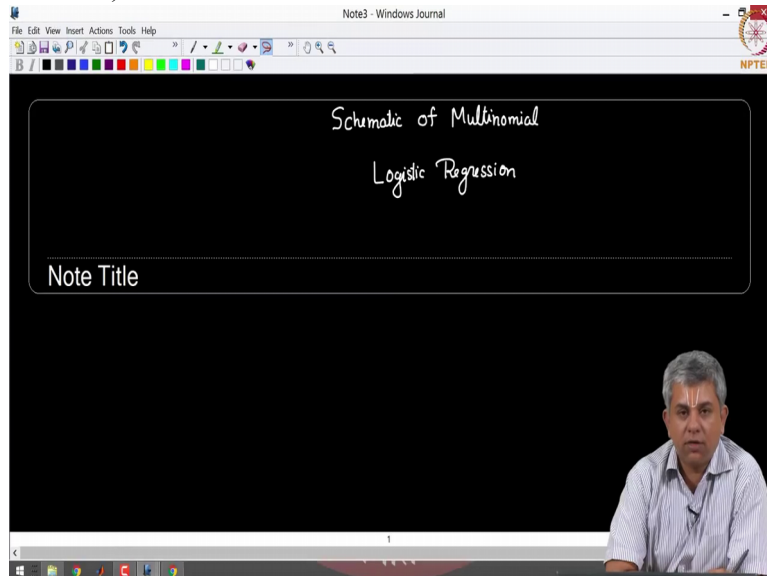


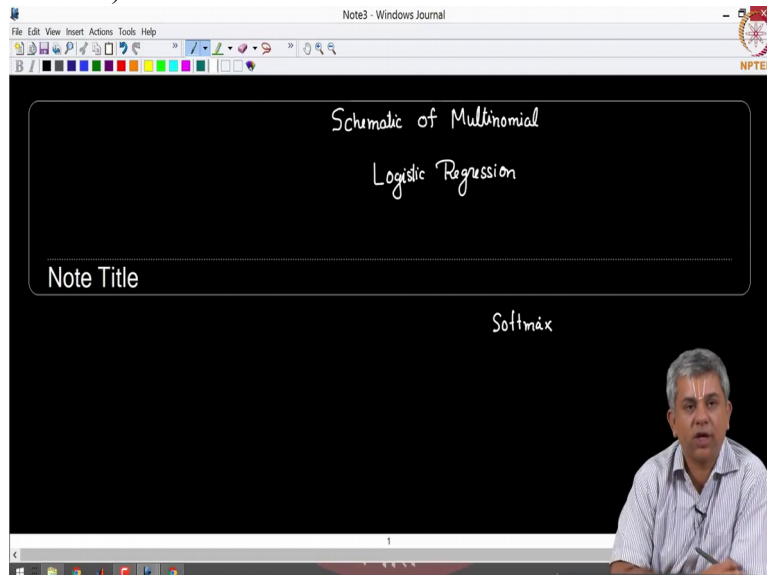
**Machine Learning for Engineering and Science Applications**  
**Professor Doctor Balaji Srinivasan**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology Madras**  
**Schematic of Multinomial logistic regression**

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Welcome back. In the previous videos we had seen how to use logistic regression for multiclass problems. We had done that using a Softmax function if you remember. We had also looked at what the corresponding

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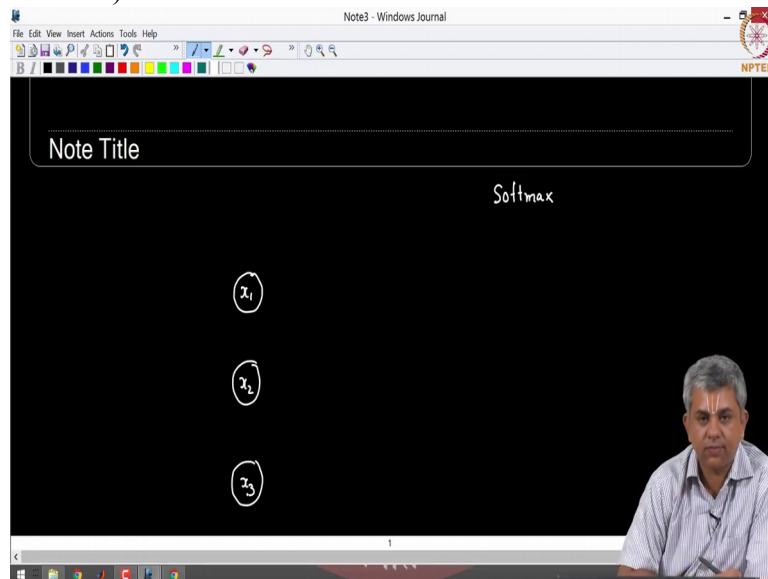


loss function was, etc.

