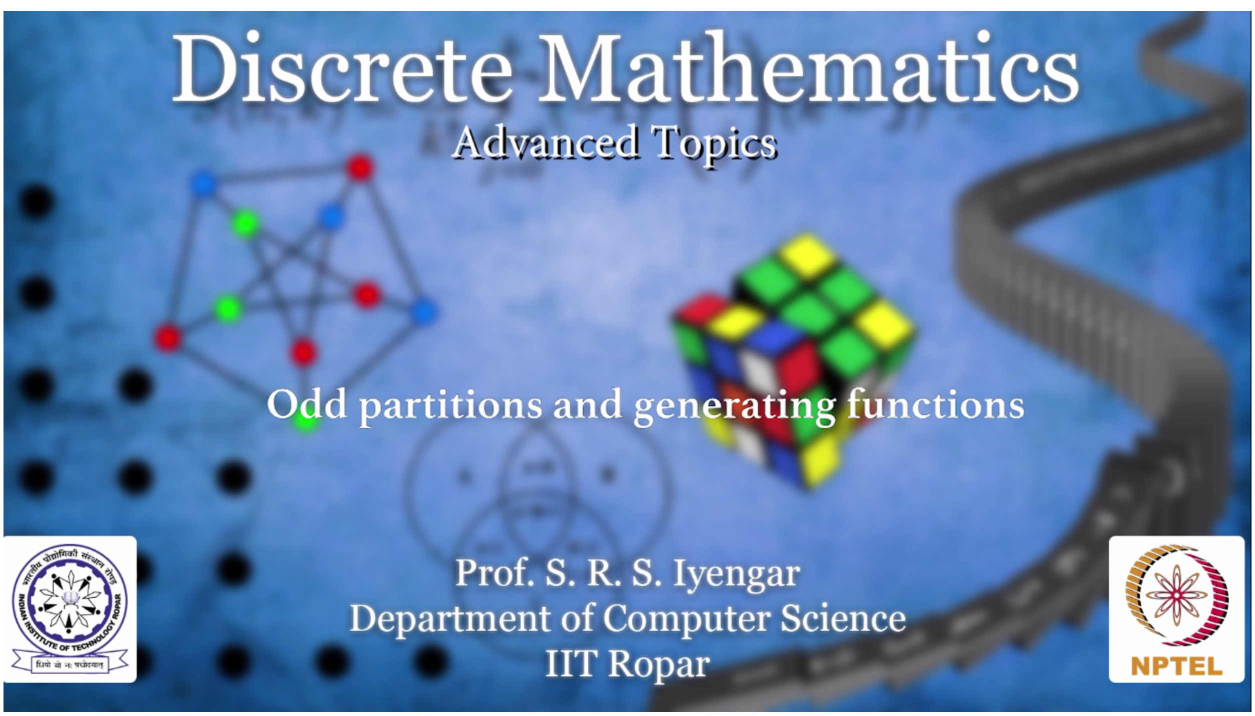
Discrete Mathematics

Functions

Advanced Topics



Odd partitions and generating functions

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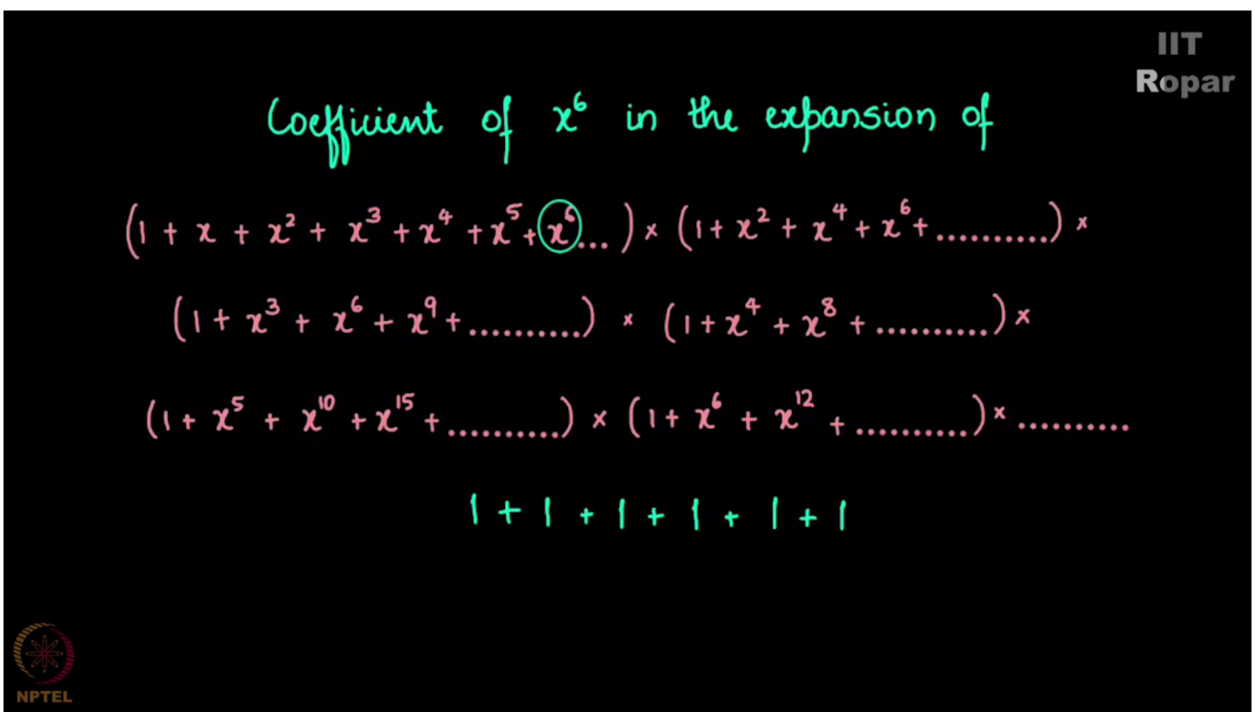
We have seen what is the generating function for Pd of n. Now the next obvious question would be what is it for Po of n. Let us see.

We will first address X to the 6 so coefficient of X to the 6 in the expansion of these polynomials we will consider these polynomials 1 plus X plus square so on. 1 plus X square X to the 4 so on. And other polynomials. We will ask for the coefficient of X to the 6.

So as we have seen earlier it is 1, 1 correspondence between the partitions and the coefficient of X to the 6 in these polynomials. So I am going to use the yard stick of partitions here. Consider this partition of 6, all 1s. Now how do we obtain the X to the 6 term of this partition. We have all 1s so it is very easy to observe that we should always go to the first house, 1s house that is. We have all 1s. How many 1s do we have? We have six 1s. And hence, what is the term in the polynomial? Yes it is X to the 6. So X to the 6 corresponds to the partition 1 plus 1 plus 1 plus 1 plus 1 plus 1. This X to the 6 is the term corresponding to this partition.



Now because it is in the 1s house we have all 1s here and therefore we went to the 1s house. Now what is it for 3 plus 3? We have two 3s here so we go 3s house and choose X to the 6. Why X to the 6 because we have two 3s. What if I had 3 only for 3 equal to 3 let's say. Then I would choose X to the 3 from 3s house.

So all 1s had X to the 6 from the 1s house and two 3s have X to the 6 from 3s house. Now what would it be for 1 plus 5? So here we have 1,1 and hence it is X and one 5 and hence it is X to the 5. So 1 is from 1s house, and 5 is from 5s house. So 1 plus 5 will correspond to X into X to the 5 which is X to the 6.

