Artificial Intelligence: Search Methods for Problem Solving

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Chapter – 08
A First Course in Artificial Intelligence
Lecture – 50
Game Playing
Game Theory

So, welcome back, today we are going to start a new topic. So, far in our course we have considered the situation where there is one agent who is solving problems in some domain and there is no one else who is making any changes in the domain essentially. But, that is not true very often in the real world, in the real world there are other players as we call them or other agents and one often has to take their actions into account essentially.

Now, there is a vast field called Game Theory, which looks at a rational behavior in social situations. We will start by looking at game theory, but then we will move on to something which is more concrete for us which is something that we can implement as a program.

And, we will move on to game playing algorithms and in doing that we will look at algorithms like chess. So, this is a section or module on Game Playing and you can find some material in chapter 8 of my book and in other sources as well.

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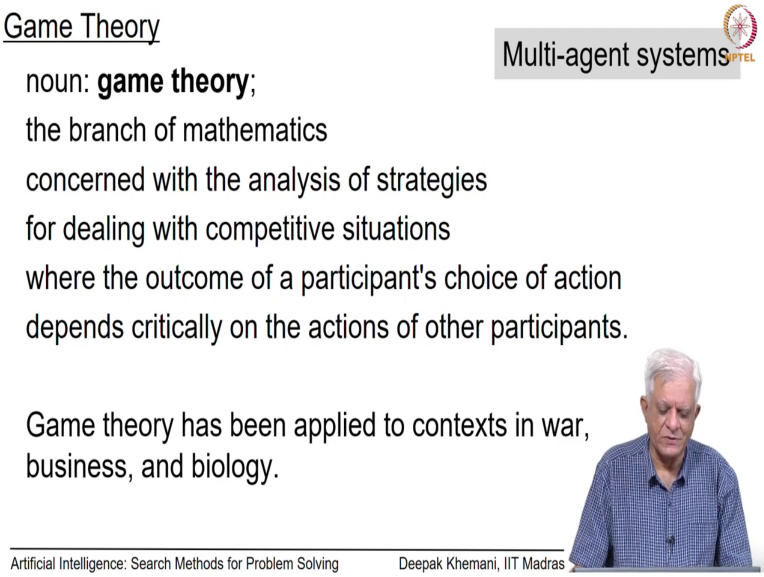

Let us start with the definition of game theory as given in a dictionary. So, game theory is the branch of mathematics concerned with the analysis of strategies for dealing with competitive situations where the outcome of the participant's choice of action depends critically on the actions of other participants as well essentially.

So, as I said its in a multi-agent scenario that we are considering how an agent should act essentially. And, essentially in game theory we assume that that agents are rational and they act completely selfishly and to the best step best of their effort essentially. Game theory has been applied to various scenarios; economics, war, business, biology and so on. And, it has been used to model such scenarios and tried to study what should be the correct actions of agents in such scenarios.

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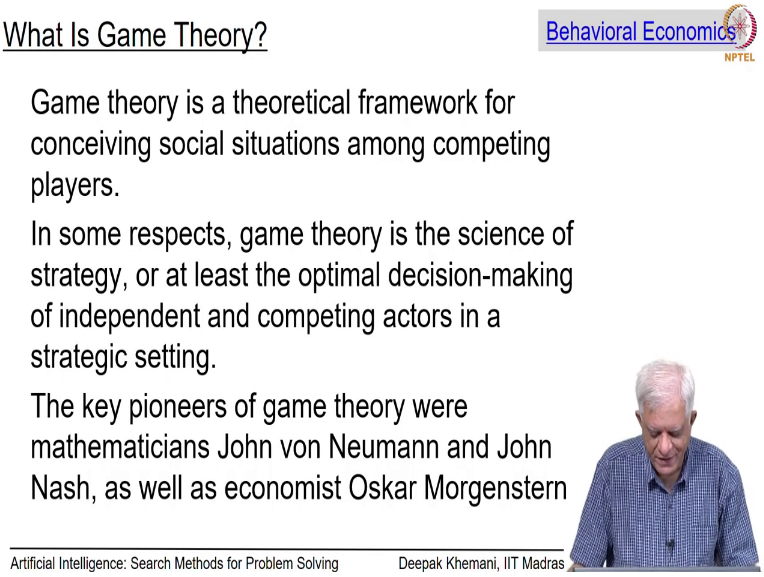

Here is another definition which we have got from this domain of Behavioral Economics. Game theory is a theoretical framework for conceiving social situations among competing players essentially. So, we are talking about how to model social situations where there are competing players. In some respects game theory is a science of strategy or at least the optimal decision making of independent and competing actors in a strategic setting.

