

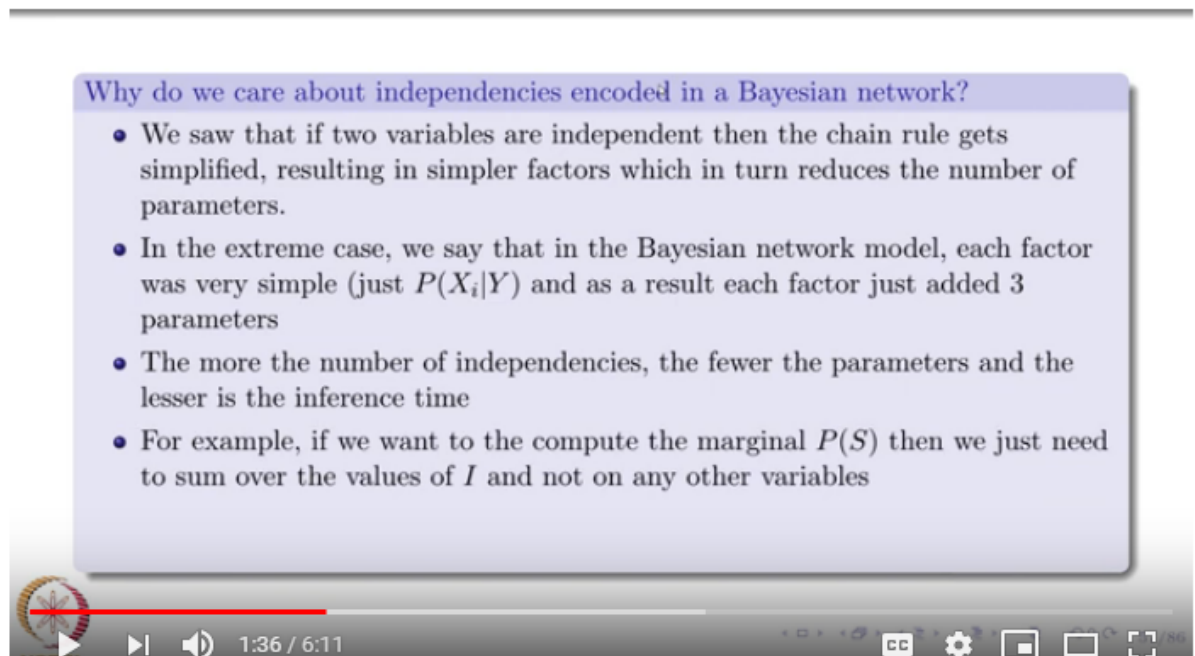
Lecture16.6

Independencies encoded by a Bayesian network

So, this was why joint distributions are important, just concretizing that idea with the help of a few examples. And now what is the other thing that we have been saying is important, which matters a lot to us in the case of a joint distribution, is local independence. So, that we care about what variables are independent of each other or what variables are independent of each other given some other variables because, if we know that then what simplifies? The chain rule simplifies, because in the chain rule you could get rid of some variables in some terms and that in turn leads, leads to what fewer parameters I will

ask this again and again and unless you answer this I'm going to keep asking this wait. So, what happens in turn you end up with everyone lesser number of parameters .Right? .Okay? And of course the computational efficiency associated with that .Okay? But that's why we are interested in knowing what are the kind of independence is encoded by a Bayesian network .Okay? So, let's look at one case which is the node and its parents which is the most obvious case.

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Why do we care about independencies encoded in a Bayesian network?

- We saw that if two variables are independent then the chain rule gets simplified, resulting in simpler factors which in turn reduces the number of parameters.
- In the extreme case, we say that in the Bayesian network model, each factor was very simple (just $P(X_i|Y)$) and as a result each factor just added 3 parameters
- The more the number of independencies, the fewer the parameters and the lesser is the inference time
- For example, if we want to compute the marginal $P(S)$ then we just need to sum over the values of I and not on any other variables

So, why do we care about independence? We saw that if two variables are independent, then the claim chain dual gets simplified, that was one thing, in the extreme case, we saw this Naive Bayes module, where we were just left with very minimalistic number of parameters because, we assumed that everything was intermittent and the more the number of independence is the fewer the number of parameters and better is the inference time and there are various such other advantages of this sight.