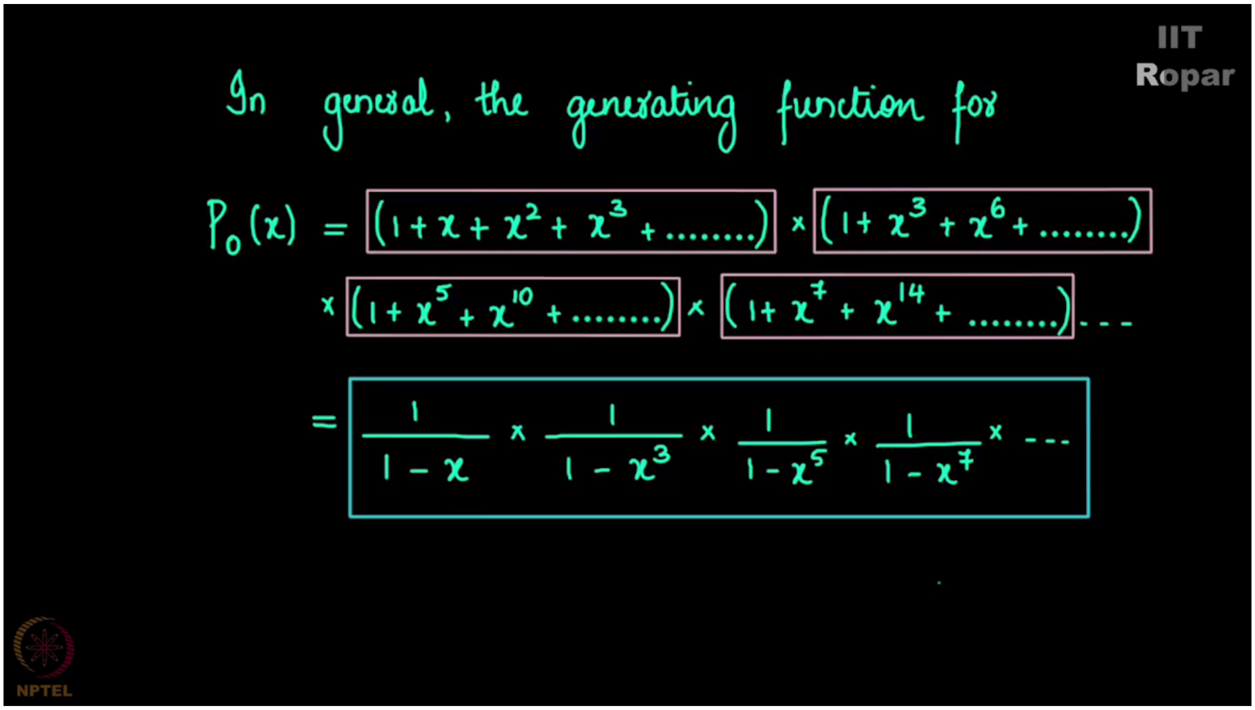
I had one 1 and hence I chose X. I had one 5 and hence I chose X to the 5. Did you observe in this process that we have always been asking for 1 all odd partitions. I have not chosen a partition which has an even summand. Now the last one would be okay we are done with everything. So now look at the boxed polynomials here. What does it correspond to? You must be jumping and tell me that these polynomials will correspond to those of the odd partitions. Those partitions which have only odd summands.

We are concerned about these boxed polynomials why not the others? Let me ask you this question. Because very easy to observe. Only if I have a summand as 2 in my partition will I go to this house? Only if I have a 4 will I go to this house. Only if I have a 6 will I go to this house.

Since I do not have a 6, or 4, or a 2 why because I am concerned only about odd summands. And hence I am not going to choose these houses. I am only concerned about these houses. So now let us see what is in general the generating function for Po of X. Po of X is odd partitions of X it is going to be 1 plus X plus X square plus X cube so on into 1 plus X cube plus X to the 6 plus so on into 1 plus X to the 5 plus X to the 10 plus so on into 1 plus X to the 7 plus X to the 14 plus so on and the product continues.

So this boxed one corresponds to 1 by 1 minus X these closed forms we have already seen them. So whenever you get a generating function which can be written as a closed form we do it for convience shake. In the earlier video where we saw the generating function for Pd of X we had seen that it is 1 plus X into 1 plus X square into 1 plus X cube so on. There is no more simplified version of it or there is no closed form and hence we left it like that. But here we see that these generating functions have closed forms. And hence we write them. And this has the closed form 1 by 1 minus X cube, this has the closed form 1 by 1 minus X to the 5 and this has the closed form 1 by 1 minus X to the 7 and so on. The further terms would be 1 by 1 minus x to the 9. 1 by 1 minus X to the 11 and so on.

What do you observe? This in general can be written as the product of 1 by 1 minus X to the I where I is all of your odd terms. 1, 3, 5, 7 and so on. So this is the generating function for Po of X that is odd partitions of X. this is the generating function.



Keep this in mind. You might want to watch this video again and also again browse through the video of Pd of X we will refer to these generating functions in the next or future videos.