Combinatorics
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Lecture - 24
Recurrence Relations – Part (2)

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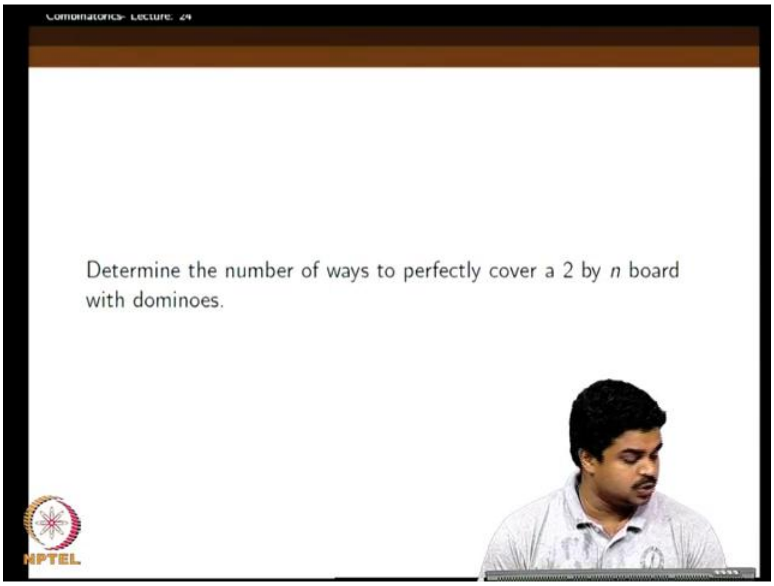

Welcome to the 24th lecture of Combinatorics. So, let us we were discussing the Fibonacci sequence in the last class as an example of recurrence relations, this is one of the most popular recurrence relations, and this therefore deserves some detailed study. So, we saw a couple of properties of this recurrence relation in the last class, and also derived an explicit formula to evaluate the value the nth Fibonacci number right value of the nth fibonacci number. Today, we will look at couple of examples, where Fibonacci numbers appears as the solution, now happens to be the solution right. So, here is one question determine the number of ways to perfectly cover a 2 by n board with dominoes, so what does this mean

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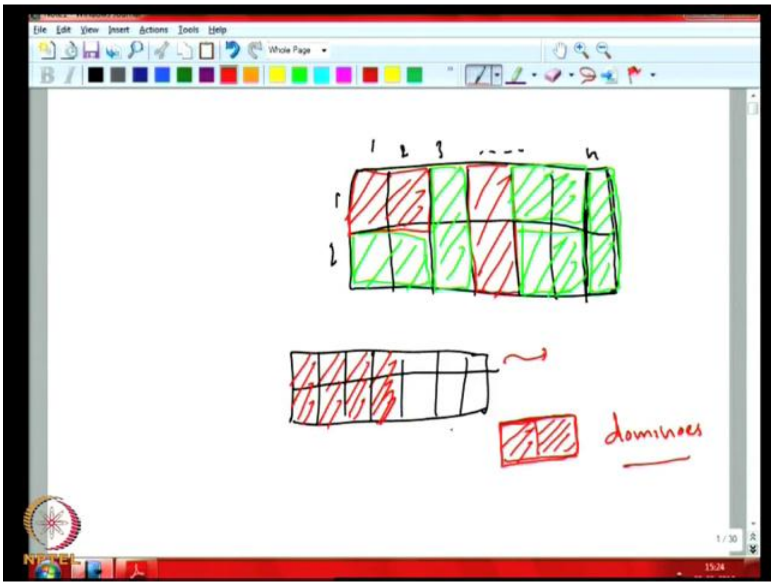

So, when we said 2 by n board, we speak of something like this so, 2 rows and there are n columns so, this is 1, 2 rows and this is 1, 2, 3 up to n, this is the 2 by n mode. Now we want to fill this board with dominoes, what do you mean by dominoes something like this say. So, this is a domino that means, it can this can fit it fit here for instance like this and this is possible or it can fit somewhere is some it can fit like this, so that means, it covers either 2 horizontal squares or 2 vertical squares.

So, this kind of blocks we have, which are called dominoes, we have to perfectly cover this 2 by n board, now the question is how many ways you can cover it for instance, I can show one covering here so, I will right. So, we can cover it like this, next one we can cover it like this and next one we can cover with this and the last one we can cover it like this.

So, this is a perfect covering of the 2 by n board and there should not be any confusion whether it is always possible to cover or not it is not always possible to cover, because if you had placed for whichever is the n. So, we can always cover each see you can that every domino can be placed like this one second like this, third like this, fourth like this. So, therefore, it is possible to cover it in this way right, because it is a 2 by n.

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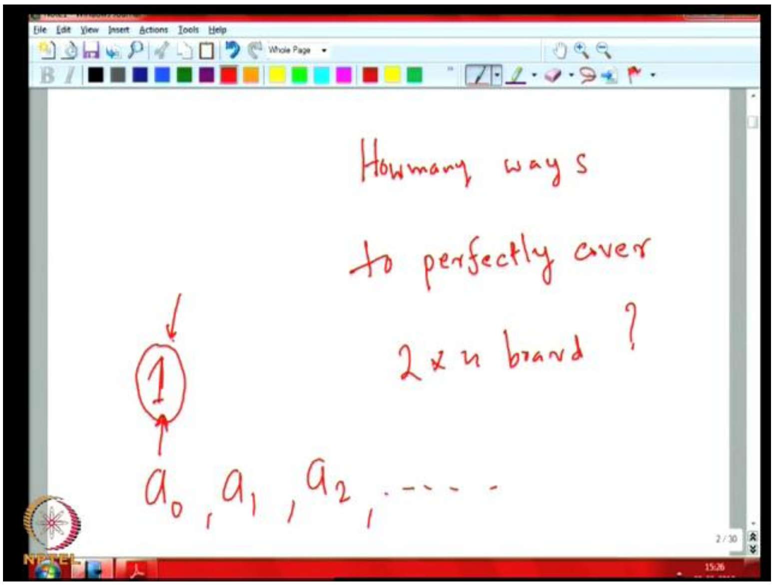

So, now the question is how many ways, you can cover it perfectly with how many ways are there to cover, to perfectly cover means no square should be left blank perfectly coveredor left uncovered cover 2 by n board. So, now the first thing is that to come up with the recurrence relation recurrence relation for this thing, what I would say is let a 0 a 1 a 2 exctera, be the sequence of numbers to represent this thing let us say a 0 is equal to 1, because as it is an n by 0 board.

So, let us say we can cover it in one way if it is a 0. So, we will say s 0 is 1 that is 1 convinent way of of defining of one may ask why do, we say a 0 is 1. So, it is and empty ball for instance, we have 2 by 0 board 2 by 0 board, we can we have an empty cover. So, one can say that, we cannot cover it, but so, let us define it as 1, because there is an empty cover for that.

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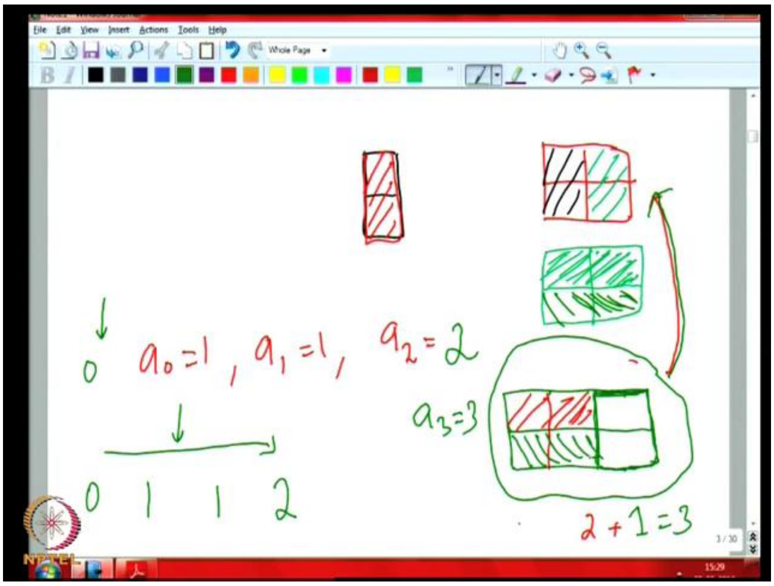

What about a 1, a 1 is a 2 by n board 2 by 1 board right sorry, 2 by 1 board and we covered in only one way, because it will be just this board. So, it can be covered, it can be covered in one way, because there is only one way, we have to place one domino on the other on that so, vertically we have to place. So, therefore, a 0 is equal to 1, the number of ways to cover the empty board is 1 a 1 number of ways to cover the 2 by 1 board, this one, the number of ways to cover 2 by 2 by 2 board for instance, this kind of a board how many ways you can cover. One persue is to place one domino here and the other domino automatically has to be place like this right, this is one way.

Another way is to say when I am drawing the board, another way is to place the first domino like this right horizontally right like this. Now, the second one has to be like this right, this also there are 2 ways of doing this, this is 2 a 2 is equal to 2, we kind of see the Fibonacci sequence here, in the first this thing, if you had 1 0, before that would be 0 1 1 2, this is the way Fibonacci sequence is also going and the next 1, if you look a 3, you should get 3, you can try it out for instance.

So, I see drawing it again and again is a little difficult, but. So, for instance this, here there are 2 ways of doing it, one is we start with this, we start this way, now we know that this corresponds to the n domino. We already have seen that it can be covered in to ways right or we could have started with we could have started with the yeah this kind of a domino right then this automatically has to be like this right now it is just one way to do it. So, that is another 1, so this will be 3.

So, a 3 will be equal into 3 and using this argument, we already got how we are doing it, we are first trying to place the first domino in the horizontal in the vertical way then we are trying to place the. And then we are counting it may be reducing the problem to a perviously solved problem and the other cases, when we place the domino horizontally and then after placing it horizontally, we are after placing it horizontally, we we analyse it and reduce it to a previous problem.

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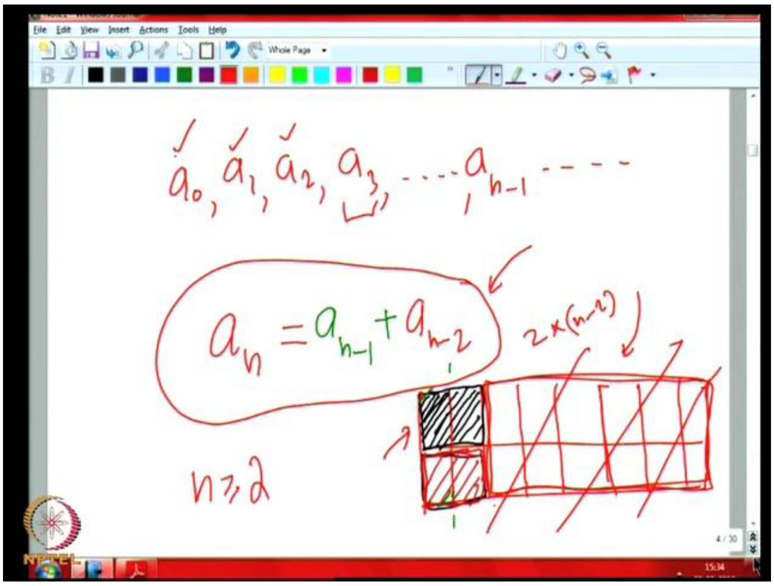

Now, we can try for the general case, of case we know the numbers a 0 a 1 a 2 a 3 upto a n minus 1, now I am interested in a n. So, this a n will be equal to how will you find it out. So, now, this is the 2 by n board 2 by n board first, we try placing the dominoes the first to domino that here, means the domino, which covers the first square like this vertically. So, if it covers vertically then the question reduces to the previous problem namely the remaining this right part now being red part now a 2 by n minus 1 dominos, how many ways you can perfectly cover it with so, 2 by n minus 1 on board.