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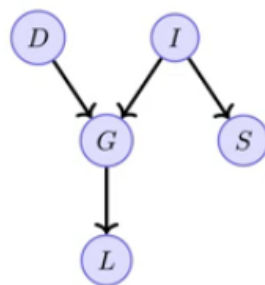
In general, given n random variables, we are interested in knowing if

- $X_i \perp X_j$
- $X_i \perp X_j | Z$, where $Z \subseteq X_1, X_2, \dots, X_n / X_i, X_j$



So, we will just go over the actual questions that we are interested in this module. Which is given n random variables we are interested in knowing if X_i is independent of X_j ? Okay? Or if X_i is independent of X_j given Z where Z could be a set of random variables from your n random variable. Right? So, the red could be the remaining n minus 2 random variable so, that's what this means X_1 to X_n minus X_i comma X_j . Okay? That's what this notation means.

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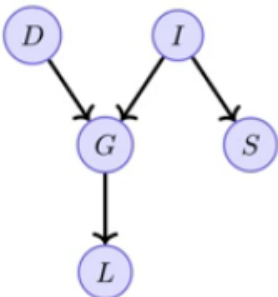


- To understand this let us return to our student example
- First, let us see some independencies which clearly do not exist in the graph
- Is $L \perp G$? (No, by construction)
- Is $G \perp D$? (No, by construction)
- Is $G \perp I$? (No, by construction)
- Is $S \perp I$? (No, by construction)
- **Rule:** A node is not independent of its parents



So, let us answer some of these questions for our Bayesian network. So, this is what will return to a student example? Now let's look at some very, very in simple independence assumptions that we have made here .Right? Or rather let's look at the ones which do not exist here. Which are the ones which do not exist which are the Independence's which do not exist in this network by design G is not independent of I and D, S is not independent of I, L is not independent of G .Right? So, by construction all of these are not independent. So, what's the general rule that you can give me? a node is not independent of its parent's .Right? So, that's the rule that, simple rule that we can come up. Right? We'll come up with two more such simple rules.

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graph TD
    D((D)) --> G((G))
    I((I)) --> G
    I --> S((S))
    G --> L((L))
  
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- No, the instructor is not going to look at the SAT score but the grade
- **Rule:** A node is not independent of its parents even when we are given the values of other variables

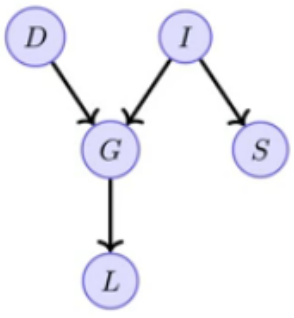
- Let us focus on G and L .
- We already know that $G \not\perp L$.
- What if we know the value of I ? Does G become independent of L ?
- No (intuitively, the student may be intelligent or not but ultimately, the letter depends on the performance in the course.)
- If we know the value of D , does G become independent of L ?
- No (intuitively, the course may be easy or hard but the letter would depend on the performance in the course)
- What if we know the value of S ? Does G become independent of L ?

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Now let us focus on G and L .Okay? You already know that G is not independent of L, what if we know the value of I? Does G become independent of L? If I know the value of I, does G become independent of L? No? But we just said that the rule was a node is independent of it's, oh sorry, so this is what would happen, if the student may be intelligent or not but ultimately the letter depends on the performance in the course. So whether if the student is going to get a good letter or not, that depends on the performance in the course. So, even if you know the value of I, L and G don't become independent of each other .Okay? If we know the value of D does G become independent of L? No. Now again the same argument write the same thing that the course is hard or difficult but once you know the grade that's completely what is going to determine them later? Even if you know the difficulty of the course nothing is going to change. Now what if we know the value of S? Does G become independent of L? No, again the letter completely depends on the grade. The instructor is not going to look at the stat score, but the grade so, what's the rule here? What's the rule? So, the previous slide we came up with a rule, that a node is not independent of its parents and I want to augment that rule, a node is not independent of its parents, even when you're given


other random variables. Right? Even when you are given the values of the other random variables .Right? So, this is the modified rule that a node is not independent of its parents even when we are given the values of the other variables. That was just as good as saying a node is not independent of its parent's, .Right? But the reason I have brought this extra clauses because, we will see that in some other cases this does not work when that's, when the parent and child relation does not hold .Okay? So, for now we have established the rule for what happens to a node and its parent's .Okay? They're always independent is what we have actually achieved? And we have just written it differently that a node is independent of its parents, not independent of its parents even when you know the other random variables .Okay?

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- The same argument can be made about the following pairs
- $G \not\perp D$ (even when other variables are given)
- $G \not\perp I$ (even when other variables are given)
- $S \not\perp I$ (even when other variables are given)

• **Rule:** A node is not independent of its parents even when we are given the values of other variables



Then the same argument can be made about which other pairs? we made about GNL what are the other pairs? That you can make this argument about first ING, DNG, INS, right all of these you will have to make the same assumption and the same rule holds here .Okay?