In this video I want you to see a simple schematic which will also tell you how exactly a matrix comes when you deal with weights with multiple classes, when we have multiple classes in multinomial logistic regression.

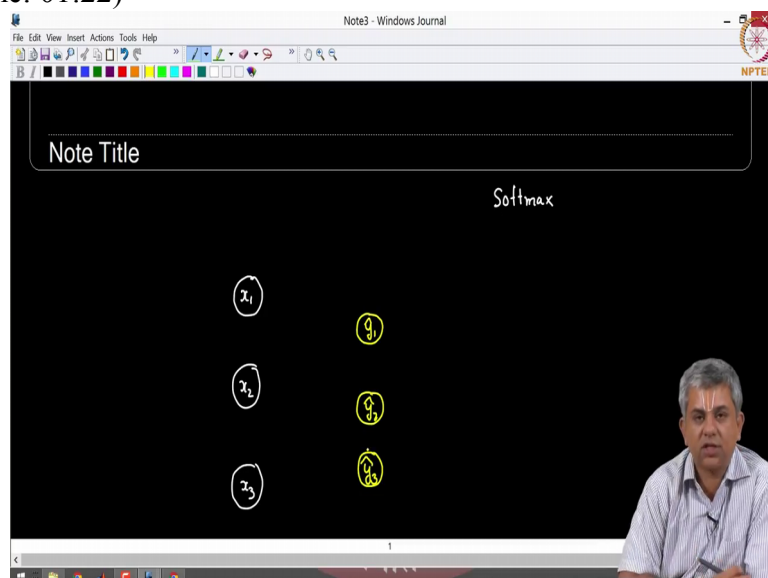
So let us consider a simple example. Let us say I have 3 input features.

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And let us say I have 3 output features also, Ok,  $y_1$  hat,  $y_2$  hat,  $y_3$  hat. So

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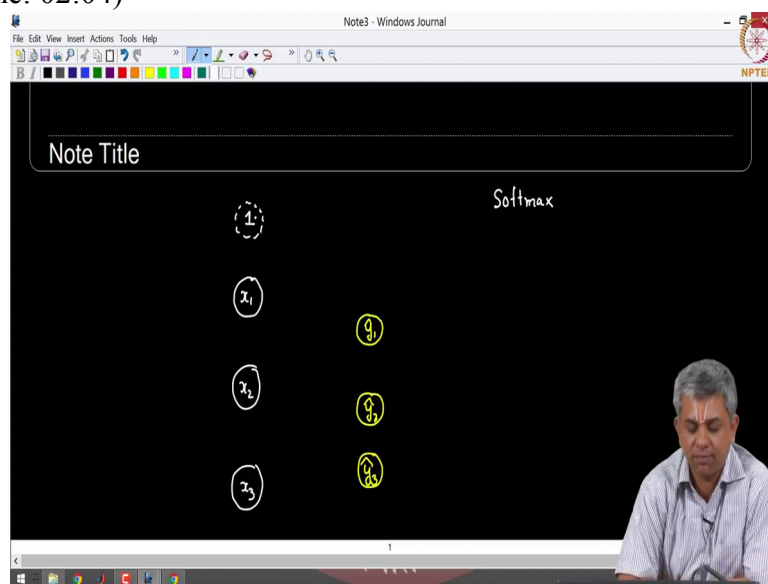


let us say this is a 3 classification problem. You can think of multiple examples for this.

For example if I give height, weight and age, suppose you want to find out whether this person has no probability of heart disease or low probability of heart disease, medium probability of heart disease or high probability of heart disease, this is not quite a classification problem but just as an example I can give you this.

We will look at several examples or at least a few examples in the examples week which will be around week 9 or so. So you can think of any convenient example for yourself. And now let us introduce our usual bias unit which is 1 or x naught

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and now we want to find out what is  $y_1$  hat,  $y_2$  hat,  $y_3$  hat.

