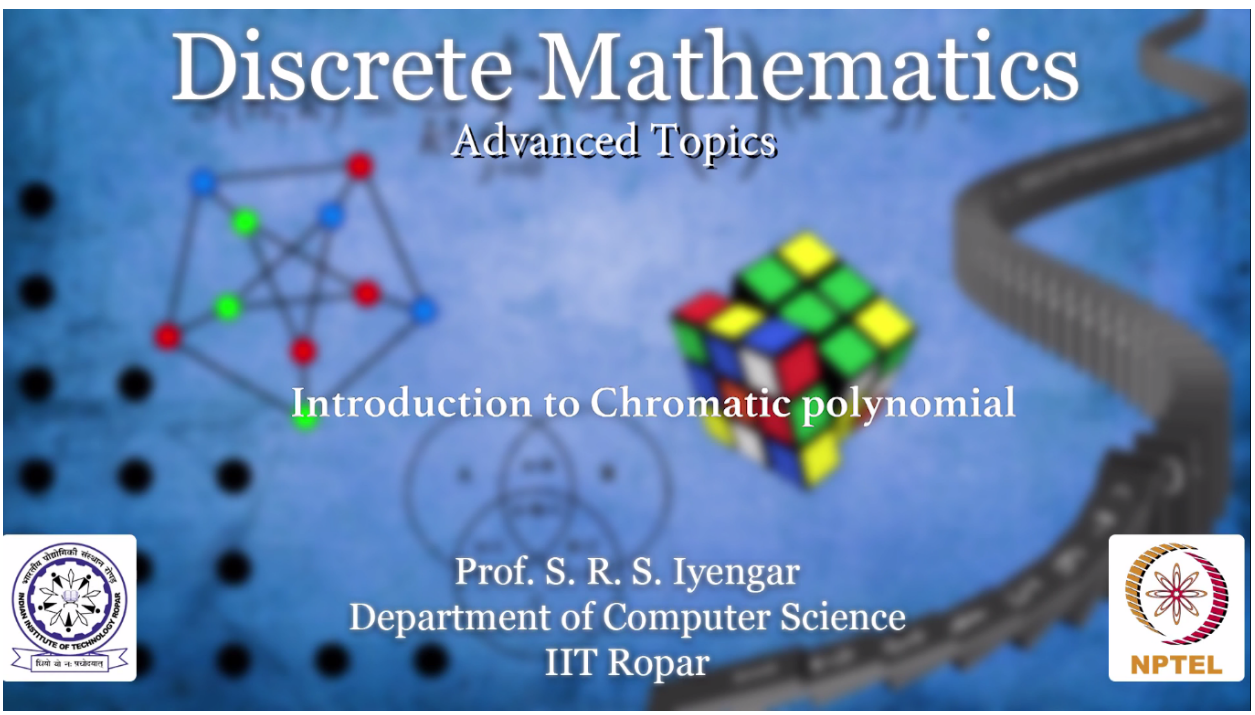
Discrete Mathematics

Functions

Advanced Topics



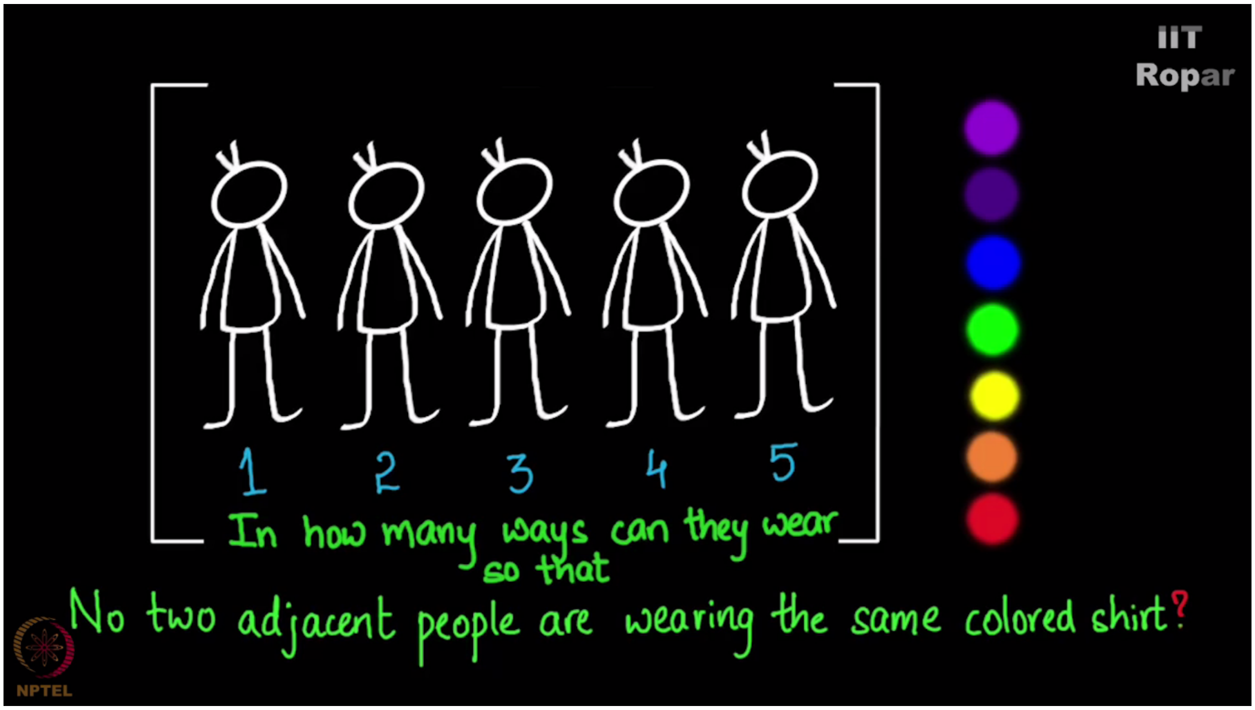
Introduction to Chromatic polynomial

Prof S.R.S. Iyengar

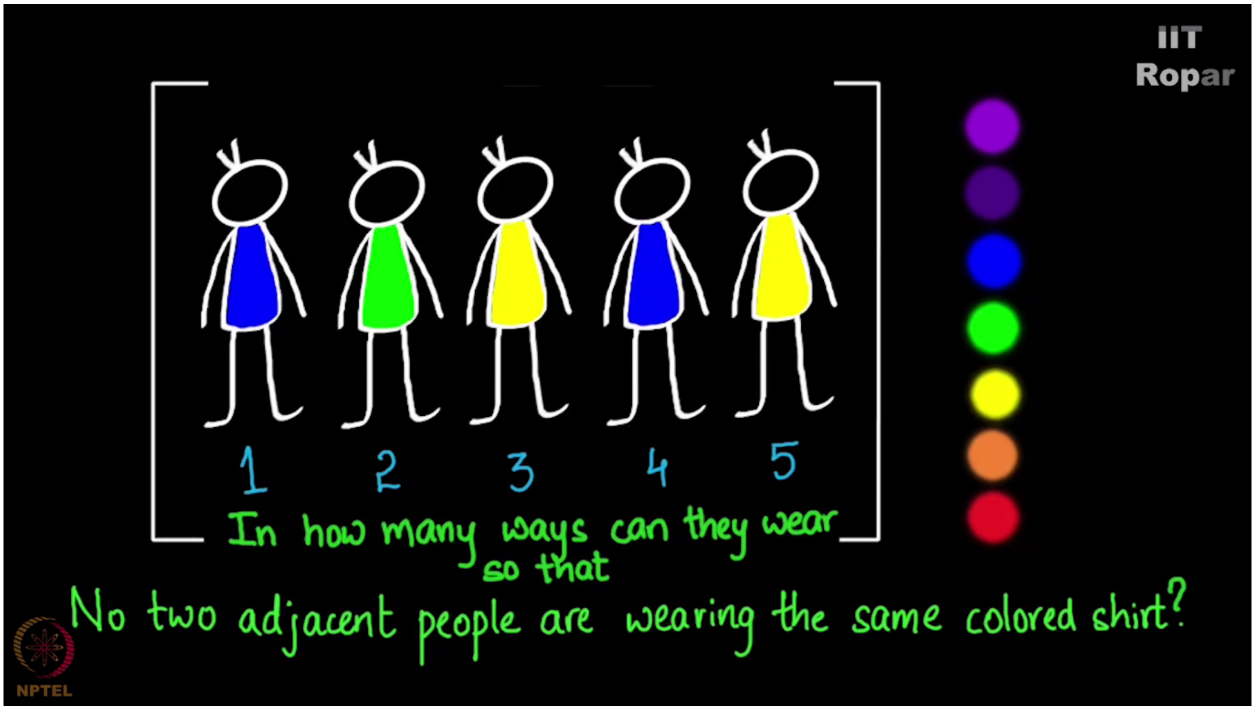
Department of Computer Science

IIT Ropar

Imagine five friends standing in a row. Friend 1, friend 2, friend 3, friend 4 and friend 5. And they decide that we will stand and take a picture by ensuring that no two adjacent people are wearing the same colored shirt. And they have choices from these seven colors. Violet, indigo, blue, green, yellow, orange, red. In how many ways can these five friends wear their shirts color in how many ways can they choose so that no two adjacent people are wearing the same color? Assume that the first person whoever he is he will stand in the first place only.



