**NPTEL**

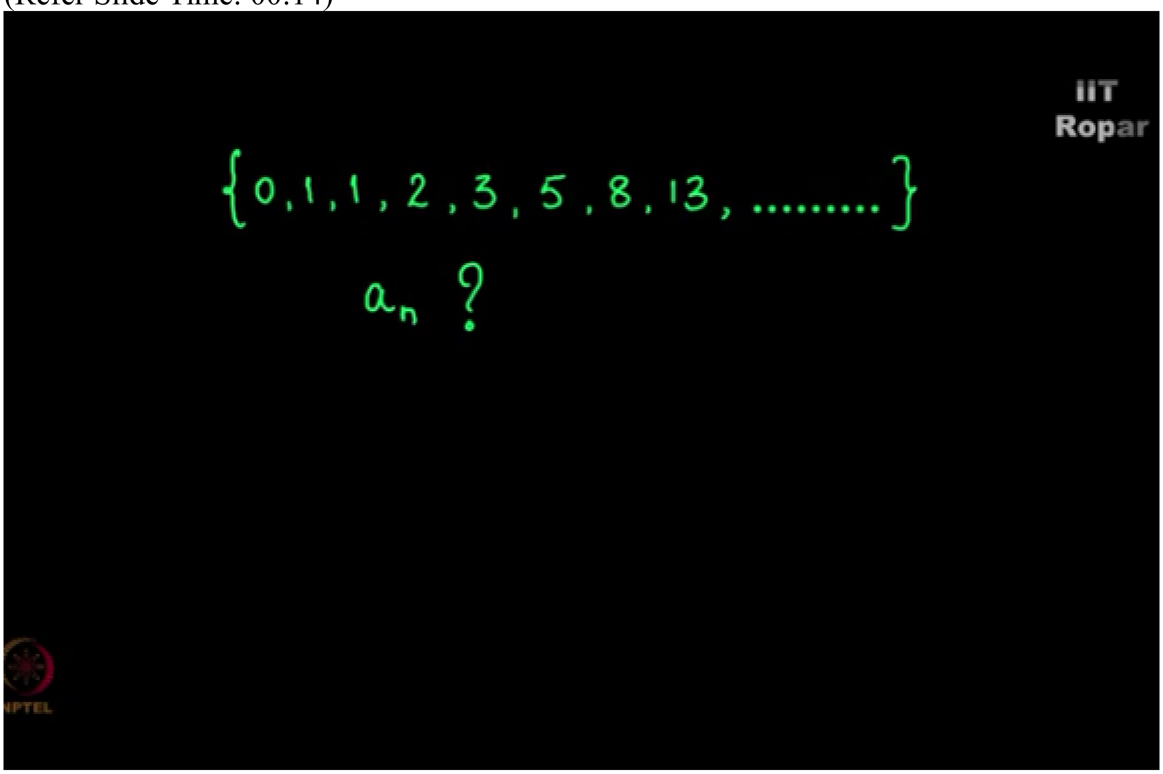
**NPTEL ONLINE CERTIFICATION COURSE**

**Discrete Mathematics**
**Recurrence Relation**

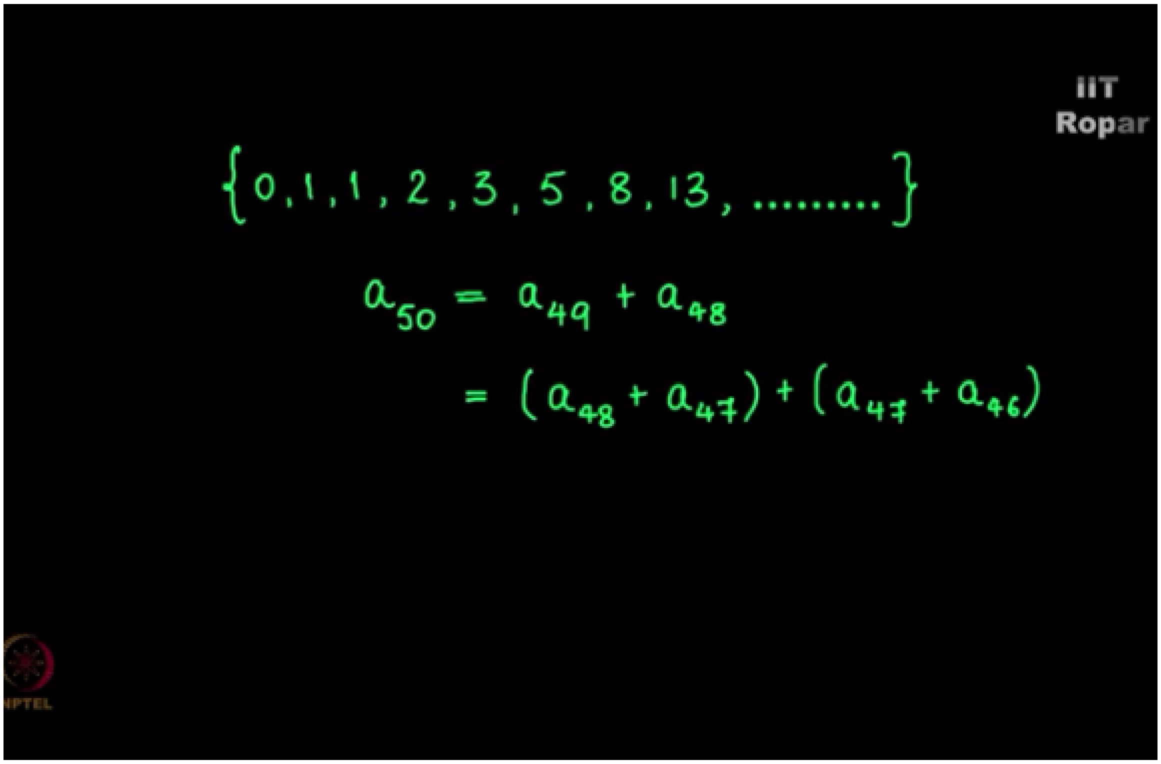
**Solution of Fibbonacci sequence**

**By**
**Prof. S.R.S Iyengar**
**Department of Computer Science**
**IIT Ropar**

So given a Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13 and so on,
(Refer Slide Time: 00:14)

