Combinatorics
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Lecture - 11
Multinomial Theorem, Combinations of Multisets - Part (1)

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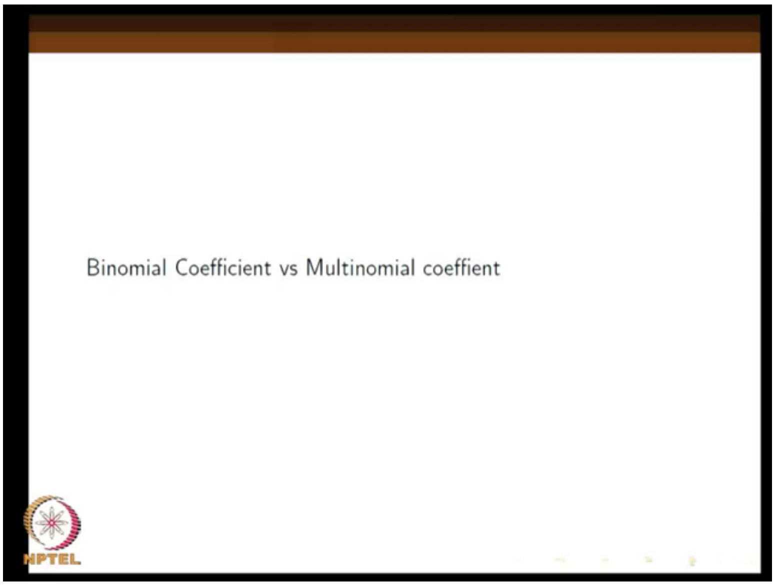

Welcome to the 11th lecture of combinatorics. So in the last class, we were discussing about the number of permutations of a multiset, multiset of cardinality n.

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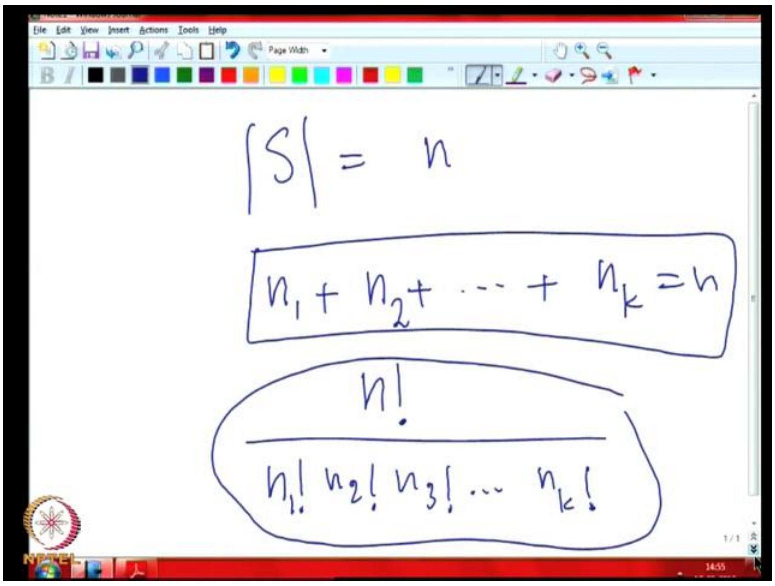

And so we see here write like this, when we say multiset, we count the the cardinality of the multiset is counting all the repetition numbers. And then the repetition numbers of each type, the first type repeats n 1 times, the second repeats n 2 times, say there are k times, n k and this is equal to n, n 1 plus n 2 plus are up to n k, this is the multiset. And this is the total number of permutations possible is n factorial by n 1 factorial into n 2 factorial into n 3 factorial into n k factorial is what we showed by several examples. Now, this coefficient has also got another name, it is called the multinomial coefficient this this ratio, how does this number this name come.

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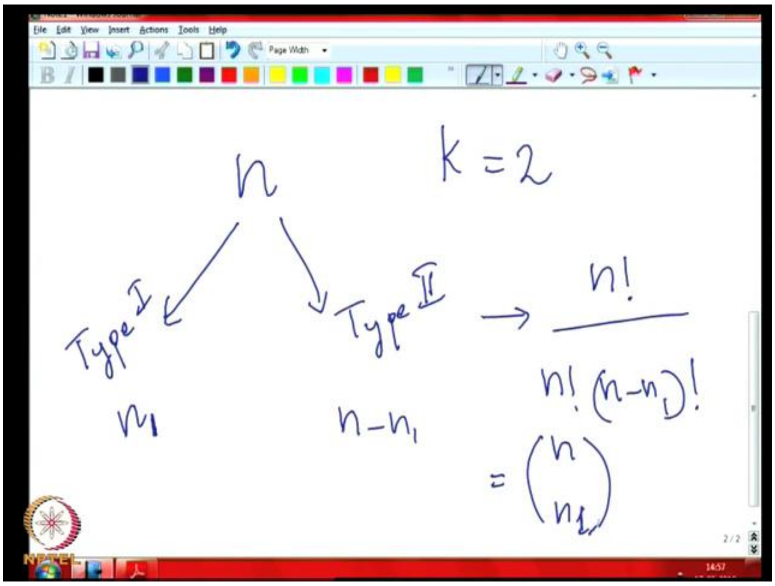

Because, we can see that suppose, there are k in this k types, k equal to 2 suppose, there only two type of things right then, there are out of n things, we have to put into two type, type 1and type 2, remaining should go into type 2right. So, type 1,we have we should have we have n 1and the type 2 we have n minus n 1right then, the multi the number of ways of permuting this multiset. This multiset contains only two type of things, the first type of thing repeats n 1 times in the multiset and the other type of thing, the second type of thing repeats n minus n 1 times. The number of possible permutations is n factorial by n 1 factorial into n minus n 1 factorial in this case right. So, here you can see that, this is same as n choose n 1right.

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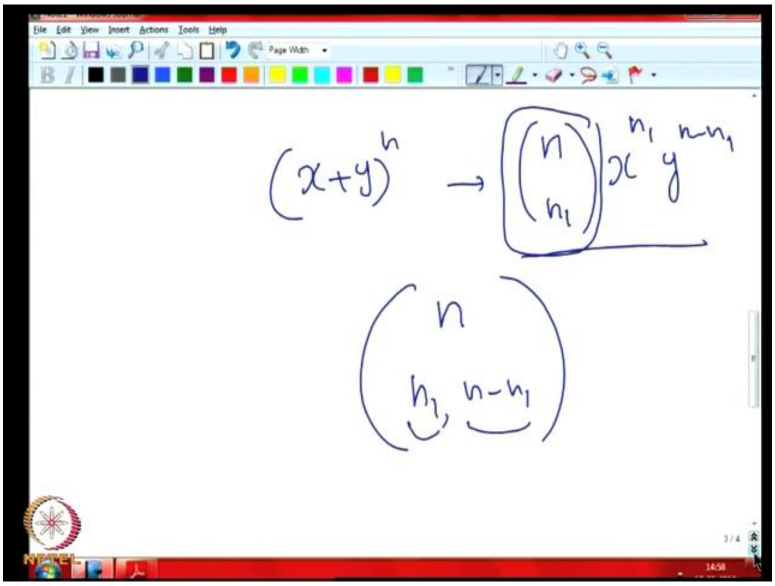

This is the binomial coefficient n, the in the expansion of x plus y raise to n, this is the n plus 1th binomial coefficient so, n plus th term will be n, n chosen raise to n, x raise to n 1, y raise to n minus n 1right. So, this coefficient only it is coming there right so because, this is binomial coefficient the term this which generalizes this. So, this is when you know, n choose n 1 comma n minus n 1 is when k is equal to 2, the n distinct objects are sorry a multiset of cardinality n with n 1 things of first type and n minus n 1 things of second type is permuted, how many ways you can do this is what but, this is essentially a selection of n 1 things out of n 1.

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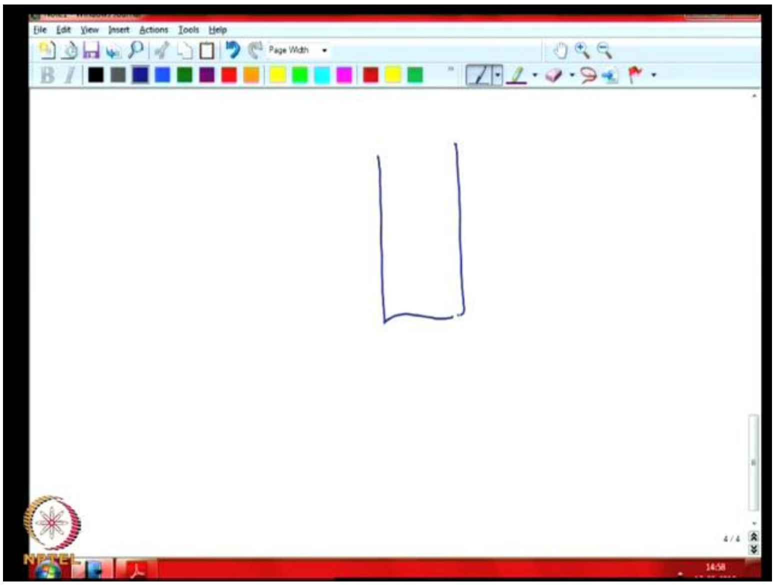

So, in the case of binomial coefficient, you can also see that, for in sense you have a if you if you had considered we had we had explained it.

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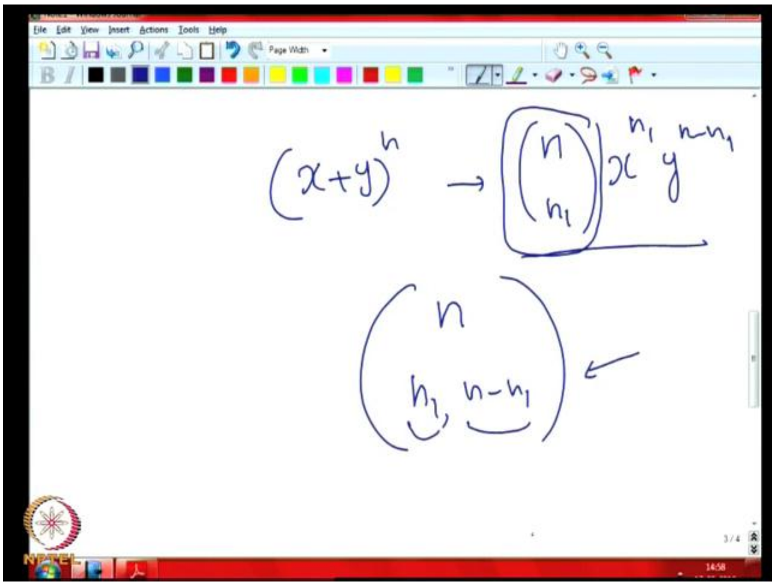

So, the distributing n 1 things into the box would so, when you when you also give the order to the boxes, in the last class we explain that, that is why this this is this is corresponding.

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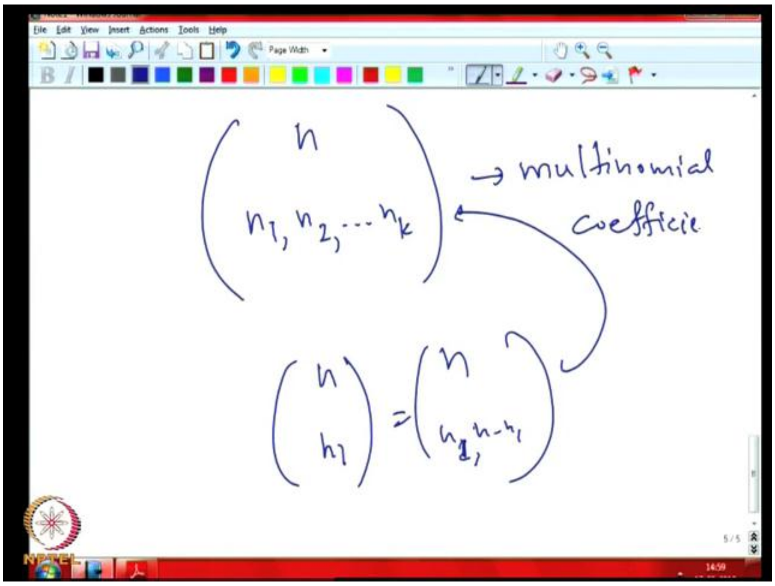

And now, we go back to the other thing so, namely n choose sorry n n1 this is the way it is written. So, we also have a notation for that because of that, we will the way we write n n 1 for we could have written, n n 1 comma n 1 comma n minus n 1 for the binomial coefficients. We generalize it and write it like this, this will be called a multinomial coefficient multinomial coefficient now the multinomial coefficient figures out in the multinomial theorem.

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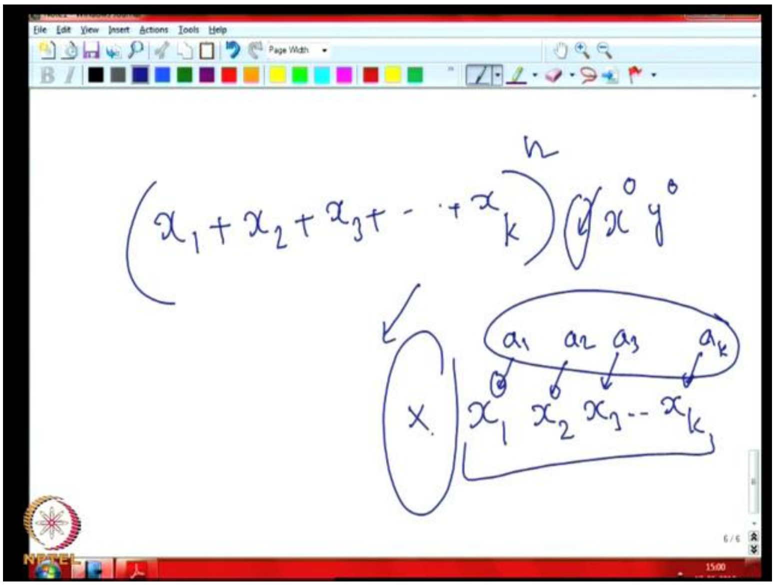

Like binomial theorem, we can have a multinomial theorem, for let us say, let us write like this suppose, we have x 1plus x 2 plus x 3 plus x t and whole power n right, there are so, we may can write k here, for sense k types. So then, you know the this is this will be expanded up to several terms, you can you can see that, what kind of terms will come. So, this will be something x 1,x 2, x 3, x k and this will be something like here, there will be some coefficient some a 1 will come, this will be there will be some coefficient here a 2 will come, this will be a 3 and this will be a k and so on.

And then, here, this kind of terms will come with some coefficient right, like in the binomial coefficients we have x raise to something into y raise to something with some coefficient this, what is happening here right. But here, in the multinomial coefficient, this kind of term with that particular coefficient will come right.