So, we try to figure out how what are the kinds of optimal decisions that that people make or players make or agents make. In game theory terminology we often use the term player essentially even though it may behave like war like situation where we are considering game theory; it does not necessarily mean they are playing a game in that sense. But, the terminology uses a term player essentially; we can use the term player and agent interchangeably.

Now, game theory was I mean one credits John von Neumann, who amongst many other things that he did in his life including giving us the stored program computer model, sequential model for computing. He also is credited with originating this field of game theory along with John Nash, who you must have heard about.

Especially if you have seen that, film called A Beautiful Mind which is about John Nash and also the economist Oskar Morgenstern, essentially. So, as you can see there are players from different communities which have kind of contributed to this field of game theory.

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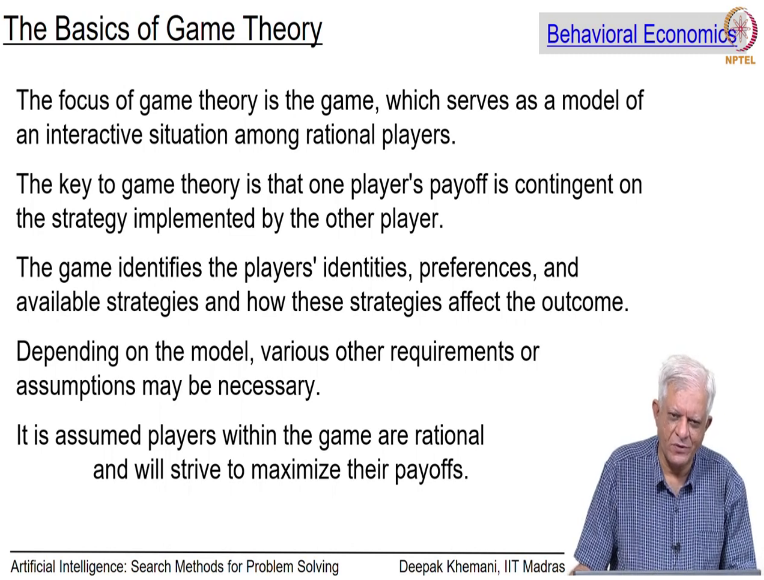

What are the basics of game theory? The focus of game theory is the game which is the model of the social situation scenario which in which this interactive situation where amongst the rational players. The keyword here is rational players. The key to game theory is that one players payoff; so, we often use the term payoff as to what do you get as a result of your choice or as a result of your action we use the term payoff.

So, the key is that one player's payoff is contingent on the strategy implemented by the other player. So, in game theory we often use the term strategy as to what is the plan behind your action or as a term says what is the strategy that you are using. The game, the modeling of the game identifies the player's identities preferences and available strategies and how these strategies affect their outcome.

So, all this we study in game theory. Depending upon the model, various other requirements or assumptions may be necessary. And, we shall look at some classification of games theory games as we go along. The key thing is that it is assumed that players within the game are rational and will strive to maximize their payoffs.

So, it is not as if you know you are working or the other participants are kind of making mistakes or acting foolishly or some such thing. We assume that everyone is rational and everyone is acting to maximize their own profits, in that sense they are selfish. And, we want to study as to what can happen in such scenarios essentially.

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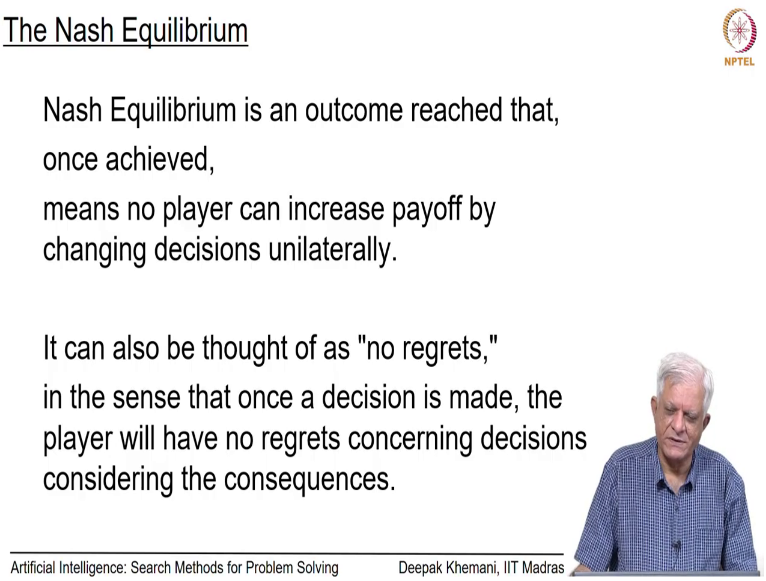

Now, there is something called the Nash equilibrium which is as you can guess named after John Nash. And, it says that Nash equilibrium is an outcome after all the players have made their decisions, that is reached which once achieved means that no player can increase their payoff by changing the decisions unilaterally.

