**Deep Learning**
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**Module – 6.3**
**Eigenvalue Decomposition**

**Lecture – 06**

And before that we will have to see something known as Eigenvalue decomposition. So, the answer to that was actually which I was hoping all of you will give because all of you have done 2 prerequisite, which is linear algebra and machine learning, both of them teach you principal component analysis. So, I was hoping that you will give that answer.

Now, can you give that answer he already of course, gave that answer, is that make sense ok? So, we relate it to that so, but before going to principle component analysis, we look at Eigen value decomposition, how many of you have seen Eigenvalue decomposition before ok? Quite a few.

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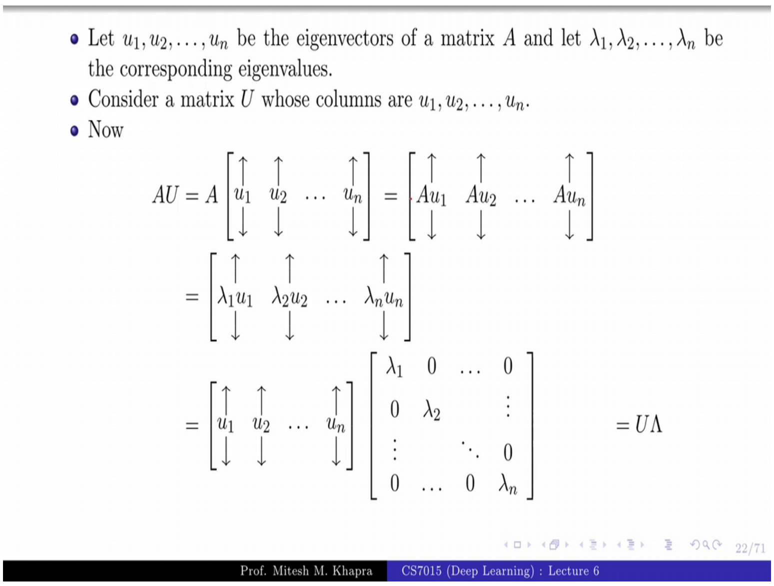


This is very straightforward. So, let u 1 to un be the Eigenvectors of a matrix A and let lambda 1 to lambda n be the corresponding Eigenvalues ok.

Now, I am going to construct a matrix U, such that the columns of U are these vectors u 1 to u n, is that fine, what u looks like. And now I am going to do this product I am taking a the product of the matrix A with the product of with the matrix U, where U is this right. It is the all the Eigen vectors tagged one after the other is this fine. The next step I am just pushing the matrix inside. If you know the 4 different ways of multiplying a matrix you will know that this is correct ok. Or else for now just thing that you can just push the matrix inside ok.

Now, what is this I can replace them by the lambda 1 u 1 lambda 2, because a u 1 is equal to lambda 1 u 1 by definition ok. Now can you write this again as a product of 2 matrices, one is of course, the matrix U and the other is.

Student: Diagonal.

Diagonal. So, the diagonal matrix will come first or the matrix U will come first? How many if you say U will come first? How many if you say the diagonal matrix will come first? The sum is never one ok.

So, it is going to be like this ok. And you can write this as U lambda. So, U is again the vector the matrix containing the Eigenvectors of A and lambda is a diagonal matrix where every diagonally element is a corresponding Eigen value.

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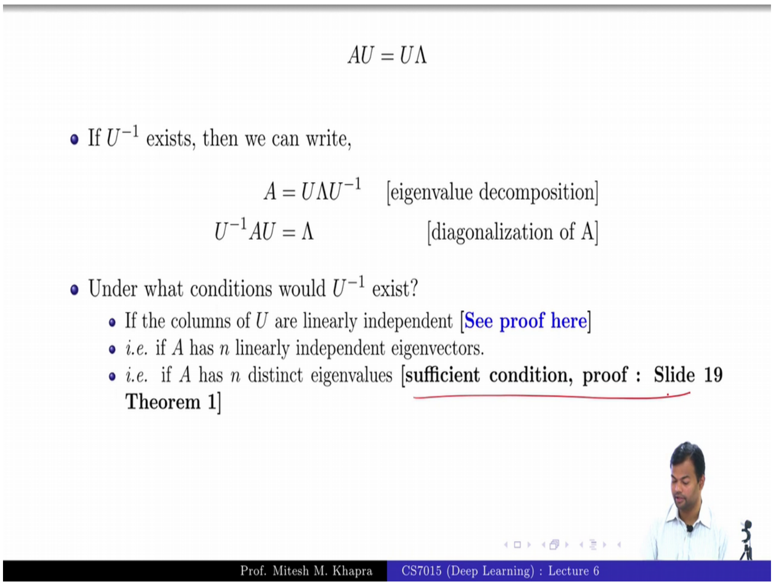


Now, this is what we have so far A into U is equal to U into lambda. Now suppose U inverse exists, I will assume that U inverse exists and later on I will tell you under what conditions it exists, then I could write it as this any of these 2 forms in one case. I am post multiplying by U inverse in the other case I am pre multiplying with universe ok.