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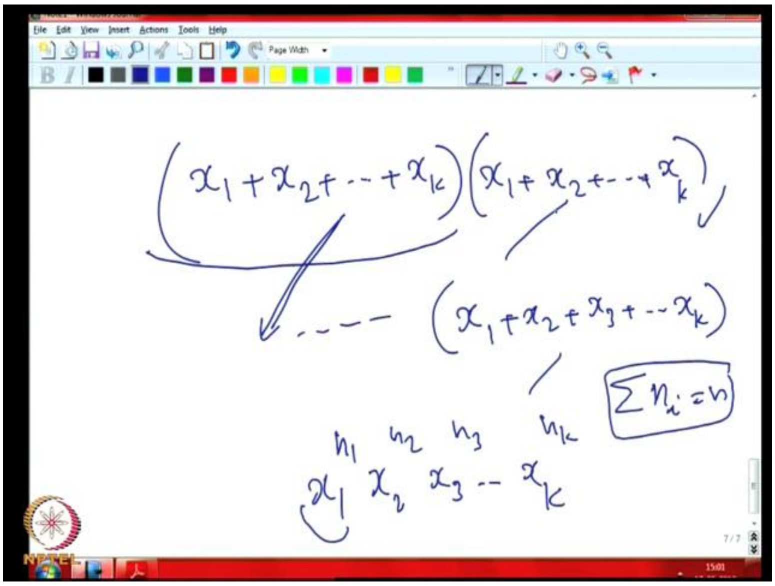

Now, how does it happen x 1 plus x 2 plus xk into x 1 plus, this is the same way we solved the binomial theorem x 2 plus sorry binomial formula we proved right so, x k and similarly, we have n times we will repeat it x 1 plus x 2 plus x 3 plus x k right. Now, a particular term right for instance, x 1right raise to a 1so, so x 1 raise to n 1let us say, into x 2 raise to n 2,x 3 raise to n 3and x k raise to n k will be a particular term where, sigma n i is equal to n right.

Because, you know, if you take x i from one here one here one here and also, there are out of n, this kind of terms this kind of terms we are, we are picking up one from here, one from here, one from here out of that, n 1 x 1's you should get, just the way we argued for the binomial theorem. So, that can be so, with this has to go to n 1 things should be selected right, that can be done in n choose n 1 ways right.

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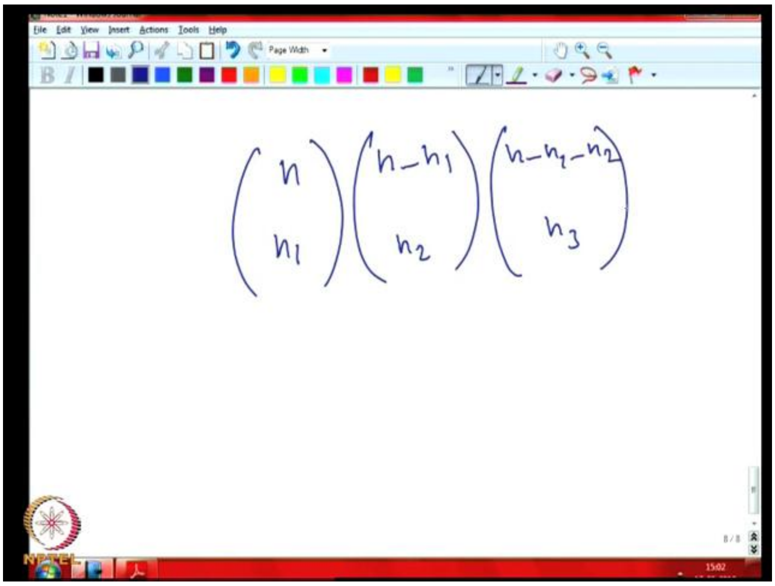

And then, from remaining n minus n 1 thing, we have to select n 2 things and from the remaining n minus n 1 minus n 2 things, we have select n 3 things and so on right.

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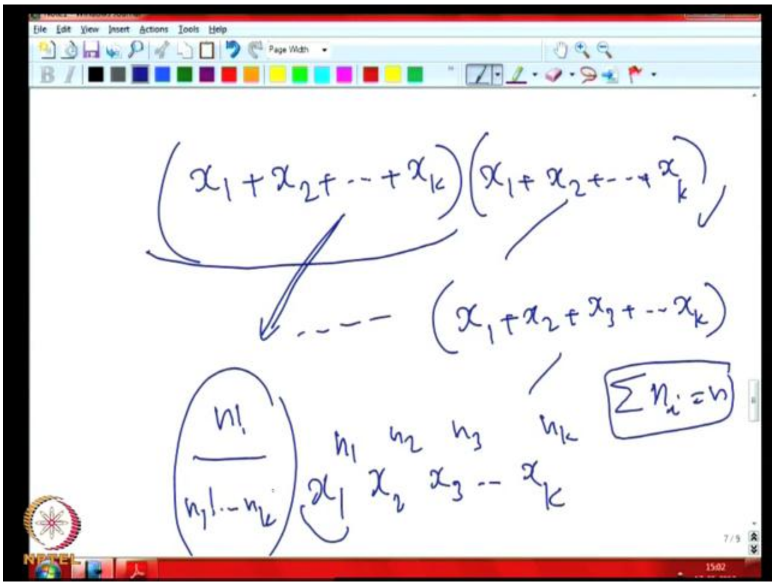

This is the same formula we wrote down for the earlier one and then, you know therefore, that is that is that will give you the same same value. The coefficient of this thing will be that n factorial by n 1 factorial into so, n k factorial which you will write as like this, n choose though it is whatever it is called so, it is a multinomial coefficients notation.

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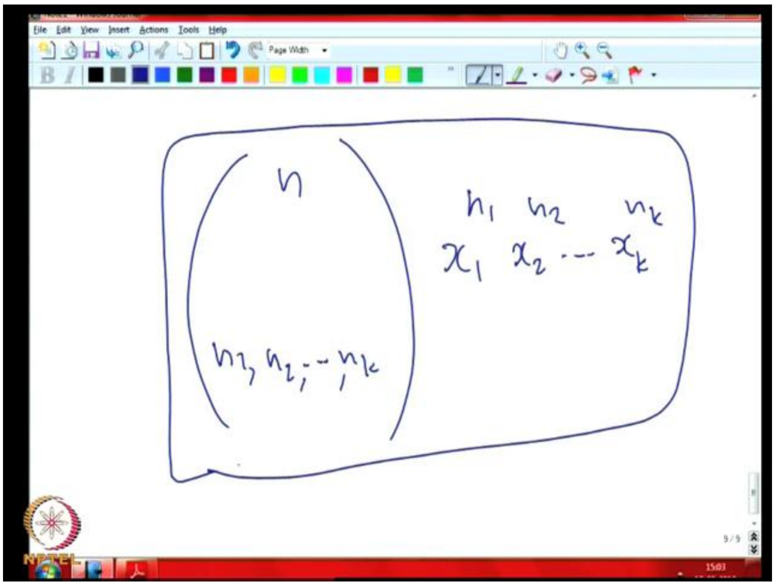

So, we will write like this n n1comma n 2 command k and x 1 raise to n 1, x 2 raise to n 2, x k raise to n k that is what right, this will be the typical term which we will get right the you can.

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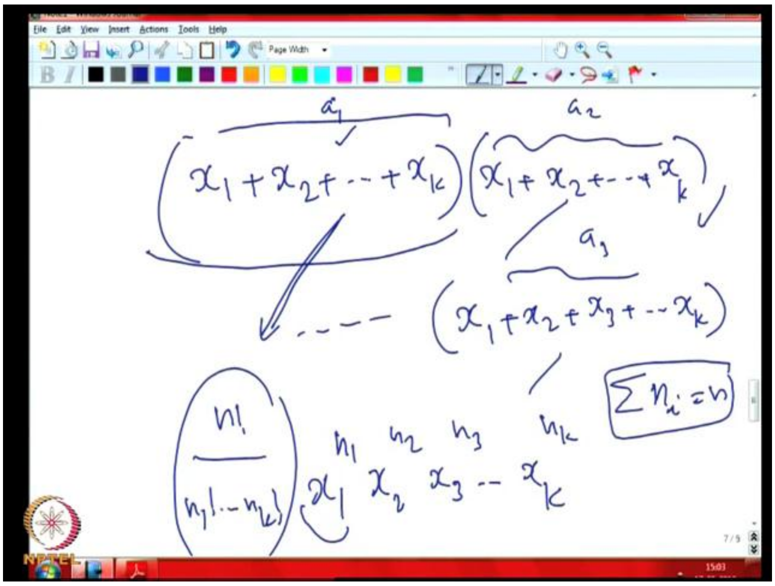

Or if you wanted to argue, this is objects different objects for instance, this is first term we can say, the first object, object number one a 1,this is object number 2 a 2 and this is number object number 3 and it is like, we are placing these objects into a bin labeled x 1.

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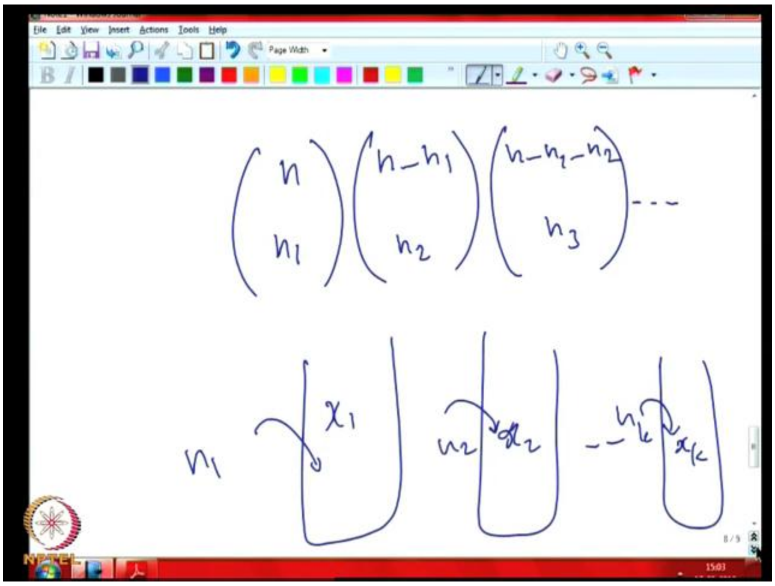

This is, the bins are labeled x 1, x 2, x k, the k different labeled bins we are placing them into this labeled bins. But we know that, n 1of those things should go into this, n 2 of those things should go into it, n k of these things should go into this, this way also we can interpret it right and the way, we we have discussed all these things in the last class.

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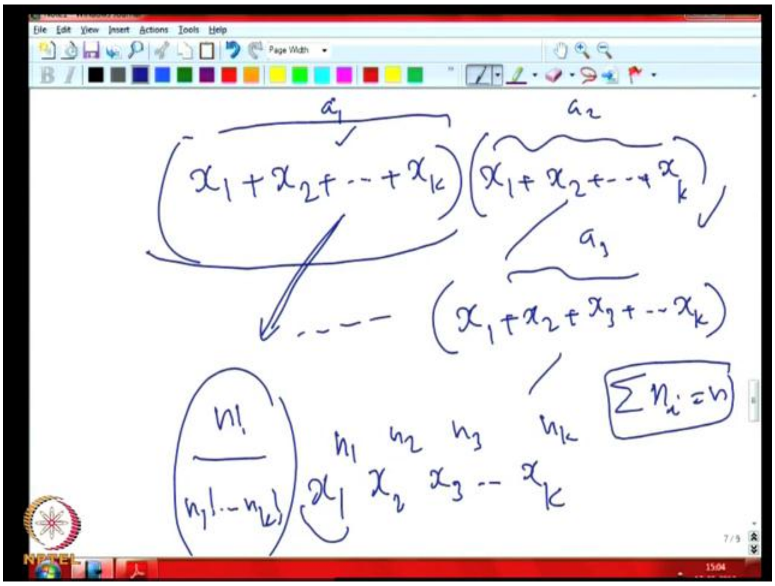

We just saying that, this is this also can be seen, you seen one of those models Bohr's and Beans model or whatever right.

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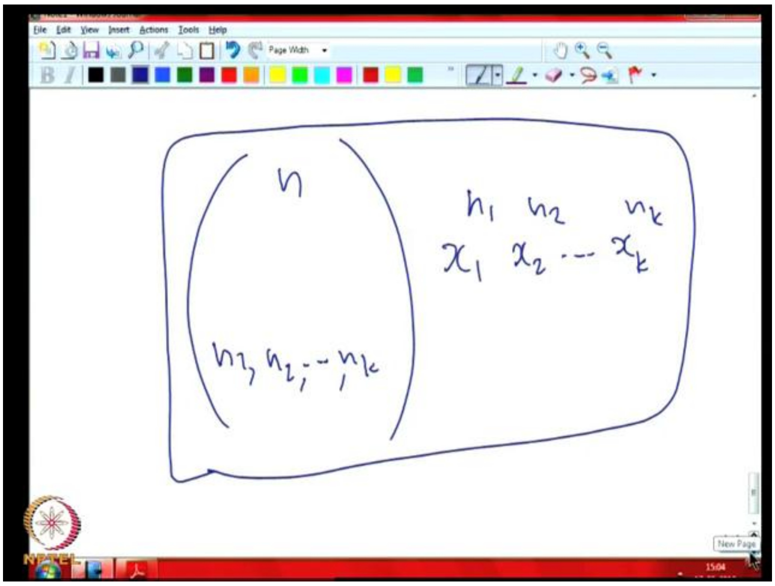

Or otherwise, we can directly argue and use this kind of a derivation once again and do it, or otherwise we can set up bijection with one of the earlier problems we have seen and then, that would also take us back to the earlier one right.

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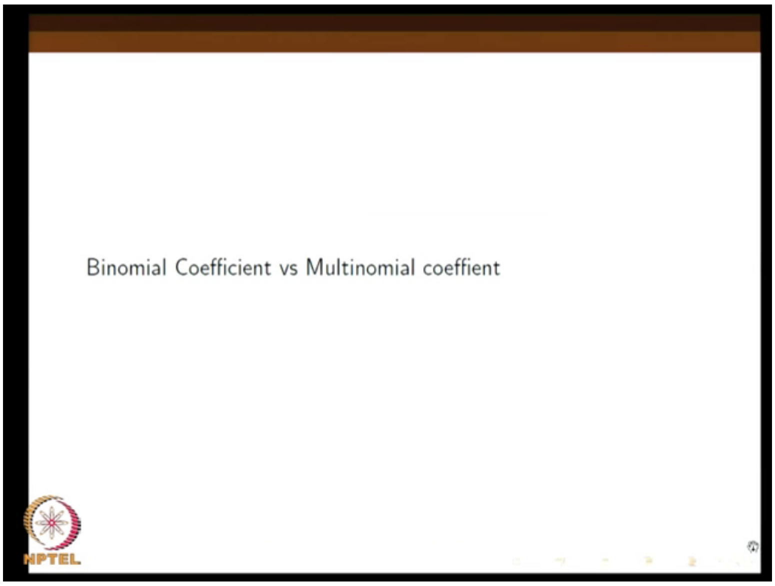

So, this was the multinomial theorem.

