

Artificial Intelligence: Search Methods for Problem Solving
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Chapter – 11
A First Course in Artificial Intelligence
Lecture – 80
Deduction as Search
First Order Logic

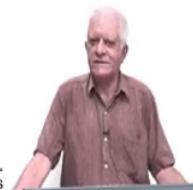
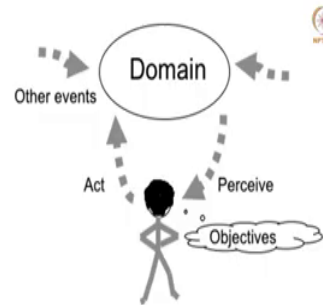
So, welcome back. So, we are starting on week 11 of our course on Search Methods for Problem Solving. And this week we are going to focus on Logic; last week we saw how rule based systems exploited, declarative, reasoning or declarative programming. And this is just one next step in that direction, because logic is primarily about representation.

And like we saw on and off; the two ways of looking at problem solving which is search and reasoning, they keep you know intertwining between each other, you know a little bit like. And we want to focus on the fact that, underlying the deductive abilities of a machine is again a search algorithm and that is what we will look at in this week here.

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What do you know?

An autonomous agent
operating in a dynamic world
creates a representation of the world
senses the world around it
makes plans to achieve its goals
monitors the execution of those plans
and constantly updates what it knows



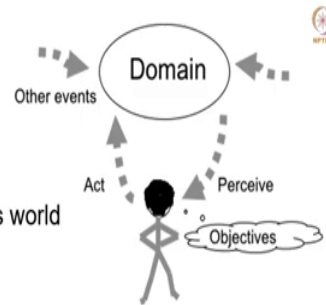
So, as we ended last week we said that, an autonomous agent operating in an dynamic world, creates a representation of the world, senses the world around it, makes plans to achieve the goals that it wants to achieve, monitors the execution of those plans and constantly updates what it knows essentially. So, you can see that there are so many things happening when a agent is operating in the real world, in which there are other agents as well; because things keep changing and the agent needs to be updated on that.

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What else do you know?

An autonomous agent
operating in a dynamic world

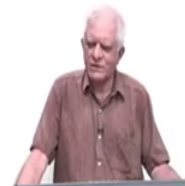
...needs to make inferences about its world



This week,

Representation and Reasoning in Logic

Another string in the bow of a problem solving agent



Then not only you have to monitor the world and keep updating what you; for example, see or hear or sense in whatever way, you have to also do something else, which is that an autonomous agent needs to make inferences about what it, about its world essentially. So, you see something and you infer something from that and that is also equally important to seeing the thing.

So, this week, we will spend some time on representation and reasoning in logic. We have chosen logic; because it is a most common language for representation; it does not have to be, but it is by far the prevalent language in this thing. And reasoning, representation of course is core; as we keep saying that an intelligent agent needs to model the world in which it is operating.

But not only does it need to model the world, it also needs to be able to reason in that world and that is what we want to look at and this is another, another string in the bow of the problem solving agent essentially.

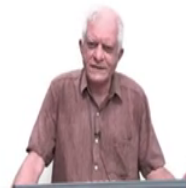
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Knowledge and Reasoning – necessary for intelligence



What does the agent **know**
and
what else does the agent **know** as
a **consequence** of what it knows?

A preview of the course
AI: Knowledge Representation & Reasoning



So, this is a thrust of this week's work. And as I have said below here, this is just the preview of a course on knowledge representation and reasoning, which I am going to offer in the next semester, and I hope some of you will come and join that course as well.

So, knowledge and reasoning these are necessary for intelligence that is a pitch we want to make. What does an agent know and what else does the agent know as a consequence of what it knows? That the second part is reasoning; the first part is sensing or knowing in some way or the other essentially.

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The Greek Syllogism



Given

All men are mortal

and

Socrates is a man

conclude

Socrates is mortal

The Socratic Argument

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So, in the logic that we are looking at, the roots of reasoning come from the Greek philosophers. And the most common form of reasoning with every student who studies logic is exposed to is the syllogism essentially.