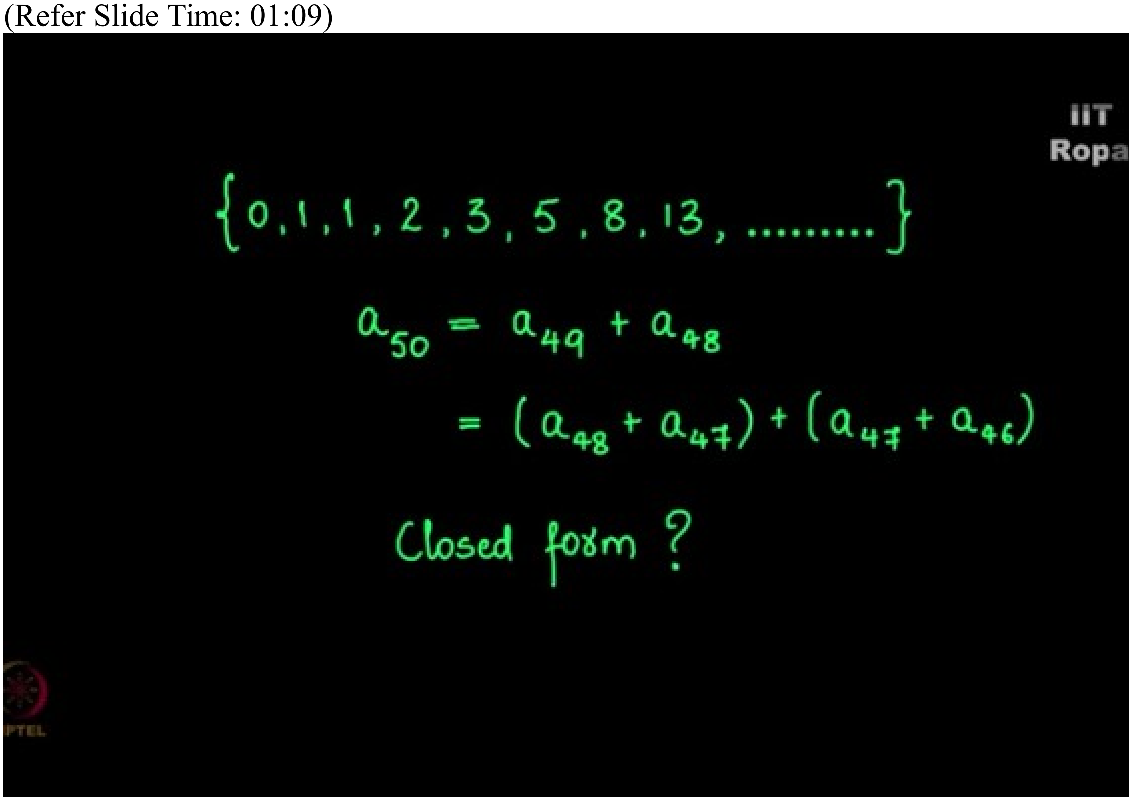



how would you compute the nth term here? How does the nth term look like? How can we even answer this question? Let's say what is A50 here, so one way to do it is simply start from 0, 1, 1, 2, 3, 5, 8, 13 and so on and then go up to 50, the other way to do it is write A50 as A49 + A48, and