NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Soving recurrence relation-Example 2

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Let us solve another problem by applying the theorem, so the recurrence relation given is AN = 2 into AN-1 + AN-2 – 2 into AN-3, so this is the recurrence relation.

Now the given conditions are A naught is 0, A1 is 1, and A2 is 1, this is the initial condition given, so what appeared just now R cube – 2R square – R + 2, this is the corresponding characteristic equation for the recurrence relation,
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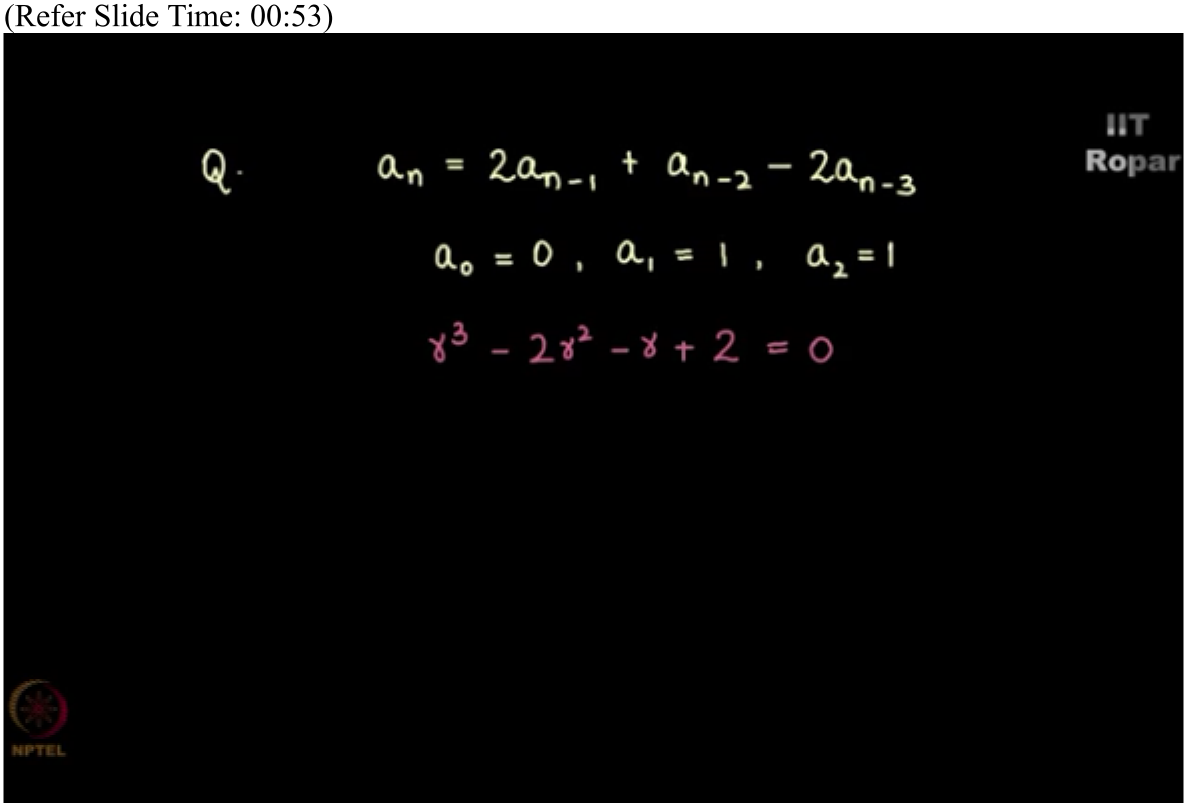

well do you observe that this has come from the given recurrence relation R cube – 2R square because this was C1 there, right, 2 into AN-1 so C1 is 2 here, AN-2 has the coefficient as +1, therefore it is –R and it is -2 into AN-3 and hence it's a +2, right, so this is the corresponding characteristic equation.