So the portion that we are doing right now is the forward model. So as usual  $y_1$  hat

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The screenshot shows a video lecture interface. At the top, there is a window titled "Note3 - Windows Journal" with a menu bar (File, Edit, View, Insert, Actions, Tools, Help) and a toolbar. Below the toolbar is a "Note Title" field. The main content area is a blackboard with the word "Softmax" written in white. On the left, a diagram shows a vertical stack of nodes: a bias node labeled '1' in a dashed circle, followed by input nodes labeled  $x_1$ ,  $x_2$ , and  $x_3$ . Lines connect these four nodes to a central hidden node labeled  $\hat{y}_1$  in a yellow circle. To the right of  $\hat{y}_1$  are two output nodes labeled  $\hat{y}_2$  and  $\hat{y}_3$  in yellow circles. In the bottom right corner, a small video inset shows a man with grey hair wearing a light blue shirt, looking towards the camera.

is equal to Softmax of the linear combinations of this,  $w_0$  plus  $w_1 x_1$  plus  $w_2 x_2$  plus  $w_3 x_3$ , Ok.

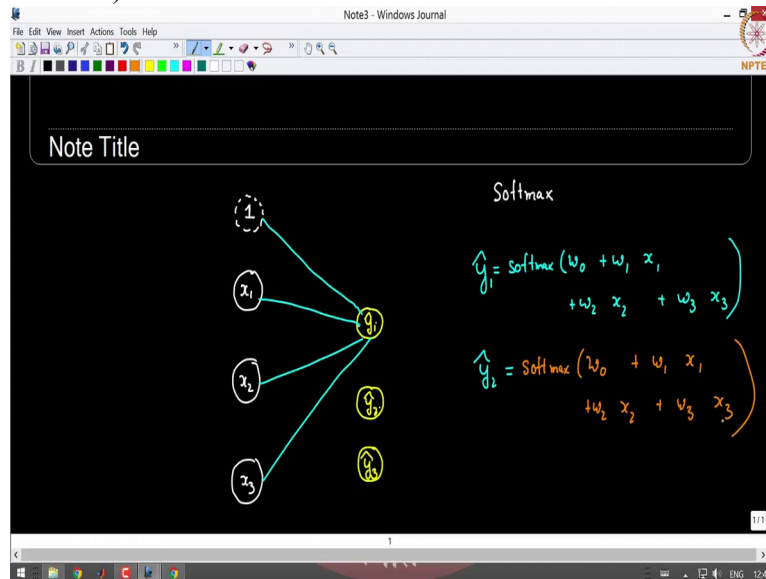
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This screenshot is similar to the previous one, showing the same neural network diagram and speaker. However, the word "Softmax" is now written in green. To the right of the hidden node  $\hat{y}_1$ , a handwritten equation in green ink reads: 
$$\hat{y}_1 = \text{Softmax} (w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3)$$

That would be  $y_1$  hat. Now suppose I have  $y_2$  hat.

Now  $y_2$  hat is also Softmax of some linear combination,  $w_0$  plus  $w_1 x_1$  plus  $w_2 x_2$  plus  $w_3 x_3$ . Now suppose

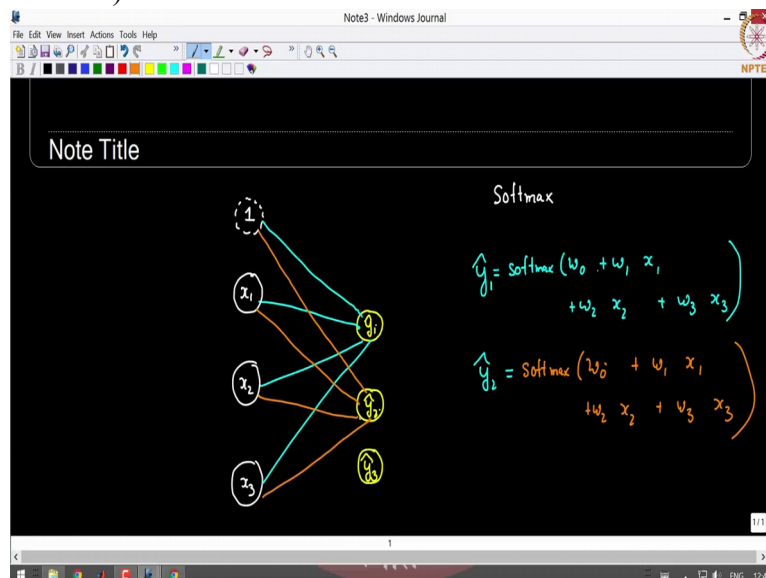
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this weight,  $w_1$ ,  $w_2$ ,  $w_3$  were the same in both these cases, obviously you are going to get the same  $\hat{y}_1$  as well as  $\hat{y}_2$ . Because otherwise the functions are identical.