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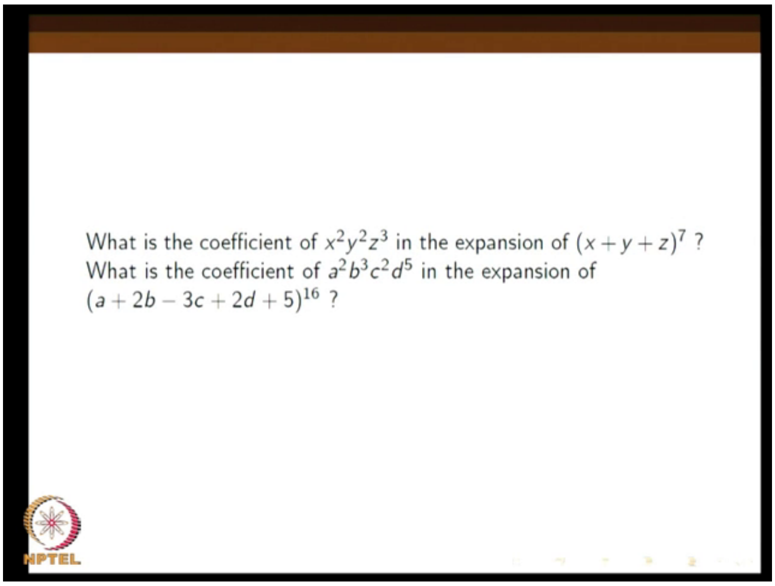

Now, we will just take a simple example, just to for practice sake. So, here, what is the coefficient of x square into y square into z cube in the expansion of x plus y raise to z right. This kind of a question this is very simple we know that, you know 7 to here, 2 plus 2 plus 3 sorry I meant 5 here right because, these things should add up to 7 sorry 5 plus 4, 2 plus 2 plus 3 is 2 plus 2 plus 3, it is indicate 7 I am made a mistake. So, therefore, we just asking for the coefficient of x square into y square into z cube in the expansion of this, that will be the multinomial coefficient 7 2 2 2 right.

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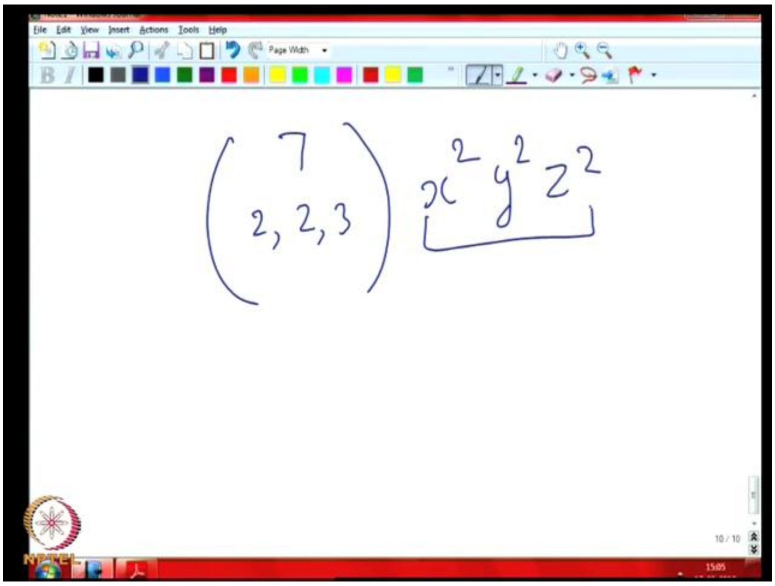

7 2 2 2 this is the way, you see 7 2 2 3 will be the coefficient of x square into y square into z square right, this is what we will get.

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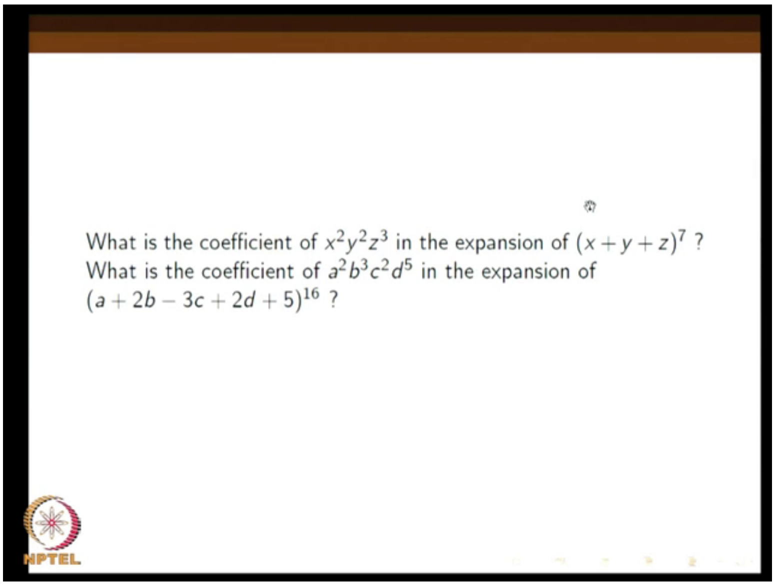

And a plus 2 b minus 3c plus2 d plus 5 raise to 16 in the expansion of this one so, that is right a plus 2 b minus 3 c plus 2 d plus 5.

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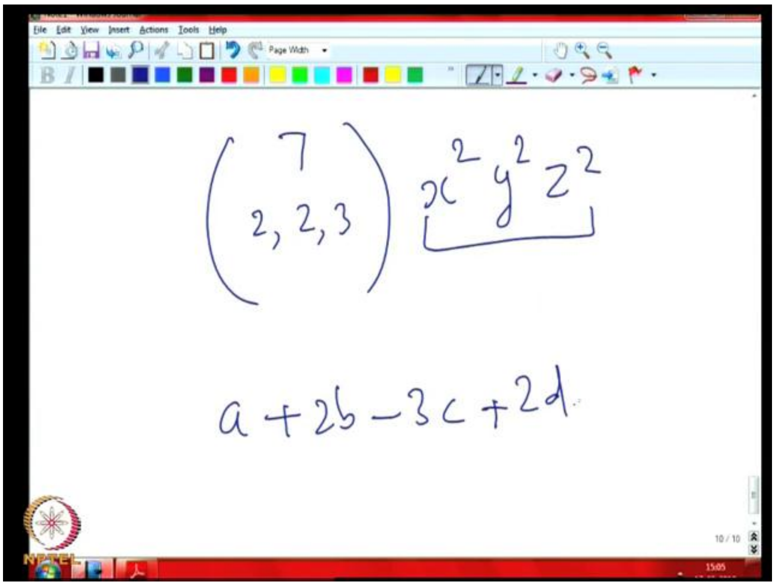

A plus 2 b minus 3 c plus 2 d plus 5 raise to16, a plus 2 b 16 in the expansion of this, what is the coefficient of you know a square b cube c square d 5.

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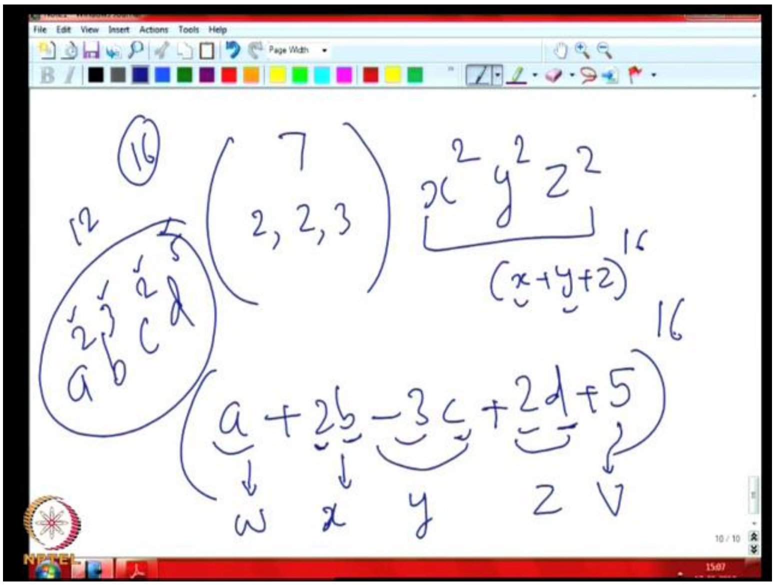

A square the coefficient, we are interested in the coefficient of a square b a square b cube c square d 5 b cube c square d 5. So, in this problem you can see that, here this is not like x plus y plus z whole raise to 16 now, where you could have a directly argued. But, here, the thing is a comes alone but, b come with a coefficient already here, c c comes with a different coefficients, b comes with a different coefficients and so on right. So, a plus b plus so, now what we do is, we convert it to some other variables, let us make it x, let us make this y and let us make this stuff z and let us make this say, w right we can w x y z we can write.

So, we can write the this is w, this is x, this is y, this can this can be z and then, one more is there of case so, that v v also we can set w x y z v right, this also corresponds to a variable suppose right. Now, the point is here, if you add up 2 plus 3 plus 2 plus 5, what we get, you only get 7 plus 10 plus,12 only we will get right.Here,16 is required so because, the total power is 16.

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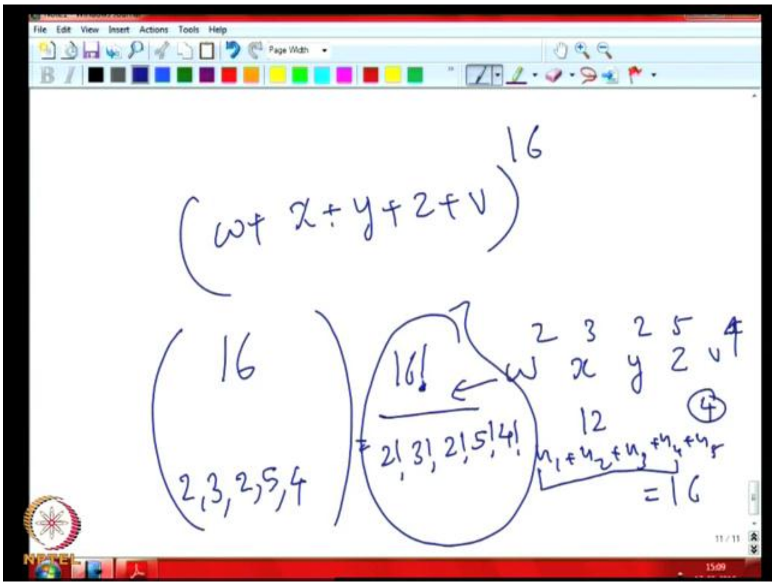

So, in the expansion of x plus sorry w plus x plus y plus z plus v whole power 16,every terms should be of this form the right w raise to n 1,x 0 is x raise to n 2,y raise to n 3 and z raise to n 4 and v raise to n 5. And then, when we add up this n 1 plus n 2 plus n 3 plus n 4 plus n 5, we should get 16 right but here, we are getting, here12 right 12.So, this has to be 4 this has to be 4 so, we are looking for the coefficient of coefficient of y a yes as we want, 2 3 2 5 2 3 2 5 and 5,10 this is total I am sorry 2 3 2 5 4, this is 4 right 2 3 2 5 4 right.

Now, this we know, is this number 16, 2 3 2 5 4 right this is 16 factorial divided by 2 factorial into 3 factorial into 2 factorial into 5 factorial into 4 factorial right, this is what we we will get right.

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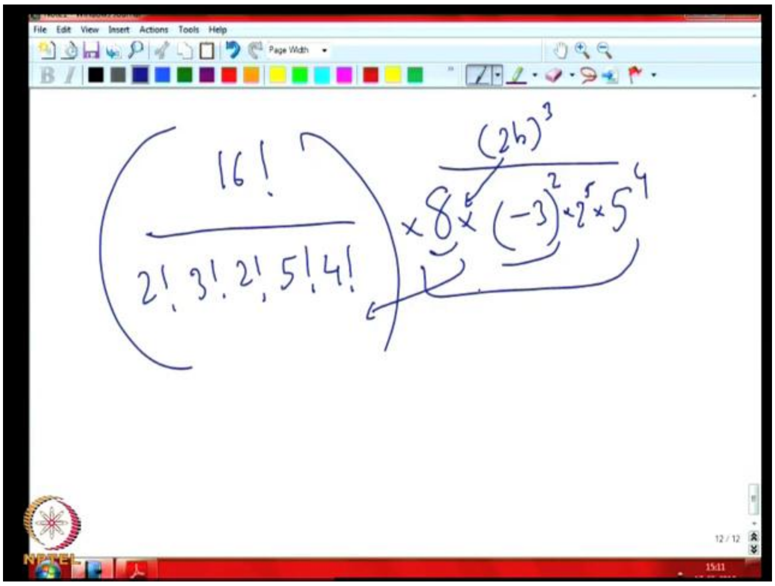

2 factorial into 3 factorial into 2 factorial into5 factorial into 4 factorial right 5 factorial into 4 factorial will be the number here. But then, this will be the coefficient of this number x raise w raise to 2 but, w as such, we know is a and that is a square we are looking for a square so, it will be like that. But, x raise to 3 will corresponds to 2 b raise to 3, that will contribute another 8 into this another 8 into this. So, another 8 will come because, this 2 b raise to 3 so, b raise to 3 and then, this 2 raise to 3 will become 8 here right.

Another y square, what y square and z raise to 5 are there, y square is y y is minus 3 square will becoming from from the y square and this 2 raise to 5 will be coming from this. And then finally, we have 5 raise to 4 also right so, that is what so, there will be a 5 raise to 4 in the end, minus 3 raise to 2 and then, 2 raise to 5 also 2 raise to 52 raise to 5, this you have to multiply with this stuff. And then, we will get the coefficient that is what, I hope I have calculated correctly so, it is it is nothing big, it is just that so, that corresponding powers of x y z and w we have here. Because, there some constants are involved there, their power should also be considered as constants and they will they will come here that is what right so, then, next one is this.

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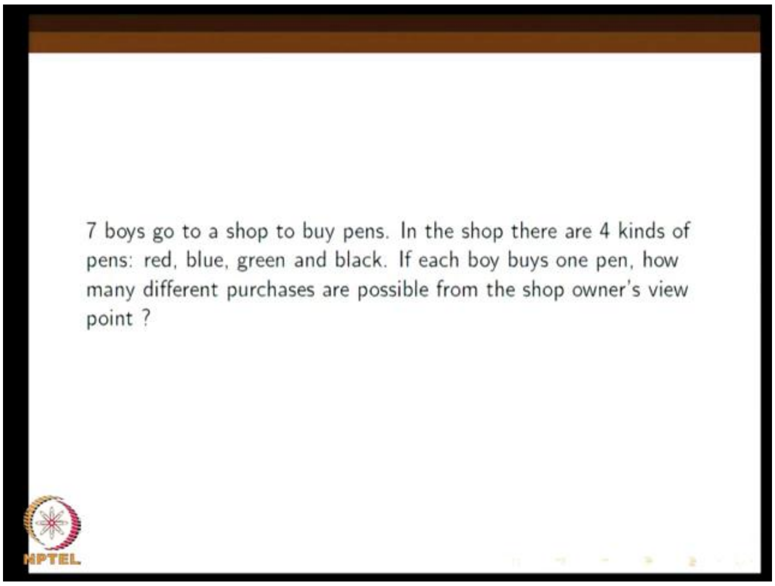

Combinations with repetitions now, we will take another topic so, let us say, just a different kind of problem. So, we have suppose,7 boys go to a shop and they all wants to buy pens, in the shop there are only 4 kinds of pens, there are there are red pens, there are blue pens and there are green pens and then, there are black pens in abandons. How many the number of pens they want is actually, how many ever pens they want in of a certain type, they will get red, blue, green and black.

If each boy buys one pen, how many different purchases are possible from the shop owners view point of view, this is what it is what say right. So, shop owners point of view means, what do shop owner thinks means, he does not it does not matter to him which boy buys which pen. So, what matters to him is, the number of pens of the type red how many are bought, number of pens of the type say, black black pens how many are bought. And then, how many blue pens are bought and how many green pens are bought, that these are the only things which he wants.