How many ways, you can cover it perfectly with dominoes, that will corresponds to a n minus 1 clearly plus there is another way of doing it namely, if you could have placed the first domino in a different way, which we so, you could have placed the first domino this way, the first domino, you could have placed in this way right. First domino is the sense that the domino, which covers the first square right.

So, the point is if you place the domino to cover the first square in this way then the 2 square's just below that has to be like this right and there is no option here. So, say it has to be like this there is no option here, it has to be like this, now we know the rest of the things are done. So, there are there is a there is a remaining things there is a 2 by n minus 2 board here to this looking from here to here, that is a 2 by n minus 2 board and we have to cover this portion perfectly in dominoes right. That is of a n minus 2 a n minus 2 therefore, we get this recurrence relation a n equal to into a n minus 1 plus a n minus 2.

So, you can see that this recurrence realtion will work right from n greater than equal to 2, because for n equal 2 case what happens is in the first case where, n minus 1 is just 1, we know that a n minus 1 is 1 that is true. So, if in the true case what has this portion this red portion, this will become empty right, the only this wont be there. So, what we have defined a 0 to be 1.

So, therefore, we can sorry this is the placing, this placing combined with empty placing for the empty part. So, therefore, it works of case you are not comfortable with that we could have taken the first 3 values and then we could have started from n greater than equal to 3. But, even with n greater than equal 2, it will work n equal to 1 case the point is you cannot see the 2 kind of placements at all the second placement is not existing.

So, this argument requires atleast 2 squares here right. So, the second type of placement right. So, therefore, what we see is here, this sequence a n namely the number of ways to perfectly cover a 2 by n board with dominoes that is n, that satisfies the recurrence relation namely a n equal to a n minus 1 plus a n minus 2 for n greater than a equal to 2 and which is same as the recurrence relations satisfied by the Fibonacci sequence.

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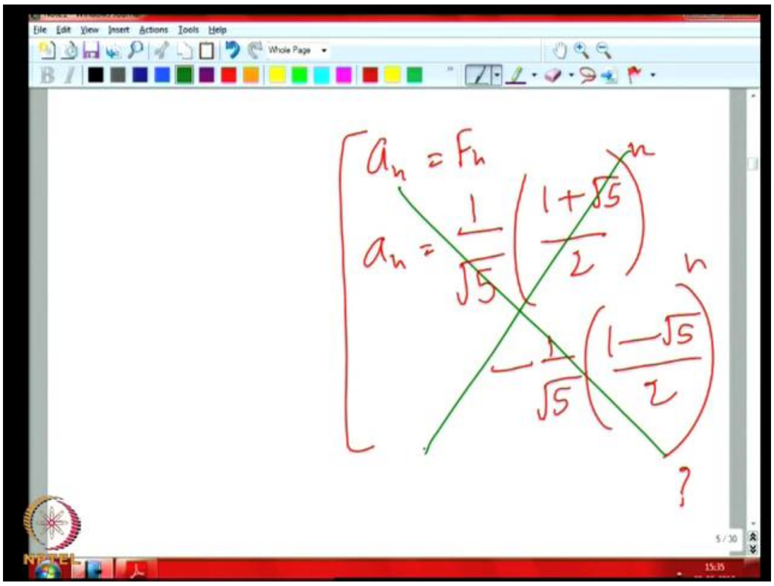

So, can we assume that then can we immediately infur that a n equal to f n write a n equal to the formula for f n may be 1 by square root of 5 right to 1 plus square root of 5 5 by 2 raise to n minus 1 by square root of 5 into 1 minus square root of 5 right it to the whole power n can we write like this. Now, because as we have seen it is not perfectly matching, this is not correct right This is not correct.

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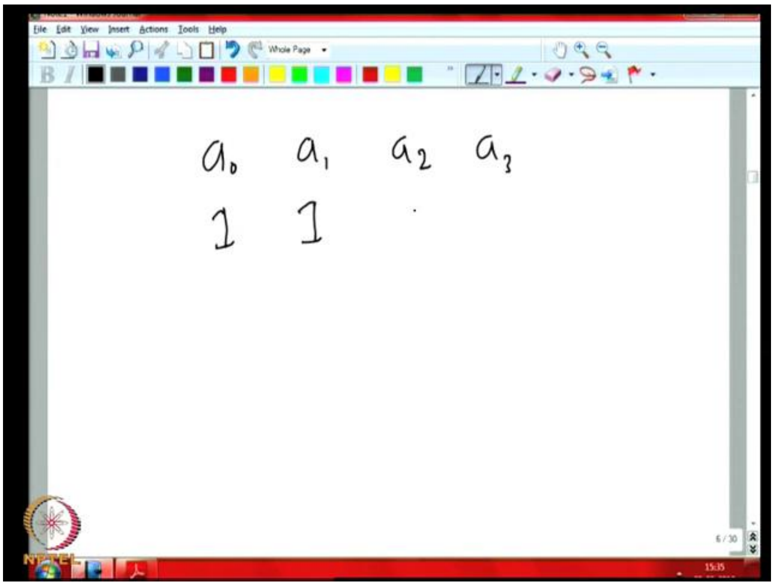

Why because, I will write down the sequence once again this is a 0 a 1 a 2 a 3 like this, this is 1 1 and then we have here, we have how much was a 2 a 2 was 2 right.

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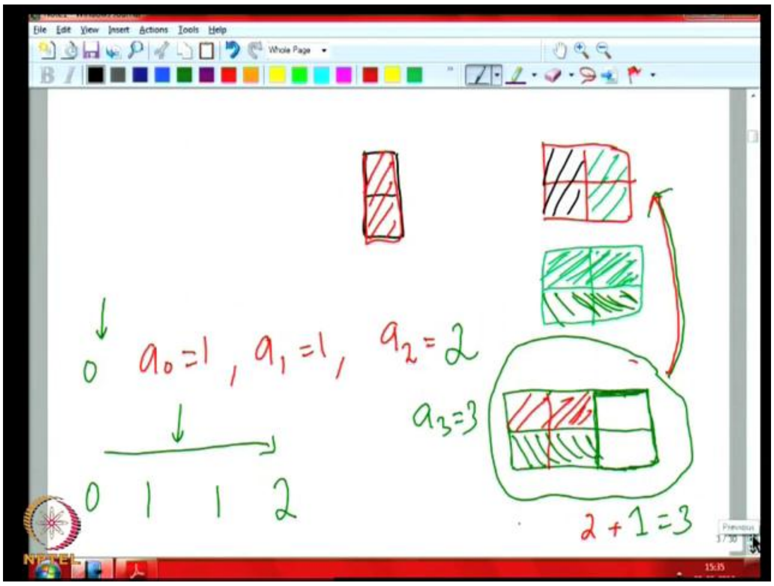

A 2 was 2, a 2 was 2, 1 1 2 3 like that only it was going. 1 2 3 and so on.

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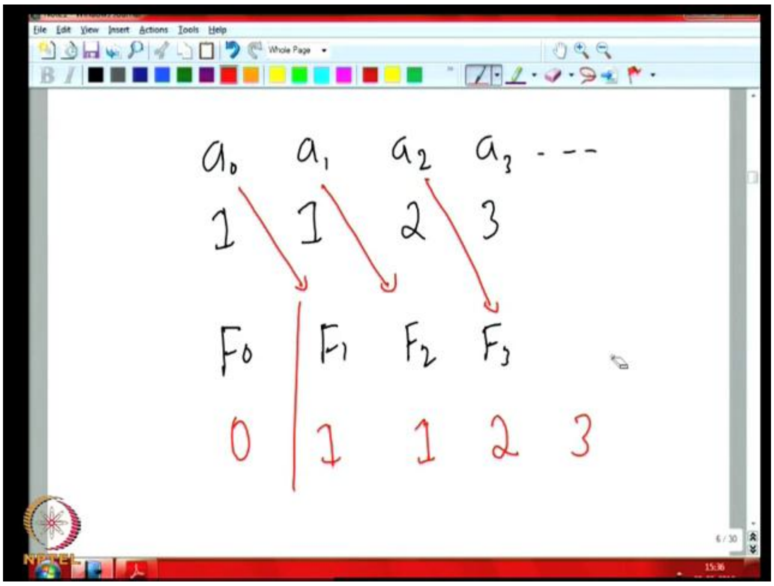

But, Fibonacci sequence was different f 0 f 1 f 2 f 3, if you take this was 0, this was 1, this was 1, this was 2 and so on and then it is correct 3. So, in other words this is obtained. So, we have to map this to this, this to this, this to this and so on. So, in other words a n is F n minus sorry, a n is f n plus 1. So, a 0 will be f 0 plus 1 that is F1, a 1 will be F 2, a 2 will be F 3 and so on.

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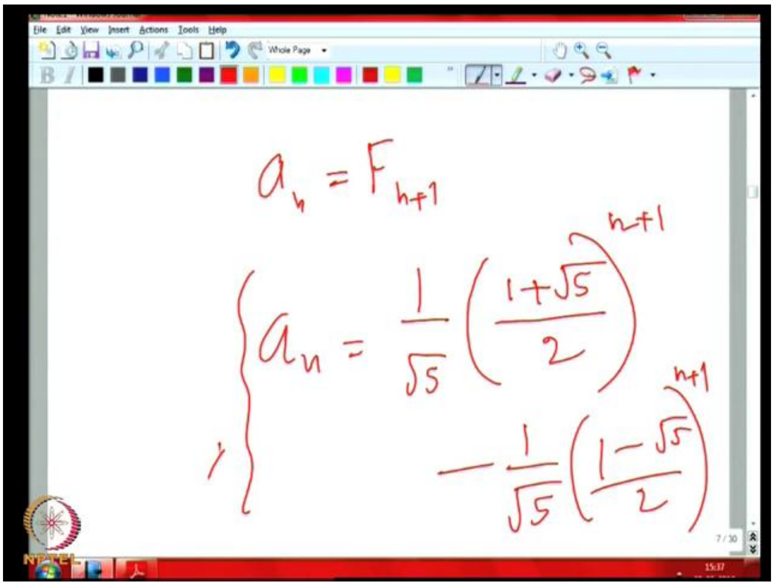

So, we get a n equal to f n plus 1. So, now, we can substitute the formula n equal to 1 by square root of 5 into 1 plus square root of 5 by 2 raise to n plus 1 minus 1 by square root of 5 1 minus square root of 5 by 2, the whole power and n plus 1 and this is true. This is because Fibonacci number sequence was true for all and greater than equal to 0, this will work. So, that is this is one instance where, the Fibonacci number was appeared. So, of course we talk about perfectly covering the 2 by n board with dominoes 1 may not really guess that there is fibanaci numbers hiding inside right. Similarly, we will look at another question now, so here.