And the argument goes like this that, given the statement that all men are mortal and given that Socrates is a man; you should conclude that Socrates is mortal. So, this is the form of reasoning that epitomizes logical reasoning that, there is a form of reasoning and we are following their form; this particular form is called Socratic argument.

So, what, what is given to you or what you know is that, all men are mortal; let us assume that somehow you have figured that out. And you know that Socrates is a man, and therefore you have no escape, but to conclude that Socrates is mortal.

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What else do you know?

True / Given	Necessarily true
Snigdha's mother is Sneha	Snigdha is Sneha's daughter
Snigdha's father is Salil	Snigdha is Sameer's sister
Sameer's mother is Sneha	Snigdha is Arushi's niece
Sameer's father is Salil	.
Salil's sister is Arushi	.
Arnav's mother is Arushi	<u>Salil is not Arushi's sister</u>
Arnav tested positive for Covid	Salil is not Sneha's sister
	Arushi is not Sneha's daughter
	.
	.
	There exists a woman who is
	Snigdha and Arnav's grandma
	.
	.

Possibly true

Sneha loves Snigdha
Snigdha loves Sneha
Snigdha loves idli-sambhar
Snigdha goes to school
Arushi is older than Salil
Snigdha is a Arnav's friend
Arnav has Covid ←



So, what else do you know is the crux of reasoning. Let us look at an example here; supposing you know something about a family essentially.

So, these are the statements that you know; we have not just spoken about the language of logic, but eventually we will move towards logic. But let us just take them in English for the moment. We know that Snigdha's mother is Sneha, Snigdha's father is Salil, Sameer's mother is Sneha and so on; we know a set of facts about a family in which there are cousins and so on.

Salil's sister is Arushi and Arushi's Arnav's mother and so on. We know this, in the addition we also know that the gender of each of the characters; I have not mentioned it here, in formal

representation everything has to be explicit. So, you should write somewhere that Snigdha is a female, that Salil is a male and so on and so forth.

Also you should know about relationships. So, there is some kind of background knowledge, this is a little bit like the long term memory that we talked about in. In rule based systems; something that you know that, if x is the father of y, then y is a child of x and that kind of relationships you know. So, you have knowledge about this relationship.

If that is the case, in this example I have put in something which is very topical which is says that, Arnav is tested positive for COVID essentially; these are COVID times as you know. If you know what is given or what is true, what is necessarily true; what you see in this second column is things which are necessarily true that is Snigdha is Sneha's daughter, because Sneha is Snigdha's mother it holds that Snigdha is Sneha's daughter.

So, we are trying to emphasize on the fact that everything has to be explicit essentially; if you say that you know that Snigdha is Sneha's daughter, then your knowledge base or your memory or your long, short term memory or whatever you want to call it, must have that statement. So, these are statements which will be true, given this the whatever the statements on the left hand side are true; then this will also be true.

Not only do you know positive things, you also know that; for example Salil is not Arushi's sister. How do you say that? Because it logically follows from the fact that, Salil is a male and to begin with Salil cannot be anybody's sister essentially; but apart from that of course, you also know that, Salil is in fact Arushi's nephew I think. So, you can conclude all kinds of things. You can even make statements like at the bottom as you can see that, there exists a woman who is Snigdha and Arnav's grandmother.

Even though it is not mentioned in that, by our knowledge of relationships; that if two people have parents who are siblings, then they must have a common grandmother. So, you can make these inferences and these kind of inferences are called directive inferences, because they are necessarily true. There are other things which are possibly true. So, we say it is

possibly true, because we do not know for certain; in a typical well knit family, you would expect these things.

But it is not, it is not, it does not follow logically from that; you know that Snigdha loves Sneha and Sneha loves Snigdha. Of course, mothers and daughters they love each other and that is a norm; but you cannot conclusively say that essentially.