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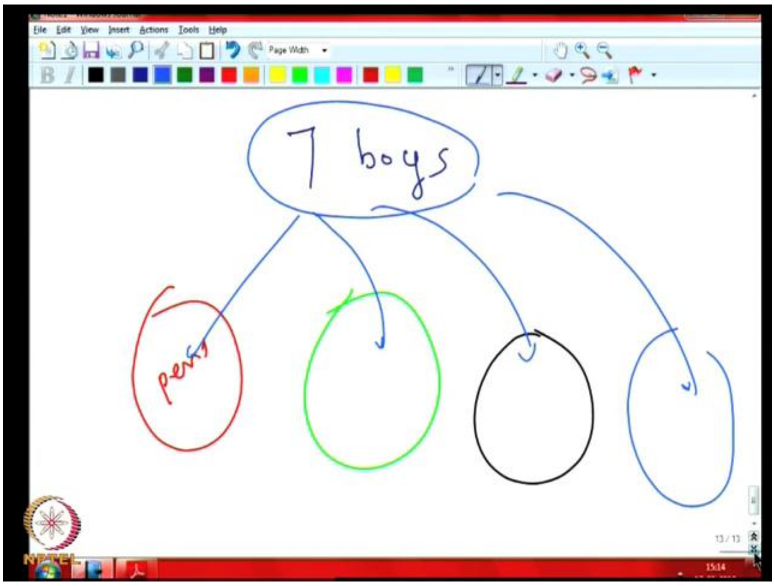

So, now, there are 7 boys, 7 boys there and then, 7 things are bought right and and the question is, there is this red pens the red pens and then here, we have sorry we have green pens and then, we have black pens and and we have black pens and we have blue pens right. And the question is, which boy goes to which pen right, buys which pen so, we can think that, the boys are indistinguishable in the sense that, over buys, it does not matter. And how many how many of the 7 boys will buy this many pen this is how, you want. So, what we do is, to solve this problem, we will set up a bijection as follows.

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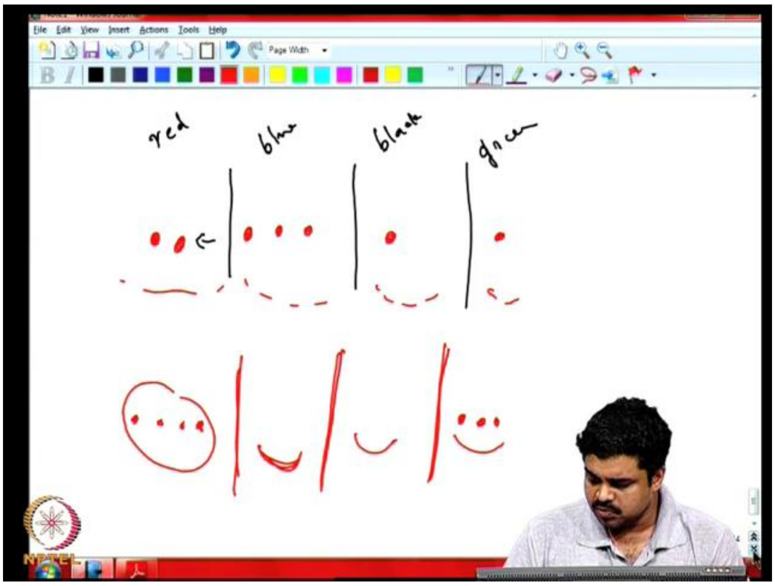

Suppose, see we will first create first draw3 bars right and we will say that, this is this correspond to red pens, on before this thing, whatever I write is the red pens and this is for blue pens and this is for black pens and this is for green pens right. Now, if 2 boys purchase red pen, we will put the 2 boys here, if some 3 boys purchase blue pen then, we will put them here.

And then so that means, total 5 and then, remaining say, 1 boy purchases this and one boy purchases, we we just put them like this, we we just separate I mean, this first part correspond to the the red pens purchased and this corresponds to the blue pens purchased and this correspond to the black pens, this can be the one configuration. It can be like write 34 pens are purchased of the red type but then, now blue pen is purchased, now black pen is purchased and 3 pens are purchased on the this is another arrangement right.

And then, similarly, any for instance, for any purchase can be put like this right and here, there will be a certain number of things and here, these bars corresponds to the regions right for instance blue pens, any blue pen purchase should be put here then, any black pen purchase should be put here as a dot and shown right. So, also if you draw a diagram of this sort where, there are7 dots and3 bars right, that will corresponds correspond to a purchasing pattern. Why because, you can just see this region and count the number of dots in that region, that is the number of pens purchased in the red category. And you look at this this region and how many number of dots I will put here, this is the number of pens purchased in the second blue type and this is the number of pens purchased in the black type and this is the number of pens purchased in the green type right. And then, this is the this is the way we can map I mean, this is the bijection we are setting up with a different kind of problem. Here, the problem is what, the problem is this.

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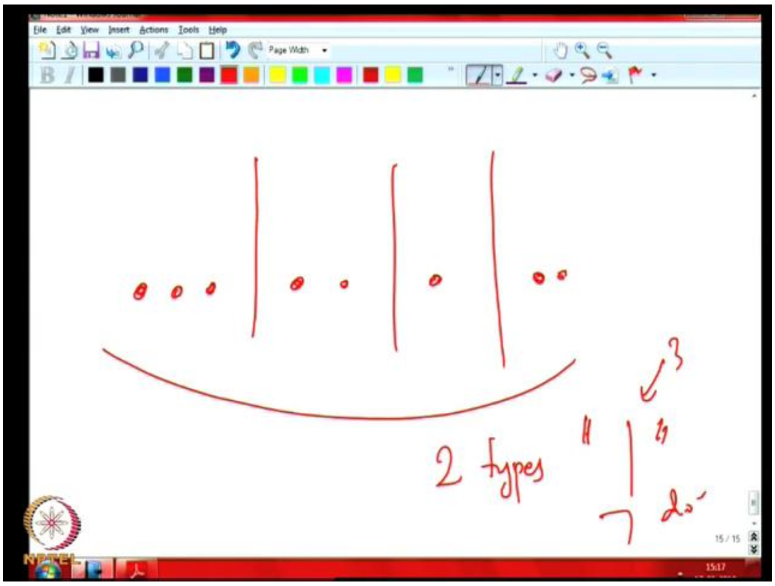

Here, the problem is just that, we are trying to arrange 3 bars and 7 dots, 7 dots in a line right 7 dots in line. So, there are in this case, the problem is that there are two type of things two types of things we have to worry about it. One is this bars right a type one type is this, there are three of them and then, the other type of thing is 7 dots right. Here, I have dot dot one more,7 dots we have to.

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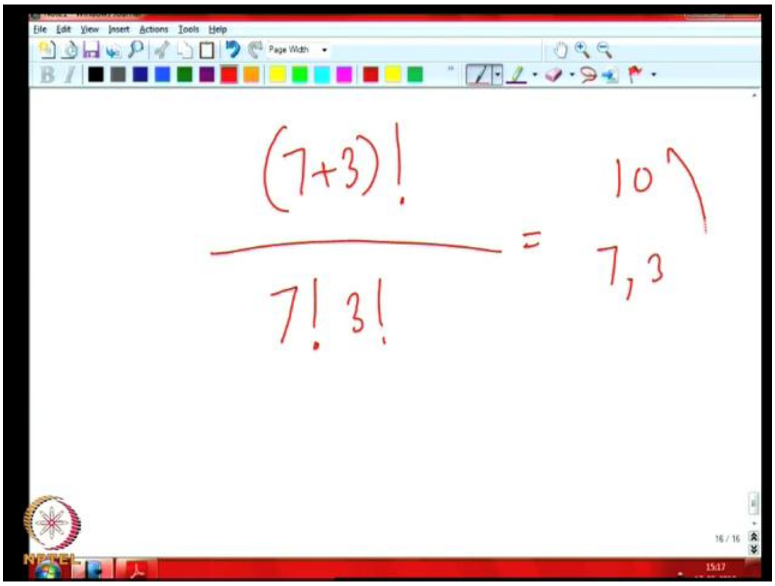

So now, we know that, the 7 dots and 3 bars can be arranged in 7 plus 3 factorial divided by 7 factorial into 3 factorial ways. This is the multinomial coefficient 1 0 7 3, which we have studied right so, that all those arrangements.

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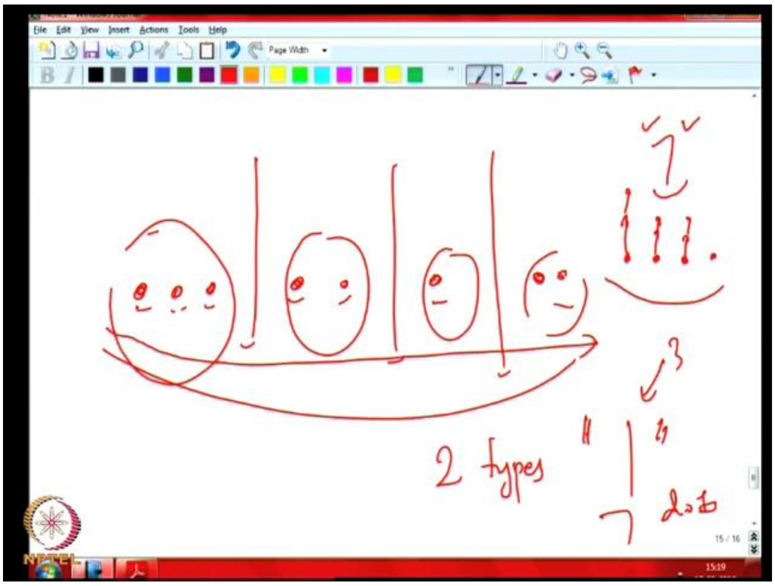

The point here is, here it is this is an arrangement order, order matters here in what in what way they will put it, just that these are indistinguishable from each other. And these things are in distinguishable from each other but, the way the relative order matters other than that right then right. So, this is this is the way, things happen right and then, on the other hand, the other problem was just that, how many ways we can distribute it to four different categories.

So, there it was we were not really bothered about the order of anything, it is just that combinations right. We are actually considering a multiset of pens right, we are we we want to create using we have to select seven things right not a multiset in fact, we are saying that, we have to select seven things but then, there are only. So, we can say that, only four things are available, red thing, blue thing, black thing and green thing.

And then, but then, the only thing is, they can be repeated, here the green thing can be repeated any number of times, blue thing can be repeated any number of times, black thing can be repeated any number of times and so on right. So, this is this is what, is what we are doing here so, therefore, we mapping into this thing, we got the number correctly.

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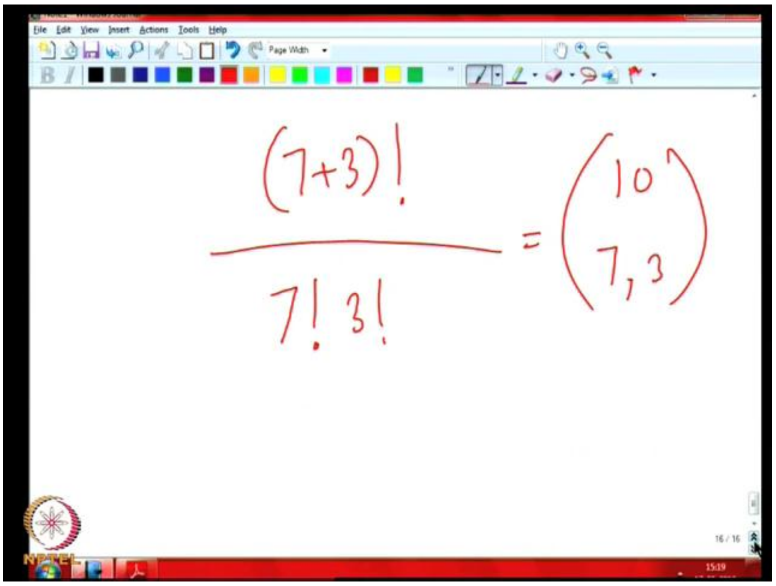

This is the multinomial coefficient 10, 7, 3 and this we can easily generalize.

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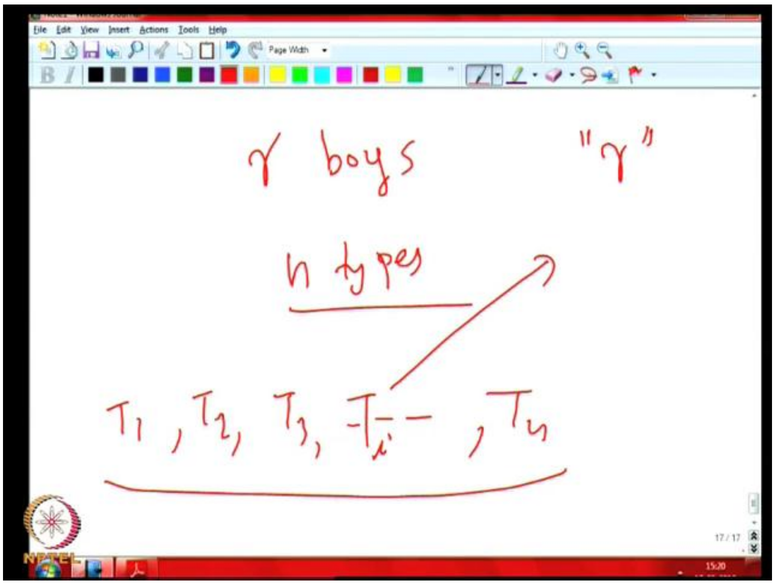

Suppose, where there were n boys right and the pens were like of k types sorry so, let us say, we can use r and this is r boys and they were the pens were of n types right, instead of a 4 type of pens, we have n type of pens here. So, the first type 1, type 2, type 3 and type n and then, this n type of pens are there right but any number, you can select any number of time for instance, type i pen can be taken any number of times, if you if you wish. Just that we have to finally make r pens, take r pens out of them, how many ways you can make it.

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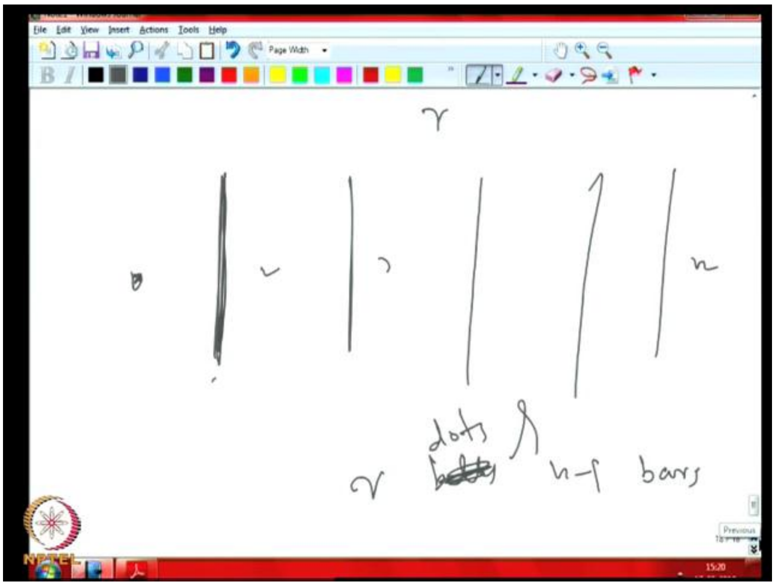

Again the question can be map to the same kind of thing here, we what for the n pens, to create n regions for pens, we can use n minus 1 bars. Because, n if you put n minus 1 bar say, region 1, region 2, region 3 so, this will be a region n so, n minus 1 bars will create n regions right. And now, in these regions, we place the balls in whatever way we want I mean, how many balls n r balls should be placed in whatever way. That will for instance, any way of placing, it will correspond to one linear ordering of ordering of r balls and n minus 1 bars so, r dots r dots and n minus 1 bars right.