So, in some sense Nash equilibrium is where the game will settle into. Because, if any player deviates from the choice that leads to the Nash equilibrium; they will actually decrease their payoff or they will not increase their payoff at least. So, it is an equilibrium where once it is reached, no player can deviate from it essentially.

It can also be thought of as “no regrets”, in the sense that you have made a choice and once a decision is made, the player will have no regrets concerning decisions considering the consequences. Because, the player knows or in some sense the player believes that it he or she cannot do better or it if it is a program.

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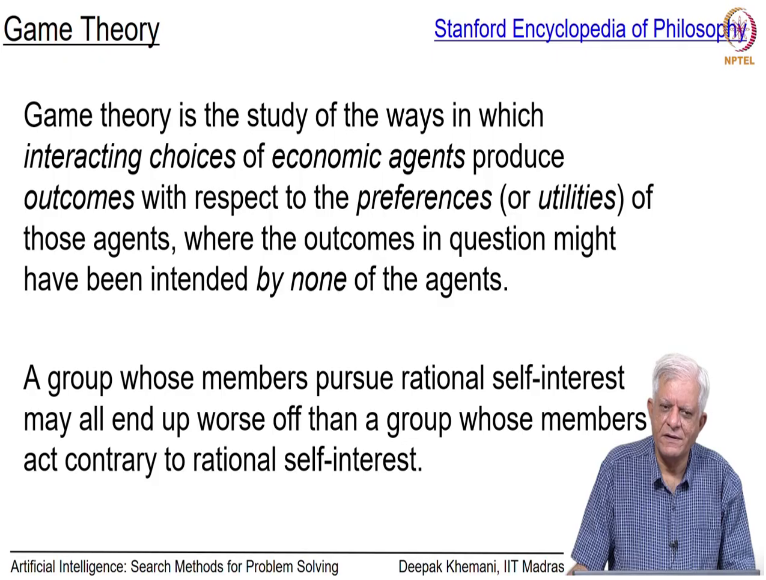

Here is another the definition of game theory from the Stanford Encyclopedia of Philosophy. Game theory is the study of ways in which interacting choices of economic agents. So, economic in the sense that you are trying to maximize your payoff. So, it is a study of ways in which interacting choices of economic agents produce outcomes with respect to the preferences or utilities.

So, we always assume that the agents have some goals or which we call as preferences or we call them as utility; what do they get out of the game? So, it is a study of ways in which interacting choices of economic agents produce outcomes with respect to the preference or utilities of those agents, where the outcomes in question might have been intended by none of the agents.

So, it is not as if you are acting unilaterally and you are driving the whole social situation as a dictator or it only depends upon your actions. It depends on other actions and it may the final outcome, the final Nash equilibrium for example, could be something that you yourself had not intended to be.

So, a group whose members pursue rational self interest; so, that is a assumption in game playing may all end up worse off than a group whose members act contrary to rational self interest. So, we will see an example of this, is that sometimes being totally selfish can end up you can end up in a situation worse than you would be otherwise.

A glaring example in today's world is our treatment of our earth and all of us act selfishly in the sense that all of us want cars and we want a good life and we want to fly in planes and do all kinds of such things. But, as a consequence all of us find ourselves in a degrading environment which people have now started waking up to. So, you can model that whole thing as a game in which each individual is acting for their own self interest, but as a consequence everyone is worse off.

