Machine Learning for Engineering and Science Applications
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Introduction to Constrained Optimization

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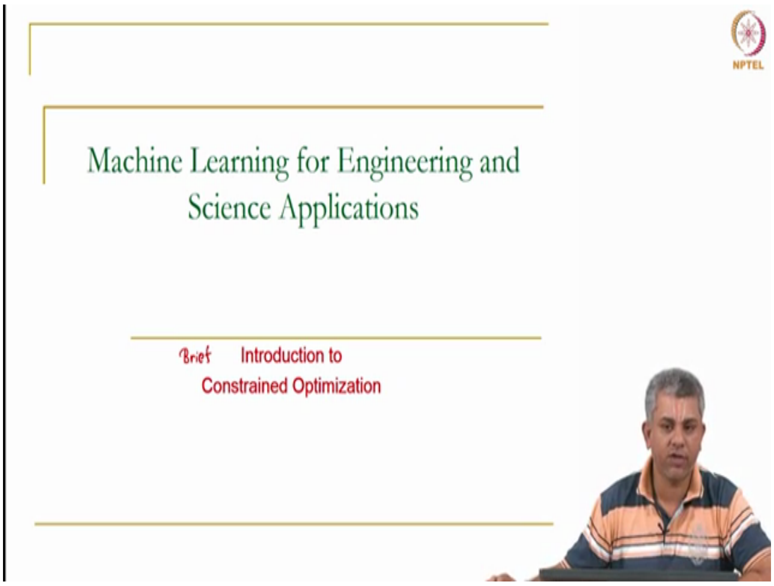

In this video we will be looking at an introduction to constrained optimization, this is a very very brief introduction because the topic is vast by itself, we looked at unconstrained optimization in the previous video, this is just to introduce you very quickly to constrained optimization we will not be doing any proofs this is just to tell you the overall idea of how constrained optimization is introduced and how it works, we will look at proofs later on week 9 or 10 of this course.

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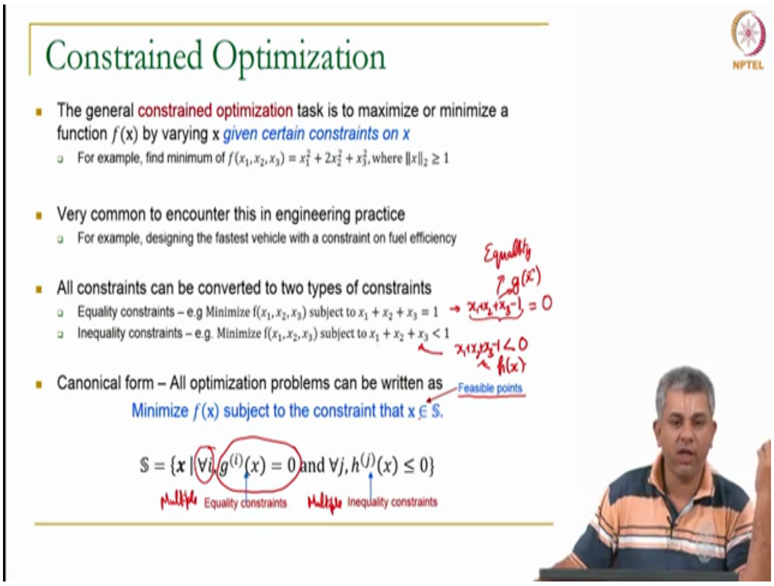

So unlike unconstrained optimization where you find out minimum of f of x for any x, for the constrained optimization task you find out f of x for certain constraints given on x. So for example you could say find out minimum of the function of x 1, x 2, x 3 which is like this where norm of x is greater than 1, we could also for example give engineering examples a theoretical example would be something like you want to design a vehicle which goes really fast but you also want to give a constrained on fuel efficiency, so you do not have unlimited fuel.

You could also say something like you know find out the weight of the best weight for this chair but at the very least you have a constrained that the weight of the chair has to be positive because mathematically you might not always be constrained by that. So such cases are very normal and in fact very very common, constrained optimization is probably the most natural occurring optimization problem.

Now there can be two types of constraints one set of constraints are equality constraints. So for example minimize f of x 1, x 2, x 3 some function given that x 1 plus x 2 plus x 3 equal to 1. For example, you could say something like given that I fixed the length of a wire, (what is the area of maximum) what is the figure of maximum or minimum area that I can make, okay.

Similarly, inequality constraint, inequality constraint is instead of giving x 1 plus x 2 plus x 3 equal to 1 you say x 1 plus x 2 plus x 3 has to be less than 1, find out my fastest going vehicle given that fuel consumption has to be less than a give value this would be an example of a constrained optimization with an inequality constraint.