So this is not a good idea. So you need different weights. So we are going to use different weights here, Ok.

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So we need some terminology in order to distinguish these two weights. So I will call it weight 1,  $w_{11}$ ,  $w_{21}$ ,  $w_{31}$  where the 1 stands for the output and the 0, 1, 2, 3 actually stand for the input.

Similarly you can easily see that now this should be weight 2,  $w_{12}$ ,  $w_{22}$  and  $w_{32}$ .

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Note Title

Softmax

$$\hat{y}_1 = \text{Softmax}(w_{01} + w_{11} x_1 + w_{21} x_2 + w_{31} x_3)$$
$$\hat{y}_2 = \text{Softmax}(w_{02} + w_{12} x_1 + w_{22} x_2 + w_{32} x_3)$$

Finally if we come here, I need another set of weights.

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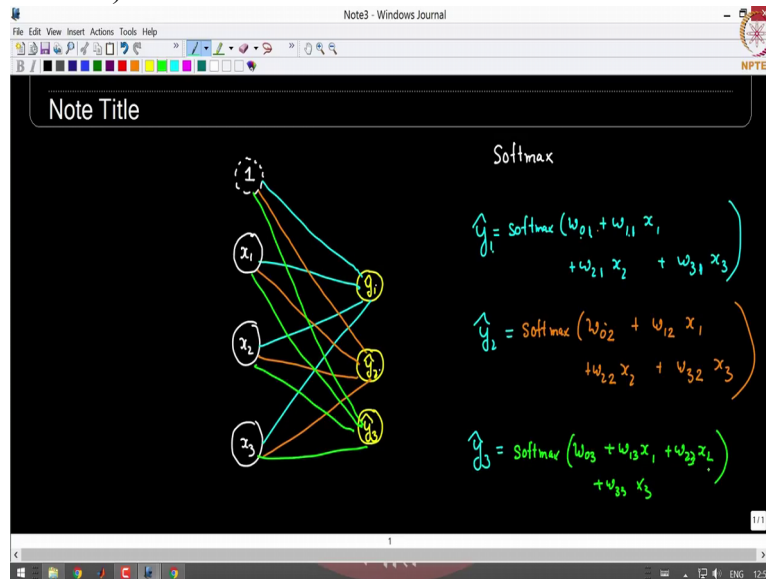
Note Title

Softmax

$$\hat{y}_1 = \text{Softmax}(w_{01} + w_{11} x_1 + w_{21} x_2 + w_{31} x_3)$$
$$\hat{y}_2 = \text{Softmax}(w_{02} + w_{12} x_1 + w_{22} x_2 + w_{32} x_3)$$

So  $\hat{y}_3$  would be Softmax of  $w_{03}$ ,  $w_{13} x_1$ ,  $w_{23} x_2$  plus  $w_{33} x_3$ .

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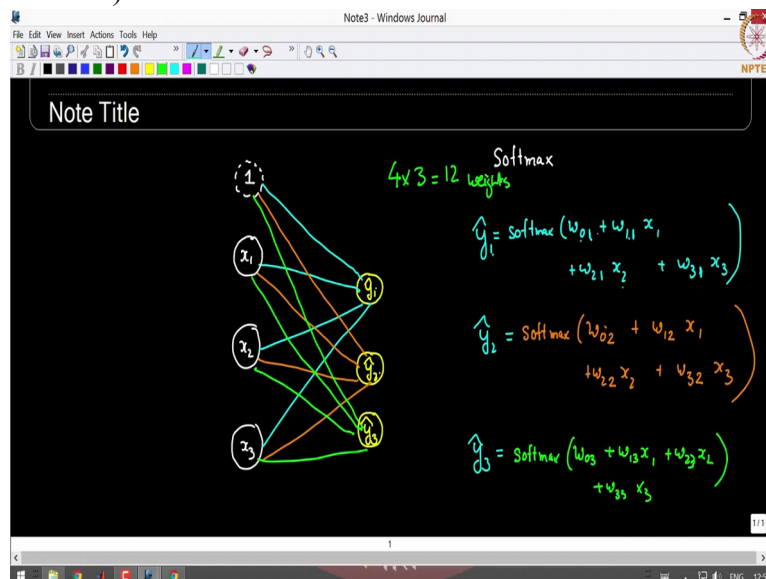
Note Title