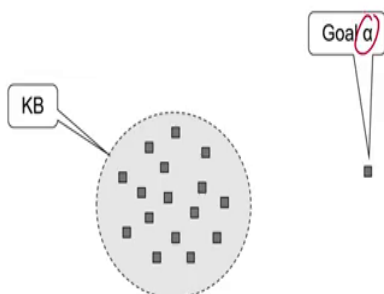
There are other things I have mentioned here, just it is possible; but you cannot say anything about that. In the sense that, that given what the knowledge base that is given to you, which is the leftmost column here; you cannot say that these other things are true that, Snigdha loves idli sambar or she goes to school or Arushi is older than Salil and so on and so forth.

You cannot also conclude that, Arnav has COVID; even though we had said earlier, that he tested positive for COVID, because. And why is this? Because we know that the COVID tests is only something like 95 percent accuracy. So, there may be false positives, there may be false negatives anything could happen. So, you cannot certainly say that he, Arnav has COVID and so.

So, we are not going to get into statements which will be possibly true, that is beyond the scope of this course; we will focus on what is necessarily true, given some knowledge base.

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Deduction: The Goal



Given a knowledge base (KB)
which is set of sentences assumed to be true,
is a given (query) sentence α necessarily true?



So, this is the goal in deduction that we have; you are given some knowledge base, which is each square here represents a sentence in some language, we will be choosing first order logic. But you have a knowledge base which is a set of sentences.

And the question that you are asking is, given a Knowledge Base KB, which is basically a set of sentences which are assumed to be true, given to be true or premises or axioms depending on what domain you are working in, but you take them to be true. Is the given query a sentence that we are calling as alpha? So, we are calling it a goal; is alpha necessarily true? That is the question we are asking and we want to look at how this can be answered in during this week essentially.

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Representation



Semiotics: A symbol is something that stands for something else

Examples:

- The "number" seven can be represented in many different ways.
- Road signs – curves, pedestrians, schools, U-turns, eating places...

All languages are semiotic systems

Biosemitotics: How complex behaviour emerges when simple systems interact with each other through signs



So, the first thing you have to talk about is representation; whenever you do computation, you have to represent things in some way essentially, ok. And in the sense that, philosophers have over the ages pondered over how does thinking happen, how does reasoning happen?

And if you remember what we studied during the introduction, there are those questions; but when we are talking about programs, we are talking of artificial intelligence or computer programs which are intelligent, representation is the first thing that you have to talk about essentially.

So, there is a whole science of representation, it is called semiotics; the science of symbols and it says that, a symbol is something that stands for something else essentially. So, the representation is a symbol; what it stands for could be something else essentially. Examples, we are all familiar with this that, for example, the notion of number seven is a notion in our

heads; we know, we have at least some vague meaning of what you understand by the number seven essentially.

But you can write it in different ways; for example you can write it as 7 which is the most common thing, some people write it as 7 with a horizontal line across. Or if you are a Roman, then you may write it like this or you may write it in English language itself or in some other language.

So, all these are symbols systems as Newell and Simon called them and they stand for something else essentially. Other examples of symbols are road signs. So, you see a road sign which says curve ahead or pedestrians or school or U turn ahead or there is a eating place ahead and these are all symbols.

Now, all languages by definition are semiotic systems. So, if you take the language of English, it is built up upon 26 characters and an alphabet of 26 and the entire language emerges out of that essentially. So, they are essentially symbol systems. We have also had a peak at bio semiotics as to how physical things can stand for something else essentially.

And we have seen that in for example, in swarm optimization or ant colony optimization or we spoke about how they signal to each other and so on. Complex behaviour can emerge out of simple systems when they interact through the science and the science are essentially physical in nature essentially.

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Reasoning



The manipulation of symbols in a *meaningful* manner.

- we can *only* manipulate *symbols*
- the *meaning* or *truth value* is in only our minds

Maths is replete with *algorithms* we use –

- Addition and multiplication of multi-digit numbers
- Long division
- Solving systems of linear equations
- Fourier transforms, convolution...



