

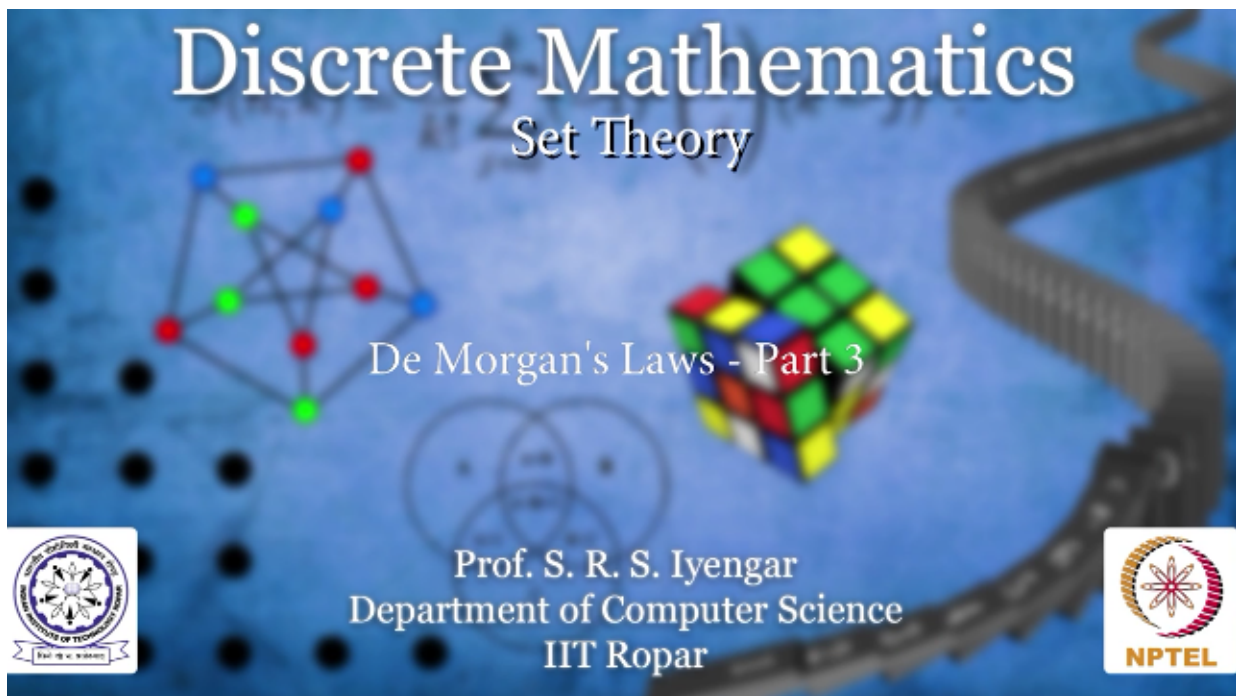
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Discrete Mathematics
Set Theory

De Morgan's Laws – Part 3

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Pick an element from the left-hand side, it's a X belongs to $A \cup B^c$, and show that X also belongs to the right-hand side, which is $A^c \cap B^c$, so whenever X belongs to $A \cup B^c$ it implies that X does not belong to $A \cap B$.

When will an element not belong to a set union another set, whenever that element does not belong to both of them, so X doesn't belong to A , and X does not belong to B . Now what does this mean? This means that X doesn't belong to A simply signifies that X actually belongs outside A , X belongs to A^c , and X belongs to B^c look at these two statements X belongs to a set as well as another set which means X should belong to a complement intersection B^c , pretty simple which implies look at this from where we started X belongs to $A \cup B^c$ we ended up with X belongs to $A^c \cap B^c$ thus proving that $A \cup B^c$ the whole complement is a subset of $A^c \cap B^c$.

Pick an element from LHS.

$$\underline{x \in (A \cup B)^c}$$

Show that $x \in A^c \cap B^c$

$$x \in (A \cup B)^c \Rightarrow x \notin A \cup B$$

$$x \notin A \text{ \& } x \notin B \quad ??$$

$$\textcircled{x \in A^c} \text{ \& } \textcircled{x \in B^c}$$

$$\underline{x \in A^c \cap B^c}$$

$$(A \cup B)^c \subset A^c \cap B^c$$



Now let us pick an element from A complement intersection B complement and show that also belongs to this left-hand side, how tough is it? It's pretty easy, why is that let's see. An element X rather let's say Y we used X already so let's say Y belongs to A complement intersection B complement, we will observe that our argument will simply be the reverse of what we saw here, Y belongs to A complement intersection B complement implies, Y belongs to A complement and Y belongs to B complement which in turn implies that Y does not belong to A, and Y does not belong to B, which means when Y neither belongs to A or B, Y certainly doesn't belong to A union B, correct, which means Y actually belongs to the complement of this set, so I start off

Pick an element from $A^c \cap B^c$.
Show that it belongs to LHS.

$$\text{Let } \underline{y \in A^c \cap B^c} \Rightarrow y \in A^c \ \& \ y \in B^c$$

$$\Rightarrow y \notin A \ \& \ y \notin B$$

$$\Rightarrow y \notin (A \cup B)$$

$$\underline{y \in (A \cup B)^c}$$

$$A^c \cap B^c \subset (A \cup B)^c$$



with Y belongs to A complement intersection B complement I ended up with Y belongs to A union B the whole complement which means A complement intersection B complement is indeed a subset of A union B the whole complement thus establishing this equality.

$$(A \cup B)^c = A^c \cap B^c$$



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