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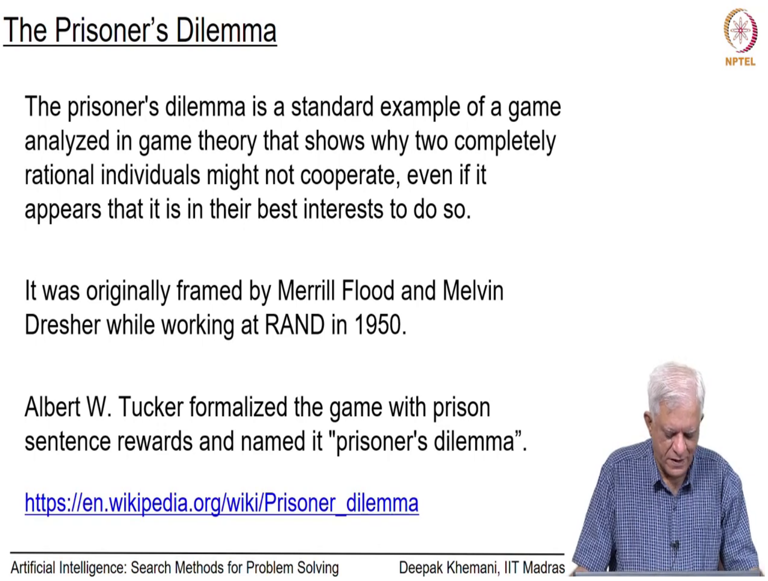

This particular thing has been kind of formalized into a situation which is now called the Prisoner's Dilemma. And, it is a standard example of a game analyzing game theory that shows that two completely rational individuals. So, this is between two players might not cooperate, even if it appears that it is in their best interest to do so.

So, it is a very nice example and we will go through that first. It was originally framed by Merrill Flood and Melvin Dresher in 1950. And, formalized into this notion of game with a prison by Albert Tucker and he gave it the name "prisoner's dilemma". You can read the read about it in many websites and this definition I have got from the Wikipedia page on prisoner's dilemma.

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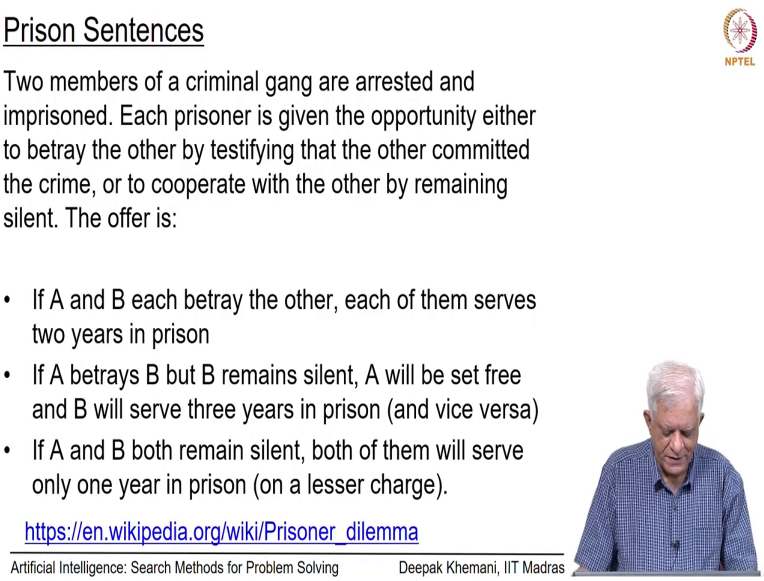

So, we are talking about prison sentences. So, just imagine that there are two members of a criminal gang are arrested and imprisoned. Each prisoner is given they are kept separately in isolated rooms and they cannot see what the other player is doing, other prisoner is doing. Each prisoner is given the opportunity either to betray the other by testifying that the other committed the crime or to cooperate with the other by remaining silent.

So, you know you must have seen films in which there are gangs which work together and they have steadfast loyalty to each other. And you know they would never betray a partner in crime and so on.

But, game theory does not make such emotional connects. We do not assume that we have the interest of partner in this scenario, only that you have your own self interest essentially. So, the offer is as follows if A and B betray each other, each of them will end up serving two years in prison essentially and both of them know that this is a offer.

If A betrays B, but B remains silent; in other in other words B cooperates with A, but A A does not cooperate with B. So, A betrays B and B remains silent, A will be set free as a deal that the police is offering them. But, B will get to serve three years in prison which as you can see is more than two years. Now, this actual figures of two and three do not matter, what really matters is that two is less than three.

And, any small gain is something that any player in a game would strive for. If A and B both remain silent; that means, they do not betray each other, they cooperate with each other; both them both of them will serve a smaller sentence of one year in prison. And, this they will do not because of the crime for which they are being investigated, but on some lesser charge.