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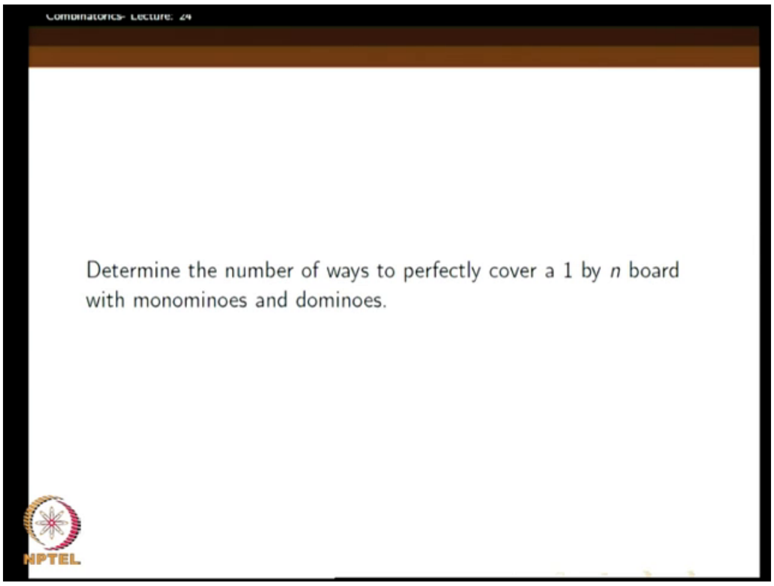

Determine the number of ways to perfectly cover 1 by n board with monominoes and dominoes of case, this is the same question put in a slightly different way so.

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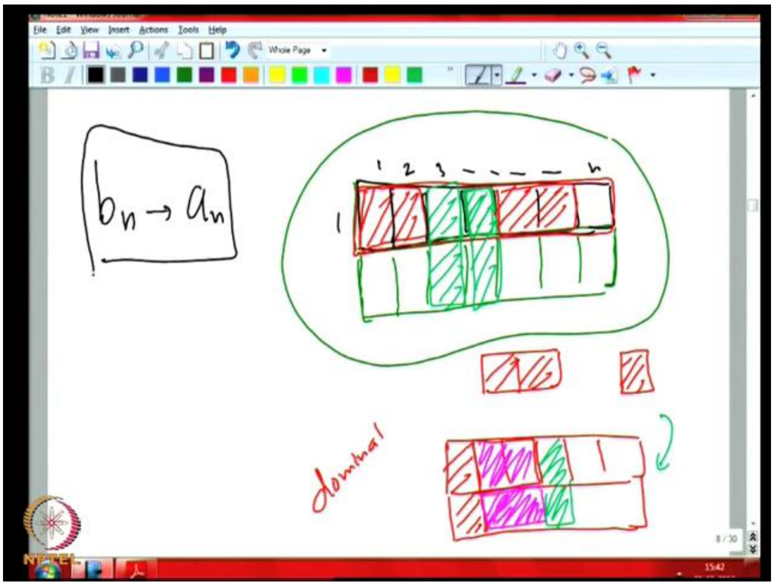

So, now we are saying that our board is only 1 by n. So, this kind of a board n, this is owned by n, now we are allowed to use not only these kind of things these are dominoes. But, also pieces like this to perfectly cover it, it is clear that we can perfectly cover it for instance, we could have done taken first a domino then another domino and then another domino until for instance, if n was an even number, we would have perfectly covered with that to a.

But, the last one, we can cover with a monomino, if it was an odd number. So, therefore, it is definitely possible to cover, now what we are interested in is in how many ways, you can cover it and I told you that the question is similar to the last one, because because we can see that. So, if we imagine suppose, we extended to a 2 by n board like this right to by n board, we just add an imaginery second row also.

And then ask so, the previous question here that means, we want to perfectly cover this monomials and that will correspond to the current question, why because, if you get a covering of this 2 by n board with dominoes. Then we have a corresponding covering of the 1 by n board including, we have consisting of only the first row here, why because, we just take the endiose to 1, because for instance, if it was a first dominoes like this then this is also a domino for the I mean another 1, for instance second domino suppose, it was like this then, we will consider here a monomial cutting of the realted part.

Similarly, if there is a monomial part here sorry, domino like this a vertical domino will give a monomial and upper part right and a horizontal domino taken will be a given domino itself on the upper part right. So, naturally there is say corresponding covering of the first row with dominoes and monominos for every cover in perfect covering of 2 by n board using dominoes. This is the correspondance wherever, we see a horizontal domino use then in the first row, we use it as that that means, it is a domino, if it was a vertical domino, which cuts out the lower part and then take a monomial in the upper part right. Similarly, the converse is that mean, we are setting up a bijection between the 2 problems as we always do. So, the suppose, we get a covering of the first row using dominoes and monomoes.

So, we can definetly extend it to 2 by n board, but whenever, we see a monomial just extend it downwards by making it, this thing whenever we see a domino here, suppose we see a domino here right then we put a corresponding domino the 2 square just below it right like this, if it is a monomial, we will extend it like this it is a domino, we will just put a domino like this below. So, therefore, we can corresponding extend to a covering of the 2 by n board using dominoes along.

So, any covering of monominoes the 1 by n board. So, only monomial and dominoes can be extended to the covering or 2 by n board using only dominoes. And the correspondance is clear for instance, it is a bijection, there is no need to explain it further, you can see from the picture that right. It is covering corresponds to that is 1 to 1 corresponding to this one before the answer is same that means, if you count now b n. Let b n denote the number of ways to cover the first row that means, 1 by n board with monominos and donominoes perfectly the same as a n right a n equal to b n. So, on the other hand, if you wanted to proceed.

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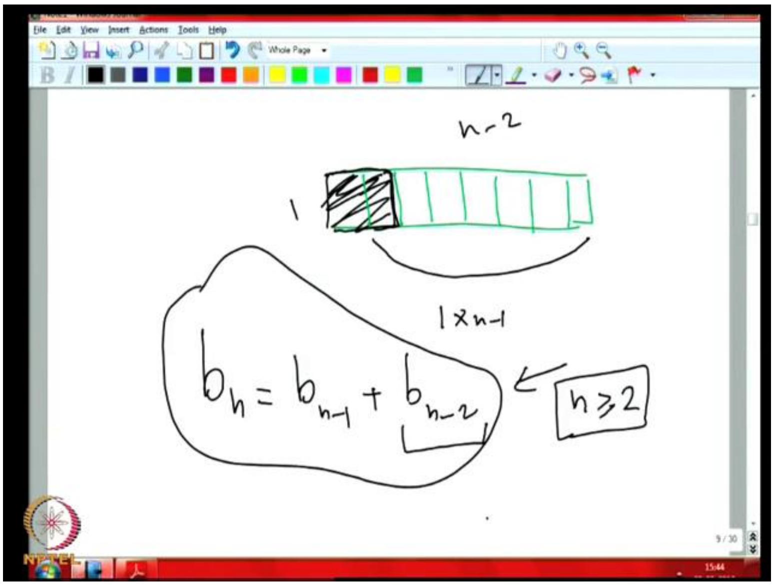

Like independently, you could have argued like the same way for instance for instance in the you just after thing the this thing, if you first values like for instance a 0 is equal to 1 a 1 is equal to 1 a 2 is equal to 2 a 2's equal to 2. Because, you can either have a domino like this or you can have 2 monominoes like this, we can have one monomino and another monomino like this, 2 monominoes like this right.

So, therefore, it is 2 right or and then you can argue to get the recurrence relation what we can argue is. So, what is b n, b n means the number of ways to perfectly cover the 1 by n board right. So, so we just concentrated on the first square here. So, if there using a monomial then the remaining n minus 1, 1 by n minus 1 board has to covered using monominos and donominoes, that is that can be done in b n minus 1.

On the other hand, if, you had decided to cover the first square in second square together by using 1 donomino then what happens is it is equivalent to b n minus 2, because the remaining portion that is 1 by n minus 1 to board it can be covered perfectly covered using monomials and donominoes b n minus 2 ways right. So, this will be the recurrence relations and this is this argument works for n greater than equal to 2 right. So, therefore, so, the same same kind of problem, but see that the fibanaci recurrence relation is coming here.

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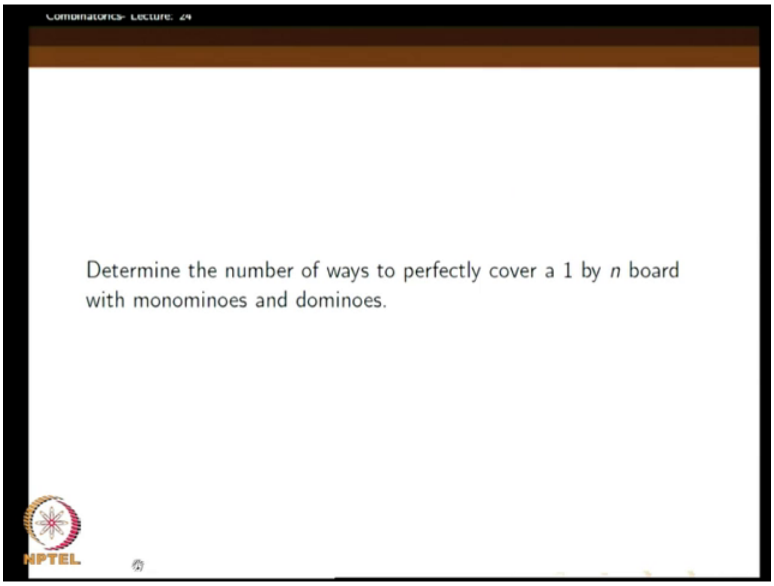

The next one is slightly different problem.

