Computation Complexity Theory
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Lecture-31
Comparison Between Randomized Complexity Classes

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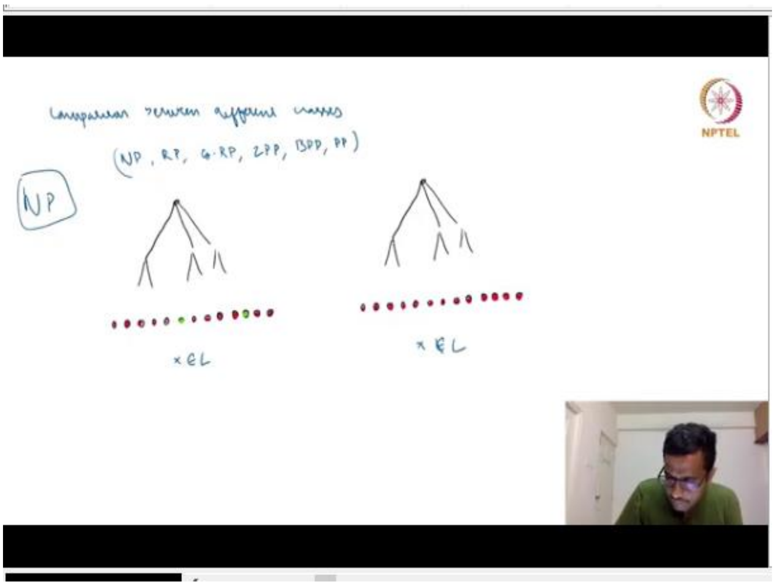

Hello and welcome to lecture 31 of the course computational complexity. In the past lectures we have seen different randomized complexity classes. In this lecture we will see a high level comparison between all of these randomized complexity classes. So, what we will do is we will start with NP as a baseline and then we will move to different randomized complexity classes.

So, let us take NP. So, this is first to begin with I will take NP and there are 2 cases when x is in the language and x is in the not in the language. So, as you know there is a computation tree and there are many non-deterministic choices made. If x is in the language at least one of these non-deterministic choices led to an acceptance. I will mark so this circles denote various outcomes.

So, we know that if x is in the language at least one of them need to acceptance perhaps let us say 2 of them need to accept. So, the acceptance I am denoting by green and for all the rest it will be red. And when x is not in the language, this will all be red because it cannot have even one accepting computation path if x is not in the language. That is the definition of NP. So, now let us see the randomized complexity classes I will just copy paste this. So, now let this move to, so can someone can we think of what co-NP would be? Co-NP is the kind of opposite of this when x is in L maybe I will just write it without drawing a figure.

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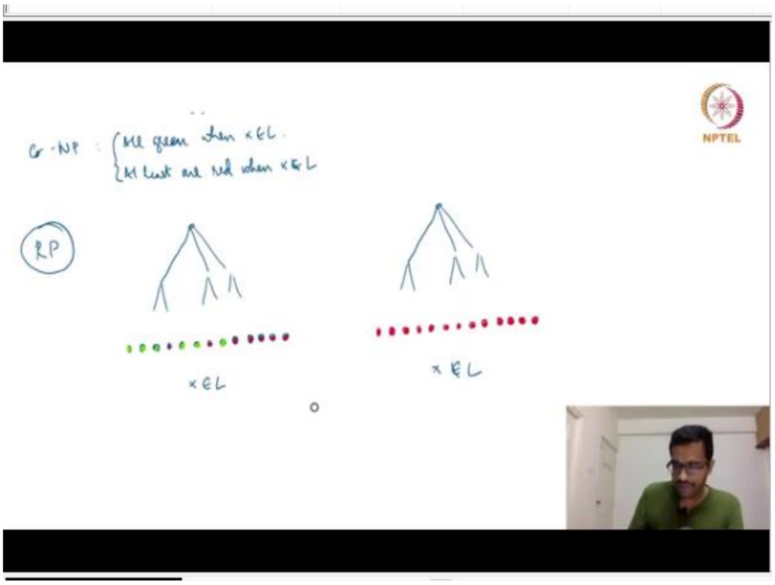

So, co-NP means all green when x is in the language and so this is just co-NP and at least one red when x is not in the language. This is the co-NP and I am not drawing a figure for it, but let us try to understand the colours all changed here that is okay. In the case of now let us move to RP. So, in the case of RP when x is in the language be at least half should be accept. So, it need not be contiguous but there are 1, 2, 3, 4, 5, 6.