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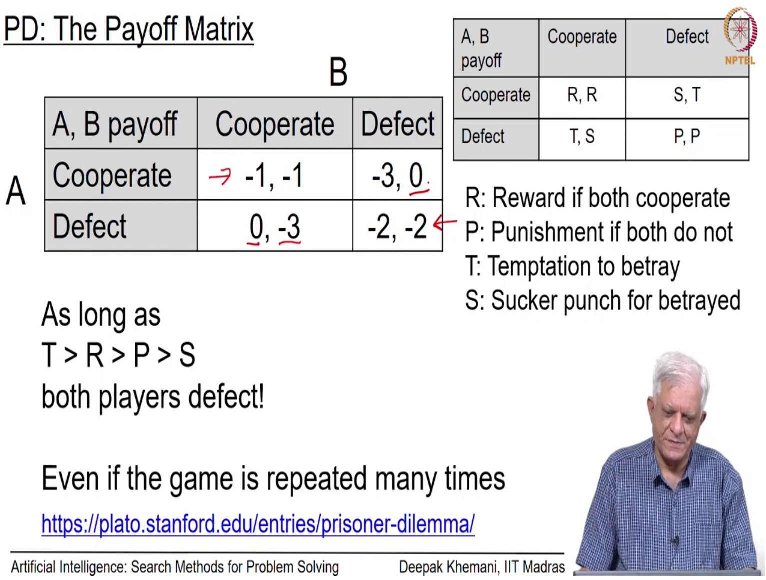

So, we can now model this business dilemma as a table, it is a payoff matrix. And, what you can see in the payoff matrix on the left hand side is the actual game that we are talking about; instead of betray we have used the term defect which we got from the Stanford site. And, you can see that if both cooperate they get 1 year each, on the lighter sentence on some related charge. You know like possessing a firearm or something like that.

If both betray each other, they get minus 2 each minus 2 each. So, the left hand side figure is for player A and the right hand side number is for player B essentially and the choices are given in the respective rows and columns. If A betrays B, then you can see that A gets off with free, but B gets a prison sentence of 3 years which we have modeled as minus 3 here, the payoff is minus 3. And, conversely if B betrays A, B gets let off with 0 and A gets a prisons sentence of minus 3.

So, we can kind of model this as you can see more generally in the payoff matrix on the right hand side. And, where you can see that either they can cooperate or they can defect and the payoffs are R, R S, T T, S and P, P. The names are chosen such that R stands for reward if they both cooperate. So, if they both cooperate then they are in some sense they both get let off lightly. P is the punishment if both do not cooperate; so, they betray each other and both of them get punished.

This is more than the punishment is more than what they would have got if they had cooperated, T is the temptation to betray in which of course, you get let off freely. And, S is a sucker punch, if you are if you are the one who is betrayed then that is the payoff that you get. And, it has been shown and studied that under the conditions that as long as T is greater than R and R is greater than P and P is greater than S; remember the T is the temptation of betraying, you get the maximum reward if you betray somebody.

R is a reward if both of you cooperate; P is a punishment if both of you betray each other and S is a reward that you get if you have been betrayed essentially. Under these conditions where, T is greater than R and R greater than P and P is greater than S; it turns out that both players defect. It is rational for them to defect and they do so even if the game is repeated many times, that is the interesting part. Let us see why this is a case.

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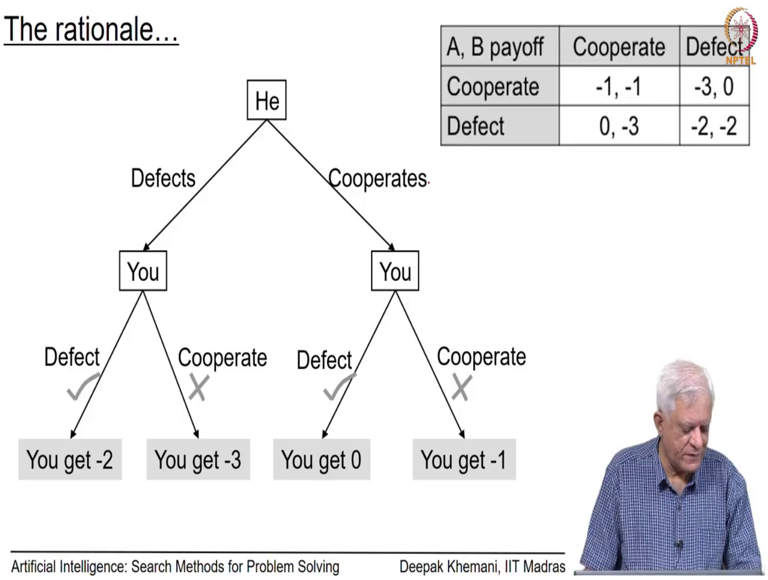

So, we can look at the rational. So, remember that this is a payoff matrix that you have. If you both cooperate you get minus 1, if you both betray or defect you get minus 2 each. If you betray the other you get 0 and the other gets minus 3 essentially. So, let us see I mean the game is actually happening simultaneously in the sense that it is not a sequential game in which one player makes a move and then the other player makes a move.