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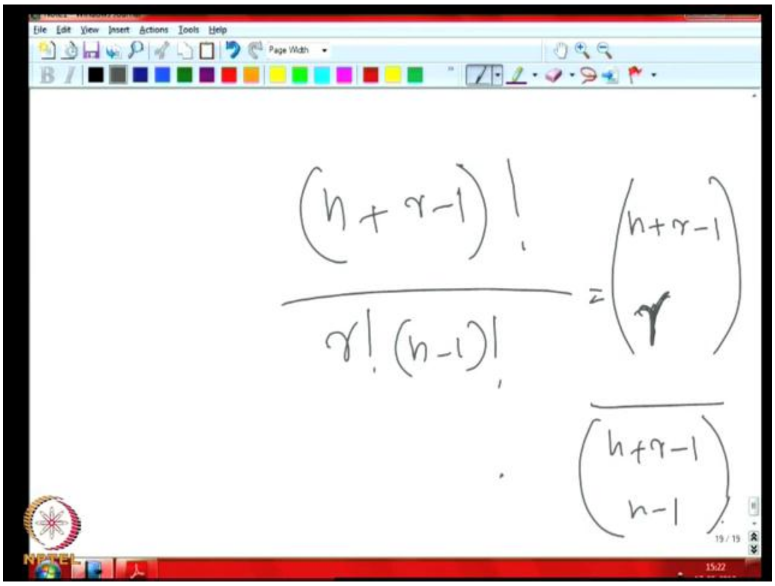

So, which means that, the total number of ways we can do this thing is n plus r minus 1 factorial divided by n factorial into r factorial r minus 1 factorial this is sorry sorry this is because, r factorial into n minus 1 factorial you see, we have n minus 1 bars and r dots right. So, this is essentially, that multinomial coefficient n plus r minus 1 r comma n minus r right this is the thing, this is. So, therefore, we can generalize it to this formula, when when there are we can tell that, there are when there are n boys sorry we we so, the it something like this.

We have to select n r pens from n type of pens right or may be there are n things and then, every each thing can be taken many times, that is what we are saying and type of things means right. So, when we have to make an r selection of size r then, we can do it in n plus r minus 1 choose sorry sorry, this this is essentially which because, it is not multinomial coefficient, this is just a binomial coefficient also because, this can be written as n plus r minus 1 choose r right, it is what.

So, as we have discuss just before this r, when we see it as a multinomial coefficient, this is n plus r minus 1 n n minus r n minus 1 because, this is add up to this. But, we all know that, this is binomial coefficient only so, n plus r minus 1 choose r right. Or it can be written as n plus r minus 1 choose n minus 1, that is also fine, this is also equally fine right and then, the next one is the next one is another example.

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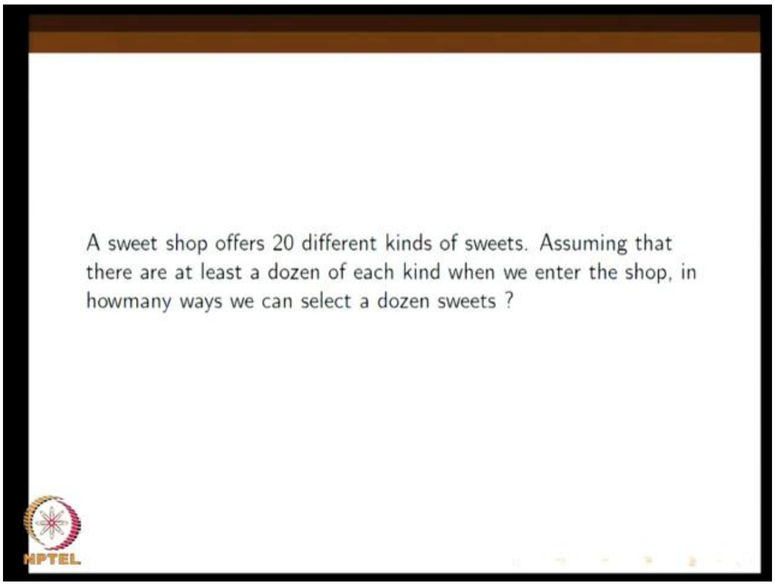

So, let us say, a sweet shop offers 20 different kinds of sweets assuming that, there are atleast a dozen of each kind when we enter the shop, in how many ways we can select a dozen sweets right. See the thing is, there are sweets, different variety of sweets, 20 different variety of sweets, the same kind of say, here we have instead of buying pens, we are buying sweets right. But, we have to buy dozen that means, 12 sweets we have to buy, see when I says 12 sweet, not 12 different type of sweets, we can buy the same sweet more than once right, we can buy 3 laddus, 5 jalebis and things like that right.

So, and total 12 we have to make but then, there are 20 different varieties, the only thing is, we are worried is for, is it possible that I can buy all of them at the same time so, is it possible for me to buy 12 laddus, we are saying that, yes it is true. So, because for each of this kind, we have atleast 12 there so, we do not have to assume that, there are infinite number of things available, that we are telling in the last thing. As long as we are only going to select r things, total r things as long as each type has r copies, that is more than enough for us right.

So, otherwise, we are we are in a trouble because, some some combinations we cannot make, some selections we cannot make. For instance, when we want to buy a dozen things and if we buy, if we try to take all laddus then, it may so happen that, there only 11 laddus and then, we cannot make 12 laddus right. So, that will decrease from the count, that is what it is so so then different so then, how will you solve this question, it is it is quite it is almost same thing that we did in the last stuff here.

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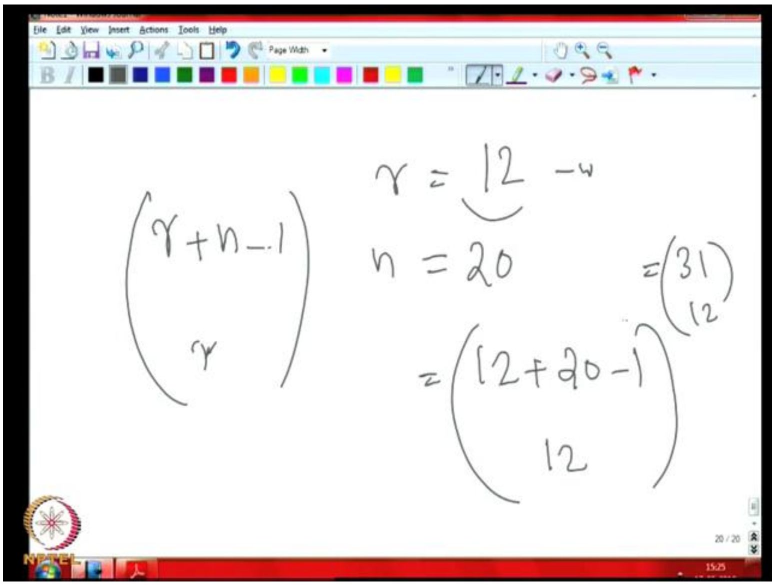

Instead of 7 pens, we have 20 sweets we have to select so, that r here is equal to 20 and then, the types that is n, n is equal to sorry we have to select 12 sweets, a dozen sweets and the types is 20, n equal to 20. So therefore, we know the number of selections we can make is r plus n minus 1 choose r that is, 12 plus 20 minus 1, choose 12 this is what we can make right that is, 31 choose 12, 31 choose 12.

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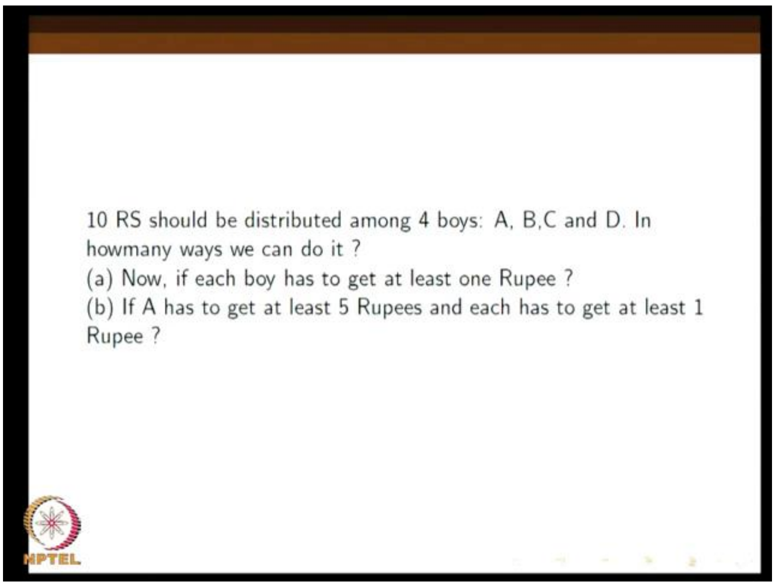

So, 10 rupees should be distributed among 4 boys a, b, c and d this is another question, in how many ways can we do this thing. This is the same question, we have we have 10 things see, asked in a different way so, earlier, we were buying sweets we are taking things from here, here we are distributing things back. For in the sense that, we are we can think that, we are when we give 5 rupees to boy a that means, we are selecting the boy a 5 times right to give first rupee 1 rupee is given to a that means, boy selected once, the second rupee is given the boy selected a second time and then, the third rupee is given he selected a third time and so on.

So, therefore, the question is same, just that we are we want to select these boys 10 times, each boy we can repeat the boy right that the some see for instance, a particular boy can be repeated several times, that is the only difference. So, therefore, here what do we see so, the question is very simple, instead of pens we have boys here, how many type of pens, this is the number of boys.

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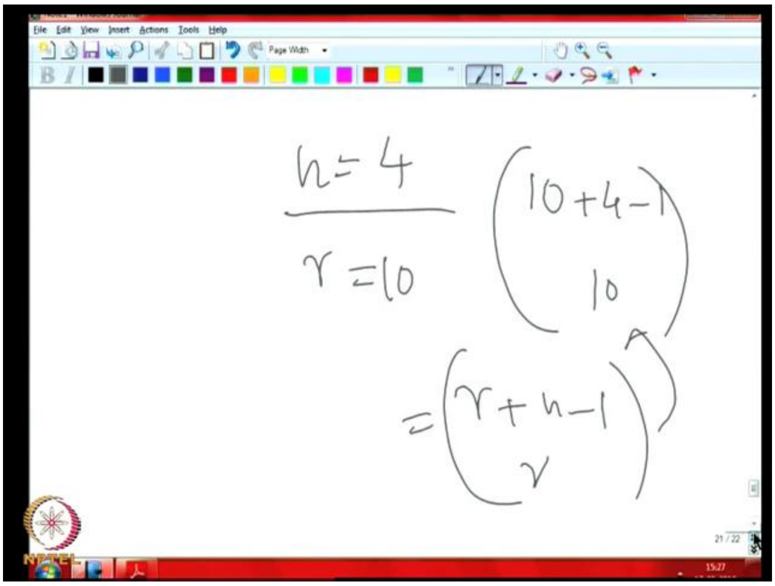

That is, n equal to 4 right and the how many pens are to be bought that means, 7 in the initial problem or dozen, 12 in the second problem, that corresponds to10 rupees here, it is right r equal to 10 here. So, in how many ways, you can distribute it that is, 10 plus 4 minus 1 choose 10 right, this is the total possible ways. And this is this is the formula r plus n minus 1 choose r, this is the possible ways right.

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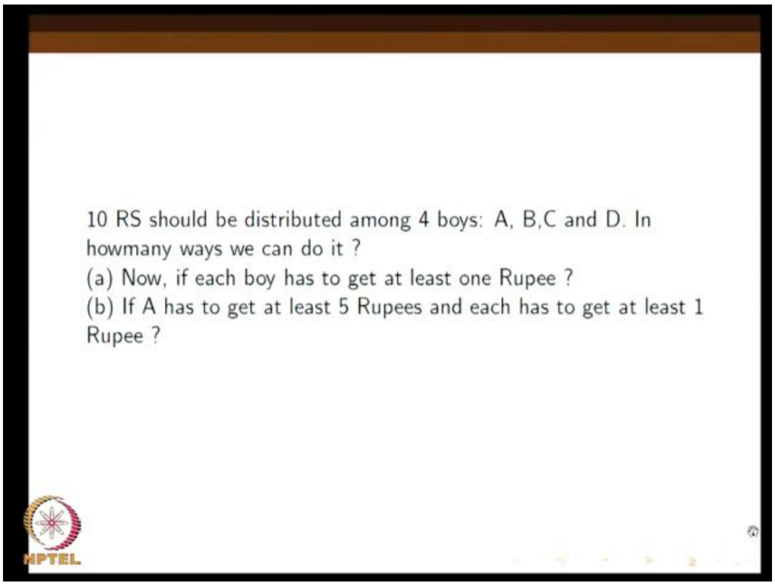

So now, we this is a question we have already seen several times now, we are asking a slightly different question. Suppose, if each boy has to get a atleast 1 rupee for instance, see we called 4 boys and then, we do not give anything to some of them, that is not good. So, we does said that, atleast 1 rupee should be given to anyone then, you can do anything you like then, how will you do it. Now, the trick is, just that also it is very easy to do this thing first first distribute.

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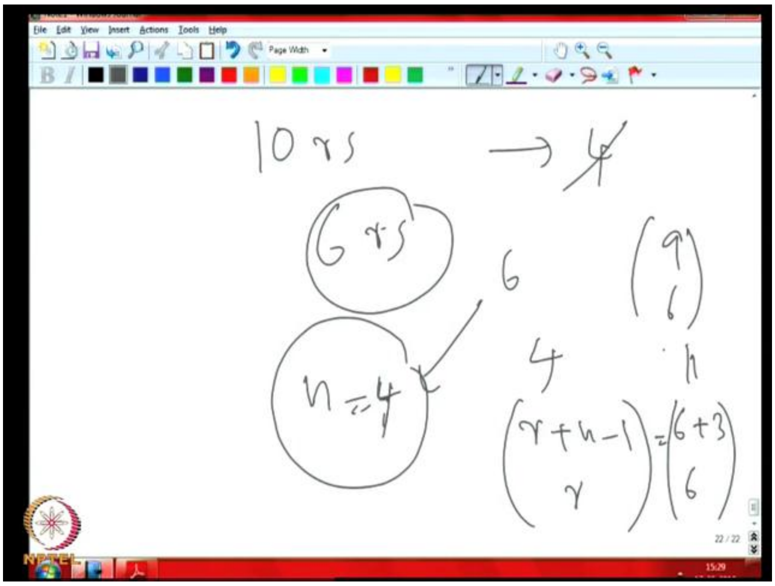

We have 10 rupees with us then, first if distribute 1 rupee to each boy that means, 4 rupees have gone right. Now, only 6 rupees are left and the question is, you can distribute this 6 rupees to 4 boys now we can also not we have the permission, not to give any rupee any money to any some boys right it is it is. So, therefore, here then, 6 and 4 are the numbers, 4 type number of type, number of type types we have is 4. That means, n equal to 4, it is like we have 4 objects, each object is allowed to repeat, 4 type of object on each object z n type is allowed to repeat.