So, around half is now green and the rest could be red when x is in the language and when x is not in the language all of it should be red so in fact let me take the easier route and do this that is the definition of RP and let us move to co-RP now.

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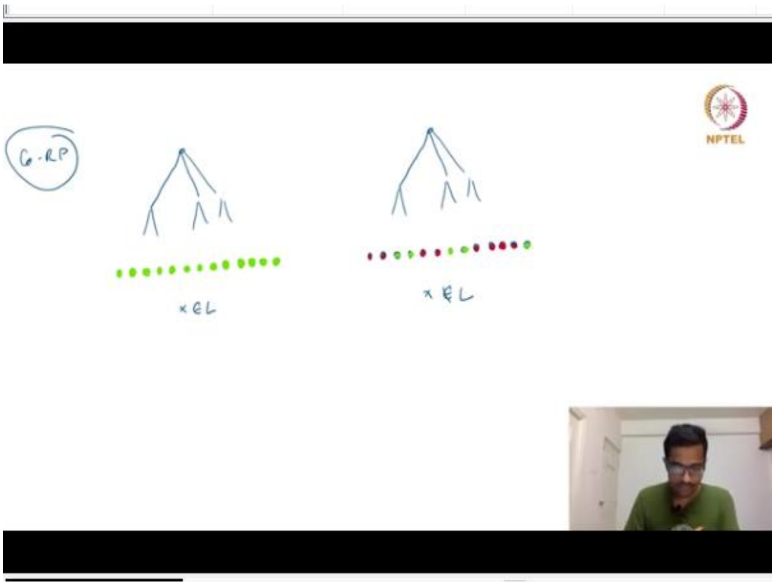

Again co-RP is a bit different when x is in the language everything should be accepted that is what co-RP says so all of them should be green, when x is not in the language at least half should reject or at most half should be green, so there should be at least half reds or at most half greens. So, maybe 7 of them could be red or maybe. So, 1, 2, 3, 4, 5, 6. So, there is an error in the RP the earlier one 1, 2, 3, 4, 5, 6, 7 and 1, 2, 3, 4, 5, 6, 7 has there are 13 dots here. In the case of co-RP when x is not in the language at least half should be reject, at most half can be accept.

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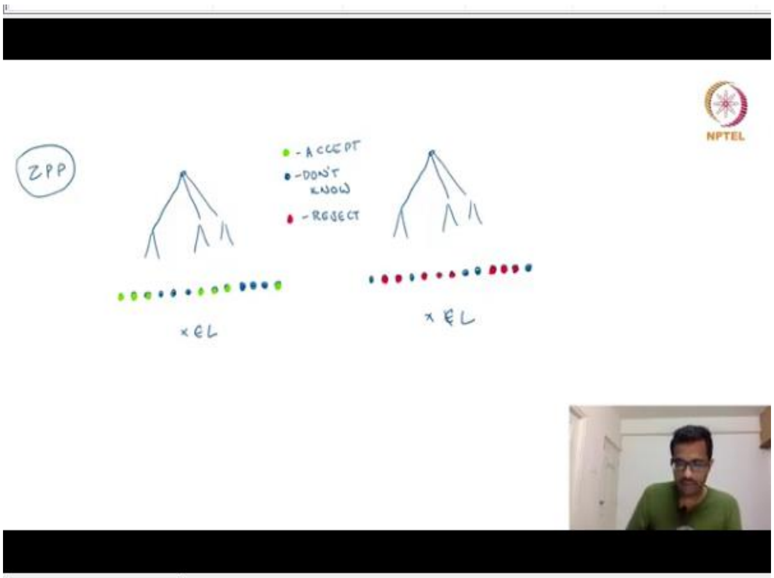

Now let us move to ZPP, again ZPP introduces a new, we will stick to the first definition that we saw where we said that there is a yes or there is a no or a do not know answer. So, green will denote yes and red will denote no as before, but let us say blue denotes do not know for both of these figures. Green denotes yes and red denotes no or accept, reject**.** So, when x is in the language there can be it rejects are not possible it has to be accepts and do not knows and at most half do not.