But, we can think of the reasoning as a sequential process in which you can consider the various options of what the other player may be doing and what should be your strategy in that situation. So, we start off by considering the situation where is the other players betrays you or defects from defects from your team so to speak; then what should you do? If you defect you will get minus 2 as well as you can see from the table, if when both players defect your payoff is minus 2.

If you cooperate with him; so, he is betrayed you and you are cooperating then he will be let off with 0 and you will get minus 3. So, clearly if he betrays or if he defects then the correct choice for you is to also defect or to betray essentially of course, you do not know whether he is betraying you or whether he is cooperating with you. But if he were to be betraying you, then your correct choice would be to also betray him. The other possibility is that he may cooperate with you.

Now, if he cooperates with you then you will have the same two options. If you defect then you get let off from the prison without any sentence so, your payoff is 0 which is a maximum payoff incidentally in this matrix; because the payoffs are negative. And, if you also cooperate then both of you will get minus 1, a 1 year sentence in the prison.

So, clearly you can see that even in this scenario for you to betray is the rational choice to me. So, we are talking about rational choices we are not talking about friendship and loyalty and other emotional parameters. We are simply talking of economic parameters, in the sense that you just want to maximize your payoff. And, you can see that now why it is logical or rational to betray the partner, if you simply want to maximize your payoff.

So, this shows that in many scenarios the Nash equilibrium that you reach is often not the optimal which sometimes people call as a Pareto optimal. In this case, the Pareto optimal would be when they both cooperate. If you sum up the total sentences that the two players receive, then in the optimal the total sentence is 2 years, in the Nash equilibrium the total sentence is 4 years.

But, in the case of one betraying the other, the total sentence is 3 years. And, you can see that the Nash equilibrium has settled into this lowest point which is not the optimal point where both of you get 2 years into prison.

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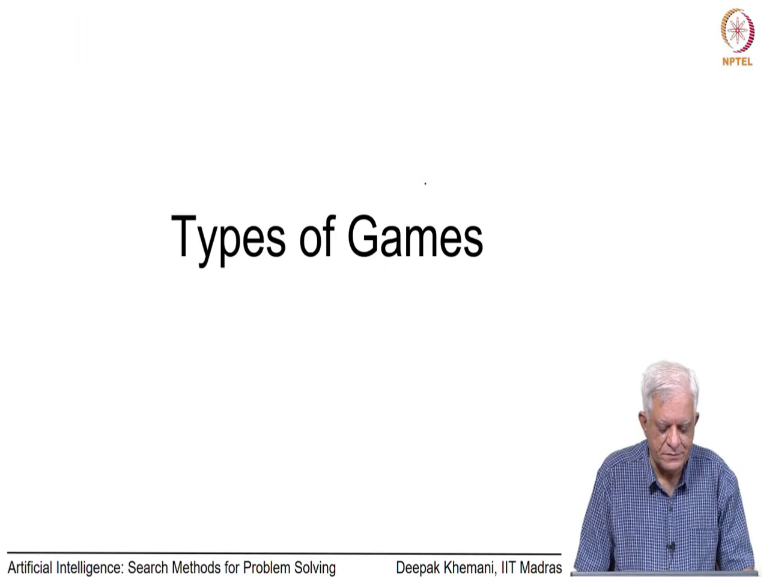

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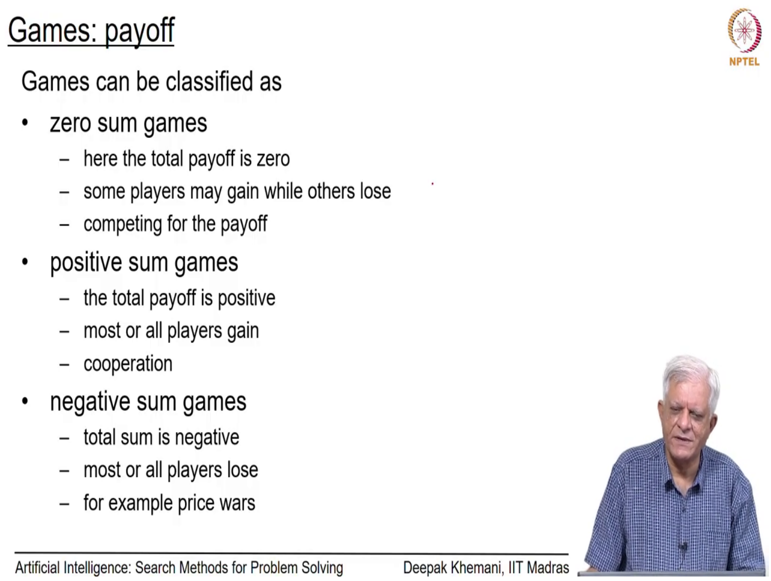