then A49 is A48 + A47, A48 is A47 + A46 and keep enumerating this and as you proceed you will only get A1's and A naught completely, and you know what that is and then you will find what is A50.
(Refer Slide Time: 01:01)



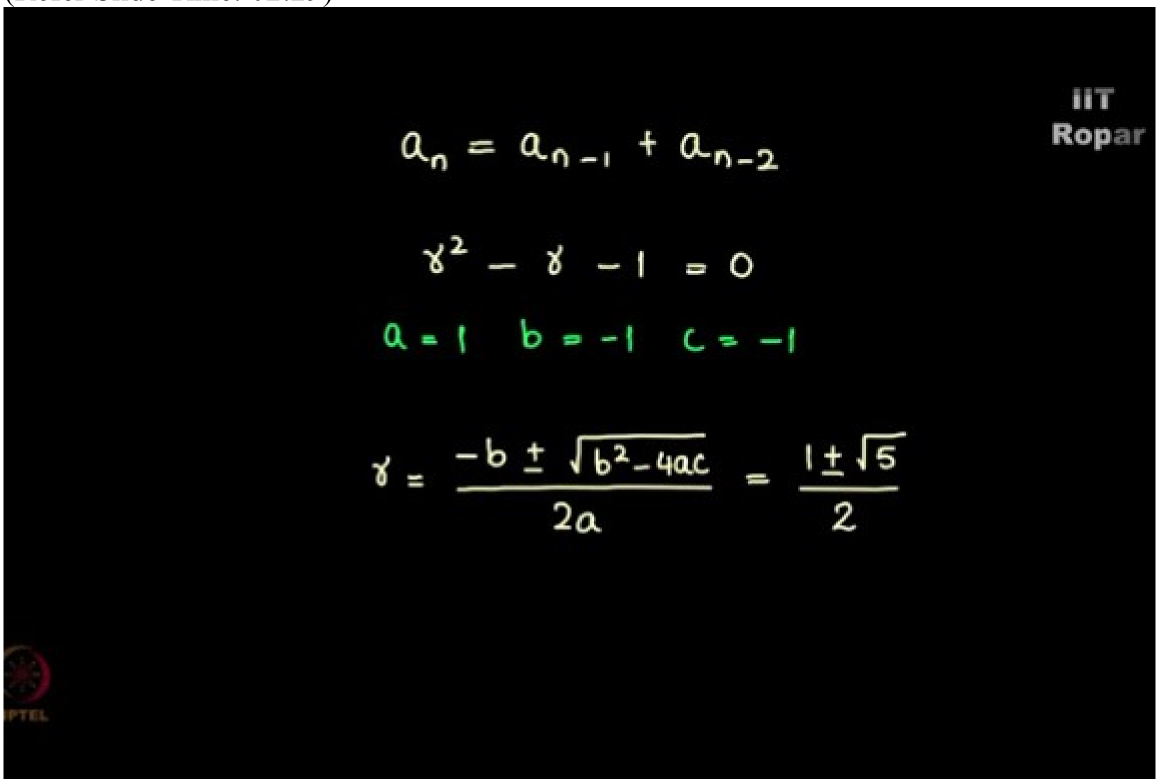

Is there a closed formula which tells us what is AN in general?