So, this is known as the Eigenvalue decomposition of a matrix. And the other way of writing it is known as diagonalization of the matrix right. You take a matrix apply some operations to it so that, the result is a diagonal matrix is this clear to all of you is very straightforward ok. And again Eigen vectors play an important role in this. Now the important question is under what conditions would U inverse exist. U inverse would exist if the columns of the matrix U are.

Student: Linearly independent.

Linearly independent ok. Do we know the columns of the matrix are linearly independent?

Student: Yes.

Yes, because it is a.

Student: (Refer Time: 03:10).

Set of Eigenvectors and we already saw the proof that the Eigen vectors are linearly independent ok. This just follows whatever I say ok. Now do we need proof for this I slide 19 we did this ok, I did not realize it fine.

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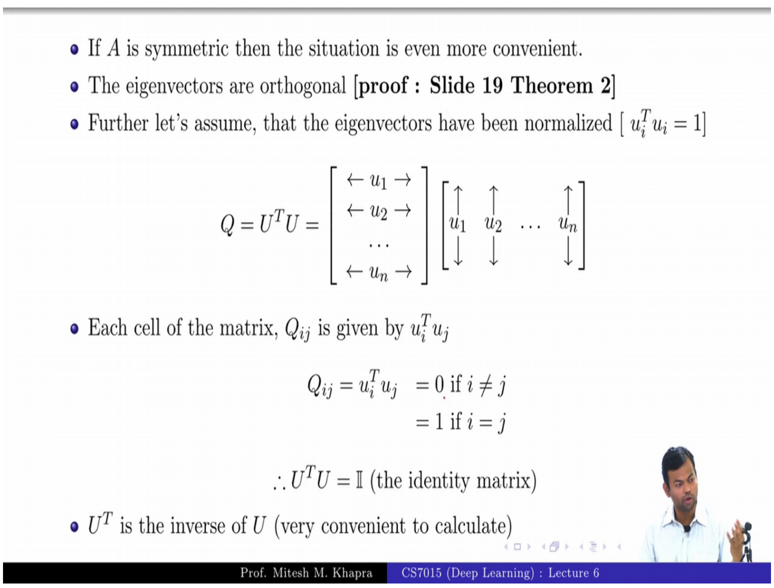

Now if A is symmetric the situation is always more convenient, why is it?

Student: (Refer Time: 03:40).

What would U be?

Student: Orthogonal matrix.

What is an orthogonal matrix actually?

Student: (Refer Time: 03:47).

So the Eigenvectors are orthogonal. So, we have this situation right. Suppose I want to do U transpose U ok. This is how that operation would look like ok. Now what is the ijth entry of the resultant matrix ?

Student: Dot product.

It is the dot product between the.

Student: ui and uj.

ui and uj. Everyone gets this right, the ijth entry of this product is going to be the dot product between ui and uj. This dot product would be dash if i is not equal to 0 or j.

Student: j.

J and there is no point in this. So, each cell of the matrix Q ij is given by the dot product and it is going to be 0 if i not equal to j, and it is going to be 1 if i is equal to j ok. So, U transpose U is equal to the identity matrix; that means, U transpose is the dash of U.

Student: (Refer Time: 04:45).

Transpose of U and of course, inverse also ok. So, U transpose is the inverse of U. And it is very convenient to calculate what is the complexity of inverse? So now, you appreciate that that is a that has high complexity and in this case if the vector if the matrix is orthogonal; that means, it is a collection of orthogonal vectors and the inverse just comes for free right ok.