Now, let us quickly look at classification of games before we move on. Games can be classified based on the payoff that you get or the payoff that the different players get. The simplest of them is zero sum games. So, in zero sum games the total payoff of all the players is zero. So, which basically means that some players may gain whereas, at the cost of others and the other players will lose.

So, this is typically a situation of competing for a payoff, it is typically a situation of pure competition or adversarial situation. And, in most of the games that we play on the field including things like football and so on are situations which are zero sum; that you win and the other side loses and so on. But, games do not have to be zero sum, they can be positive sum games. These essentially model cooperation and the total payoff is positive in this situation most or all the players gain.

So, a typical example of cooperation is for example, when two students study together and help to clear each other's doubts; then they can both benefit from it or if two researchers collaborate with each other sharing methods and results, then both of them can benefit. So, games can be positive sum games as well, but games can also be negative sum games where both the sides lose essentially.

So, the total sum or total payoff here is negative and most of the players end up losing in the game. So, environment is a situation like that as you can imagine, but also simpler things like price wars. If there are two companies which are selling a product and both of them try to gain market share by reducing the price of their product; then both of them end up losing revenue or decreasing their payoff and the total payoff is negative. So, these are negative sum games.

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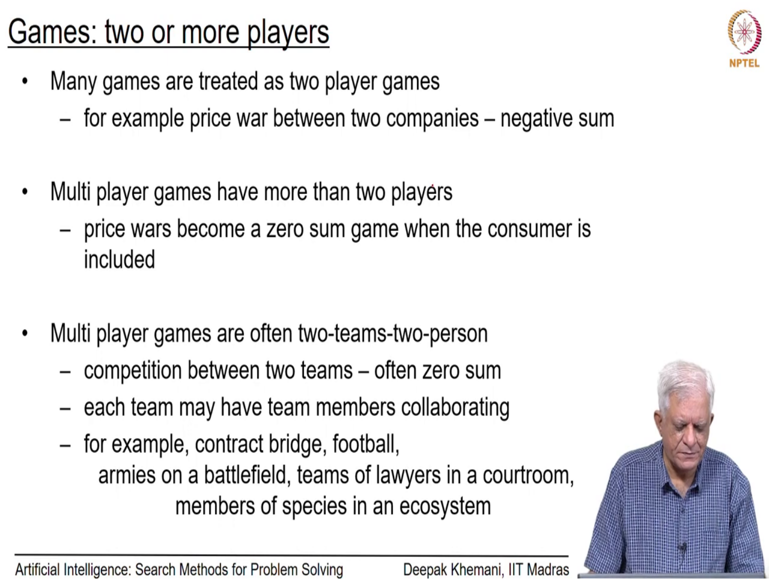

Games can be classified by the number of players as well. So, many games are treated as two player game. So, it is and many of the games that we will be interested in will be two players game. But, as we said the price war can be modeled as a game between two companies negative sum. But, it can also be modeled as a multiplayer game. So, multiplayer games have more than two players and price wars become a zero sum game when you include the consumer.

So, if there are two companies which is which are indulging in a price war, then the consumer games and the total payoff is zero in a sense. What the companies lose, the consumer's game. Multiplayer games are often team games. So, I have written this as two-teams-two-person as an example.

So, it is competition between two teams which is often zero sum for example, two football teams. There are two teams of 11 players each, the total sum is zero, but each team may have members which collaborate with each other.

Now, of course, you might imagine that there are situations where individuals in team games try to strive for individual glory; sometimes at the cost of the team. But, we are imagining here that the team members cooperate with each other. Examples of such team games are contract bridge, football, armies on a battle field, teams of lawyers in courtroom which are fighting against another team of lawyers.

Members of a species in an ecosystem you know where different species are competing for the same resources. But, as we know there is competition within the species as well as competition between the species. So, we have studied that when we looked at genetic algorithms.