Softmax

$$\hat{y}_1 = \text{Softmax}(w_{01} + w_{11}x_1 + w_{21}x_2 + w_{31}x_3)$$
$$\hat{y}_2 = \text{Softmax}(w_{02} + w_{12}x_1 + w_{22}x_2 + w_{32}x_3)$$
$$\hat{y}_3 = \text{Softmax}(w_{03} + w_{13}x_1 + w_{23}x_2 + w_{33}x_3)$$

So how many weights do we have? 4 unique weights in each one of these, so you have 4 into 3,

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Note Title

Softmax

$4 \times 3 = 12$  weights

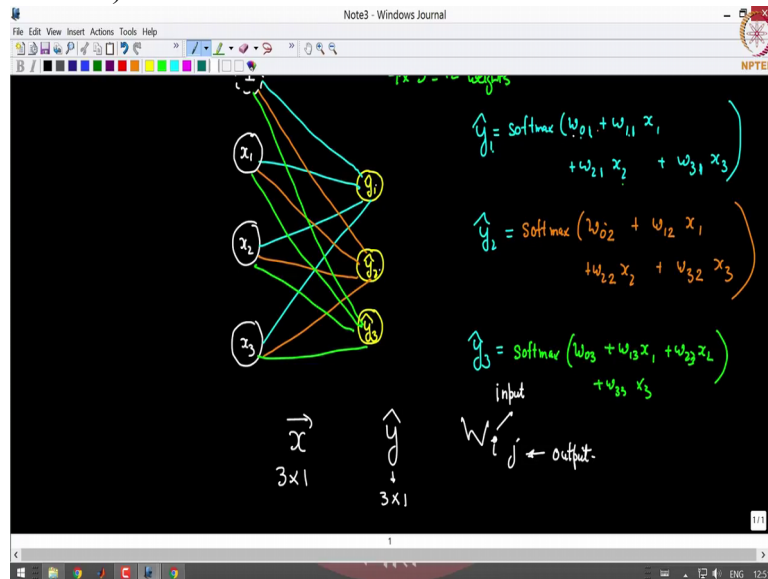
$$\hat{y}_1 = \text{Softmax}(w_{01} + w_{11}x_1 + w_{21}x_2 + w_{31}x_3)$$
$$\hat{y}_2 = \text{Softmax}(w_{02} + w_{12}x_1 + w_{22}x_2 + w_{32}x_3)$$
$$\hat{y}_3 = \text{Softmax}(w_{03} + w_{13}x_1 + w_{23}x_2 + w_{33}x_3)$$

12 weights in order to account for the bias term also.

So how would we write this matrix wise? So we have x vector which was 3 cross 1, we have y hat which is also 3 cross 1 and we have w which is now a weight matrix.

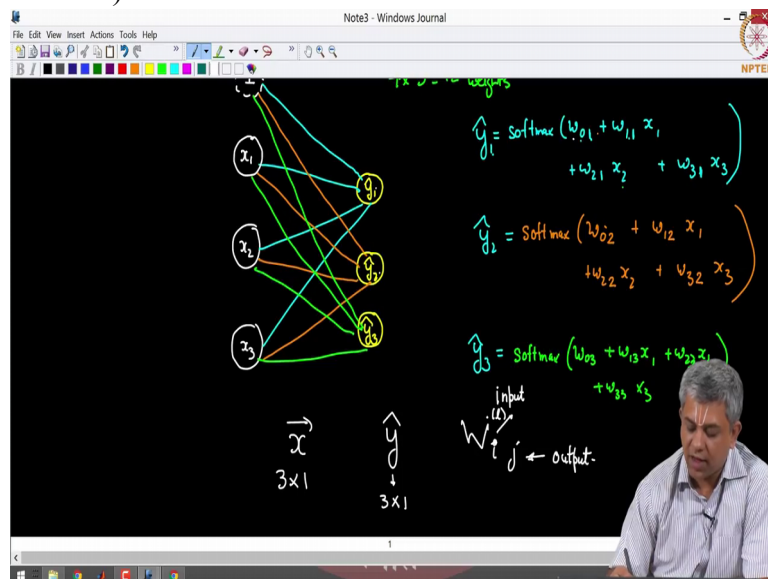
You can see w has 2 indices. w i j

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has  $i$  as the input feature,  $j$  is the output feature. You had also seen in the earlier video with XOR that you could have more than 1 layer. In that case typically

