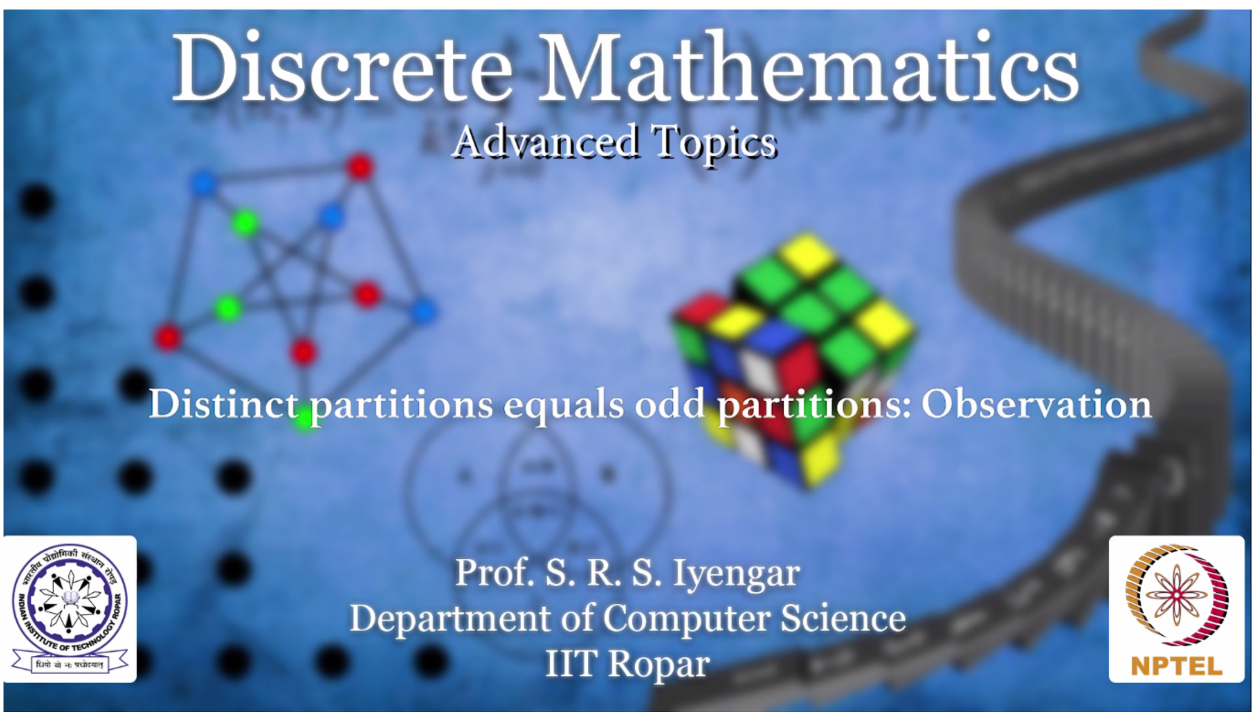
Discrete Mathematics

Functions

Advanced Topics



Distinct partitions equals odd partitions Observation

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Let us see something very interesting. I am going to write these three columns; n, Pd of n, and Po of n. You will see why. At the end of the video you will really appreciate it.

Let me consider n as 1. What are the partitions? 1 is 1. 1 equals 1. this is the only partition. N is 1 let's say I am going to keep filling up the table as and when I compute the partitions. What is the distinct partition of n? How many are there? Just 1. Odd is also 1. I can consider this as 1 partition. What if I take 2, n equals 2. 2 is 2. 2 is 1 plus 1. these are the two partitions of 2.