And then, we have to take it 6 times you know, though rupees are given to the boys, it is like we can think that, 6 boys are selected for each rupee the boys are selected to be that that rupee to be given that is all. So then, the answer is, again r plus n minus 1 choose r, this is 6 plus 6 plus 4 minus 1 is 3 choose 6 right 9 choose 6, this is the answer for this right.

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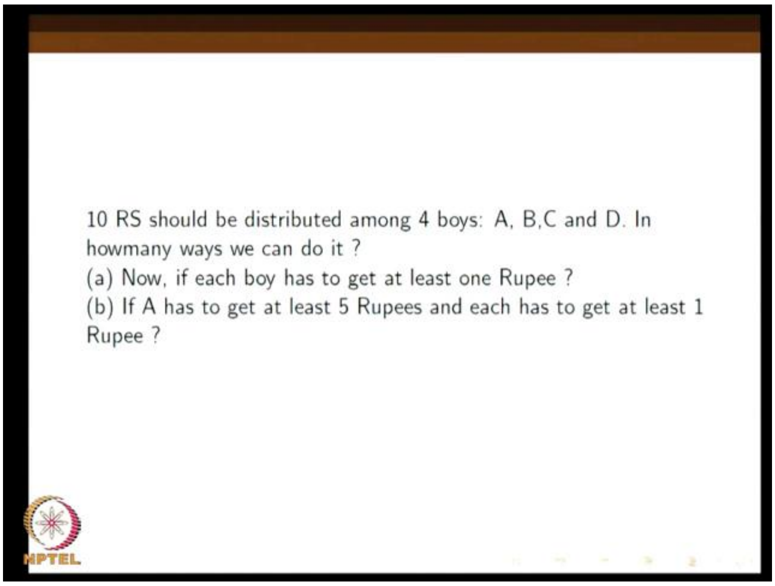

And then, next question we want to consider is a similar one but, a little more sophistication add added here. If a has to get atleast 5 rupees and each has to get atleast one rupee if a has to get at least 5 rupee andeach has to get at least 1 rupee then, what.

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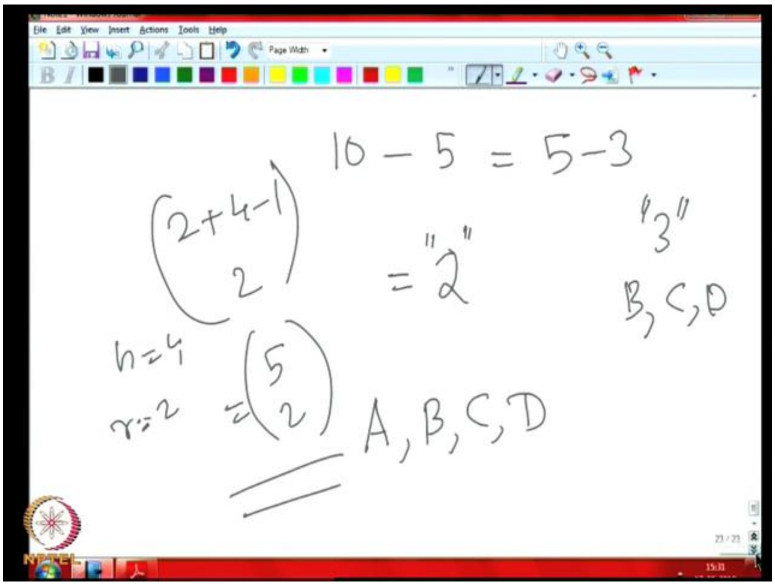

So then, the same idea can be used so, a has to be given 5 rupees so, give it to him so, out of 10, we have already finished 5 rupees now, we have only 5 rupees left. Now, each has to get 1 rupees, a already got 5 rupee then, we do not have to worry about them, there are 3 more boys b, c and d so, they should get 1 rupee each. So, give them, the 3 rupees is used to that use for that then, we have 2 rupees left. Now, the question is, how do you distribute this 2 rupees among 4children where, we are allowed not to give any money to some boys right some boys.

So, a, b, c, d it is clear that, we have to do that now, because, 2 only 2 rupees and 4 boys, some two of them atleast will not get anything right. So but, this can be done in how many ways so because, here the number of types is again n equal to 4, r equal to 2. So, we have 2 plus 4 minus 1 choose 2 ways of doing this thing that is, 5 choose 2 ways of doing this right 5 choose 2 ways of doing this right. so, right.

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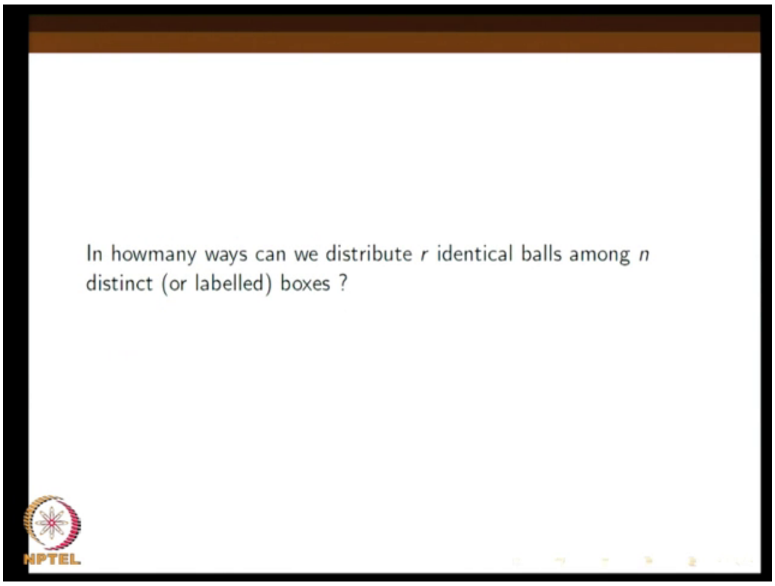

Now, the next one so, we this this this problem has some idea here in the sense that, we are we can refresh the, see the question like in the earlier one, we can say like this suppose, we have to select n things out of n type of things right with repetition. In the sense that, we are allowed to that certain type we can repeat any number of times, we we are not restricted that, we can take it again and again.

But, we have to make r of them right and then, but, we put the extra put the extra condition that, we put the extra condition that, atleast one should be given to each sorry would be taken of each type atleast one should be taken of each type then, what will happen, there are total r things to be selected right.

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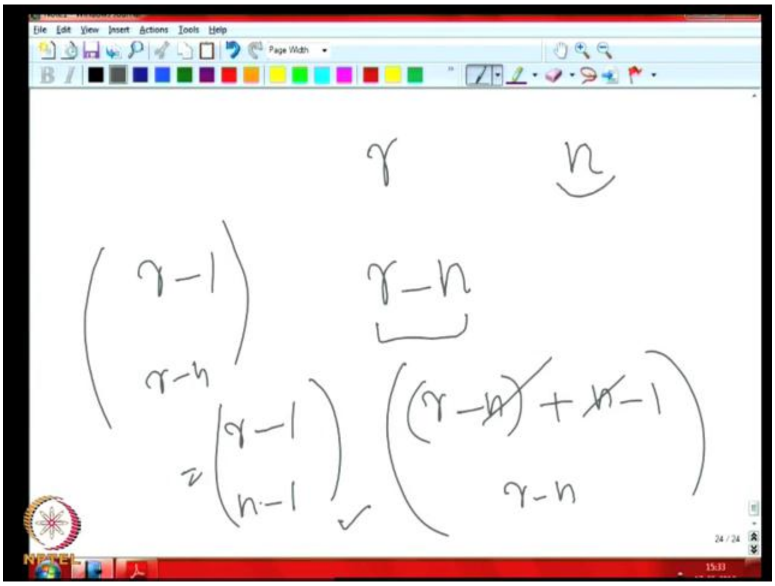

N types are there then, you know because, for each type we have to select one so, we have to already we have to anyway select it. So, we will we will after that selection, we have only r minus n things to be selected but, this r minus n things can be selected in such a way that, we can avoid some type also, we can take or not take right it does not matters, even 0 number of things can be taken of a certain type. So, that will be r minus n instead of r, we have r minus n plus now, number of times is n n, n minus 1 choose r minus 1 right r minus n right, how much is this.

So, here n will cancel out, that will just become r minus 1 choose r minus n sorry so, we can we can also put it as r minus 1 choose n minus 1, if you want and because, if you add this thing, we will get the r minus 1 right. By symmetry pool, we can also write it like this so, it is a this is a Nizer formula because, it comes it looks Nizer atleast.

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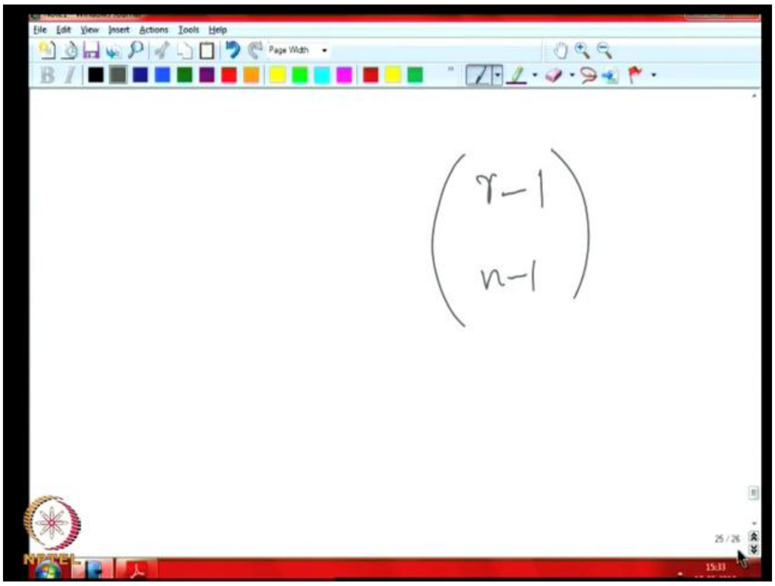

R minus 1 choose n minus 1 right here, the only condition is that, every type has to be taken atleast once, that is the only extra finishing deport right. We know the technique is there, we also so, even more complicated conditions can be put, though we do not attempt to get a general formula for all these things all those things right.

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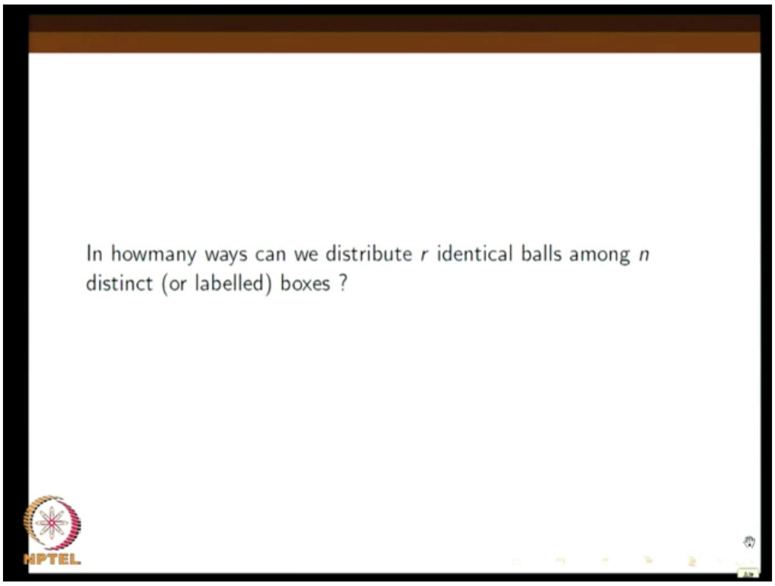

And then now, let us see a different formulation of this problem so, which is also, yes I was telling in combinatorics, it is very common to talk of balls and beans problems and so many. So, here, this is the formulation in terms of balls and beans, we have r identical balls and n distinct labeled boxes, in how many ways we can distribute this r identical boxes among n distinct labeled boxes right. So, this r identical boxes now, corresponds to the selection of r that is what, and and n distinct labeled boxes corresponds to the types, that is what we want to tell. So, we will show that, this is the same problem.

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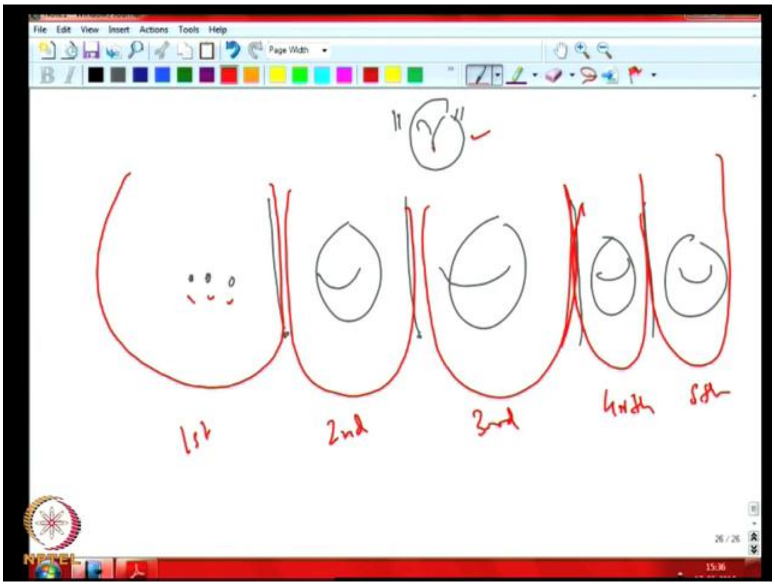

Because, you know when we wanted to select r things from n different type, what we did is, we drew n minus 1 bars. So, that we create it total n regions here and this r selections, it depending on how many selections are taken from this type this type, this type, this type and this type, we we we decided to put so many dots here right. Now, when we put these dots, we can also think of see now, so many dots are put so, like we can think that, for instance, we can create say, this is a box and this is another box, this is another box we can imagine like this, this is another box, this is another box right.

So, in each region can be considered as a box and these are labeled boxes because, this is the first box, this is the second box, this is the third box, this is the fourth box and this is the fifth box and so on right. Now, we want to put this r identical because, here it does not matter because, they are just dots in a, we just have r dots, that r dots should be placed in some way, it is just r identical balls being placed in these boxes, that is what it says right it does not so, only the relative numbers which goes to the boxes that.

So, you can clearly see that, there is a bijection between this and this problem, the earlier problem. Therefore naturally, this question of placing r identical box into n different boxes, distinct boxes distinct boxes means, the boxes are labeled, they can be distinguish from each other. It can be modeled using this r identical boxes and and then, distinct boxes, we can we can model it using this this bar and dot thing and that same the earlier problem also was modeled like this. Actually both have a bijection to this so therefore, both problems are same.