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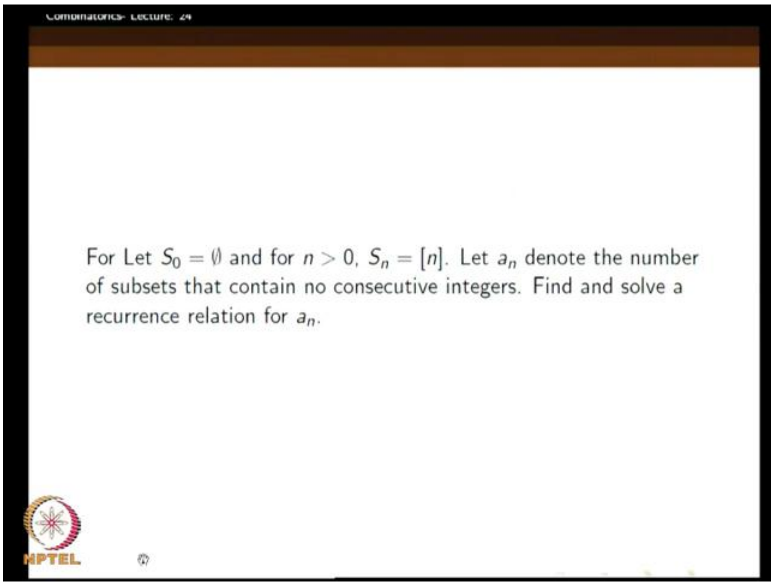

That also has this Fibonacci numbers, you done it. So, let S 0 is equal to the empty set and for n greater than 0, let us define S n is equal to 1 2 3 up to n, the first n positive integers. Now, let a denote the number of subsets, that contain subsets of S n, that contain non consecutive integers find and solve the recurrence relations for n for means, what is this number that is what we are asking.

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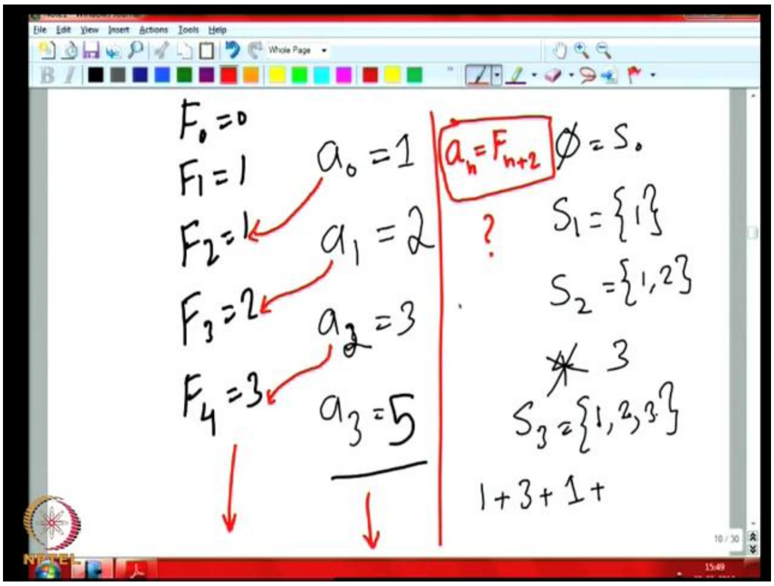

Now, here for instance a 0 is what empty set, how many subsets of empty set is such that there are no consecutive integers, we can take as 1, because that empty set is like that there is nothing in it. So, how can any consecutive integers be present in it, now for S 1, S 1 equal to S 0 S 1 S 1 is this 1 just 1. So, you can take there are 2 subsets to S 1 namely empty set and that itself, both are because, they do not have consecutive integers. Because, maximum 1 integer is there in it therefore, they cannot have 2 consecutive intergers. So, a 1 is equal to 2, but when I say S 2 S 2, we have to be more careful this is S 2.

So, there are 4 subsets of it empty set that is fine. Because, it dosent have any consecutive integers in it of case any of the single tone subset are find, because single subsets cannot have consecutive 2 integers. But, then you cannot the whole full set because, 1 2 has consecutive integers in it 1 and 2 right. So, therefore, it is not four it is 3. So, a 3 equal to 3 and now if you look for 4 right. So, we will see that, I mean S sorry sorry, this was a 2, a 2 equal to 3.

So, now this is a 3 now look at a 3, this 3 equal to a 3 1 2 3 they want to count how many are there right. So, definitely empty set is 5 all singletons set are fine. Now, if you take 2 element sets up to the 3 things, 1 2 is not allowed, because it is consecutive 2 3 is not allowed, but 1 3 is allowed, you get 1 there. Now, the full set is not allowed 1 2 3 has consecutive integers in it, so this much is the answer.

So, that is 4 plus 1 5, 5 is the answer 5 is the answer 5 is the answer, now you see the fibanaci number here, just that Fibonacci numbers f 0 is 0, f 1 equal to 1, f 2 equal to 1, f 3 equal to 2, f 4 equal to 3 and so on. So, this part you can see that here the mapping, if at all the fibanaci numbers nothing has to be here a 0 equal to f 2, a 1 equal to f 3 and a 2 equal to a 3 and so on.

The question is this will it be different later as of now it seems to be, because upto here atleast the 3 numbers are verified. So, later will it be different from the corresponding Fibonacci numbers, we are seeing, we are we would like the conject say that a n is equal to f n plus 2 right as we see from here a 0 is f 2, a 1 is f 3, a 2 is f 4 and so on. So, I would like to conject a and is f n plus 2 is this correct. So, we want to prove that, this is correct.

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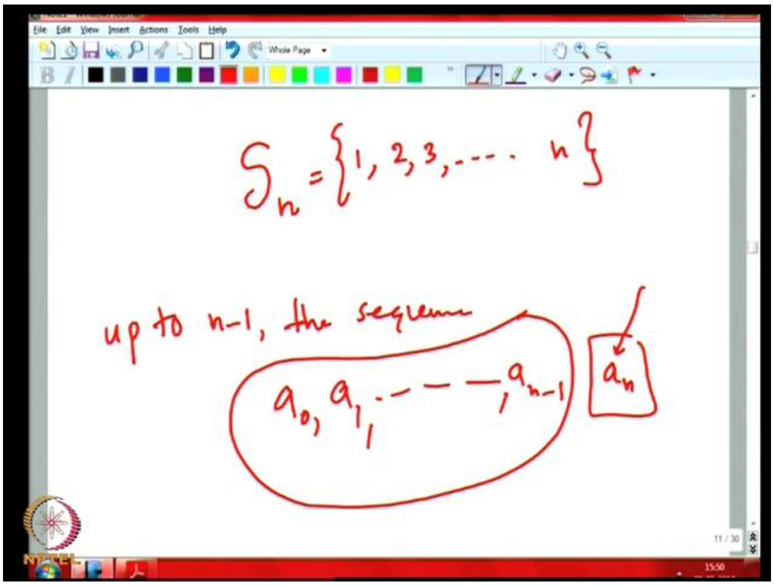

Now how will you prove how will you prove it to prove this, we consider a S n, now S n is like this 1 2 3 upto n. Now, lets say upto n minus 1, we have figured out the numbers that means, n minus 1, we know the sequence the sequence a 0 a 1 upto a n minus 1 is non right. Now, we want to get a relation mean to a n express in terms of this previous numbers right (( )).

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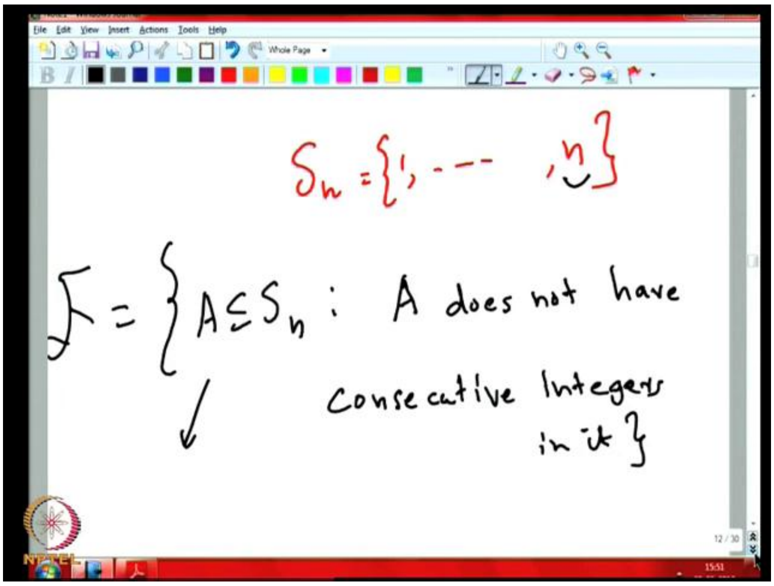

So, this S n being 1 to n. So, we can consider 2 cases a counting the subsets a, so that means, if you are counting a element a subset of S n such that such that a does not have consecutive integers in it. Now, these sets suppose this is of family f these sets, we have 2 types of A'S, A'S that contain n and A'S that do not contain n. So, let us say, we will put it as f dash and f double dash. So, f dash this is also a subset of f this is also a subset of f.

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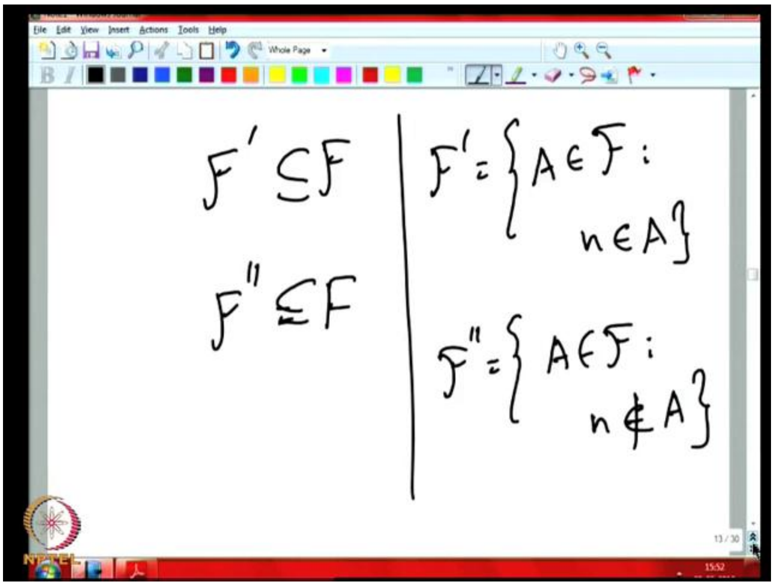

So, we put it as f dash equal to this is equal to the A is from f such that n element of A and f double dash is such that A'S form f with n not element of A, you can definitely partician f into f dash and that to much this is exhaustive. So, mutually they dichurn also that is if an A is A f dash then that a cannot be in f dash and vice versa right and every A will be one of these type right may have n or may not have n that is all. Now lets try to estimate f dash and f double dash, so the cardinality of f dash and f double dash and then submit up, we get the cardinality of the f.