So, that is about representation. What about reasoning essentially? Now, reasoning if you remember when we are discussing about what the cart was worried about you know the mind body dualism as to, the mind and the body and things like that. One of the questions that we had asked was that, reasoning or logical reasoning is the manipulation of symbols in a meaningful fashion.

That you know there should be something meaningful about that and otherwise what is the whole point of playing around with symbols. But you have to emphasize the fact that, we can only manipulate symbols; we cannot manipulate the meanings behind the symbols essentially. The meaning or the truth value as we will see in the case of logic is only in our minds essentially.

So, if I say Sneha is a girl, it is just a symbol or a sentence in some language; but conceptually in my head, it maps into some ontology in which I know what is the girl and I know the Sneha refers is a name of a person and all that kind of thing which goes on in the background is a meaning. But that meaning is in the mind, but the sentence itself may simply says, Sneha is a girl.

Maths as every child knows is replete with algorithms, that we use all the time addition, multiplication, long division and that of some you know; children have struggled to such things. Older students have worried about how to solve linear equations or how to calculate the Fourier transform or convolution and so on. These are symbolic techniques we use in the sense that, these are algorithms that we use that we have somehow learnt from at some point.

And internalized in some way that, we can do some symbol manipulation and in the end we do, end up doing something meaningful. So, this meaningfulness has to be tied to what are those symbols representing and how are you manipulating them and we will spend some time looking at that.

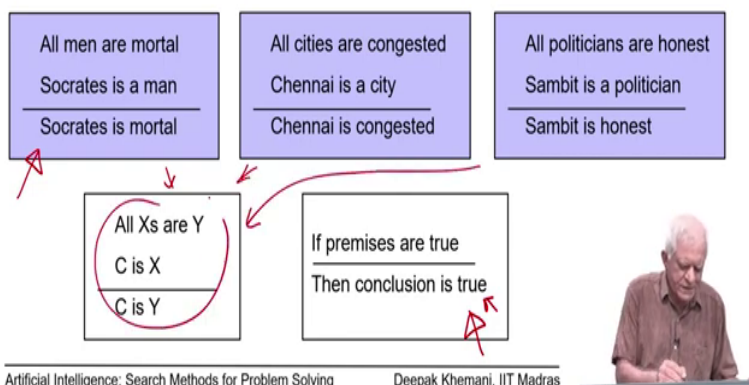
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The Syllogism



The Greek syllogism embodies the notion of **formal logic**

An argument is valid if it conforms to a **valid form**



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Let us get back to the syllogism. The Greek syllogism embodies the notion of formal logic. So, this is something that I like to keep emphasizing is that, logic is essentially formal essentially; it is based on form and it is not based on content essentially.

So, a pattern of logical reasoning which is given at the bottom here essentially says that, this pattern is a good pattern or a meaningful pattern if you want to say. Or in the terms of logic we would say, it is a sound rule essentially; is that if the premises are given to you, you can happily go and add the conclusion to your knowledge base essentially. So, somebody gives you the premises and you say that, you can add the conclusion to the knowledge base.

Now, we see three such arguments here; the first one of course, we started with what was the Socratic argument here which says that, all men are mortal, Socrates is a man and Socrates is mortal. The second argument has a similar form, but very different content essentially; it says

all cities are congested, Chennai is a city and therefore Chennai is congested. You will see that both these and in fact all three of them, they correspond to this pattern which says that, you are given a statement of the form all X's or Y's.

Then you are given that C is an X and you are forced to conclude that C is a Y essentially. So, if somebody told you that all politicians are honest, let us say we will not question that fact. And that Sambit is a politician, you are forced to conclude that Sambit is an, Sambit is honest.

So, the point about logic, the syllogism which we have inherited from the Greeks is that, it connects true statements or true statements; that if you are given true statements, it will tell you what are the other true statements.