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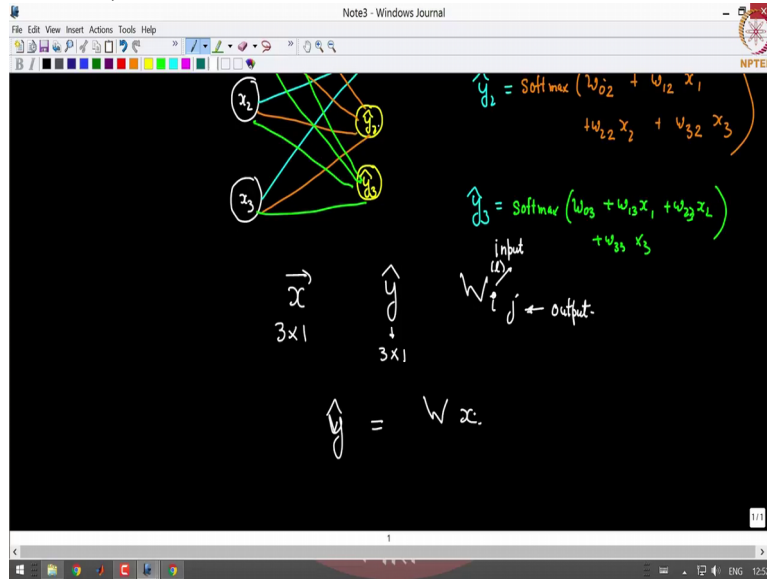
we add an  $L$  here which denotes the level. So you could have  $w_{ij1}$ ,  $w_{ij2}$ ,  $w_{ij3}$  etc, etc.

So you will have multiple weights. So this is the large number of weights as you will see in next the, in the next, the videos in the next week when we come to convolutional neural networks, you have billions and billions of parameters in usual practical neural networks that sit in today which is why they are extremely powerful, Ok.

Coming back to this, if we want to write  $\hat{y}$  as  $w$  times  $x$ ,

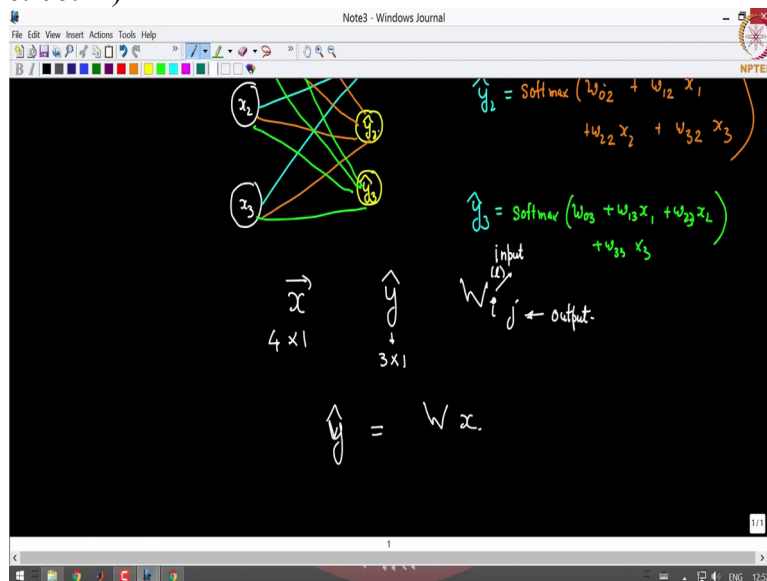


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so just for this case I will make x as 4 cross 1 so that

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the 4 includes our bias unit also.

So you can write  $\hat{y}$  as  $W x$ .  $x$  will be 4 cross 1,  $y$  will be 3 cross 1. So if you want an appropriate  $W$ , please imagine what  $W$  should be. This should be

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Diagram illustrating a neural network with 4 input nodes and 3 output nodes. The input vector is  $x$  ( $4 \times 1$ ) and the output vector is  $\hat{y}$  ( $3 \times 1$ ). The weight matrix is  $W$  ( $3 \times 4$ ). The output is calculated as  $\hat{y} = Wx$ . The activation function is given as  $\hat{y}_j = \text{Softmax}(w_{j0} + w_{j1}x_1 + w_{j2}x_2 + w_{j3}x_3)$ .