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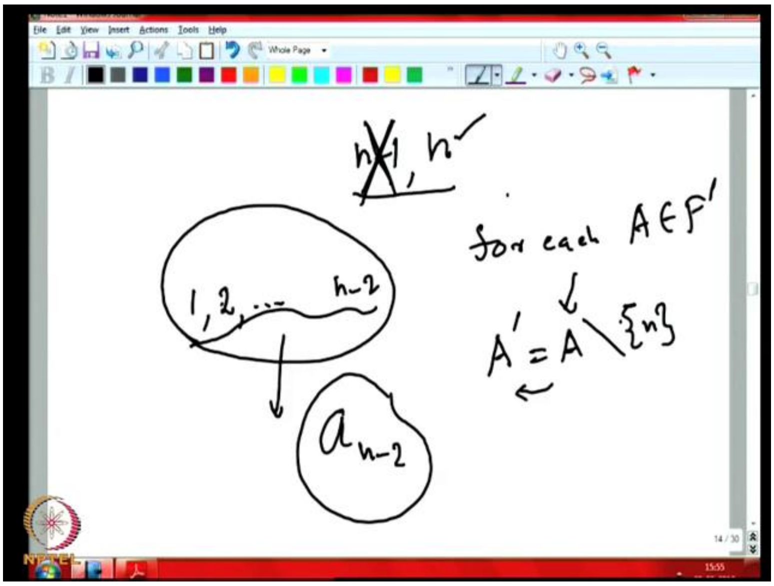

Now, these f dash that means, when n is there in A then what can happen is. So, what is clear is because n is there n minus 1 cannnot be there, because otherwise n minus 1 and n together is consecutive integers then that cannot be present in a valid A. So, that is that is our condition, so we have upto. So, the remaining part of a are coming from 1 2 3 upto n minus 2. So, these sets are should also be such that they should not have consecutive way. So, what we define is from f dash for, we define f say for each for each A element of f dash, we can consider A dash, such that A that is A, A bar n right, we just remove n from it each a in f dash has n in it, we just remove it and we get A, A dash n it is the cardinality of such those sets are same.

Because, it is not possible to get the same A'S where in we take 2 different sets in f dash and remove n from it, we will not end up with this same A dash why because each of these members in f dash add n. Now, if you remove n from each of them and end of the same set that means, original 2 sets are also the same that means, the cardinality of f dash can be found by finding the number of A dashes.

And those a dashes contain only members from 1 2 3 upto n minus 1, because n we have removed n minus 1 cannot be there because because originally, we had the consecutive the property the consecutive integers were, not present n is present and minus 1 cannot be present. So, now, its coming from 1 to n minus 2 and now consecutive integers are there in any of the and naturally, this number is a n minus 2, because this is the number of sets, you can form from 1 2 3 upto n minus 2 without having any consecutive integers, any pair of consecutive integers in it.

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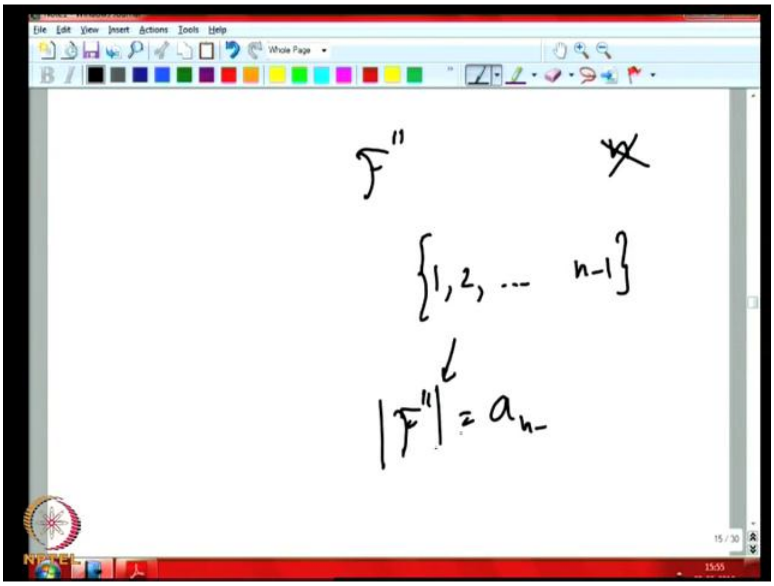

On the other hand to estimate f double dash what we can do is notice that n is not there. So, if n is not there, it is coming from 1 2 upto n minus 1 right. Now, we could as well save here selecting sets from 1 2 upto n minus 1, 1 2 3 upto n minus 1, such that no consecutive integers are present that number is clearly this number f double dash cardinality will be a n minus 1.

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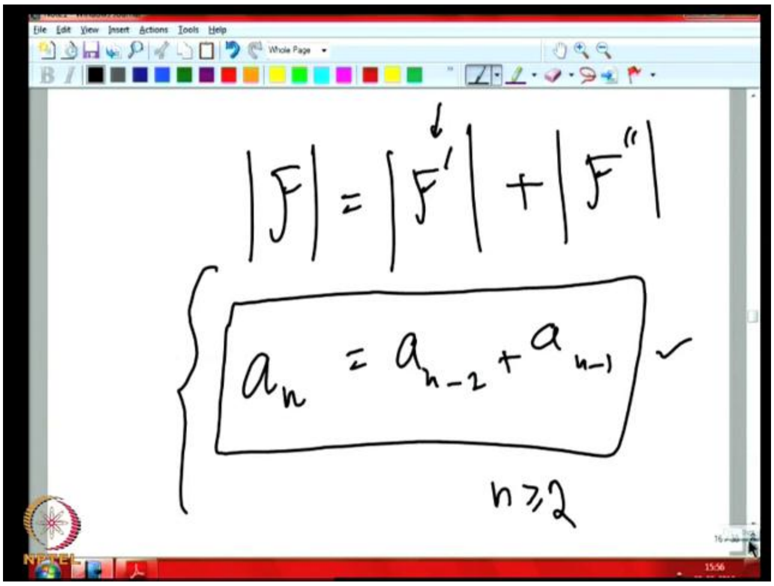

So, therefore, cardinality of of f is cardinality of f dash plus cardinality of f double dash this is a n, this is a n minus 2 plus a n minus 1 as we have seen. So, this satisfies the fibanaci recurrence relation and now under, we are seeing the pattern the and this you can see that this argument is true for n greater than equal to 2. Because, in the first argument when, you found out that, we removed n and n plus 1 automatically went away and now we have the empty set right. So, the other thing, we just removed n.

So, we have because, n is 2 then we will have 1 number then, we can take n minus 1. So, that is fine. So, therefore, it is this recurrence relation is valid for n greater than equal to 2 and as we have seen the correspondance the first initial values, when you check, we see that the recurrence relation is valid. But, this initial values are a little shifted in the sense that the a 0 correspond to f 2.

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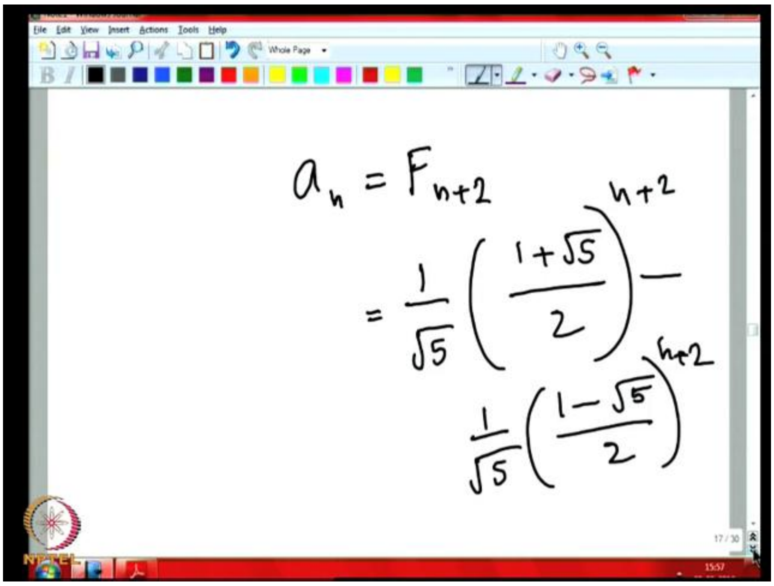

So, therefore, we can infer that a n equal to f n plus 2 as we conjucted right and this is we know the value of this thing, this is 1 by root 5 into 1 plus root 5 by 2 whole power n minus 1 by root 5 1 minus root 5 by 2 whole power. So, this n plus 2 sorry n plus 2. So, that is. So, here also, we can see again, I repeat the problem was that, we have taken the integers 1 to n and then, we asked to find the number of subsets of this set 1 2 3 upto n, such that no consecutive integer is present in it.

So, looking at this at this problem 1 may not think that, it has something to do with the Fibonacci numbers, but the answer happens to be the Fibonacci numbers, this there itself is shifted. So, n'th Fibonacci umber for for n natural numbers, we are getting the n plus second n plus 2 Fibonacci number mentioned. So, like that Fibonacci numbers have several interesting properties.

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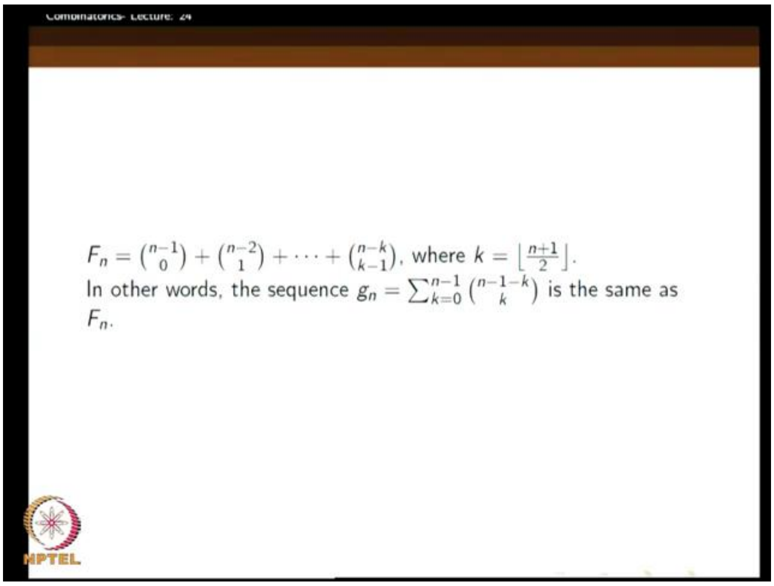

So, here is one property, which is a little the slightly different thing, but it is interesting I can state it, but probably they will not prove it, that's it. So, it is not because, it is difficult to prove, but to save time. So, it says f n equal to n minus 1 choose 0 plus n minus 2 choose 1 plus upto n minus k choose n minus k minus 1 where, k is equal to n plus 1 2.

