

**Module 17 - Week 4 - Lecture 18.08 - Motivation for Sampling - Part – 02**

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$$\frac{\partial \mathcal{L}(\theta)}{\partial w_{ij}} = \mathbb{E}_{p(H|V)}[v_i h_j] - \mathbb{E}_{p(V,H)}[v_i h_j]$$

$$Z = \sum_V \sum_H \left( \prod_i \prod_j \phi_{ij}(v_i, h_j) \prod_i \psi_i(v_i) \prod_j \xi_j(h_j) \right)$$

- That is where the problem lies!
- To draw a sample  $(V, H)$ , we need to know its probability  $P(V, H)$
- And of course, we also need this  $P(V, H)$  to compute the expectation
- But, unfortunately computing  $P(V, H)$  is intractable because of the partition function  $Z$
- Hence, approximating the summation by using a few samples is not straightforward! (or rather drawing a few samples from the distribution is hard!)

Where we're actually talking about, training RBMs and in particular we were interested in the, we were interested in the gradient of the log likelihood, with respect to a parameter, in any parameter. And we saw that this is actually is a sum of two expectations. And it's hard to, compute these expeng, expectations, because you have, a summation over an exponential number of, terms in this expectation. Right? So then you're making a case for, how we need sampling to do this, where we'll actually, approximate the true expectation, by a sample expectation or by an empirical expectation. Right? And then we gave this analogy that we could, do this routinely, if you're trying to find the, average weight of a population or average height of a population, you could randomly pick them. Because when we are assuming that, all samples are equally likely. But in our case where we have images, in this high dimensional space, there are only a few points which are actually legitimate, the rest them are just, noise. Right?

So we need to just sample points, when we are estimating something, based on their probability in the given space. Right? So that's why we want, we had to draw samples, based on probability, instead of just, drawing them randomly. Yes, so that's where we have ended. So our quest is still the same. We want a good way of approximating this intractable expectation, by computing a, sample expectation or computing a, empirical expectation. Right? So that's what we want to do. And then

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### The story so far

- Conclusion: Okay, I get it that drawing samples from this distribution  $P$  is hard.
- Question: Is it possible to draw samples from an easier distribution (say,  $Q$ ) as long as I am sure that if I keep drawing samples from  $Q$  eventually my samples will start looking as if they were drawn from  $P$ !
- Answer: Well if you can actually prove this then why not? (and that's what we do in Gibbs Sampling)

we ended with this, slight hint into, what we are going to do, today, which is, 'Gibbs Sampling'. And where were we're interested in is that, okay, we understand that this distribution

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$$Z = \sum_V \sum_H \left( \prod_i \prod_j \phi_{ij}(v_i, h_j) \prod_i \psi_i(v_i) \prod_j \xi_j(h_j) \right)$$

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is hard to, draw samples from. Because to compute that distribution, I need to compute this intractable quantity. So that's why, I will try to come up with some other distribution, which is more tractable. I'll try

to see, if drawing samples from that distribution, is the same as, drawing samples from this hard distribution. Right? So that's what we're going to,

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#### The story so far

- Conclusion: Okay, I get it that drawing samples from this distribution  $P$  is hard.
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- Answer: Well if you can actually prove this then why not? (and that's what we do in Gibbs Sampling)

try to, do today, doing something known as, 'Gibbs Sampling'.