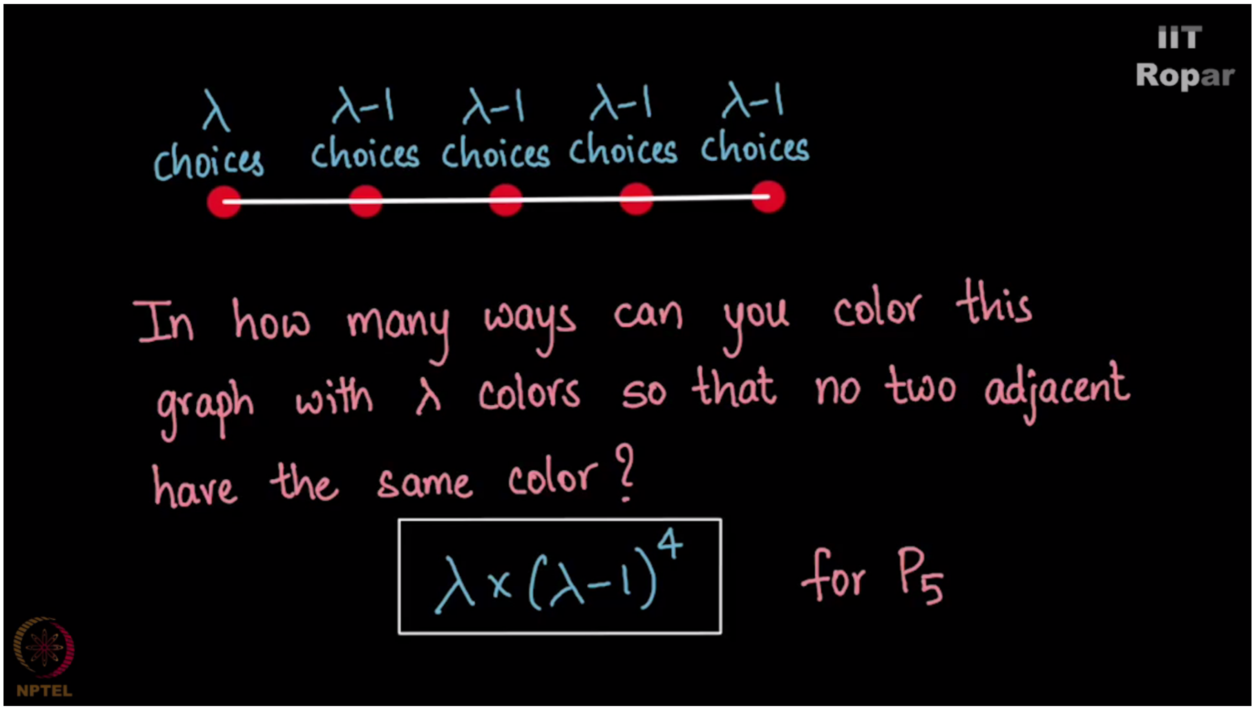
I am not talking about the ways in which they can shuffle their places. All I am saying is violet, indigo, violet indigo, violet is one way. Violet, indigo, blue, green, yellow is another way. Blue, green, yellow, blue, yellow is another way and so on. In how many ways can you make these five places colored so that no two adjacent places have the same color.



Let us try answering this question. The first person has seven ways. The second person has one way less than the first person because first person has chosen a color which second person cannot choose. Think about it. So he has six ways of choosing a color. Third person need not worry about the first person is wearing but should worry what second person is wearning because the rule is your adjacent person should not be wearing the same color so three the third person will have one color less namely the color that the second person is wearing. And hence three will have six colors.

And four will again have six colors similarly. And five again apart from the color that four is wearing five has six colors. So the answer to this question is 7X6X6X6X6 happens to 9072. Well let's keep this question aside and ask some other question.