Similarly when x is not in the language there can be only rejects and do not knows, we cannot have accepts and there can be at most half do not knows, is the case here so there are 8 rejects and 5 do not knows. This is the ZPP.

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So, now let us see BPP we will stick to the first definition that we said which was in the terms of two thirds, one third. So, when x is in the language at least two thirds should be accept which means the rejects will be a small number something like this and when x is not in the language there can be at most one third accept. So, something like this. So, out of 13 4 is less than one third, this is the case with BPP. Finally let us come to PP.

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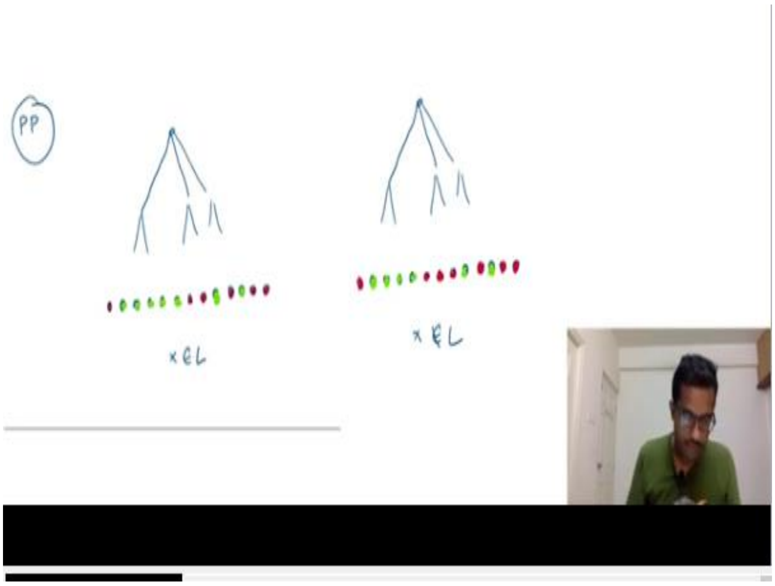

When x is in the language we said it can have again PP is also just accept, reject there is no, do not know. So, we said it can have at most at least half accept, so 7 now this will be considered as a YES instance and when x is not in the language at least half rejects again because we have only 13 dots here it is not possibly the lack of a gap is not really that apparent you may think 7 and 6 is relatively large when we are dealing with 13 outcomes.

But the point is that there is no requirement of a gap even if you had 1 million outcomes. So, it could have 500000 accepts and 500000 rejects for an YES instance and 4 lakh 9999 4.9999 accepts when x is not in the language also. So, there is really no gap required for PP. So, that is a situation with PP. Whereas in the case of BPP you have there is a certain gap. So, when there is yes it is at least or significantly more than half accept and when it is in NO instance significantly less than half or significantly more than half reject.

And ZPP allows the do not knows and at least half should be a accept; reject output which is the correct output. Co-RP when x is in the language all of it should be accept and x is not in the language at most half should be accept. RP when x is in the language at least half should be accept, when x is not in the language all should be reject. Again NP requires at most or exact at least 1 when x is in the language 1 accept.

And when x is not in the language it has to be all reject, so this is a kind of visual or pictorial summary of how each of these randomized complexity classes are supposed to behave and all of these are useful perhaps except for PP and hope this kind of gives you a good idea of how the probabilities work. Sometimes it is not easy to recall the definitions and exact numbers and this kind of images tend to stick to one's mind. So, that is all I want to share for this lecture, thank you.