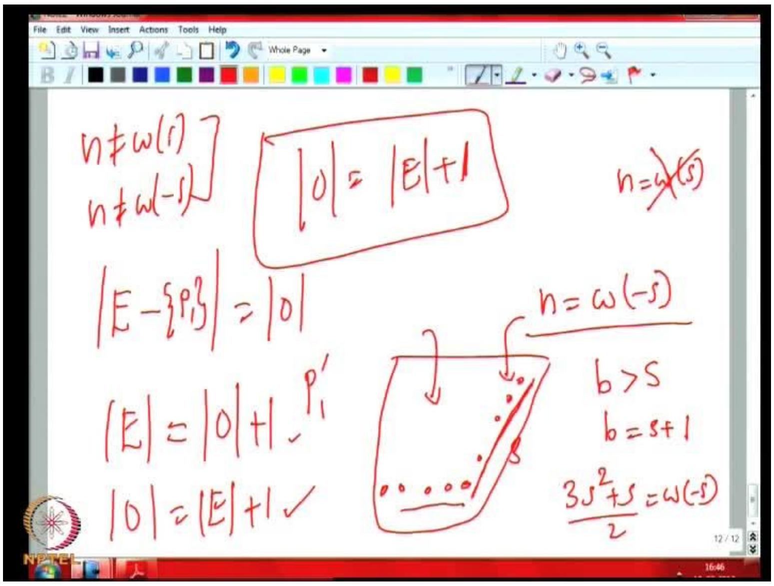

This is obvious right the same argument can be this, assuming that that problem partition belong to b then remove that O minus p 1 is considered and then, we can set up a bijection between O minus p 1 and E. Now, what about if n is omega of minus s then the first operation is safe because which problem case comes only when n equal to omega of s. If n equal to omega of minus s, is not equal to omega of s. If n equal to omega of s then we look at the problem partition will look like this.

So, we have a base and see you know b is actually greater than s but, is actually just 1 more than s, b equal to s plus 1 right. So, s will be something like this. Again the number of rows is s, b is just s plus 1 and we know that. If it is like that for whichever slope, we will, if a partition is like that the total number of what is that has to be 3 into omega s square plus s by 2 that is omega of minus s and you know for a given n there is only one as like that which completely determines that Ferrer's diagram. So, therefore only one problem case here also lets say that is p dash 1, right?

Now, what you will look if s is an even number that partition belongs to E. Now, as usual, as earlier we can remove that partition from E 1 and for all other partitions, we can apply both the operations so it follows that the bijection can be set up between that. We will get this is equal to cardinality of O. So, if s is even then we get that cardinality of E is equal to 1 more than the cardinality of O. On the other hand if s was odd, we will get the cardinality of O is equal to 1 more than the cardinality of E this is what both case will get.

So, to sum up what we have done we should show that when n is not equal to omega of s for any s, when n is not equal to omega of minus s for any s then we have cardinality of E is equal to cardinality of O. If n equal to omega of s depending on omega s or omega of minus s depending on whether s is even or odd we will they have either E is bigger or O is bigger. We will continue in the next class.