3 cross 4.

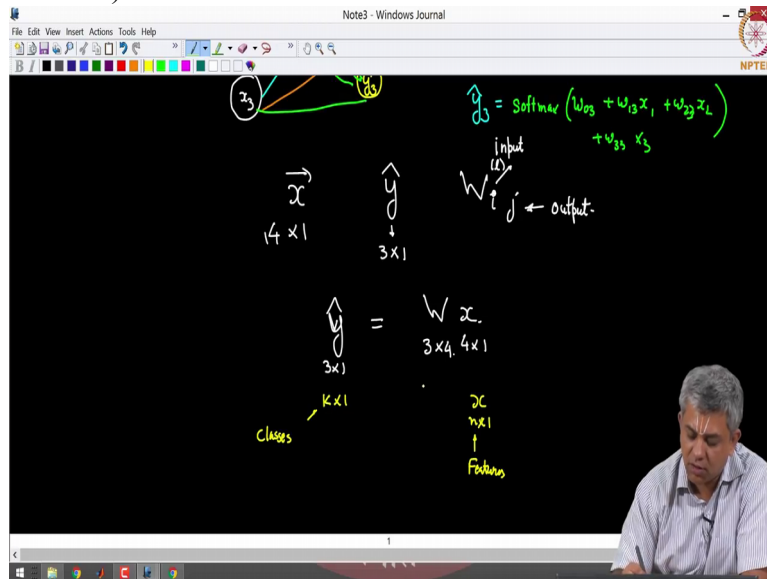
In the general case, if  $\hat{y}$  is  $k$  cross 1 where  $k$  is the number of classes

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Diagram illustrating a neural network with 4 input nodes and  $k$  output nodes. The input vector is  $x$  ( $4 \times 1$ ) and the output vector is  $\hat{y}$  ( $k \times 1$ ). The weight matrix is  $W$  ( $k \times 4$ ). The output is calculated as  $\hat{y} = Wx$ . The activation function is given as  $\hat{y}_j = \text{Softmax}(w_{j0} + w_{j1}x_1 + w_{j2}x_2 + w_{j3}x_3)$ .

and  $x$  is  $n$  cross 1 where  $n$  is the number of features.

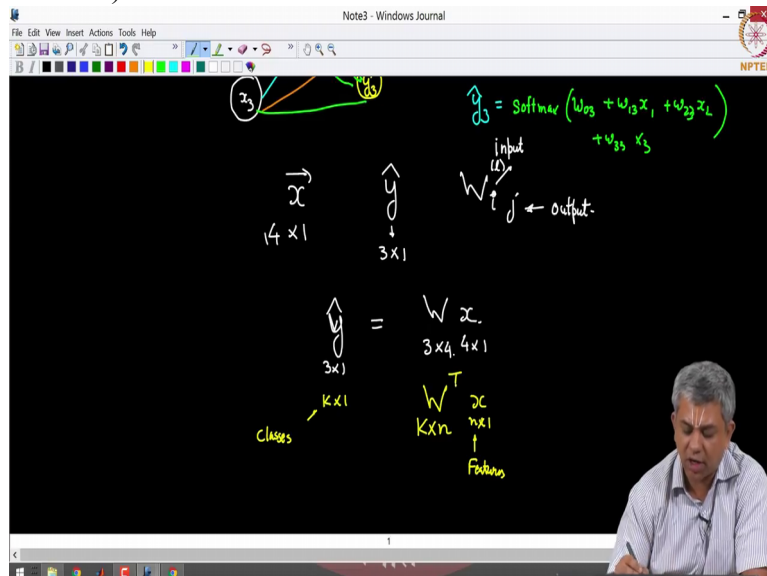
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Then  $w$  should have the size  $k$  cross  $n$ , Ok.