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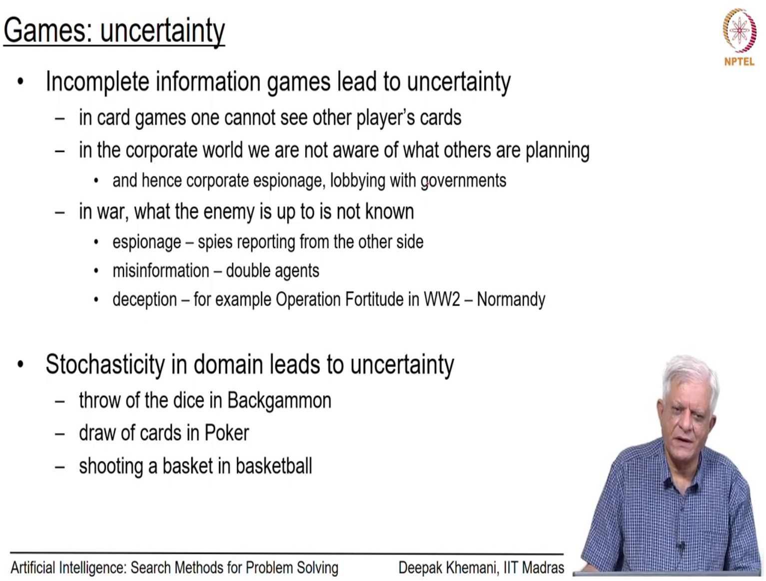

Games can be also modeled on how much information is available by the amount of uncertainty in the situation. So, incomplete information games lead to uncertainty. If you do not have enough information about the world, then clearly there is going to be uncertainty. And card games are typical examples, because you cannot see the other players cards.

In the corporate world we are not aware of what other companies are planning and very often you hear cases of corporate espionage or lobbying with governments to you know favor their own company. In a war like scenario we do not know what the enemy is up to, again espionage - spies reporting from the other side or use, spreading misinformation via double agents.

Deception is a key part of incomplete information situations. For example, in the World War 2, the allied forces they fooled the Germans into thinking that they were landing in one side whereas, they were landing in another side. So, this has to do with the Normandy landings. Uncertainty can also come due to stochasticity in domains that you are operating in.

So, for example, if you are playing a dice game like backgammon or snakes and ladders or something like that, then you do not know what the dice will show up with. And therefore, that leads to uncertainty. If you are playing a game like poker, you are drawing cards and really the strength of your hand depends on the cards that you draw or even if you are doing actions in a stochastic world.

So, for example, if you are shooting a basket into a basketball hoop in a basketball game, there is a certain amount of stochastic nature of this action. Because, it is never sure even for the best players that when they throw a ball into the basket it actually lands into a basket. So, that can also lead to uncertainty in games.

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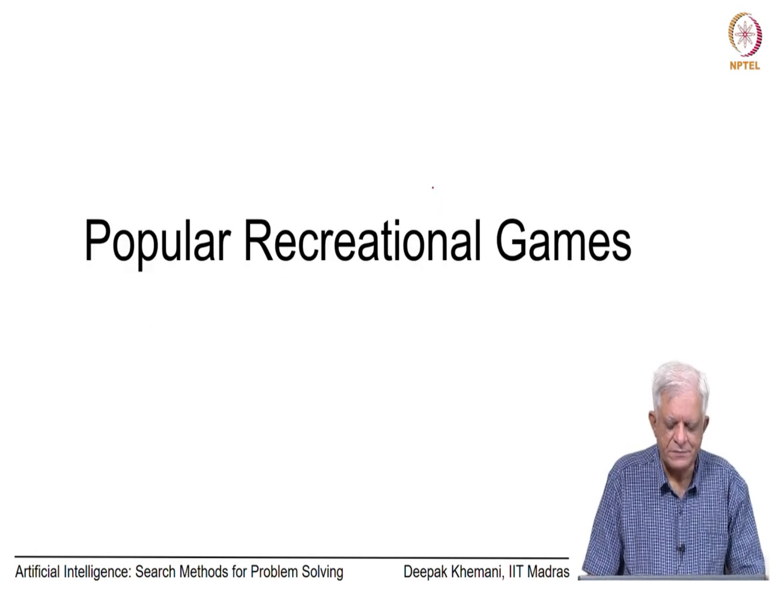

So, this is was a study of a game theory in general. And we just did this so, that we get our foothold into the games that we are interested in which are going to be smaller games; in the sense that their domains are much smaller. And, we will in the next session look at some popular recreational games essentially.