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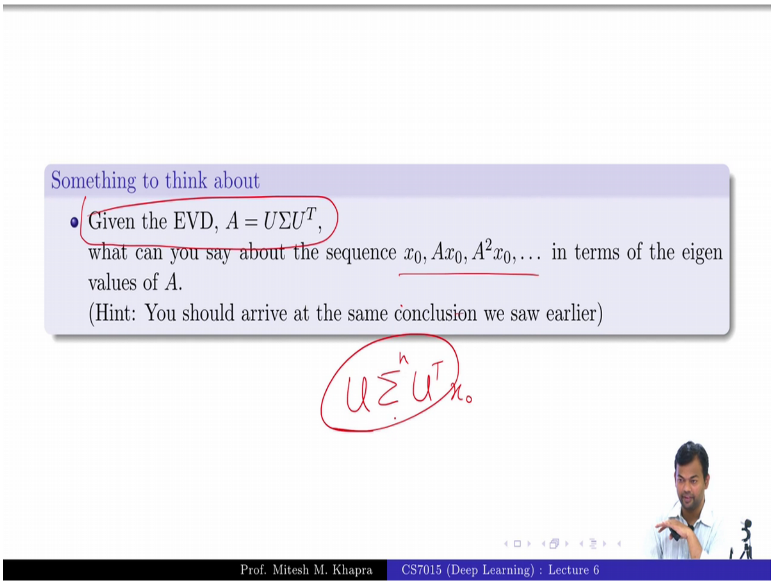


So now given this situation, and do not read the hint as if this is going to help, but yeah, what can you now say about the sequence? The same sequence that you saw earlier. So, I have given you that the EVD of a is equal to U sigma U transpose, where u is the collection of the Eigenvectors and sigma is the Eigen values the diagonal matrix containing the Eigenvalues.

Now, what given this and ignoring the knowledge of the first section of this lecture can you tell me something about this series? What would be the nth element of the series?

Student: U sigma power n.

U sigma.

Student: Power n.

Power n.

Student: U transpose.

U transpose and you arrive at the same conclusion right? Where I was talking about this operation right. So, if we can say something about this matrix then we can say something about this series what can you say about this matrix, if the largest Eigenvalue is greater than 1 as you keep raising it is power that value is going to explode. And hence, the entire product is going to explode less than 1. That product is going to vanish and everything else would be less than that right remember is the dominant Eigen value ok.

So, everything would be less than that. So, that product will vanish ok. So, the same conclusions you can arrive at right. So, that is why I want to do these sections again. So, you would have done these in linear algebra, but you would have not arrived at these conclusions from a very different interpretation, but I want to focus on the interpretations that I care about. I do not, how many of you have seen this series in the course on linear algebra? You have ok, but I do not see why anyone else would teach this is not required is only required for some things that I want to do in the course right, that is why; I wanted to do this section.

So, everyone is comfortable with Eigenvalue decomposition it is a very simple stuff right I mean there is no proof or anything involved there we just use some properties of Eigenvectors and Eigenvalues and do it ok.

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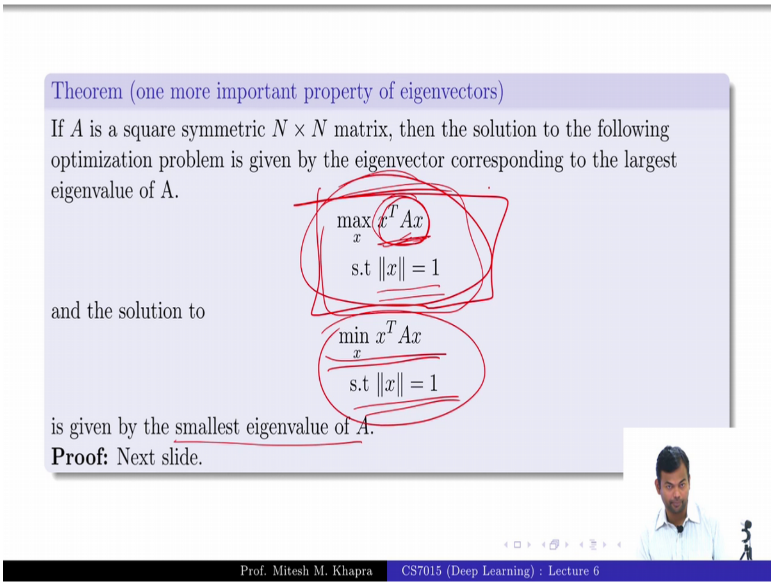


Now, there is one more important property of Eigenvectors, which well use today. So, let us see what this means right. You have a matrix A which is an n cross n matrix ok. And your import interested in computing this value, x transpose A x where x belongs to Rn x belongs to Rn ok.

So, what am I trying to do here of all these vectors possible in Rn, I want that vector which maximizes this quantity. What is this quantity scalar, vector, matrix, tensor?

