NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics Recurrence Relation

Solution for the recurrence relation of Binary search

By Prof. S.R.S. Iyengar Department of Computer Science IIT Ropar

We are now going to see the solution for the recurrence relation of binary search, so the recurrence relation goes like Tn is Tn/2 + 1 and the initial condition is given by T1 = 0, we know that Tn/2 can be written as Tn/4 + 1, and +1 remains as it is.

Now 4 can be written as 2 square and hence this becomes Tn/2 square + 2, I have opened the bracket and added the 2 once, N/2 square if I expand it as I had done earlier Tn/2 square can be written as Tn/2 cube + 1, and +2 remains as it is, again opening the bracket I can write this as Tn/2 cube + 3, now what happens at the Rth step, when we write it, it becomes Tn is Tn/2 to the R + R,

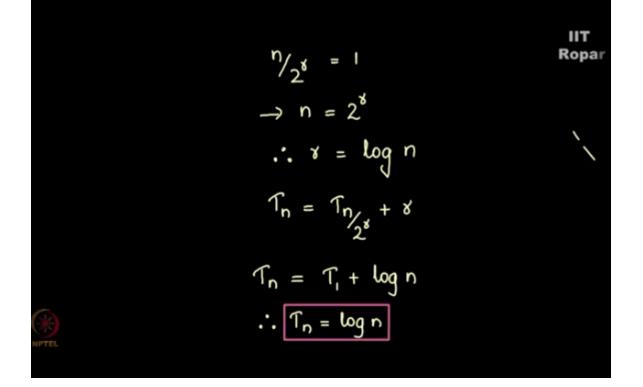
(Refer Slide Time: 01:10)

$$\begin{aligned} \mathcal{T}_{n} &= \mathcal{T}_{n} \mathcal{I}_{2} + 1 \quad , \ \mathcal{T}_{1} = 0 \quad \text{Ropar} \\ \mathcal{T}_{n} &= \left(\mathcal{T}_{n} \mathcal{I}_{4} + 1\right) + 1 \\ \mathcal{T}_{n} &= \left(\mathcal{T}_{n} \mathcal{I}_{2}^{2} + 2\right) \\ \mathcal{T}_{n} &= \left(\mathcal{T}_{n} \mathcal{I}_{2}^{3} + 1\right) + 2 = \mathcal{T}_{n} \mathcal{I}_{2}^{3} + 3 \\ \text{At sth} \text{ step}, \quad \mathcal{T}_{n} &= \mathcal{T}_{n} \mathcal{I}_{2}^{3} + 3 \end{aligned}$$

you might be wondering here how did I get this, observe the previous step you might find the pattern and hence Tn is Tn/2 to the R + R.

Now we know that T1 is 0, and we have here Tn/2 to the R, so N/2 to the R = 1, I am going to write it like this which implies N is 2 to the R, now if N is 2 to the R, what does R become? R = log N, and Tn becomes Tn/2 to the R + R substituting for Tn/2 to the R, and R we get Tn is T1 $+\log n$, T1 as we know is 0 and hence Tn becomes $\log n$, so this is the solution for the recurrence relation of binary search.

(Refer Slide Time: 02:05)



IIT MADRAS PRODUCTION

Founded by Department of Higher Education Ministry of Human Resources Development Government of India

www.nptel.iitm.ac.in

Copyrights Reserved