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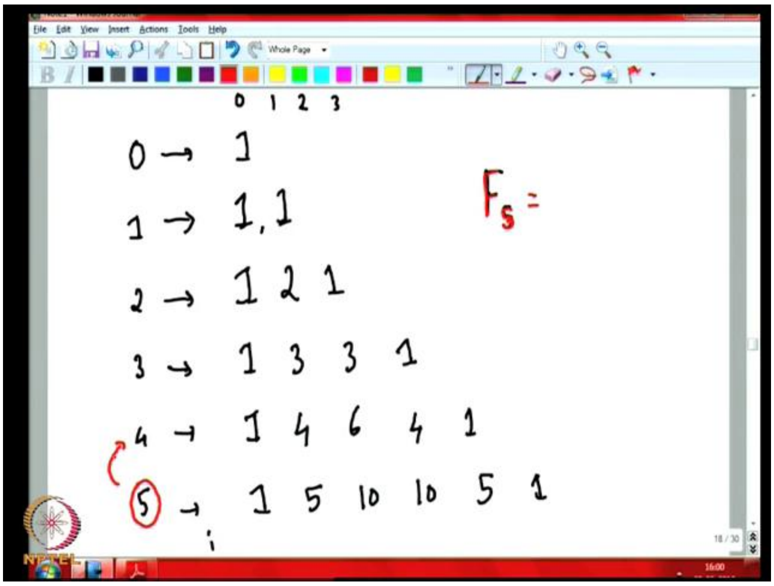

So, let us say, so this is it says, you remember the pascal's triangle. So, pascal's triangle was written like 1. So, for instance here 0 through of the pascal's triangle the 0, through of the pascal's triangle contained just 1 n choose 0, 0 choose 0. This is the 0th column and say the first column, second column, third column right. So, now, first row of the pascal's triangle contained 1, I mean 1 choose 0 and 1 choose 1 1 and 1 right 1 and 1 and then the second row the pascal's triangle contained 2 choose 0 1 and 2 choose 1 2 into choose 1 and 3, it is 1 3 3 and 1 right and 4th it was 1 4 6 4 and 1 5, it is 1 5 10 10 5 1 like this. So, now, it will keep on going like this right, now the formula says means the Fibonacci number fifth Fibonacci number. So, the fifth Fibonacci number right, it says it is equal to you start from the previous row. So, the fifth Fibonacci number is a 5 here.

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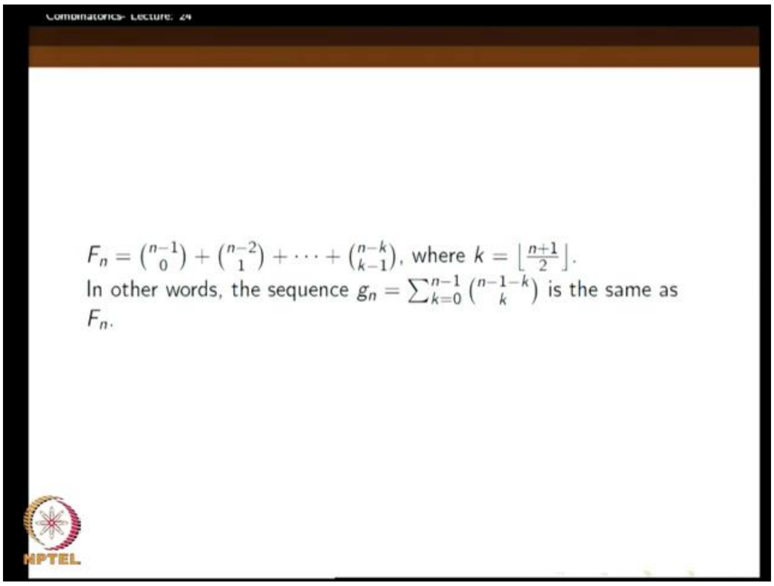

Then you go to the previous row and then here you start from n minus 1 choose 0 and then go backward in the row and go backward in row wise that make it increasing the column numbers. That means, along the diagonal you go upward that what it says right.

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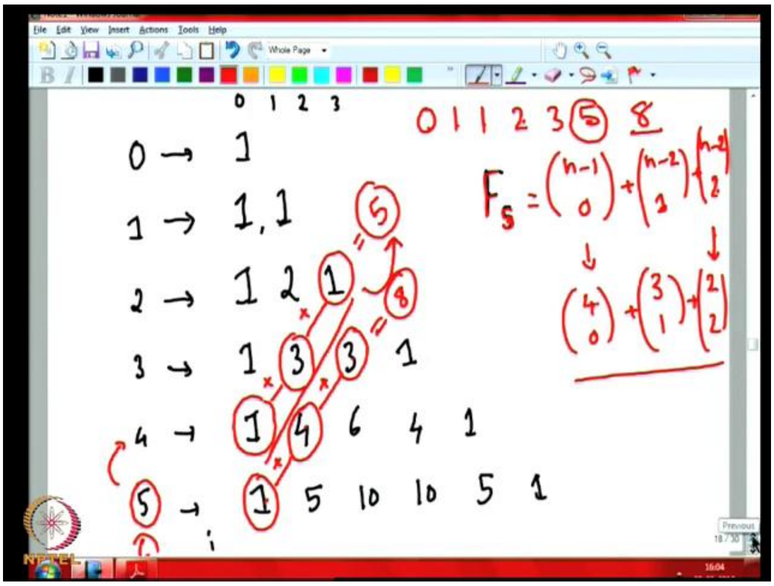

Along the diagonal, you go upward. So, the previous row the first number. So, you can this is the first number, which is n minus 1 n minus 1 choose 0, n minus 1 choose 0, which is 4 choose 0, that is the begin with plus n minus 2 choose 1 right, which will be 3 choose 1 right. That means, here 3 choose 1 and then here, we will take n minus 2 choose 2, that will corresponds to 2 choose 2, here the column numbers are increasing 0 1 2 like that and the row numbers are decreasing 0 1 2 like that. So, here it is like that. So, it is a long diagonal. So, next one what happens is this here 2 minus 1 is 1 and 2 plus 1 is 1 here. So, 1 choose 3 will come from now 1, it will be 0 upto here only.

So, what is this, this is 3 plus 1 plus 1 that is 5, if you sum up these things right, we will get 5 and the fifth Fibonacci number is indeed 5, because the first Fibonacci number 0 1 1 2 3 and 5 right this is the fifth row. So, we wrote first second third fourth fifth fifth Fibonacci number is this are this are adding upto 5, it is a indeed an interesting property right and so, next next Fibonacci number has to be 8 right. So, let me see. So, for instance when I go for the sixth one where. So, to start from here then it will come here, it will come here and take this thing.

So, when you add up all these things 1 plus 4 plus 3 that means, 6 5 plus 3 8 it is working out. So, what we claim is that just go to the previous row for instance, if you want to find the n’th Fibonacci number as a sum of n’th (( )) the pascals triangle. So, n’th row, we are in. So, we go to the n minus 1th row start with the 0th column and then go backward go upward right that means, the rows are decremented 1 by 1and columns are incremented this is some diagonal way you go upward right.

So, and add up to when you will hit after sometime they will be all 0s then you just add up to here upto here you get 8. So, as I get the corresponding number and this proof does not use anything complicated, you just have to do some manipualtions. So, what I do is I leave it to the student to figure out, because if I spend time on that the like I spend a lot of time in on that. So, I would rather leave it to the student.

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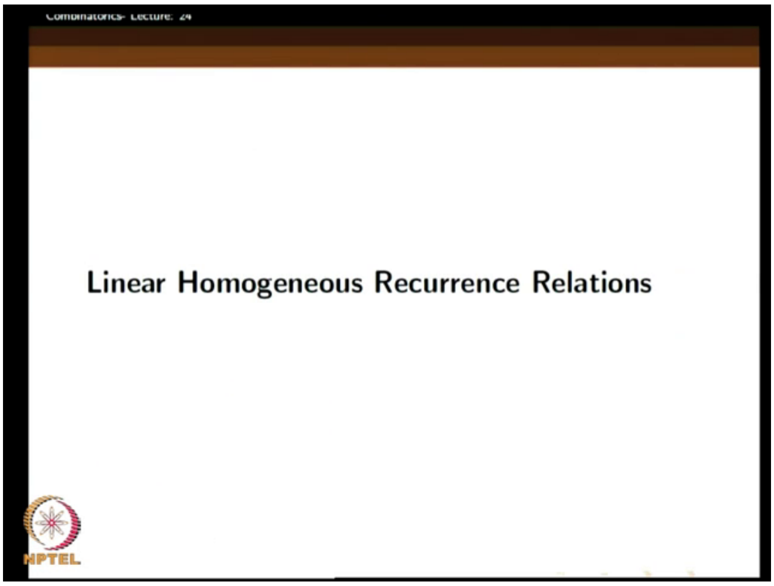

Now, I will go further to formerly consider as some I mean more general kind of recurrence relation. So, we just studied fibanaci recurrence relations here. So, now, we will consider the same type of recurrence relation, but in a generalized setting. So, we will consider this thing called the linear homogenous recurrence relations.

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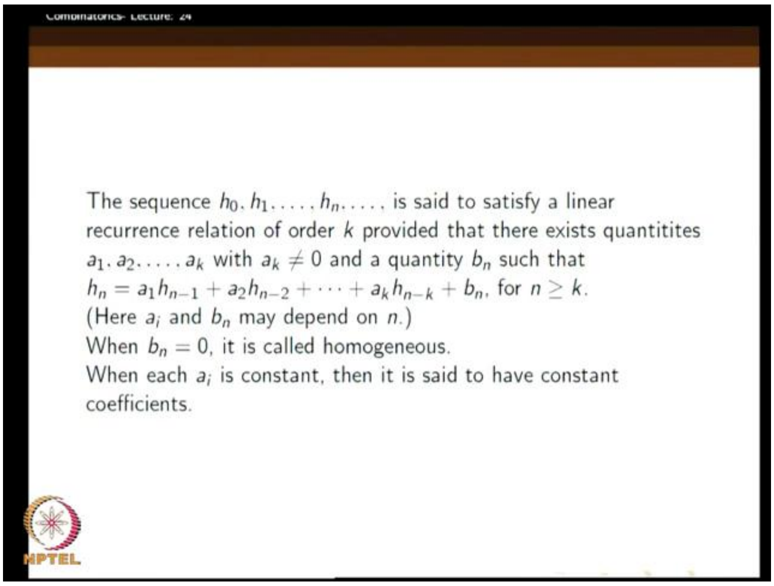

So, what is this linear homogenous recurrence realations.

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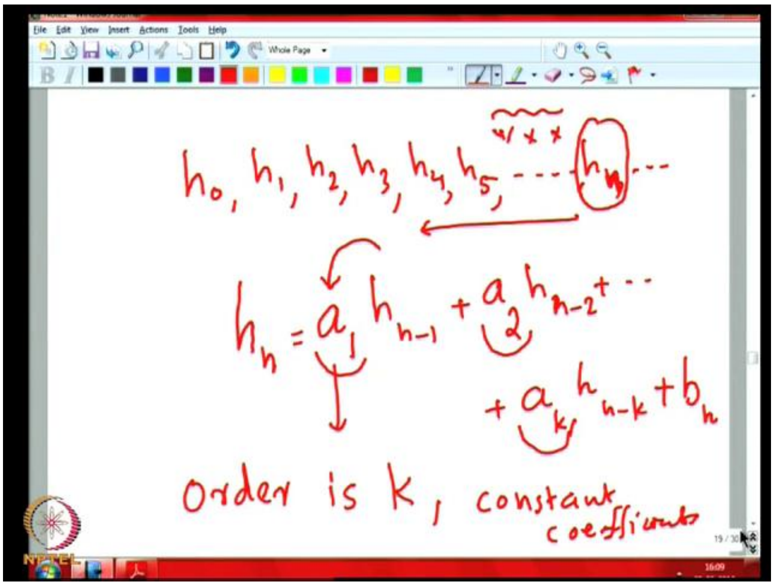

So, we have some sequence h 0 h 1 h 2 h 3 h 4 h 5 like this h n like this, now suppose you can find a formula for h n in terms of the previous things. There we can we have a formula such that h n equal to something times let us say a 1 times h n minus 1 plus something times let us say a 2 times h n minus n minus 2 plus up to a k minus a k minus 1 sorry, a k h n minus k in other words, we are combining the pervious k terms.

