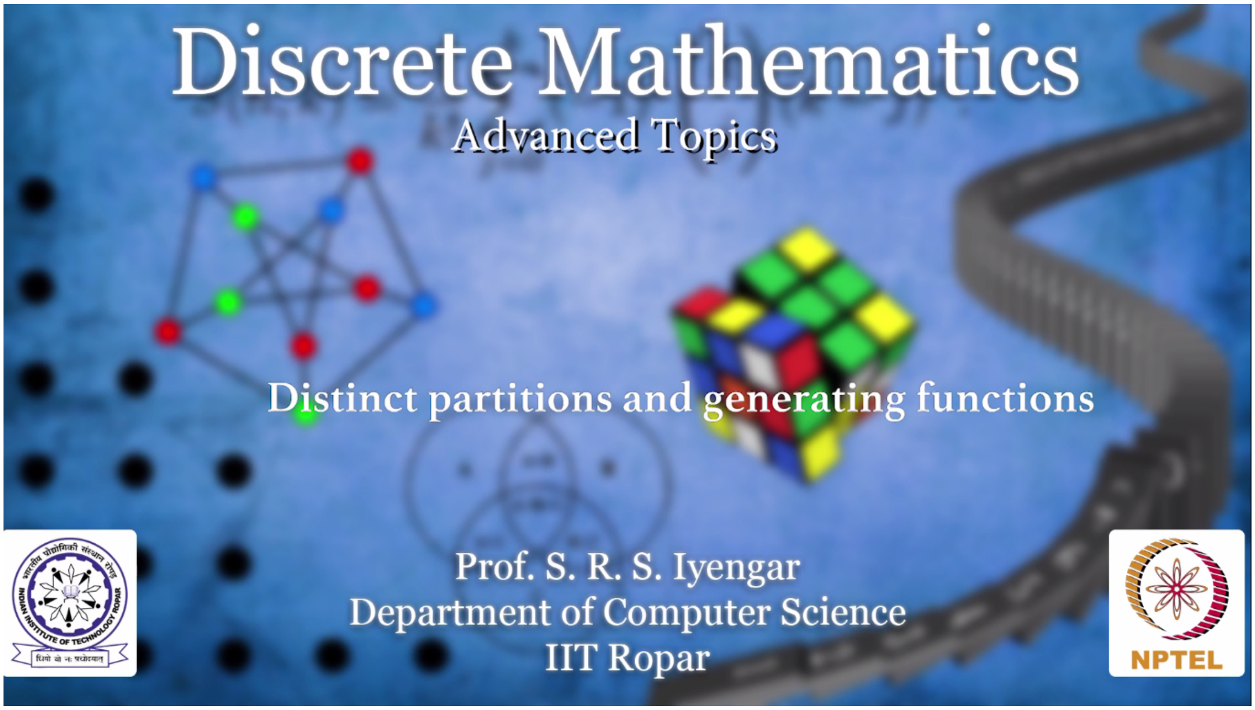
Discrete Mathematics

Functions

Advanced Topics

Distinct partitions and generating functions



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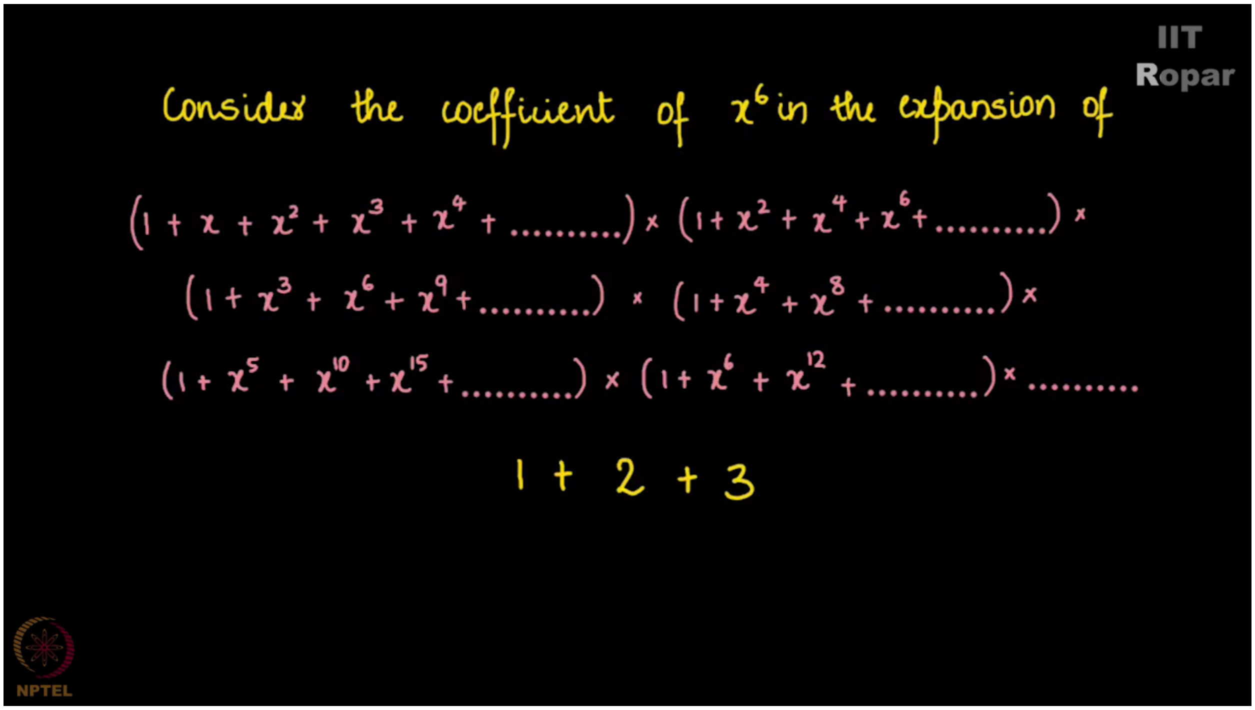
Consider the coefficient of X to the 6 in the expansion of the following. We have already seen these polynomials. I am not going to read out all of them. Rather explain all of them you all know what are these polynomials.

We had earlier stopped upto 1 plus X to the 4 so on I have just continued till 1 plus X to the 5 plus X to the 10 so on into 1 plus X to the 6 plus X to the 12 so on.