(Refer Slide Time: 01:09)



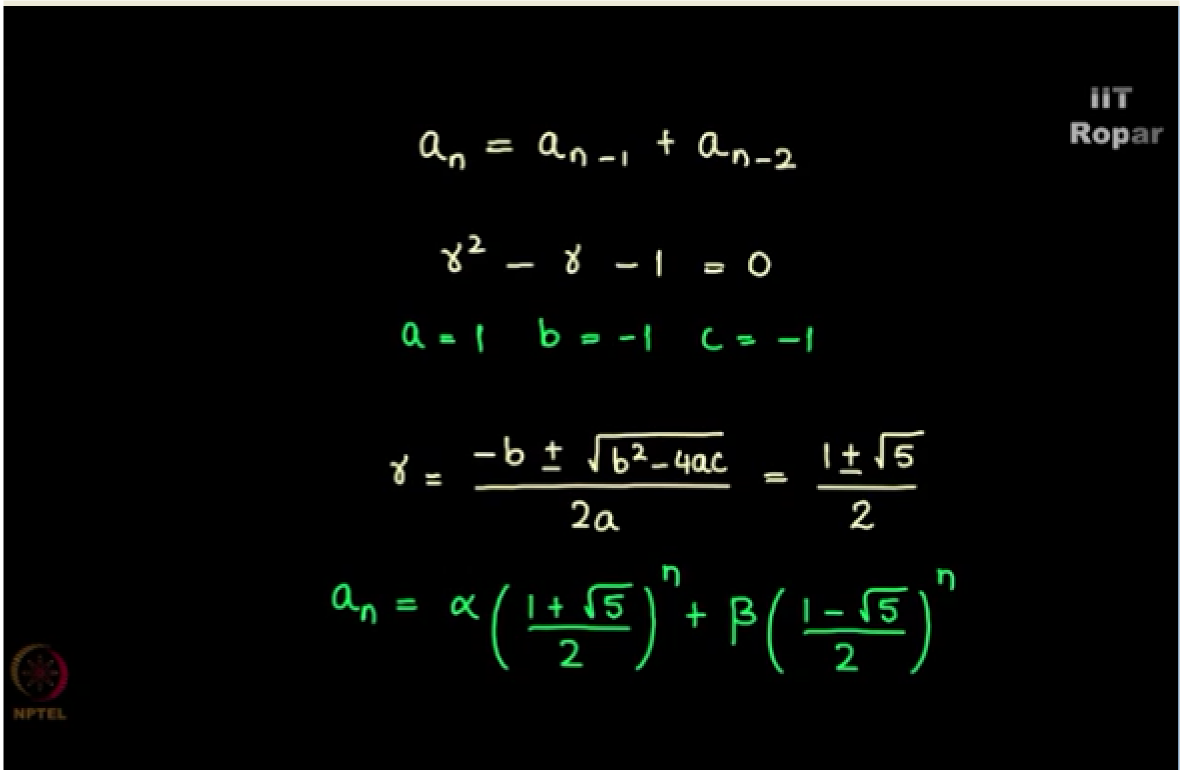

Yes, there is, let's see how to do that, let's recall our just now refresh knowledge on recurrence relations, so let me write down the recurrence relation for Fibonacci sequence, AN is AN-1 + AN-2, you see we know how to solve this, we write down what is called the corresponding quadratic equation R2 – R – 1 = 0, and then we find out that this is a quadratic equation where the coefficients A is 1, B is -1, C is -1 and I plug in the formula –B + or – square root of B square – 4AC divided by 2A, and substitute for ABC, and I'll get AN is equal to, I mean I'm sorry, the root R = 1 + square root of 5 whole by 2, and the second root is as I said + or – square root of B square – 4AC which is 5 basically, B square – 4AC, so second root is 1 – square root of 5 whole by 2,

(Refer Slide Time: 02:29)




these are the two roots of this quadratic equation and we know how to write what is called the general solution, so AN will be equal to alpha times the first root + beta times the second root, and these root should be N power 2N, so AN will be alpha times 1 + root 5/2 whole to the N + beta times 1 – root 5/2 whole to the N.