So, when you want to get this h n when you want to get this h n, we are combining the previous k terms 1 2 3 upto k terms here and in some way that it is coefficents are like capturing that. So, whatever I am talking about some way what is so, that is in a I can say in a linear way right. And because linear way because in our we do not write h i into h j here right, we do not multiply 2 different previous terms together. They are separate but, just that they get a some other multiplier may be a constant may be a function of n this a 1 can be a function of n in general. But, never it it will never involve another previous term it is not that a 1 contains some h i, which was before right.

So, in that sense it is linear and finally, we also add some term, which does not even contain it is not even multiplier this thing, here we say that this is a linear recurrence relation and also we say that order is k, it is a linear recurrence relation of order k linear recurrence relation of order k and then as I mentioned this coefficent, you are using a 1 to a k are can be functions of n. They can be constants also, if they are constants then we say that the linear recurrence relation with constant coefficents order k and constant coefficents. So, it is not necessary that is always the constants are coefficents, but it can be constants also in the simple cases.

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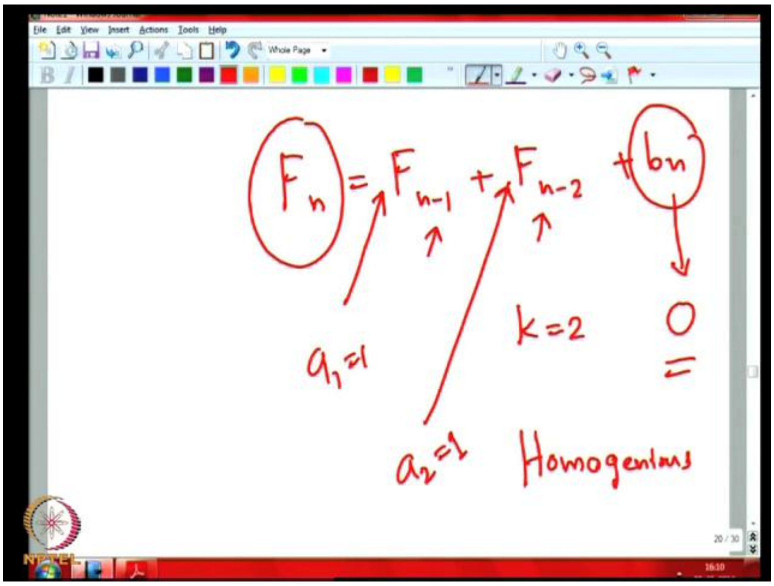

Then for instance, I can give you some simple examples for instance our fibanaci recurrance relation, it was like f n equal to f n minus 1 plus f n minus 2 can clearly see that here k equal to 2. This is a second order linear recurrance relation, I mean because 2 terms 2 previous terms, we are combining to make the n’th term right and here our a 1 is equal to 1. Similarly a 2 is equal to 1 and there is no b n, b n is 0 right, similarly we can consider see not that when b n is 0, we can also have one another qualifier it is called homogenous homogenous recurrence relation. Homogenous linear recurrence relation and b n is equal to 0 called homogenous recurrence relation and the other terms are linear homogenous recurrence realtion right.

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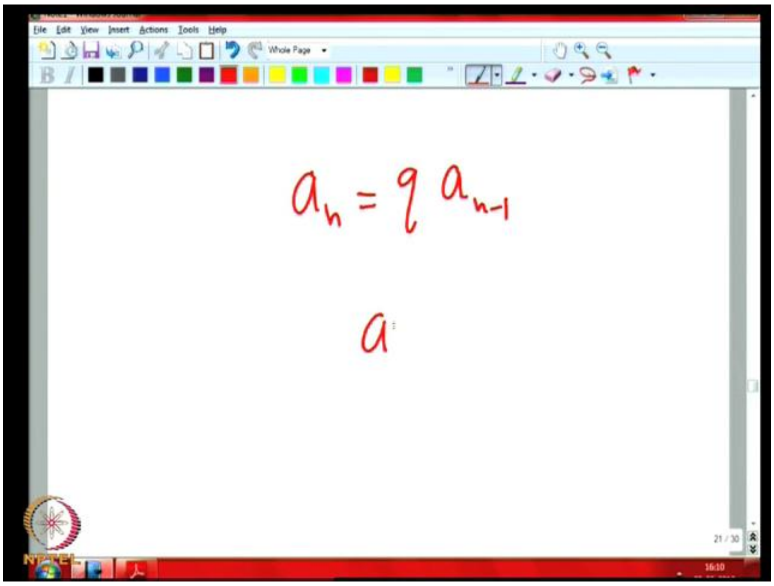

Now for instance you can consider this this recurrence relation, which appears in the geometric progression namely say a n equal to q times a n minus 1. Here you see the first order 1, because there is only one previous term, we are using to get the next term and a 1 sorry. So, I am sure. So, you should have.

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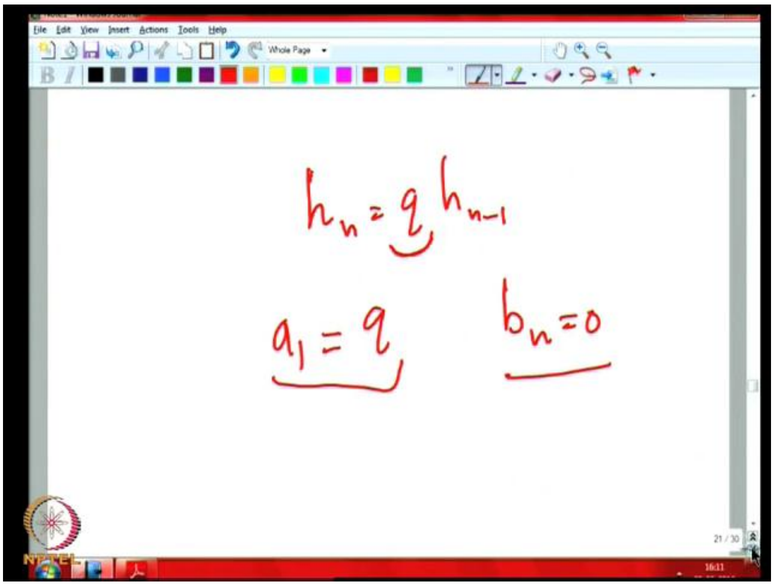

So, lets say, we will this h n equal to q times h n plus 1. So, this was first order order is 1, this a 1, the first coefficent is 1 q only and then b n equal to 0 here, that is a homogenous 1 also. This is of case, if q is a constant with we can also say that with constant coefficents, the fibanaci recurrence relation, we had constant coefficents.

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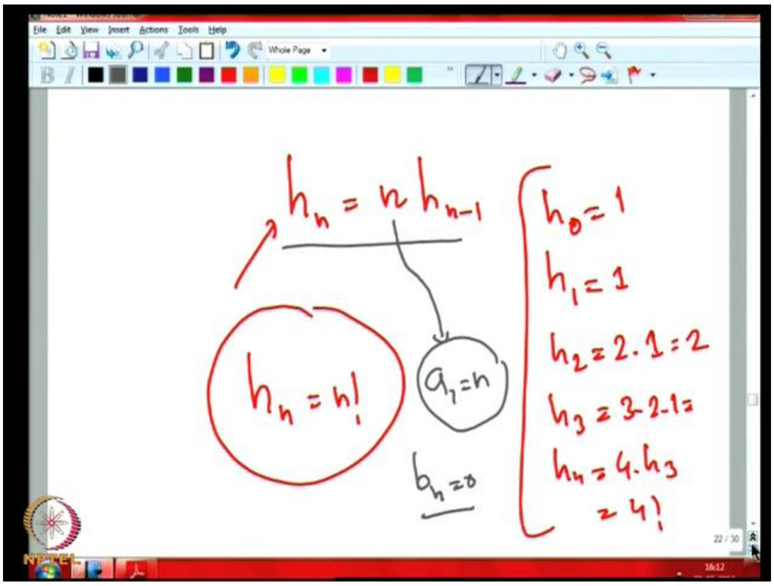

Now, another one may be, we can consider this recurrence relation n into h n minus 1 right what does this give, this is h n, we can easily see is n factorial, why because, if we put h 1 equal to 1 x 0 equal to 1 h 1 equal to 1. So, so h 0 equal to 1 then h 1 equal to 1 into 1 h 2 will be equal to 2 into 1, that is 2 h 3 will be equal to 3 into 2 into 1. So, h 4 will be equal to 4 into h 3, it is 4 factorial and so on. This is the recurrence relation for n factorial.

So, here this is also a linear recurrence relation jus that. So, of case here our the first order wont be because there is only one previous term, we are using and this a 1 is n here, this is not a constant right. This is a function of n and we do not have any b n, b n equal to 0. So, homogenous 1 of this. So, there are several such examples, you can consider, this is such to make sure that the student understands the keywords.

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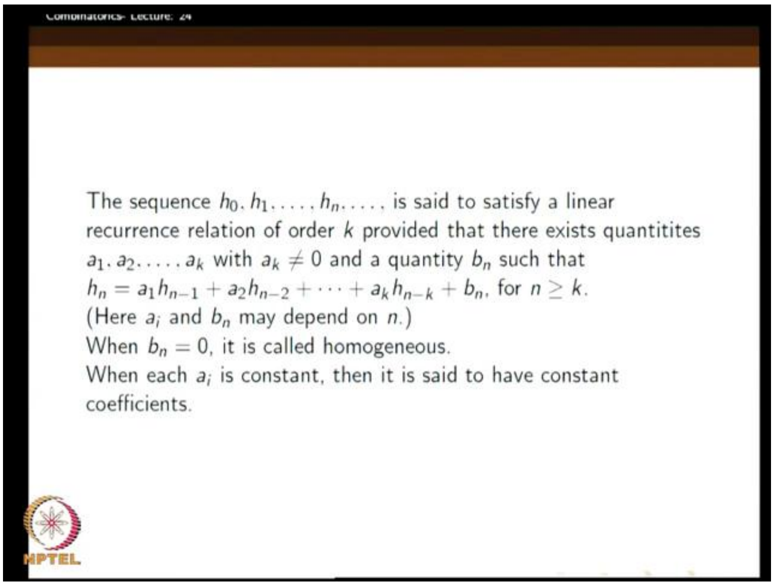

Now, we quickly move on to the important thing, one thing of case, so we can write this relation. So, whatever I have written here.