It does not question the original truth value of the given statements. Now, if you have studied Indian logic and nyaya sastra and such things; you will see that they go a little bit deeper into this they, they are more worried about truth of the conclusion as opposed to the Greek syllogism which says that, the truth of the conclusion is contingent on the truth of the premises. Logic says that, if the premises are true, then the conclusion is true; it is not primarily concerned with whether the premises are true or no essentially.

So, we have seen that, that logic is essentially formal in nature it depends upon the form. And it is not concerned with the truth value so much as the relation between true statements and the consequent true statements essentially.

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Formal Logic



Logic is a formal system

Logical reasoning is only concerned ONLY with the FORM of the argument, and not with CONTENT.

If the form is valid AND If the antecedents are true
THEN the conclusion is true.

Thus, the conclusion holds only if the antecedents are true.

Logic does not concern itself with the truth of antecedents
OR

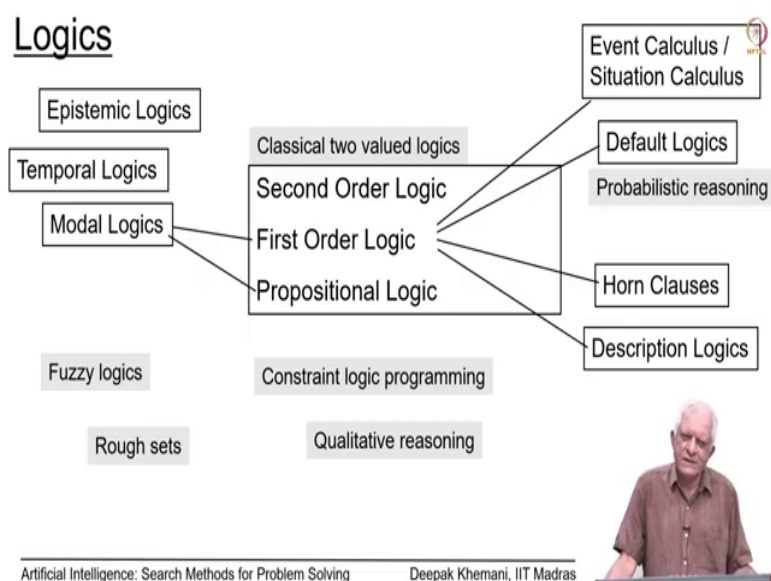
what the sentences are talking about (content).



So, logic is a formal system, it is concerned only with the form of the argument and not with the content. If the form is valid and we will see that there are valid forms and there are not non valid forms; if the antecedents are true, then the conclusion is always true.

The conclusions hold if the antecedents are true; logic does not concern itself with the truth of the antecedents and it also does not concern itself with the meaning of the sentence, as so what is it the sentence are is are talking about. Or what is the content of the sentence, it is only concerned with valid forms of argument.

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Now, it turns out that there are many logics that mathematicians have studied; these are the three of the simplest ones, which are called classical two valued logics. They are classical in the sense of being mathematical and true valued in the sense that every statement can either be true or false, and can have no other value essentially. The people have worried about other kinds of logic in which we can say possibly true or do not know and other kinds of stuff.

Here you must be familiar with some of these logics. So, propositional logic is a simplest logic one learns in early discrete maths education, where you have a set of statements given to you PQRST, we will see an example also later. And you have some means of you know arriving at proofs essentially. In first order logic, we introduced the notion of variables and we introduced the notion of quantifiers.

So, we can make statements like for all x something is true or there exists x , such that something is true and so on. We will again look at first order logic. In fact, first order logic is the core of most of computing; all programming languages can be seen to be equivalent to doing reasoning in first order logic essentially. Second logic, second order logic is more expressive and it allows you to introduce variable relations essentially.

So, instead of saying that brother is a relation, you can have say that, let p be a relation or something like that. And then you can make statements like for all relations or something like that, some statement is true essentially. So, first order logic quantifies over variables objects in the domain; second order logic quantifies over relations in the domain or predicates and so on.