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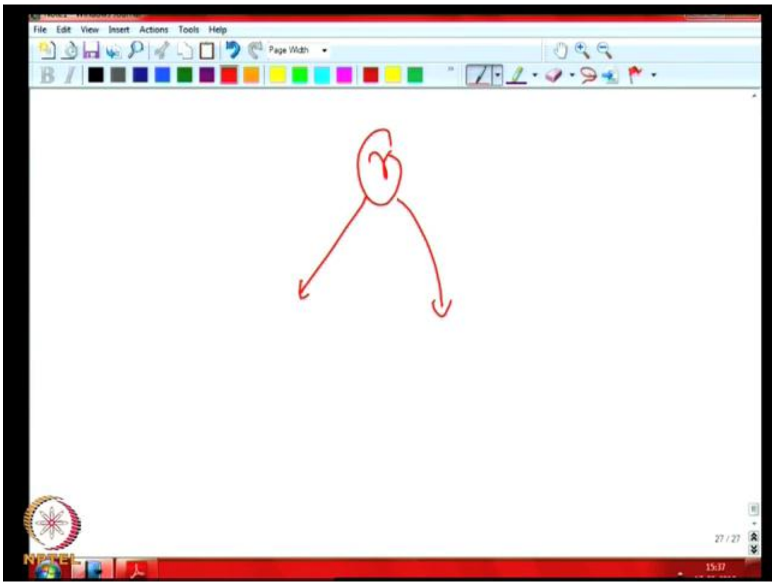

Let us insert so, this here, yes I pointed out, this r identical balls, the number of identical balls corresponds to the size of the selection we want to make, how many things we have to select. And the boxes correspond to the types, each box correspond to a certain type and r balls are placed in 2 boxes r balls are placed in 2 boxes right, that is the correspondence because, it is like when we are taking a certain type, it is as good as placing that ball into that box right that is the that is the way, we are mapping those two concepts.

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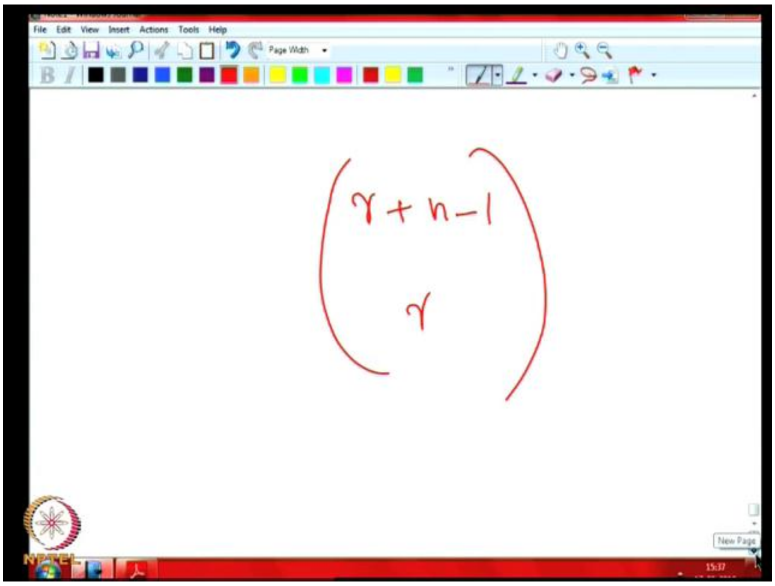

So therefore, this number is also r plus n minus 1 choose r, like the same number as before and the earlier problem will correspond to like this, when we say that, a type a particular type of object should be taken at least once, each type of object should be taken at least once. Here, we are saying that, each box should contain atleast one thing, we should there should not be any non empty box is the corresponding formation. Then, the number will be as we have seen, r minus 1 choose n minus 1, it is also we can remember right.

So, So this is the the balls and beans way talking about the same problem, the problem is same just just that, we we we when we see something like this also, we we have to identify it. At one look it may look different, the two problems may look different but, we think for some time, we will realize that both are one and the same.

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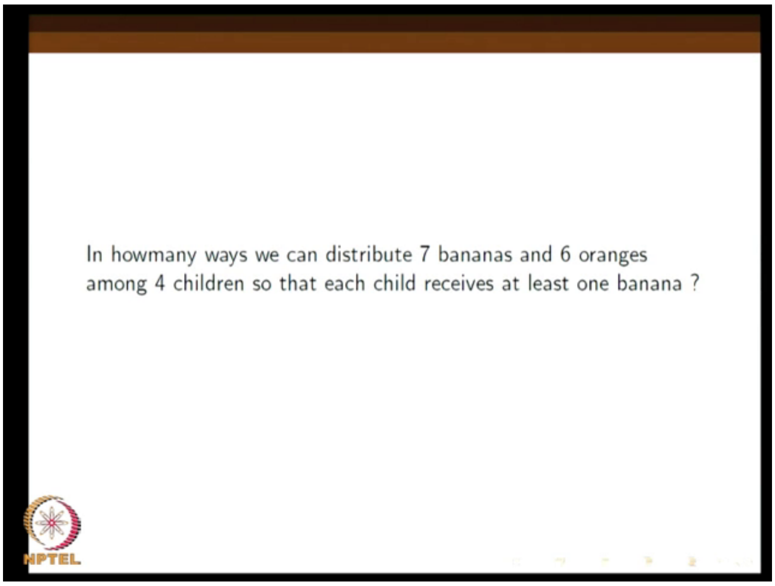

So, it is good to good to remember all the variations of the the the different ways and which it is formulated right, the problems are same, problem is asked right. Sometimes, it will be easier to think in terms of this one or sometimes it can be easier to think in terms of the other way. So now, the next question, in how many ways can we distribute 7 bananas and 6 oranges among 4 children so that, each child receives atleast 1 banana, this is the next question right receives atleast 1 banana.

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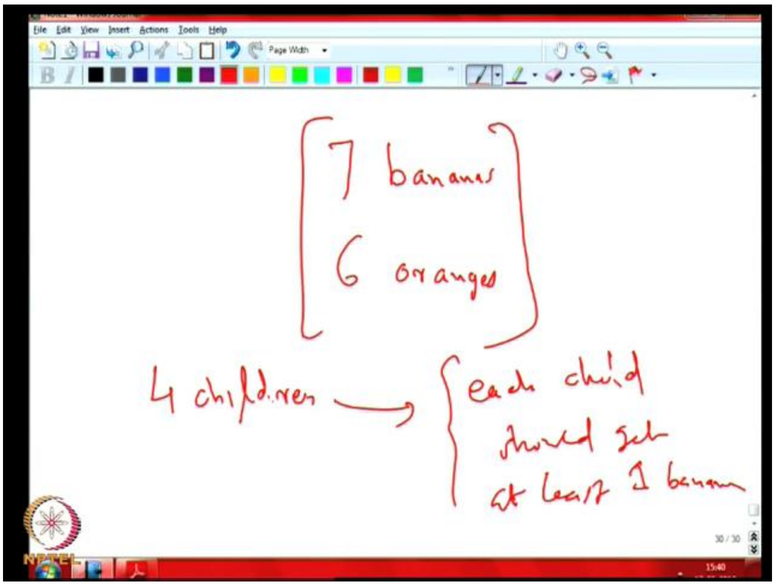

So, this also the same kind of problem but here, we have 7 bananas right and 6 oranges and how many children, 4 children are there right 4 children. And but, the only condition 4 children are there the only condition we have placed is that, each child should get atleast 1 banana right each child should get at least 1 banana right. So, we can start with the bananas so, this question can be attack like this, first we will see how many ways we can distribute the bananas then, we will see in how many ways we can distribute the oranges.

And then, multiply the total number of ways to distribute the bananas with the total number of ways to distribute the oranges. This will give you the total number of ways to distribute 7 bananas and 6 oranges.

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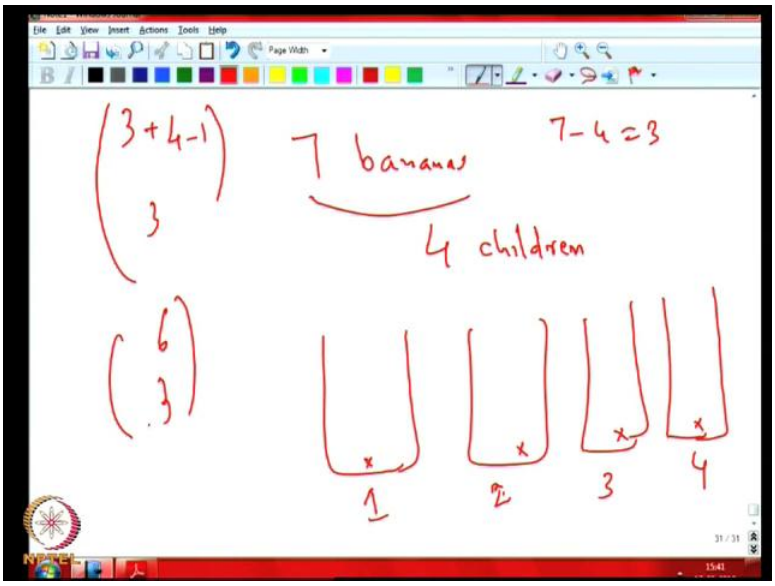

This is the application of multiplication principle, first question 7 bananas should be given to 4 children right. Now, see if each child can now be thought of a of a container or box like the child 1 right child 2. So, I am using the this previous problem to map this one to so, each child, this is a 4 children. And then, 7 bananas but, only thing is, we have to make sure that, all of them get at least 1 so that means, one should go here, one should go here, one should go here, one should go here, one should go here.

So, 7 minus 4, 3 only is left now, 3 should be distributed to 4 children now we know, 3 equal to r right, the r identical balls because, bananas are identical here. The bananas corresponds to the identical balls and the children correspond to the beans right. So, 4 plus 3 plus 4 minus 1 choose 3 that means, 6 choose 3 ways.

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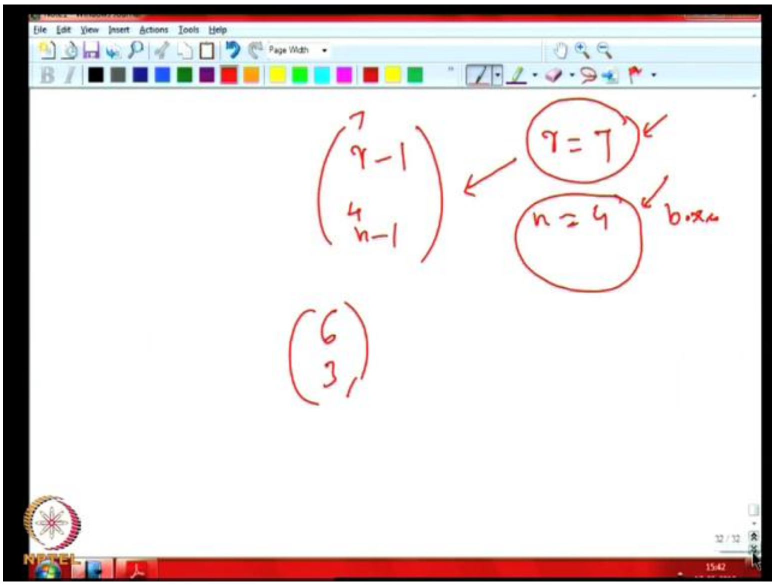

So, we or we could have directly use the previous formula r minus 1 choose n minus 1 also where, r (( )) the 7 bananas n correspond to 4 children. And we are looking at a situation of this 7 identical balls, balls correspond to the bananas and then, children correspond to the boxes, 4 boxes. So that, no boxes empty atleast 1 child 1 banana is given to each child, that would have here we would substitute 7, we would substitute 4 here, that would have given a 6 choose 3 anyway right.

If you if you wanted to use the formula directly, write the formula we derived in the last problem. So, this is 6 choose 3, if you remember it then, other other one the other problem is 6 oranges should be distributed to the 4 children right 6 oranges should be distributed 6 oranges. Now, this is easier because, there is no no extra condition so, 4 children should be considered a 4 boxes now right.

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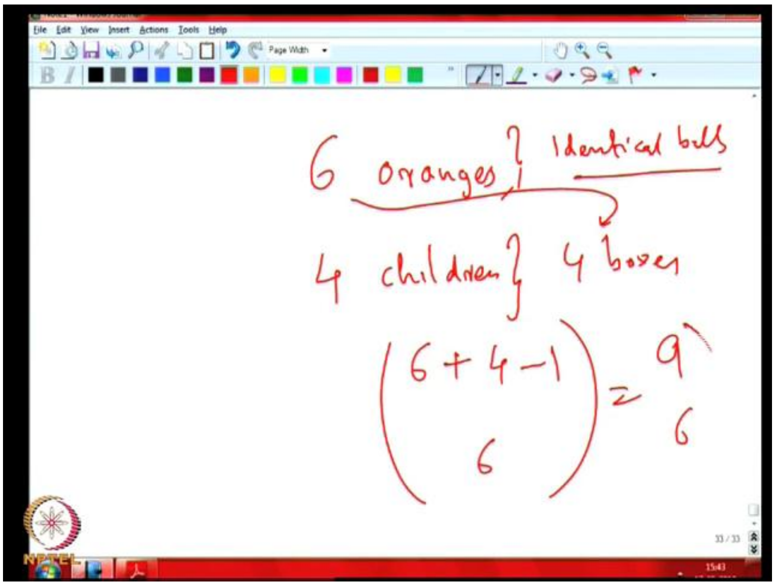

And then, these 6 oranges are the 6 identical balls identical balls right now, 6 identical balls have to be distributed to 4 boxes and that is 6, 6 is r. So, 4 minus 1 choose 6 that is, 9 choose 6 this is the number of ways to do this thing.

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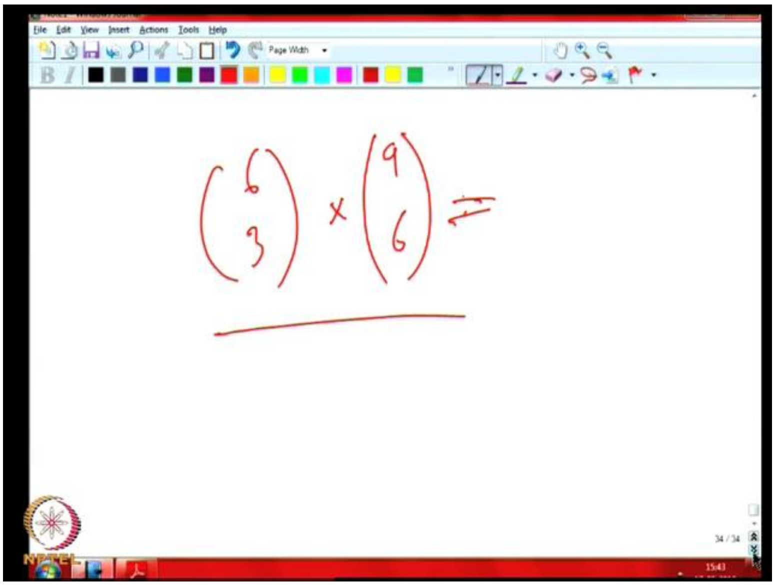

So, that is 6 choose 3 into 9 choose 9 choose 6, 6 choose 3, 9 choose 6, this will be the total number of ways to distribute the bananas and oranges. So, typical problem is that, we have used all the concepts we have studied right and now the next one.