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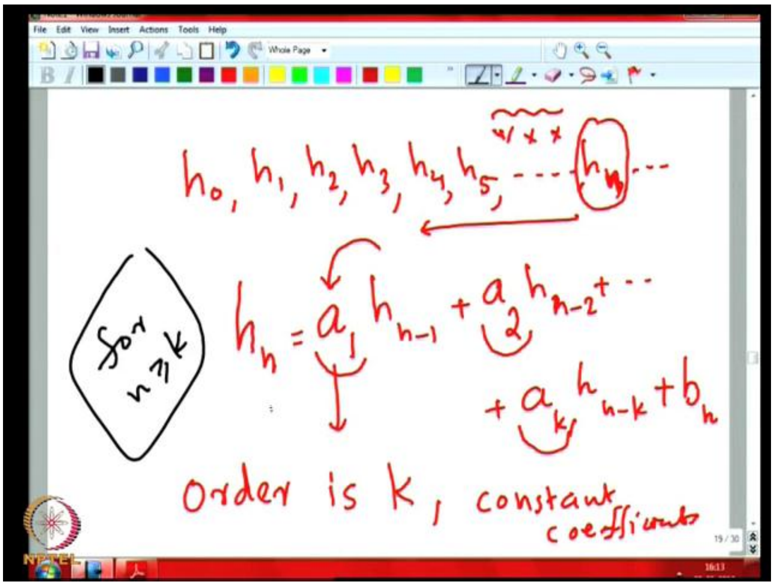

This relation for only for n greater than equal to k why is it. So, because you know when need k previous terms to define this thing, otherwise there is no point saying that h k.

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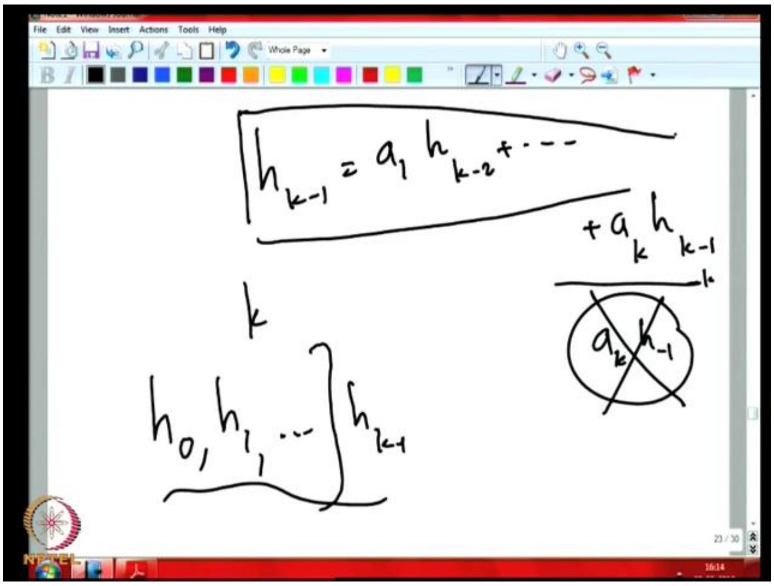

What is h k, h k minus 1 is equal to a 1 into h k minus 2 plus like that in the end what happens is a k into h k minus k will come sorry, k minus 1 k will come that is a k into h minus 1 will come, which is not defined. We have that sequence starts from h 0 h 0 h 1 up to h k minus 1, we have k terms right. But, if you are only taking up to h k minus 2, we have only k minus n terms, but we have defined this thing for k terms of case one more thing, we have to be careful about.

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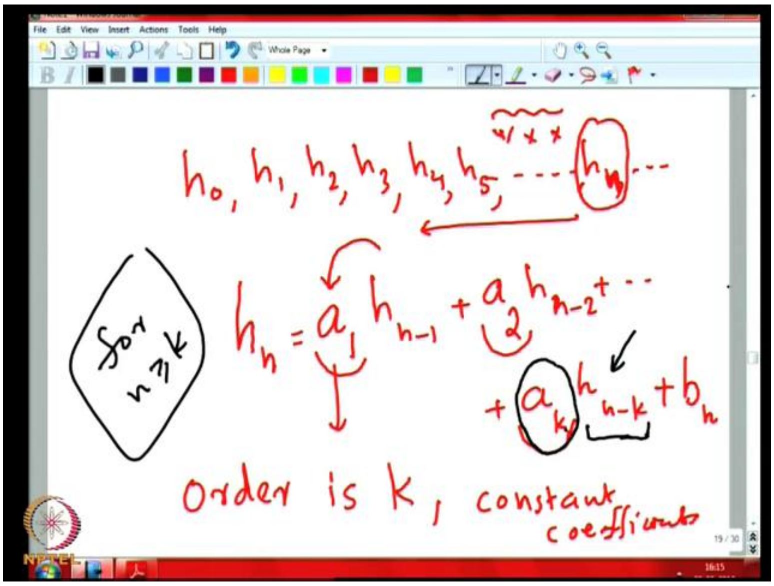

When I say order is order is came it is important to note that, we are indeed using a k as we are we are claiming that a k is non zero, because h n minus k, we are going all the way down to the n minus k'th term. Means k terms down, we are going, but inbetween terms, we may not use, but we are indeed using the lowest term, which is told here that is of a a k is assumed to be non zero here, this defination a k is assume to be. Otherwise, we cannot say it is a k'th order, because see if this was 0 then what will you do is we would just stop in the previous term and a k minus 1 h n minus 1 k, you can as well say that is k minus 1 right order is k minus 1, that is also important.

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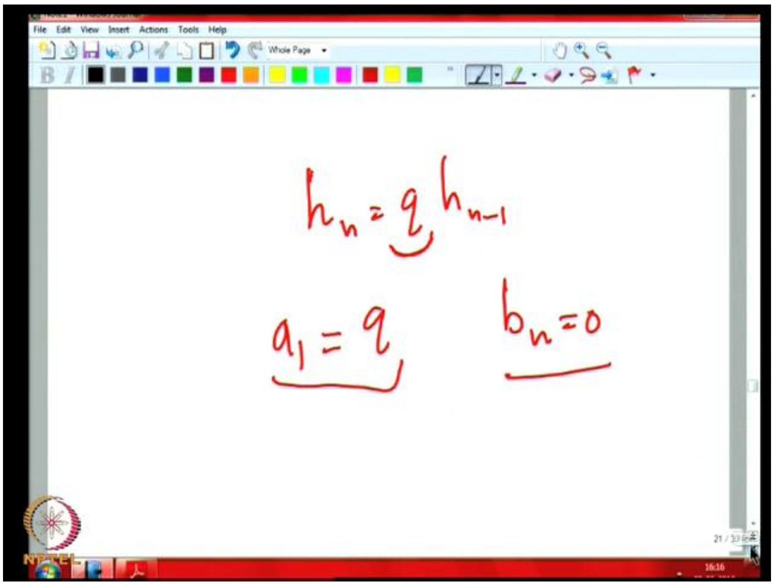

So, the order means, so its its important to note that, we are not really worried about all the a i's being non zero, the first a i is 0, it is for us. So, what we are worried about just go down from the highest term downward right the least term, we are taking the least term, we are taking should have a non zero coefficent the least term, we are taking should have a non zero coefficent right, that is why that is where, the k'th order is coming, because k previous terms are used is coming in from there right.

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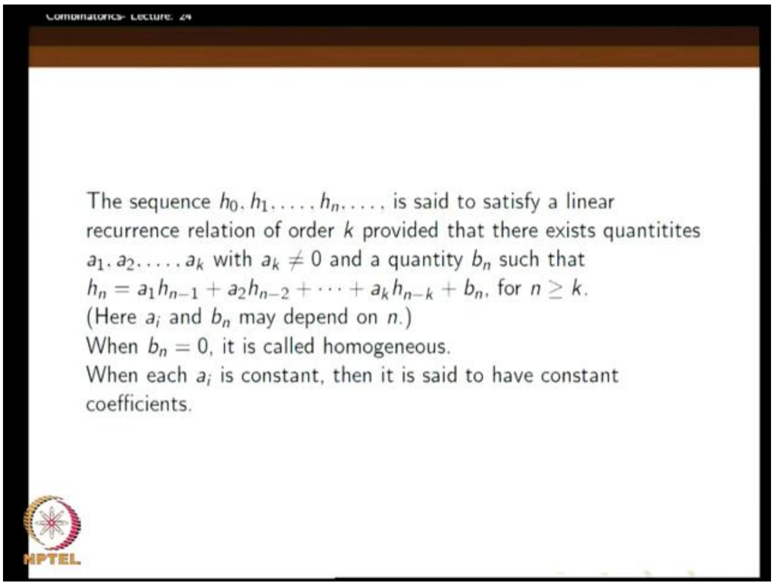

Otherwise, we would have thought that, we are going only k minus 1 terms downwards. And so, the order what is the thing of defined the k'th order homogenous recurrence relation a k is not equal to 0 and the quantity is these are the points and then.

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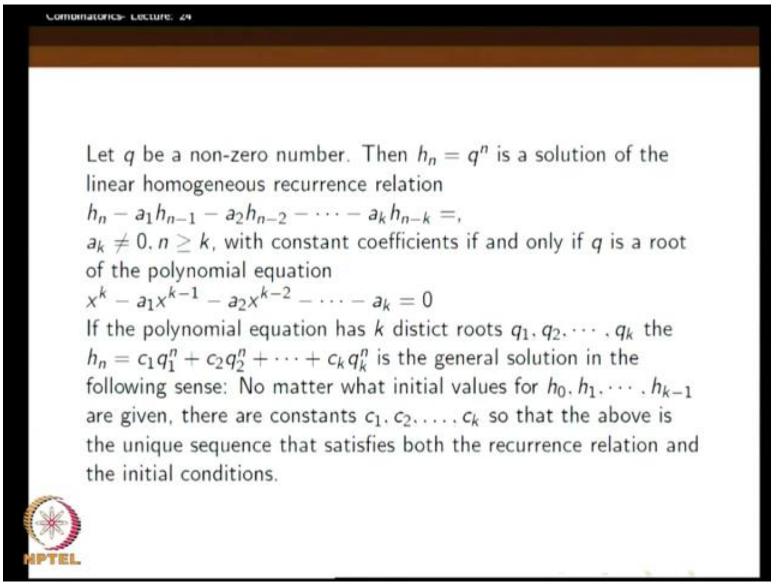

Now, the next thing is a method to solve general, I mean to get a general solution for this kind of homogenous recurrence relation, that is what out aim is right, of case we have seen that any interesting recurrence relation is falling into this category. So, this worthwhile to attempt to get a strategy to get a general solution for this recurrence relation, we will do it in the next class.

Thank you.