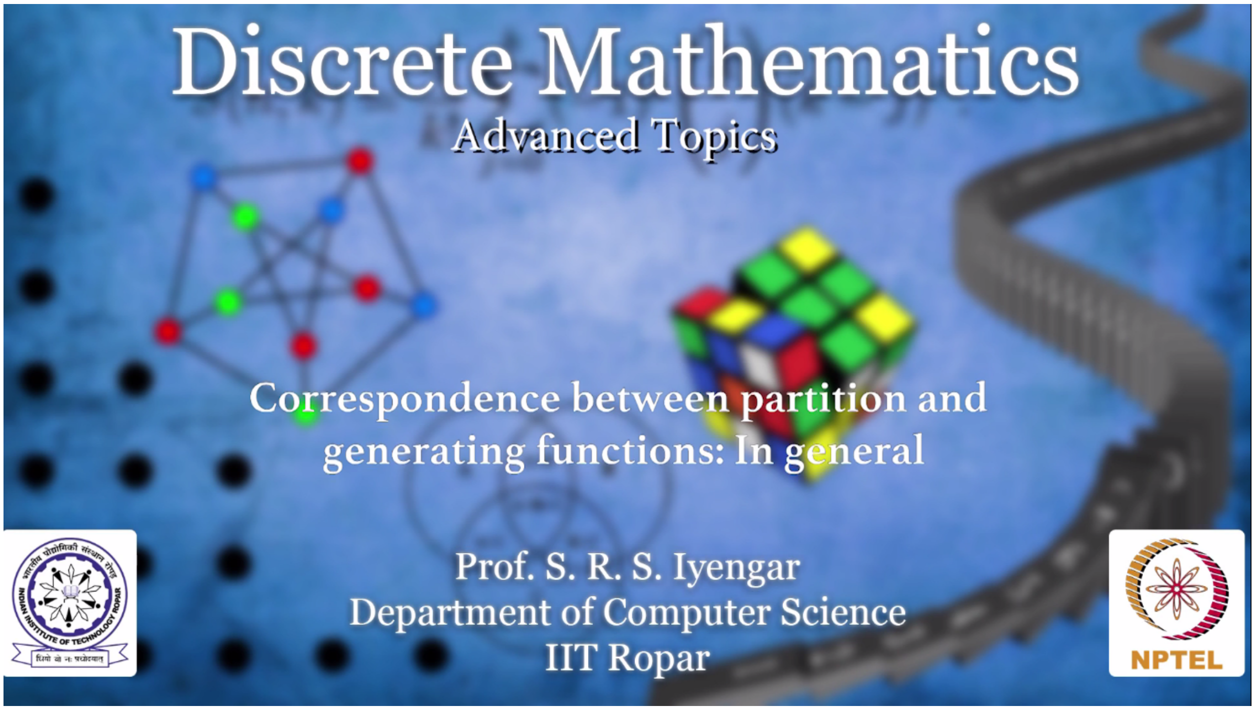
Discrete Mathematics

Functions

Advanced Topics

Correspondence between partition and generating functions In general



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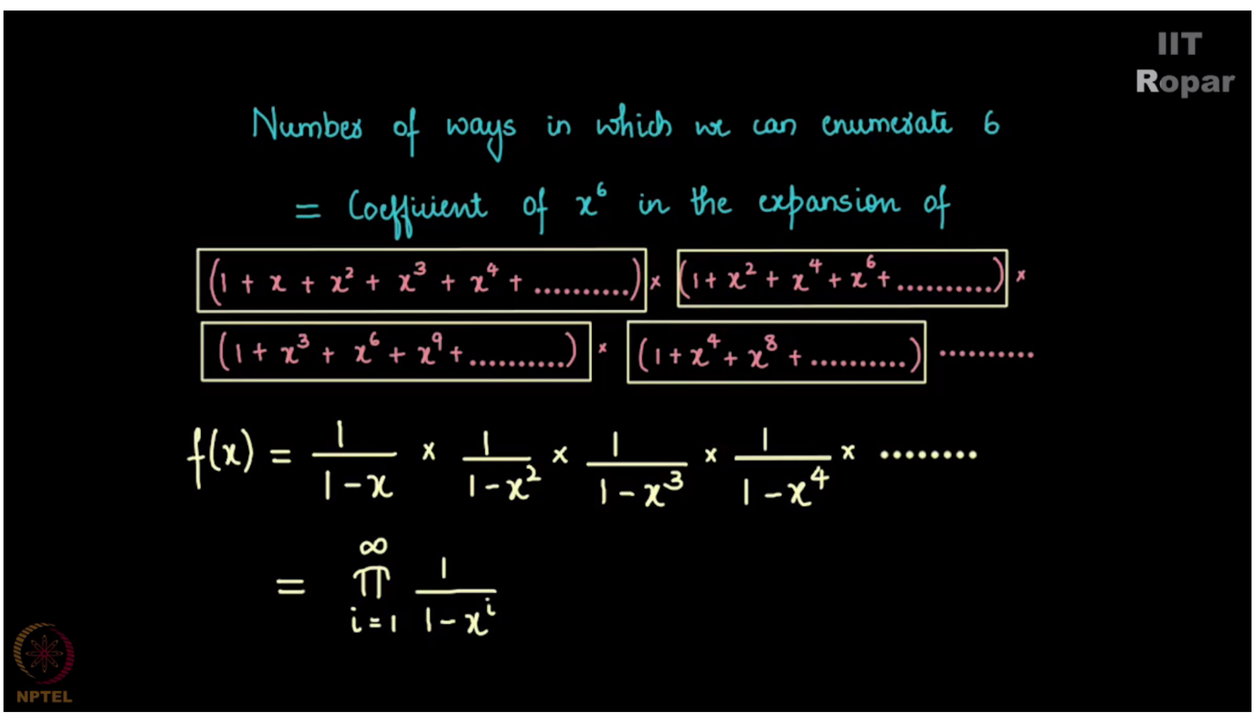
I am now going to paraphrase what the professor has taught in the previous video. We have seen that the number of ways in which we can enumerate 6 that is the partitions of 6, the number of ways in which we can do that is the coefficient of X to the 6 in the expansion of 1 plus X plus X square plus X cube plus X to the 4 so on into 1 plus X square plus X to the 4 plus X to the 6 so on into 1 plus X cube plus X to the 6 plus X to the 9 so on into 1 plus X to the 4 plus X to the 8 plus so on into the product continues. We had seen that there is 1,1 correspondence but when the number of ways or the number of partitions of 6 and the number of ways in which we can get X to the 6. There is 1, 1 corresponds between these two.

Now let me show you a nice observation here. What is this? This can be written as we have earlier seen in the chapter of generating functions that this is nothing but the closed form can be written as 1 by 1 minus 6. 1 by 1 minus X is the closed form of the boxed generating function.

Now into this is what? This is nothing but 1 by 1 minus X square. We have studied all of this. And 1 plus X cube plus X to the 6 this one is actually oen1 by 1 minus X cube. And this one is nothing but yes you must be guessing by now it is 1 by 1 minus X to the 4 and so on as we continue the generating functions you will get 1 by 1 minus X to the 5 into 1 by 1 minus X to the 6 and so on.

Now if I write this product as f of X if I represent it as f of X then in general it can be written as product from I equal to 1 to infinity, 1 by 1 minus X to the I. You are seeing that that I am multiplying X to the I so here I is varying from 1 to infinity and that's a product. So this is f of X.

So now you have seen what did I do just now. I am telling that the coefficient of 6 in the expansion of these products can be written in a closed form as this f of X. What is f of X? Product of 1 by 1 minus X to the I.



So in general the coefficient of X to the n in product I from 1 to infinity 1 by 1 minus X to the I is the number of partitions of n. I hope it is clear to all of you. We had earlier seen what is the coefficient of X to the 6. Now in general I am telling you that the coefficient of X to the n is nothing but it will give you the number of partition of n in this product.