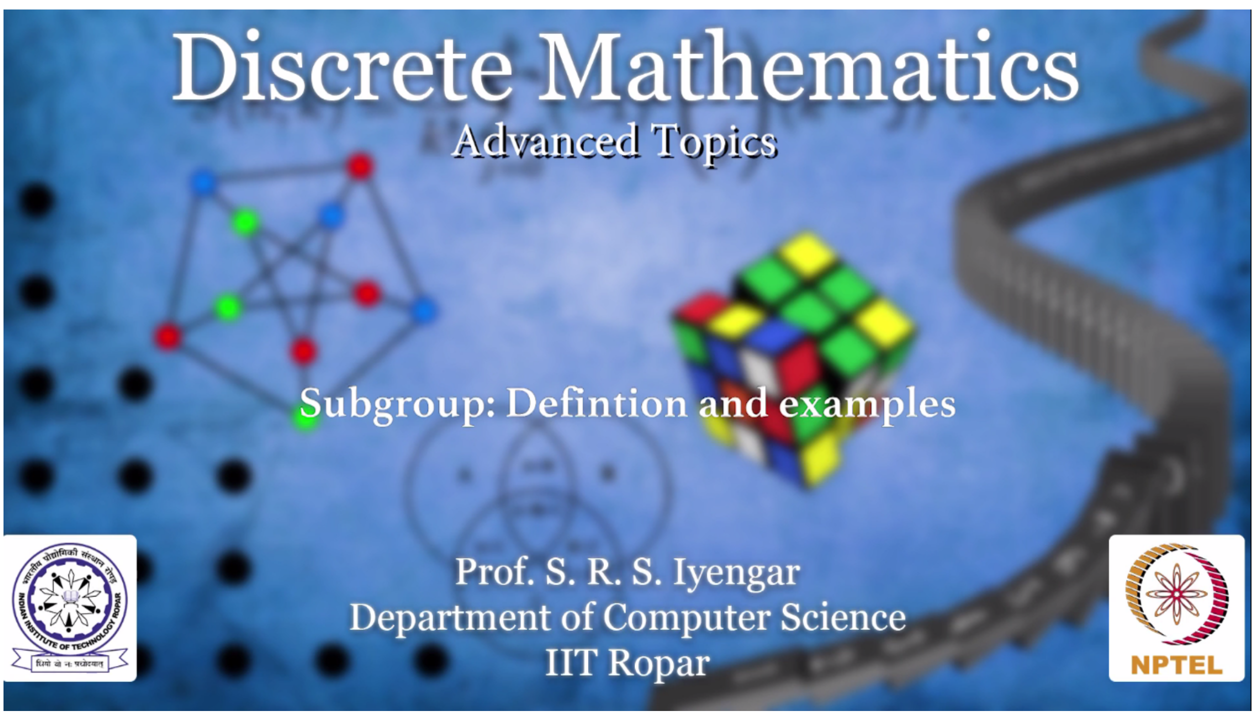
Discrete Mathematics

Functions

Advanced Topics



Subgroup Defintion and examples

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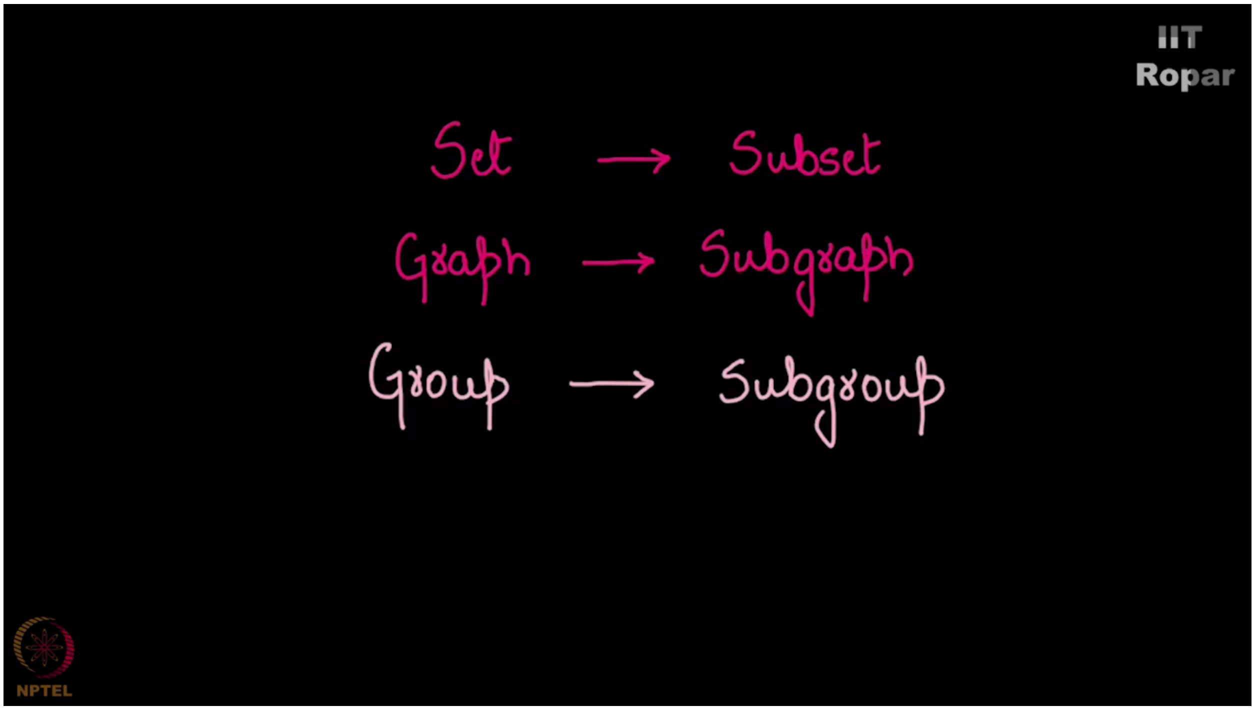
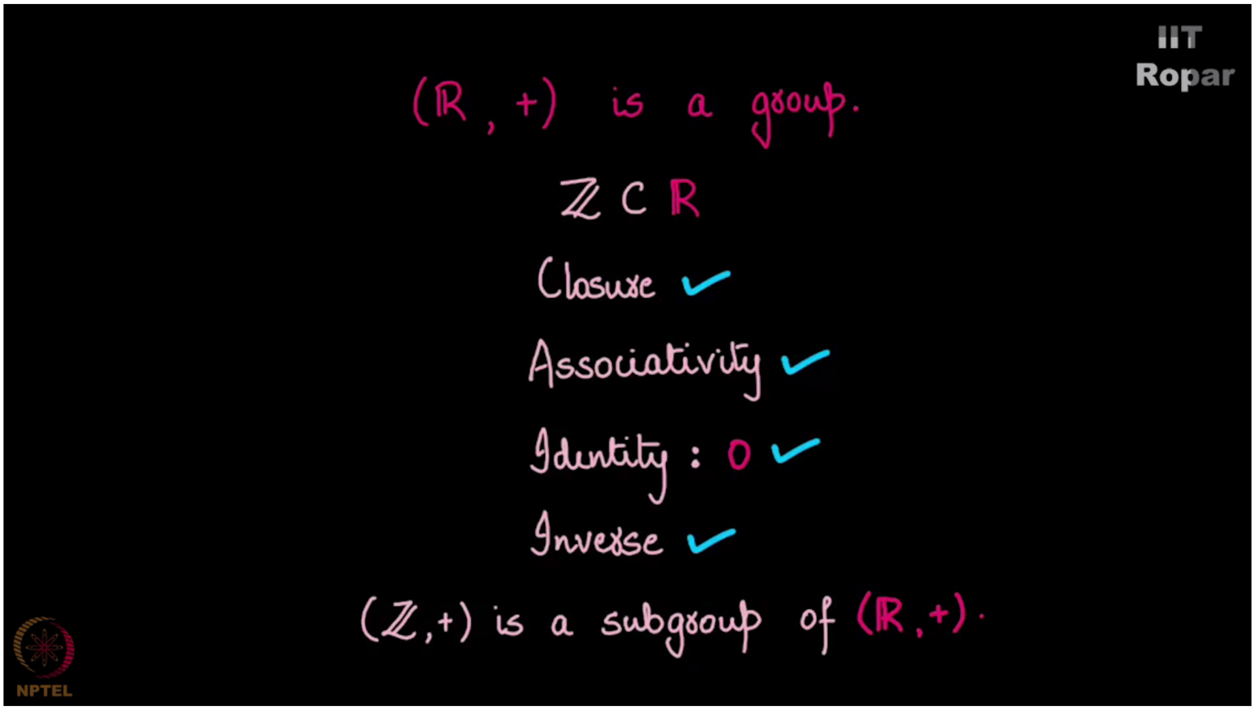
In week two we had seen what is a set and we had seen what is a subset. We took a few elements from the set and we formed what is called the subset. It was a sub structure. Then we moved ahead and we studied graphs. We particularly selected a few vertices and a few edges from the original graph and formed what is called the sub graph. We had a graph. We got a sub graph. We had a set we got a subset. Now we have discussed what are groups. we can also tell here what is a sub group. We can form a sub structure and also define what is a sub group. It holds true here to let us see more about this.