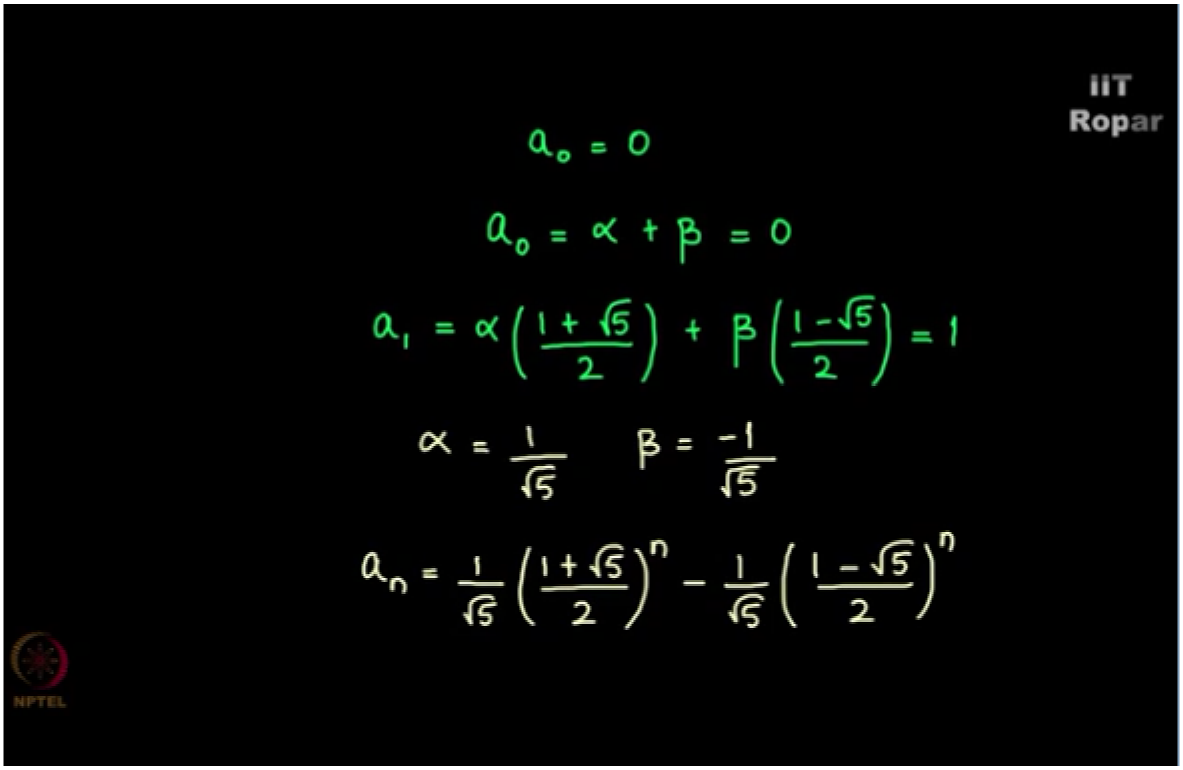
(Refer Slide Time: 02:54)




What is this alpha and beta? Now look at this, what is A dot? In Fibonacci sequence A naught is 0, but A naught in this equation happens to be alpha 1 + alpha 2, correct, when you put N = 0 these terms become and you're left with alpha + beta, so A naught = alpha + beta which is equal to 0, correct, and then A1 will be equal to, put an equals 1 you get A1 = alpha times 1 + root 5/2 + beta times 1 – root 5/2 and this is going to be A1 which is 1, now this are simultaneous equations as you can see.

So solving it a little bit of jugglery as you can see gets me to alpha being equal to 1 over root 5, simultaneous equation, eta is a unique solution, and beta is -1/root 5, now I have a complete solution for my AN, just plug these alpha and beta there and you will get AN = 1 over root 5 times 1 + root 5/2 whole to the N – 1 over root 5 into 1 – root 5/2 whole to the N, now A50 is easy to compute, simply put N = 50 you will get the answer for A50 which is the 50th element of a Fibonacci sequence,

(Refer Slide Time: 04:19)




pause and think how, a simple looking Fibonacci sequence which is given by AN = AN-1 + AN-2 can involve this much of math.

**IIT MADRAS PRODUCTION**

**Founded by**
**Department of Higher Education**
**Ministry of Human Resources Development**
**Government of India**

**www.nptel.iitm.ac.in**