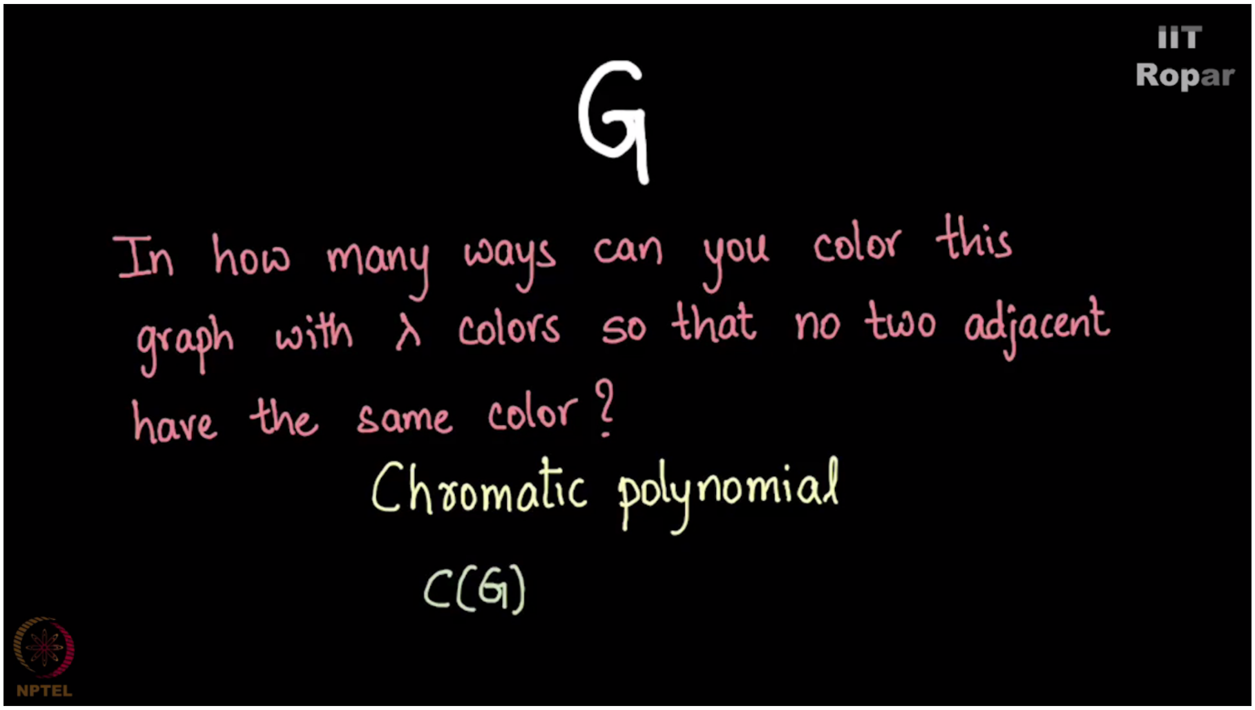
Given this graph look at this path with five vertices in how many ways can you color them with lamda number of colors so that no two adjacent vertices have the same color? Sounds familiar. This is the same question as the previous one. So the first one will have lamda colors for it's choice. Second one cannot have the same color as the first one so it will be lamda minus one. Similarly lamda minus one for the third vertex lamda minus one for the fourth. Lamda minus one for the fifth. So it is lamda times lamda minus one whole to the four for a path with five vertices given lamda colors. So the question in how many ways can you color a given graph G which is the path with five vertices with lamda colors such that no two adjacent vertices are the same color the answer is lamda times lamda minus one to the power of four.



A point of confusion, you are probably wondering why am I writing seven for the first person in the previous example. That's because he can choose it in seven ways. Why is it six for the next person for each way that he has chosen this person has six ways. So it is 7 into 6. it is the rule of product that we have been discussing. And this could be a point of confusion get this clarified by thinking about it for a few minutes before going further.

So all in all we have shown that given a path this is the total number of ways in whichu can color the given graph properly. Properly means we have discussed this. No two vertices which are adjacent, two notes that are adjacent cannot have the same color. In how many ways can you color with this constraint is what we mean by proper color. Right.

Now if I give you in general a graph G and if I ask you given lamda colors in how many ways can you color that given graph G that's a very non-trivial question. Not at all easy to answer. It is not a simple as the path graph that we just saw. It's going to be deep. It's going to very subtle. It's going to invoke some advance thinking. Let's go ahead and see that for a general graph how do we find out what is the total number of ways in which you can color a given graph G with lamda colors.



Anyways, a definition lamda into lamda minus one whole to the four is called the chromatic polynomial of the given graph G. So we denote it by C of G, C stands for chromatic polynomial of G and that is equal to lamda times lamda minus one to the power of four in the case of a path with five vertices.