Student: Scalar.

Scalar ok, such that x is equal to1. This is the problem that I have been given to solve why it is not clear as of now, but suppose this is a problem I am trying to solve, or the inverse of this which is minimize the same thing, of all the vectors in Rn find the vector which minimizes this quantity, subject to these constraints. Then the solution for this is given by the smallest or largest the solution is the smallest Eigen value of A.

And x is the Eigenvector corresponding to that. So, if you are trying to minimize and the solution is a smallest Eigenvalue, we need to clarify that if you are trying to maximize and the solution is the largest Eigenvalue is that clear and the value of x would be the corresponding Eigen value. So, largest Eigen vector is the same as something that we have defined today dominant Eigen vector right.

So, let me just repeat. So, that there is no confusion. Let us focus on this problem. The solution to this problem that is the x which will give me the maximum which will maximize this is the dominant Eigen vector of the matrix A right, is that clear? Fine ok. And if you want to minimize it is going to be the smallest Eigen vector; that means, the inverse of the dominant ok.

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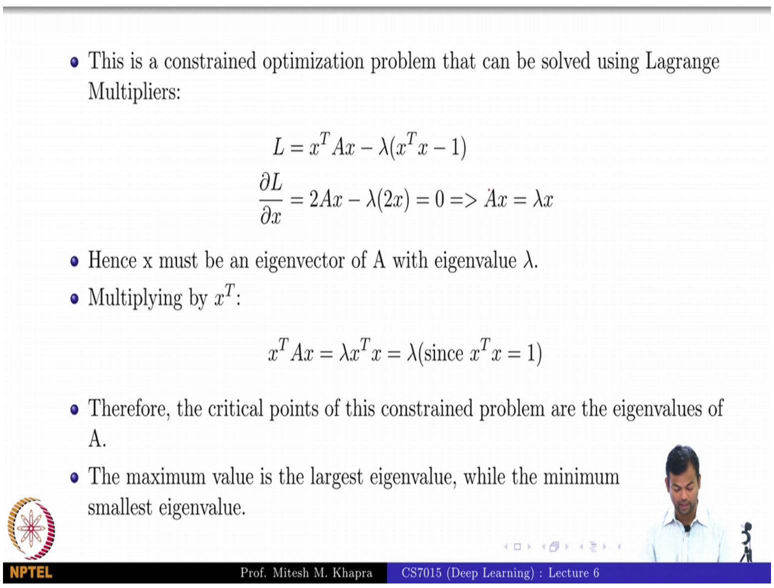

So, there is a proof for that I will not go over the proof you can take a look at it at your own leisure.

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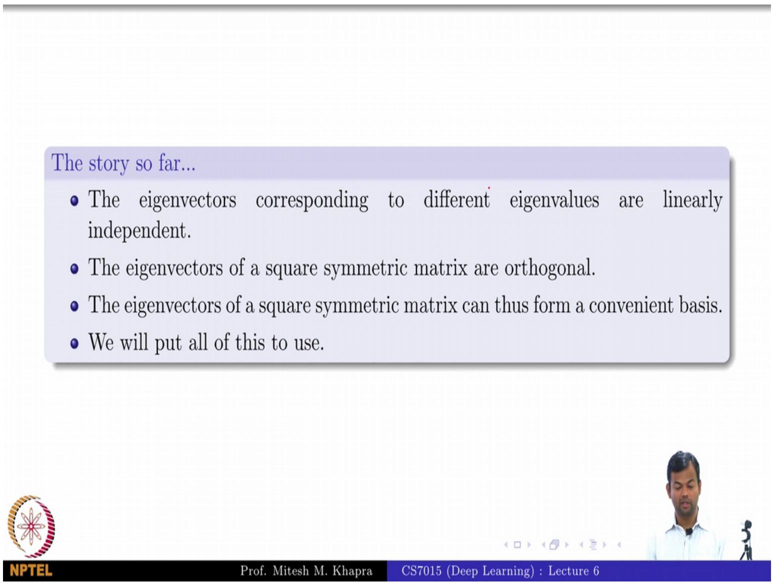


So, what has been the story so far. The story has been that the Eigenvectors corresponding to different Eigen values are linearly independent ok.

If you are dealing with the square symmetric matrix, which is something that we will deal with soon. Then things are even more convenient because the Eigen vectors are actually orthogonal ok. And they form a very convenient basis, and now we are going to put this to use when we talk about principal component.