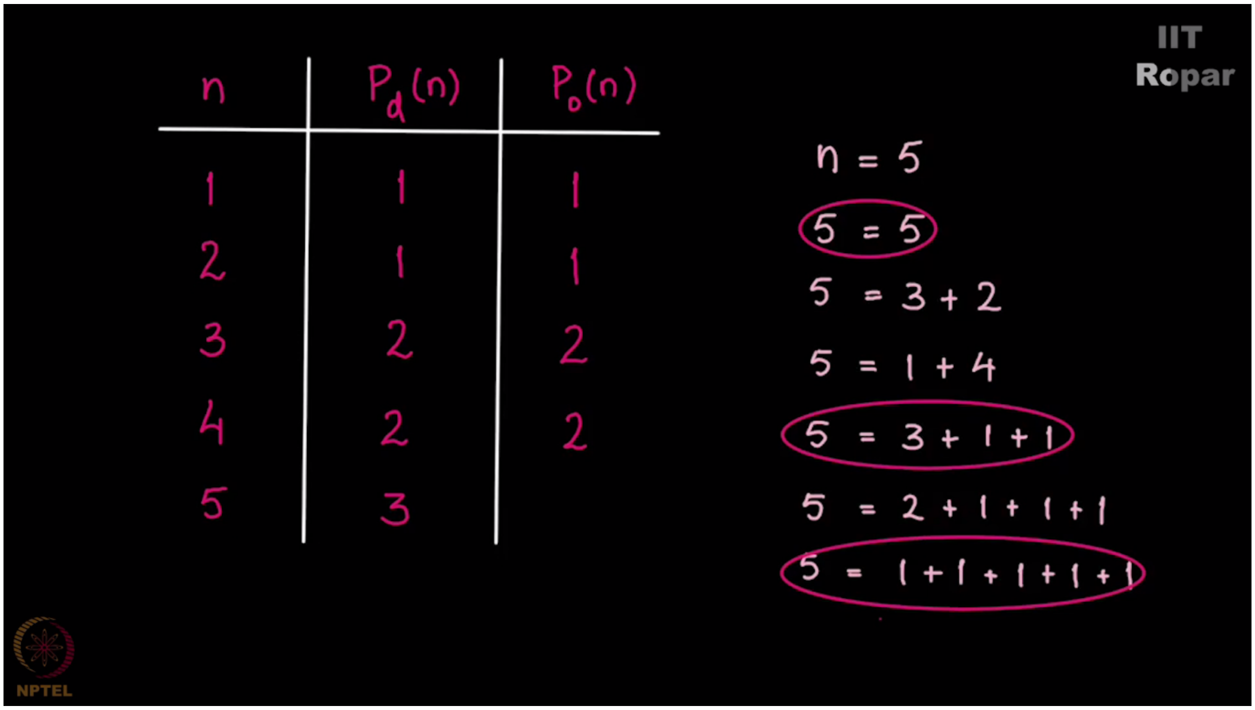
So let me fill up the table. N as 2. Pd of n what are the distinct ones? Only 1. and what are the odd partitions? Only 1. So this was the distinct one and this was the odd one. I cannot consider this to be a distinct one because 1 plus 1 it is not distinct. It comes under odd and not distinct. You have one distinct partition and one odd partition here for n equals 2.

Now let us go ahead and see what it is for n equal 3. So for n equals 3 let us see what are the partitions. So let me see 3 equals 3 is one such. 3 equals 2 plus 1. 3 equals 1 plus 1 plus 1. So these are the partitions of 3. n equals 3 observe these partitions, 3 equals 3 and 3 equals 2 plus 1, yes these are the distinct ones. So Pd of n is 2 and now what is the odd one. Observae these ones. So these are the odd ones. Odd partitions. So they are two in number. 3 and 1 plus 1 plus 1.

now let us go further for n equals 4. 4 equals 4. 4 equals 2 plus plus. 4 equals 3 plus 1. 4 equals 1 plus 1 plus 1 plus1. All 1s. Now these are the partitions, n equals 4 observe these two. 4 and 3 plus 1. They are the distinct ones. So Pd of 4 is two.

Now observe these partitions; 3 plus 1 and 1 plus 1 plus 1 plus 1. They are the odd ones. Yes it is very obvious. So we have again P of 4 as Po 4 as two. Two distinct and two odd.

The last one let me consider as 5 and we will stop here. So for 5. 5 equals 5. 5 equals 3 plus 2. 5 equals 1 plus 4. 5 equals 3 plus 1 plus 1. 5 equals 2 plus 1 plus 1 plus 1 and all 1s. These are the six partitions of 5.



Now observe these ones. These partitions in circle. For n equals 5 they are the distinct partitions. So Pd of 5 is three. Observe these partitions. 5, 3 plus 1 plus 1 and all 1s. Yes they are the odd partitions. So for Po of 5 it is 3. The rest of the ones contain one summand which is even. 3 plus 2 has 2 even. 1 plus 4 has 4 as even. And 2 plus 1 plus 1 plus 1 has 2 as even number even summand and hence I consider the rest of them as odd partitions.

Now so we have computed here Pd of n and Po of n for 1, 2, 3, 4, 5. We see some very neat observation. Let us do observe these two columns. 1, 1. 2, 2. 3 it is for n equals 1 it is 1, 1. For 2 equals n equals 2 it is 1, 1. N equals 3, 2, 2. N equals 4 2, 2 and n equals 5 3, 3.

What do you observe? Yes Pd of n equals Po of n. This is the observation which we are able to make from this given table. Well if it true let us try to prove it. In the next video we will give you the proof.