And as logics become more expressive, they also become harder in the in the computational sense and you must have, some of you must have heard about Gödel's incompleteness theorem, which basically talks about second order logic. And it says that no logic can be complete; we will look at the notion of completeness very briefly in this course as well. Gödel also show that first order logics are complete, so that is why they are the most popular vehicle for reasoning in computer science.

They may be complete, but first order logics are not tractable; you know their proofs can be typically exponentially long, and therefore people have looked at restricted versions of first order logic. So, one is called horn clause logic and the other are called description logics; these are restricted forms of first order or subsets of first order logic statements, which are computationally tractable essentially. But we have also looked at things which are more complex in first order logic.

So, we have looked at something called event calculus or situation calculus, which talks about event and time and change and things like that. Or we have talked about default logics, we have talked; we have mentioned a little bit of default reasoning when we are talking about rules, but there is a whole logic which is built upon default reasoning.

There are other kinds of logics like epistemic logics, temporal logics, modal logics; modal logics are the generalisations of epistemic and temporary logics. Epistemic logics talk about multi agent situation; if there are multiple agents, what do they know about what other agents know and that kind of thing. There are fuzzy logics in which the truth value is continuously variable; rough sets in which there are statements which are necessarily true and in which statements that are possibly true.

There is probabilistic reasoning in the real world; you often end up doing probabilistic reasoning that if somebody has passed the COVID test or somebody has a positive COVID test, then that person, what is the probability that the person has COVID.

So, people often use probabilistic reasoning in such situation. And there are things like constraint logic programming and qualitative reasoning. And all these come under the umbrella of logical reasoning, representation and reasoning essentially.

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Classical Logics



- Mathematical Logic
- Once a statement is true it remains true
 - for example the Pythagoras Theorem
 - no notion of time, events, or statements becoming false
- The RHS of a rule has only ADD actions
- Non-monotonic logics
 - Fluents: sentences which can flip their truth value
 - Knowledge and Belief
 - Default Reasoning
 - Inheritance

AI: Knowledge Representation and Reasoning



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We will focus in this course on classical logics, classical logics are also called as mathematical logics; because once a statement is true, it is always true essentially. For example, the Pythagoras theorem; once it is true, you can not say that it is true today and it is not true tomorrow or something has change in the world that makes it not true.

Of course, what can change are the axioms, the basic axioms behind mathematics about the notion of lines, straight lines and that kind of stuff; if that changes, then Pythagoras theorem may change. For example, on a spherical surface, the Pythagoras theorem does not hold I think.

In fact, on a spherical surface, you can have a triangle which has three right angles, on that does not make sense on the plane. But in any case, given a mathematical statement based on a set of axioms; it does not change. The right hand side of a rule and we are seeing this from the

perspective of rule base systems that we saw, in which we had left inside had a pattern and the right hand side had some actions.

It has only one action, which is that you can add something. So, essentially what are you going to add? So, just like in the syllogism, we can; we will add the consequent or the consequence of the premises. And once you have added, in classical logics you can not do anything about it; if it is true, it is true essentially. And we will be focusing on classical logics in this courses essentially.

There are other things called non monotonic logics, in which we have something called fluents sentences, which can become true now and false, false a little bit later. For example, I am holding this pen, it is true at this moment; but if I put it down, then it is no longer true. So, with time and change. So, I talked about event calculus and there you are allowed to reason about statements which can change in their truth values.

We can talk about knowledge and beliefs; you know that people have all kinds of beliefs for example, about the politicians. Some people feel that Trump would be the best president in his second term, some people feel that no Biden is a better option. By the time you are seeing this lectures, the elections were probably be over; but when I am recording them, this is all in the domain of beliefs essentially, people have beliefs about other things.

Default reasoning we have already seen a little bit and inheritance also we have seen a little bit. Again to make a little bit of a pitch for my course on this thing; we cover all these things in more details in the course on knowledge, representation and reasoning.

So, we will focus on classical logics in this, in this course here and we will take a short break now. And after the break we will come back and see, what is the process by which you can arrive at new true statements, given a set of true statements or given a knowledge base or given a set of premises? So, we will do that in the next video.