Now there are some people who will denote this  $w$  as

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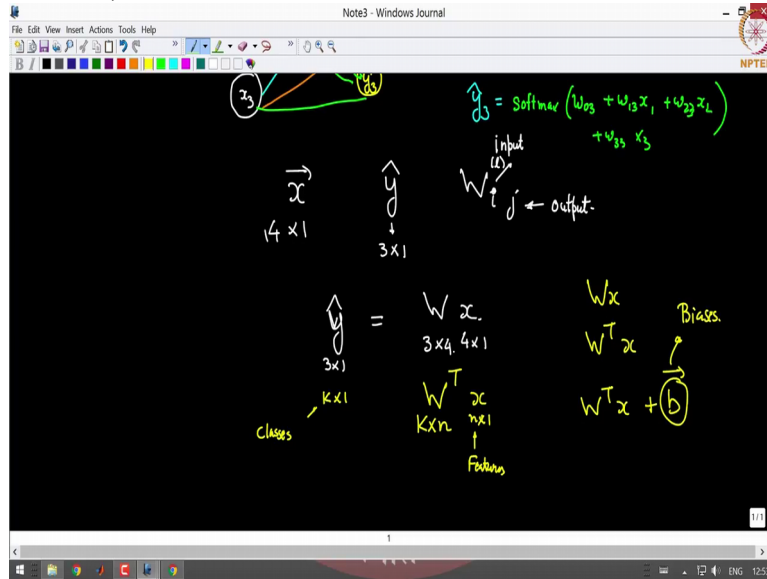


$W$  transpose so as to be consistent with the notation I have used here.

So you might see this at multiple places sometimes, you will see  $w \cdot x$ , sometimes you will see  $w^T \cdot x$ . Sometimes you will also see  $w^T \cdot x + B$  where  $B$  is a vector, the vector of biases.

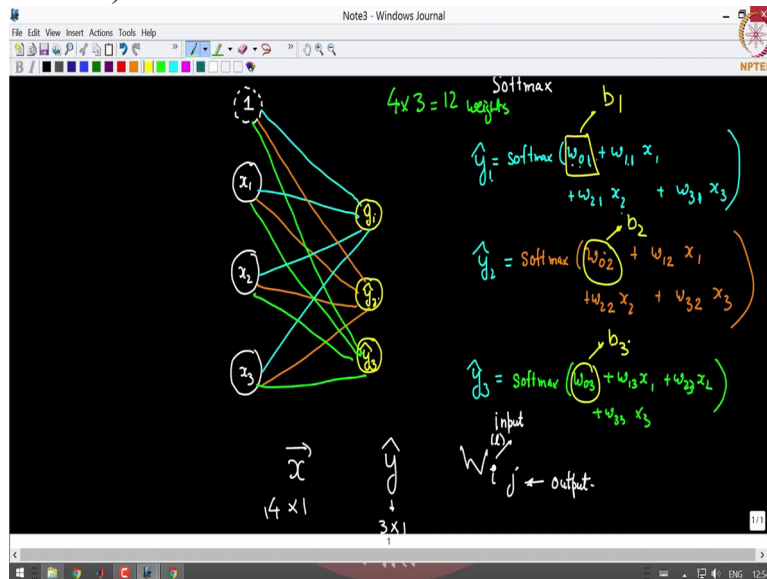
So just to

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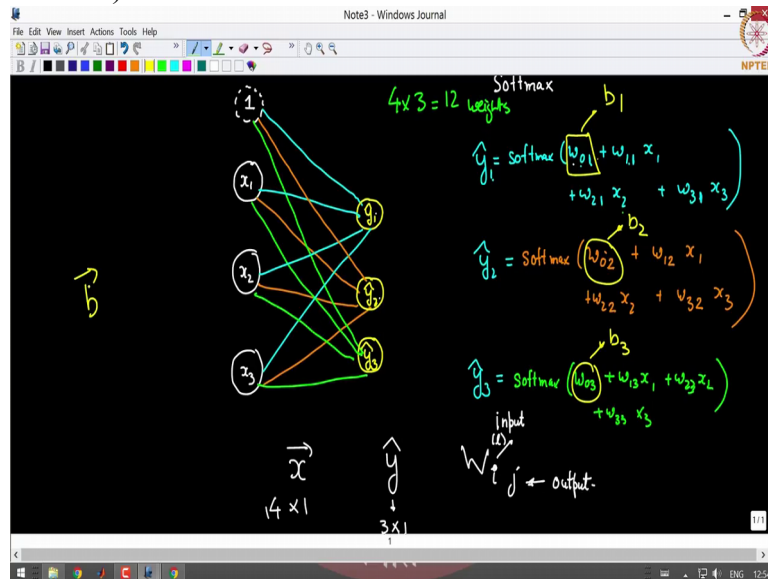
clarify this notation for you, please notice, if we remove the bias separately this will become b 1, this will become b 2 and this will become b 3.

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b vector

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is separated and  $w x$  is separated in such notations.

So we will be using this kind of notation. As I said before we will be using this interchangeably especially when it comes to future videos and future weeks, thank you.