

NPTEL

NPTEL ONLINE CERTIFICATION COURSE

**Discrete Mathematics
Recurrence Relation**

Example: Door knocks example solution

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So how many knocks happen? Let us analyze it slowly, the first person knocks n doors, let $T(n)$ be the total number of knocks that happen, if there were to be n doors, so which is first person does n knocks, so I'll say $T(n) = n +$ second person does another n knocks, why he starts from 1, 2, 3, goes up to n , realizes someone is there in the n th door and settles down in $n-1$, the next person will go up to $n-1$, he knocks $n-1$ doors and settles down in $n-2$ door, correct, so on and little bit of thinking patiently that is will make you realize that $T(n) = n$ knocks by the first person + n knocks by the second person + $n-1$ knocks with the third person + $n-2$ knocks by the fourth person and so on,
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How many knocks happen?

1st person : n doors

$T(n)$ - Total number of knocks if there are n doors.

$$T(n) = \overset{1}{n} + \overset{2}{n} + \overset{3}{n-1} + \overset{4}{n-2} + \dots$$



actually recurrence relation for this would be $T(n) = n + T(n-1)$, right, so it's actually, so let's not write the recurrence relation for this it's pretty straight forward, so $T(n) = n + n + n-1 + n-2 + n-3 + n-4 + n-5$ up to 1, you see this is nothing else but the sum of the first n natural numbers, if you reverse this and then write it $T(n)$ will be $1 + 2 + 3 + 4$ up to n + one more n, let's write that down no problem.

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How many knocks happen?

1st person : n doors

$T(n)$ - Total number of knocks if there are n doors.

$$T(n) = \overset{1}{n} + \overset{2}{n} + \overset{3}{n-1} + \overset{4}{n-2} + n-3 + n-4 + n-5 + \dots + 1$$



So this is equal to $n(n+1)/2 + n$, and what is this? $N^2 + n + 2n/2$, correct, (Refer Slide Time: 01:58)

How many knocks happen?

1st person : n doors

$T(n)$ - Total number of knocks if there are n doors.

$$T(n) = \overset{1}{n} + \overset{2}{n} + \overset{3}{n-1} + \overset{4}{n-2} + n-3 + n-4 + n-5 + \dots + 1$$

$$T(n) = \frac{n(n+1)}{2} + n = \frac{n^2 + n + 2n}{2}$$



who cares what they see, it's all that matters to us, remember is that it is big O of n square, so it's of the order n square, the number of knocks happen the order n square, what we just now did is actually that time taken for bubble sort algorithm, so what's a bubble sort algorithm? (Refer Slide Time: 02:21)

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How many knocks happen?


1st person : n doors

$T(n)$ - Total number of knocks if there are n doors.

$$T(n) = \overset{1}{n} + \overset{2}{n} + \overset{3}{n-1} + \overset{4}{n-2} + n-3 + n-4 + n-5 + \dots + 1$$

$$T(n) = \frac{n(n+1)}{2} + n = \frac{n^2 + n + 2n}{2}$$

Time taken for Bubble Sort

$$= O(n^2)$$


Look it up, it's a little out of scope of our discussion right now, I'll not discuss maybe I'll assign this to you as little homework, Google for what is bubble sort and understand it and try to see how bubble sort and knocking of the doors are similar, in terms of what? In terms of the number of transactions that you make, remember binary search the number of transactions you make is of the order log n, the number of transactions you make in bubble sort is of the order n square, why? Bubble sort resembles the knocking on the door question, (Refer Slide Time: 02:59)

Number of transactions in
binary search — $O(\log n)$

Number of transactions in
bubble sort — $O(n^2)$



think about it, so don't worry much about bubble sort if it's difficult for you, just look at the total number of knocks puzzle you will realize that $T(n)$ is simply $n + n + n-1 + n-2 + n-3$ up to 1 which is of the order n square to summarize.

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