Now we come to something called the canonical form just like all optimization problems could be written as minimization problems it turns out all constrained optimization problems can be written in a particular way. So the (overall method is very) overall expression is very simple, minimize f of x remember I can always (minimize) maximize by putting minus f, so minimize f of x subject to the constraint that x belongs to a given set S, okay.

So the expression for S will look like this at first side this will look quite complicated but it is actually fairly straight forward. So instead of giving one constraint, I will give multiple constraints. So let us say I have a whole bunch of equality constraints and a whole bunch of inequality constraints, this set S for x which satisfy this constraint are called feasible points.

So for example let us say somebody comes with a design of a car and you had set a fuel consumption rate of saying atleast it should give me you know 10 miles per litre, but they give a design of a car which gives you 3 miles per litre and then you will say this is outside, this is not a feasible design, okay. So the constraint is what decides whether your final design or final optimum is feasible or infeasible.

So x that satisfies the constraint is called a feasible point and we have written this big expression but these are actually fairly simple expressions. So instead of having one equality constraint let us say you have multiple equality constraints. So you are designing a room you could say something like length has to be this much, breadth has to be this much, but you are free to decide on the height. In that case so your length and breadth would now be equality constraints.

You could also have bunch of inequality constraints, so the equality constraints are simply written (of) as g of i or g i of x is 0. For example, this can be rewritten as x 1 plus x 2 plus x 3 minus 1 equal to 0. So any equation by bringing the constants on one side you can write it as an equality constraint and now this is your new g of x, g of x is x 1 plus x 2 plus x 3 minus 1.

Similarly, all inequality constraints can be written as something less than equal to 0, this can be written as x 1 plus x 2 plus x 3 minus 1 is less than 0 and now your h of x would be x 1 plus x 2 plus x 3 minus 1. So the feasibility set is usually a combination of equality constraints and inequality constraints.

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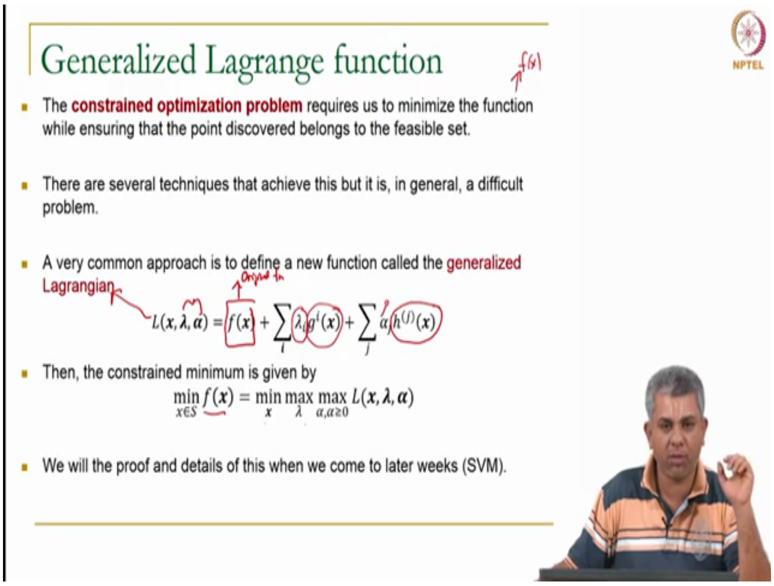

Now how do we solve this problem? The expression I am going to write right now is actually I am going to write it just of the (())(5:44) it will look a little bit complicated, even if your familiar with what is called Lagrangian functions, but we will see this this is actually finally turns out it is a fairly simple function, we will see details of this in week 9 or 10. So remember that our constrained optimization problem wants us to minimize the function f of x while ensuring that the point discovered belongs to the feasible set this is in general a difficult problem.

A very common approach for this is what is called the generalized Lagrangian, what is a generalized Lagrangian? This implies a standard trick in a lot of mathematics. Suppose your finding a problem which is difficult to solve, you can actually simplify the problem by adding extra variables, this is sort of like in geometry you made some extra constructions in order to make proofs it is similar to that, you add a few things which are now there.

So we add a couple of things we add these two variables called lambda and alpha both are vectors, here this was our original function and you create a new function called the Lagrangian L this is the original function plus your equality functions multiplied by some arbitrary constant plus your inequality function multiplied by another arbitrary constant, okay. So you just sum this up and make a giant new function called L, which is f plus lambda times g plus alpha times h.

Then it turns out please once again before seeing the expression, the expression looks complicated but it is actually fairly straight forward as we will see later in week 8 or 9 till then you will not be using constrained optimization problem. It turns out that the minimum of f is exactly this minimum of maximum of maximum of L, so we will see details of this later when we do what is called support vector machines.

For the extent of this all we would like you to recognize is to know that there is something called constrained optimization which is different from unconstrained optimization, thank you.