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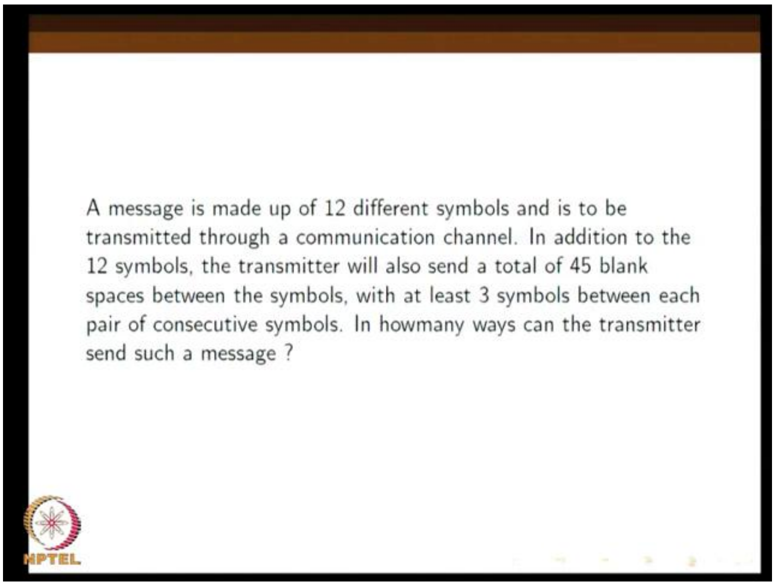

So, here is little longer problem, a message is made up of 12 different symbols and it is to be transmitted to by communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 blanks spaces between the symbols with atleast 3 symbols between each pair of consecutive symbols, in how many ways can the transmitter send a message right.

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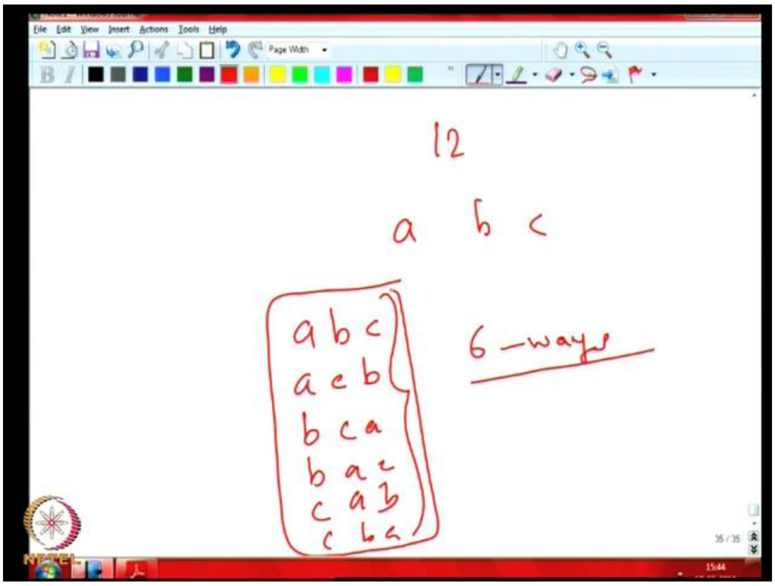

So, what we are asking is suppose, say instead of 12 symbols, let us say 3 symbols a, b, c, this somehow how may messages you can make. So, you may say that, you can make a b c, a c b, these are all possible messages. So, b c a, b a c and c a b, c b a these are the 6 possible ways, 6 ways we can send the, in which we can send the message right, make the messages right.

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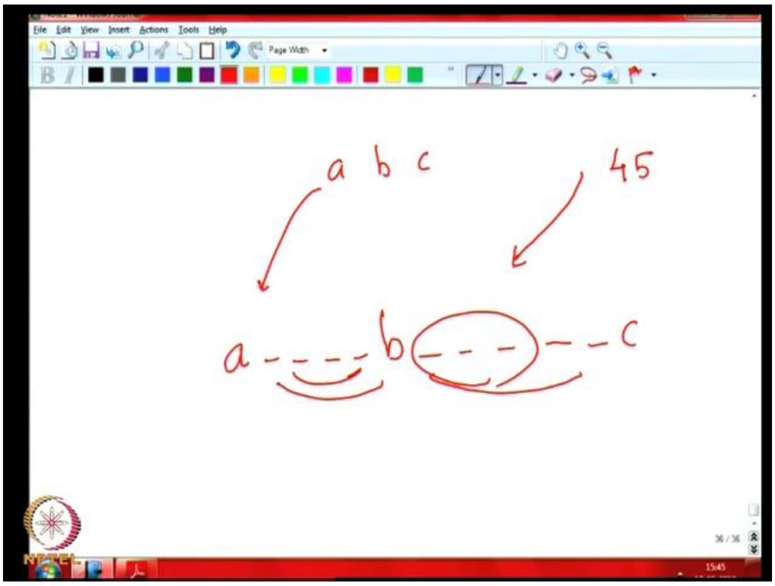

For each message but, the complication is that suppose say, if a b c is a message, the the system, the transmitter will add lot of blank spaces right. So, it is something like, the a is sent first then, the transmitter may add some blank spaces three, four of them and then, b sent then, the transmitter may add a few blank spaces then, again this is like that so, after the last last, this thing is a nothing. So, of the total number of blank spaces send by the transmitter is 45 and the condition is that, atleast 3 blank space will be there between any of them.

It can be from starting from 3 onwards upto 45 not because, you know everybody has to get 3, like after ensuring 3 in between each of them you can see, how many will be left that can be added arbitrarily by the transmitter between any pair of letters.

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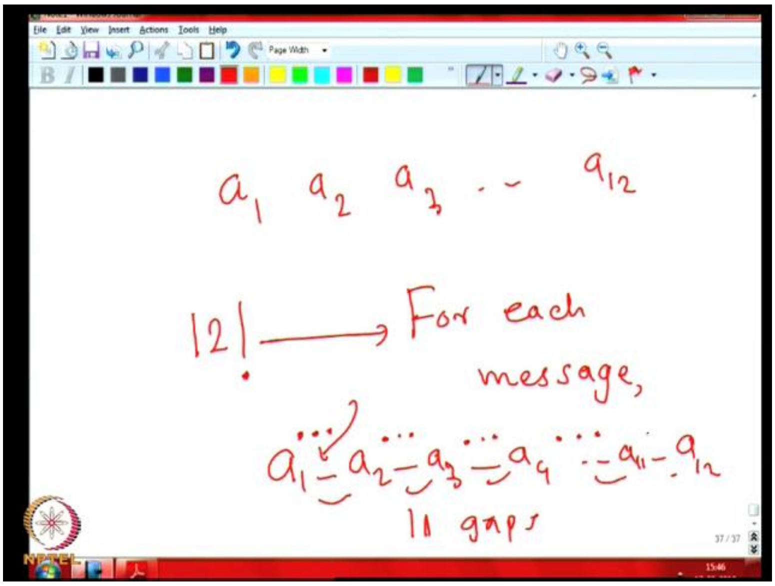

So, this is let us say, a 1, a 2, a 3 so, these are the letters a 12 are the letters we know that, 12 factorial messages can be formed because, there are 12 symbols right. So, for each message for each message, how many gaps are there, how many between the letters, how many gaps are there. See for instance, this is a 1 between a 1 and a 2 there is a gap, a 2 and a 3 there is a gap and between a 3 and a 4 there is a gap and between a 11 and a 12 there is a gap, there are 11 gaps right. 11 spaces between 2 letters and then these spaces, each of them will anyway get 3 blank spaces see, 3 blank spaces will be obtained by each of them right.

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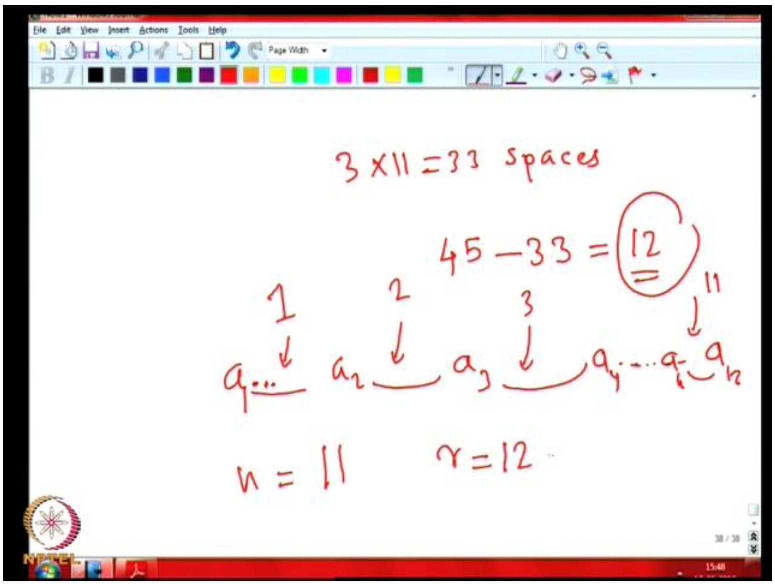

So that means, 3 into 11 3 into 11 this equal to 33 spaces will be anyway use space like that now but, we have total 45 blank spaces that is, 45 minus 33spaces. That means how many, it is 7 plus 5, 12 12 extra spaces has to find some position right and this position has to be say between the message messages. For instance, this is a 1, a 2, a 3, a 4 4 as you can see, till a 12 a 11 a 12 so, as you can see here, this can be consider as boxes right the box 1, this is box 2, this is box 3 and this is box 11 right there are 11 boxes and each box in each box, we can put either 0 or 1 or upto even if necessary all of this 12 spaces right.

Because, you know 3 already we have given and on top of that, we are saying that, all the 12 can be placed in the same gap or 1 can be place or 0 can be placed or any number can be placed out these 12 right. We know that, this is now labeled boxes, there are how many labeled boxes 11, that will be n and the number of identical balls correspond to the number of spaces, that is r equal to 12.

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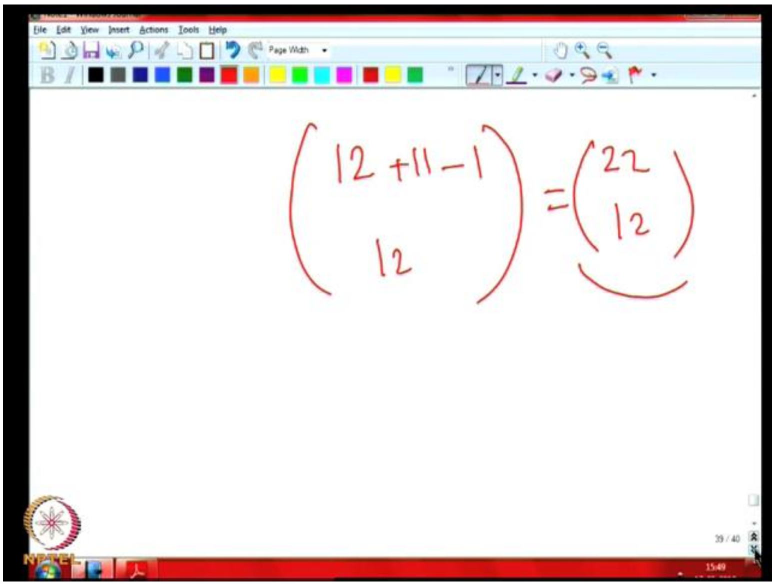

So, the number of ways we can do is 12 plus 11 minus 1 choose 12 that is, 12 plus 10 that is, 22 choose 12 right, this is the this is the number of space we can do that right right. So, if we want to like go back to the original problem of selecting r things from n different type of things, we we we will have to consider this 12, for this 12 blank spaces, we have to select the gaps, which gap should it correspond to it should go into, that is a way, either this one or this one or this one or this one or this one should be selected for each blank space to be put right.

So, we have to select these objects, these 11 objects is a repetition is allowed for for instance, a particular object can be repeated many times but, total we have to select 12 of them. Because, there are 12 spaces that is why that is how, we interpret the situation in terms of the first problem, the original problem we considered. That means, the selecting r things out of m things where, each thing can repeat right. And now, the next one we will consider fine we have no completed the answer here. What we have done is, this is the way we can arrange the blank spaces the number of…

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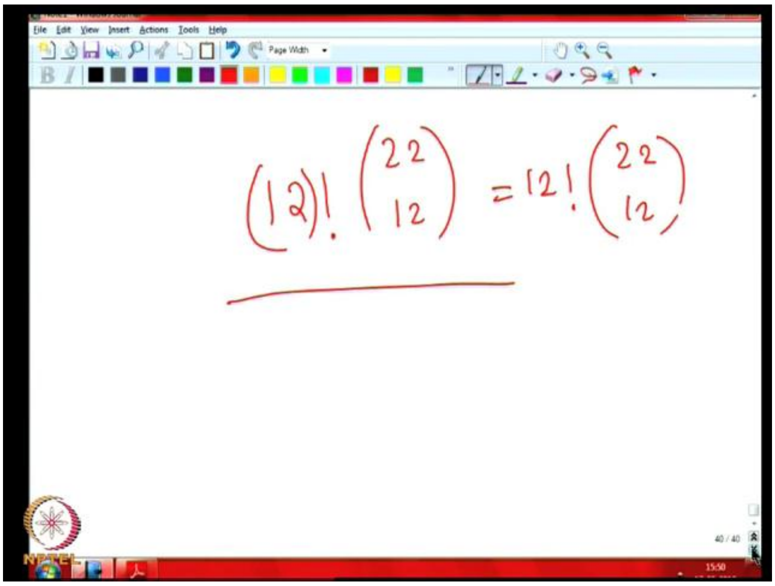

But then, you remember there were 12 factorial messages, for each messages we can do this distribution of blank spaces among the gaps that is, total of 22 choose 12 22 choose 12 right. These many ways we can do this thing, 12 factorial into 22, it choose 12 ways (No audio from 56:08 to 56:16).

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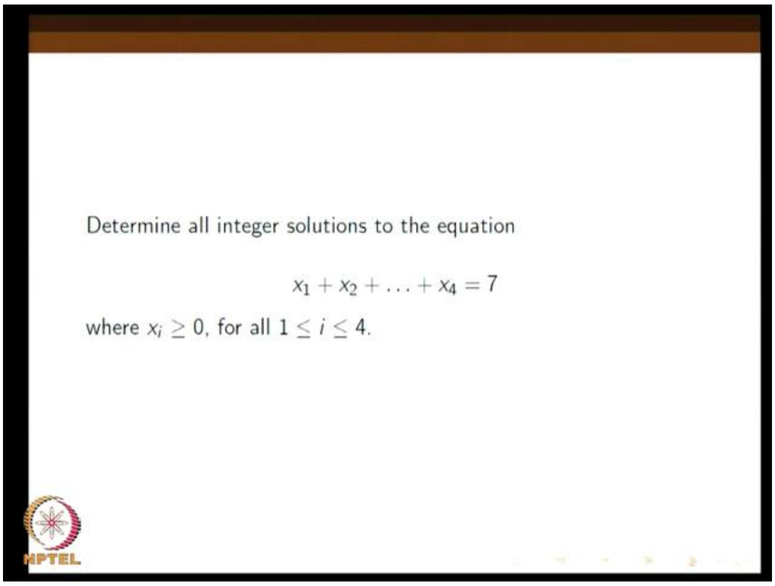

Now, the next question so, the the next we will consider a slightly different way of looking at the same thing or using number theoretic question. So, we want to same, find the integer solutions to this equation x 1 plus x 2 plus x 3 plus x 4 is equal to 7. But, the only thing is, each x i has to be atleast 0, 0, 1 or 2 or 3 or 4 because, we are only up going up to 7. So, x i has to be greater than 0 that means, this can only go up to 7 for each of them, how many ways, how many solutions are there for this thing right, this question we will discuss in the next class.