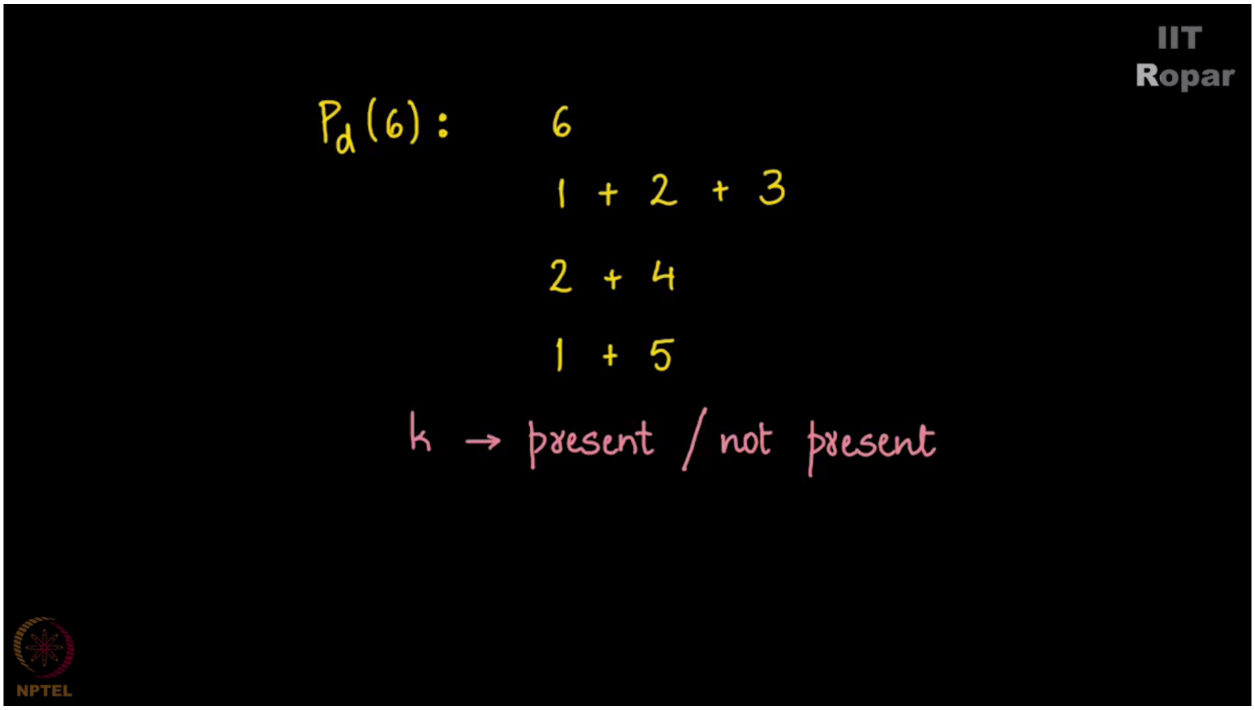
Now how do I get X to this coefficient here? Well before that let us take a look at the partition of 6 this one. 1 plus 2 plus 3. How do I get 1 here? It is equivalent to taking X here from the first house because I have only one 1. I have just one 2 therefore I will select X square here. And one 3 therefore I will select X cube here. And X into X square into X cube will give me X to the 6. That is equivalent to the partition 1 plus 2 plus 3.

Now how will I get X to the 6 if I consider the partition 1 plus 5. 1 corresponds to picking an X from the first house we have only one 1 and 1, 5 represents picking X to the 5 from the fifth house. There is only one 5 and hence it is X to the 5.

Now what does this represents 6? it is directly picking X to the 6 from the sixth house. 6 is equal to 6 we have only one 6 and hence it is equivalent to picking X to the 6 from the sixth house. Now what if I take 2 plus 4? We have already seen this. It is equivalent to picking X square from the second house and X to the 4 from the fourth house. Each is X square and X 4 the reason is because we have one 2 and one 4 we have already seen this.

Now the question is look at this. Pd of 6 distinct partitions I was addressing all this while. We saw 2 plus 4. we saw 5 plus 1. 6. 1 plus 2 plus 3. and so on.

We have seen these four partitions of 6. All of them were distinct and I showed you how do we get an equivalent of X to the 6. How do we by multiplying X, X square, X cube we got X to the 6. Similarly for the other partitions as well.



Now when you take a summation K either it is present or it is not present in the partition. You chose 1, either it is there or it is not there and this happens what is the statement mean either it is present or it is not present. It means since we are addressing distinct partitions it doesn't repeat. So either it is present or it is not present and it is the same for all the summands for any K and therefore it is represented as 1 plus X to the K. The function is equivalent to writing 1 plus X to the K. So the generating function for Pd of X is given by 1 plus X into 1 plus X square into 1 plus X cube into 1 plus X to the four so on and therefore Pd of X is product of I from 1 to infinity 1 plus X to the I.

I hope it is clear to all of you. If I take the summation 2 then the corresponding generating function will be 1 plus X square. Now corresponding to 3 it is 1 plus X cube. So the generating function or Pd of X is 1 plus X into 1 plus X square so on. This product will give the generating function.

Then what is the coefficient of X to the n? Pd of n is the coefficient of X to the n in the expansion of 1 plus X 1 plus X square. 1 plus X cube so on upto 1 plus X to the n. Box this and keep. It is very important.

We have earlier seen how do we get P of n is the coefficient of X to the n. Now in particular we study how do we get the coefficient of X that is Pd of n is the coefficient. We have got more stronger observation now. Pd of n is the coefficient of X to the n in this product. You might want to watch the video once or twice more for better understanding.