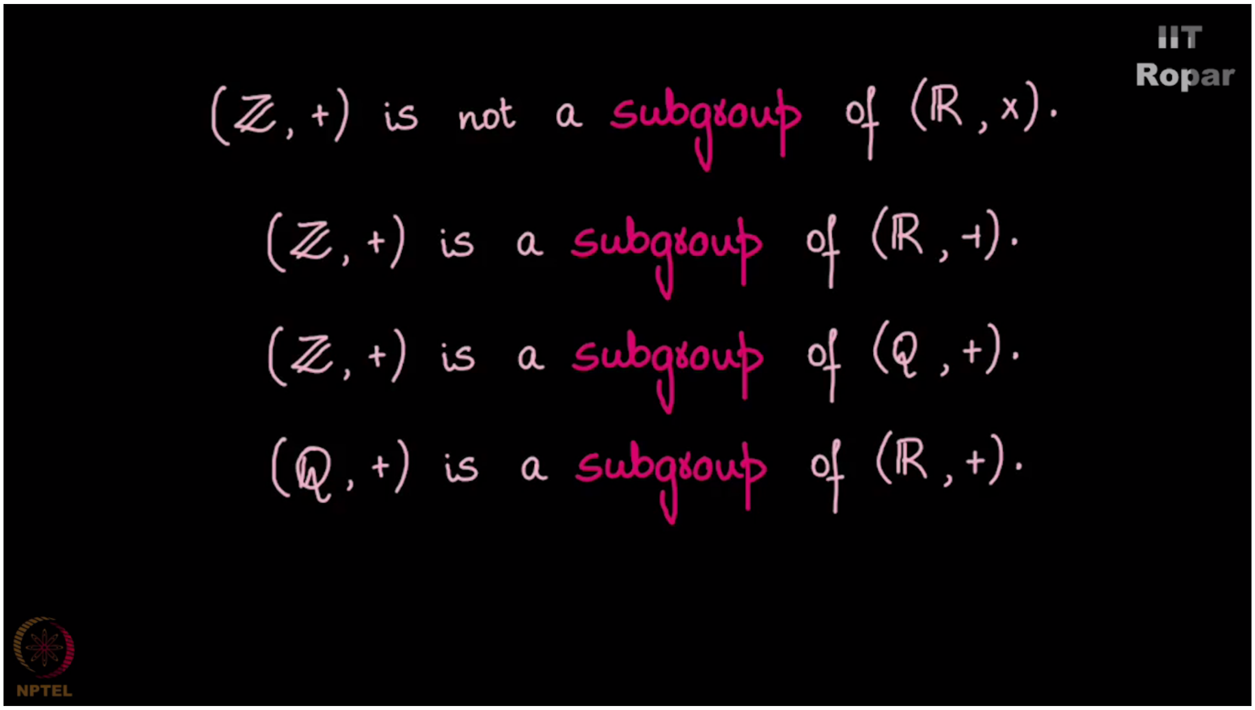


We know that R plus is a group real numbers under addition is a group. Now integers is a subset of R we all know that. Now integers if I take and if I give a plus operation to it closure satisfies here, associativity satisfies, identity exists which is 0, and inverse also exists, and hence under the addition operation Z plus is a subgroup of R plus. If your understanding well and good otherwise please listen. All four properties hold true here. It was a plus sign of the group. We were doing a plus operation addition operation on R and we saw that it is a group. Now I took a subset of R. I am giving the same operation to it. What do I observe all the properties hold true. Please note identity here is the same as the identity which was in the original group and hence I can tell that integers under addition is the subgroup of R plus.



Now let us see the formal definition of a subgroup. If G, * is a group it is a binary operation and H is a subset of G. I took a group and I take a subset of G then H is said to be a subgroup of G if H,* is a group. Now I take a group I take a subset and I say that H forms a subgroup if it is given the same binary operation as the original group. This is the formal definition. Now please note whatever operation you are giving on the group it must be the same on the subgroup as well. So group and subgroup must have the same operation or I can also tell it as subgroup must be given the same binary operation which is therefore the group. It cannot be different.

Now Z plus is not a subgroup of R under multiplication. We cannot compare like this. Why? Because R has multiplication and integers have addition. But it is a subgroup of R under addition. This holds true. We have seen that now it is also a subgroup of rational numbers under addition. Why? You can probably see the four properties write it down and you will be able to verify it yourself. Also rational numbers are a subgroup of real numbers under addition. Both of them under addition. Now we have seen all this for addition. Let us see for multiplication. Z under multiplication well it is not a subgroup and then under multiplication Q is a subgroup of R under multiplication. Why? Because in Q you have inverses which are there in R as well; that was the only concern why integers were not a subgroup. We see that Q under multiplication is a subgroup of R under multiplication.



Well these were some of the examples of subgroups.