Now let me write this as R cube – R square – R square – 2R + R + 2, I'm just expanding -2R square and I'm writing –R as -2R + R, so this is how I am writing the characteristic equation.
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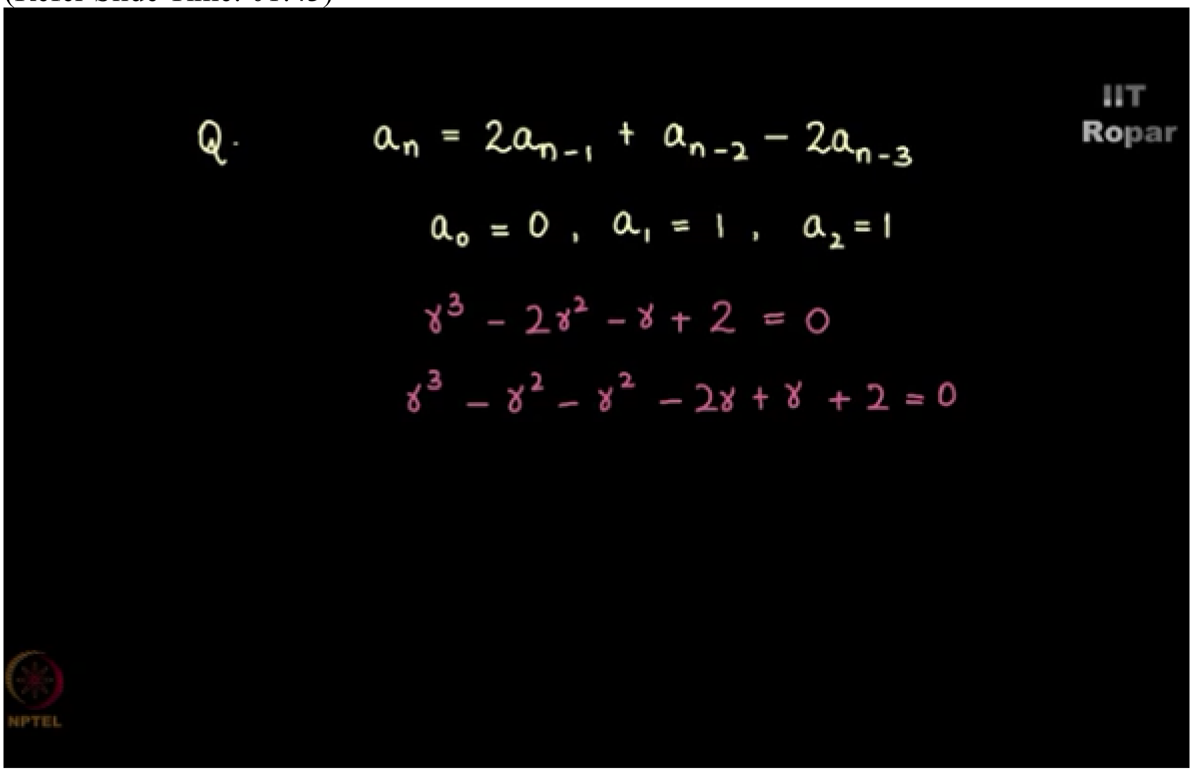

Now what I observe is R square – R – 2, if I can take it out common that is I have done some jugglery and I have got this R square – R – 2 into R-1 = 0, so if you multiply these two polynomials you will get the given equation which is R cube – 2R square – R + 2 = 0,
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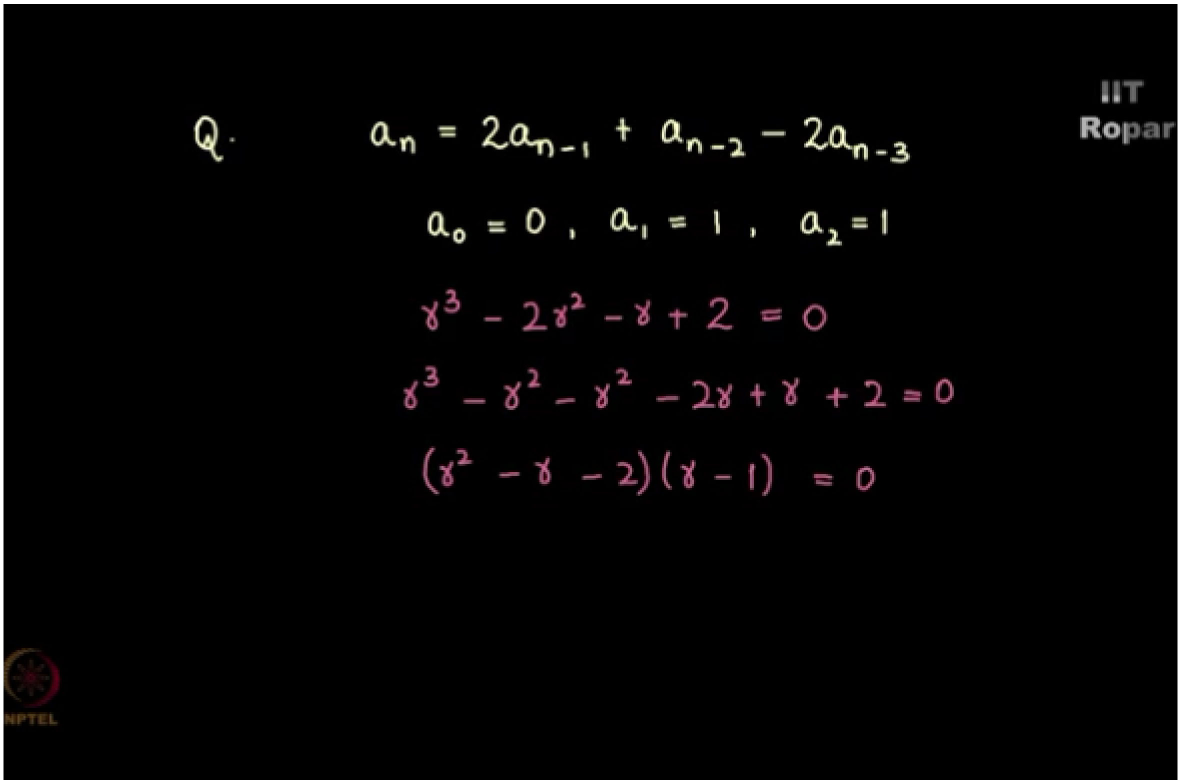

so now by solving this quadratic equation we observe that it is R + 1 into R - 2 into R – 1 = 0, now this will give the distinct roots of the characteristic equation which is R = 1 – 1 and 2, so these are the distinct roots of the characteristic equation.

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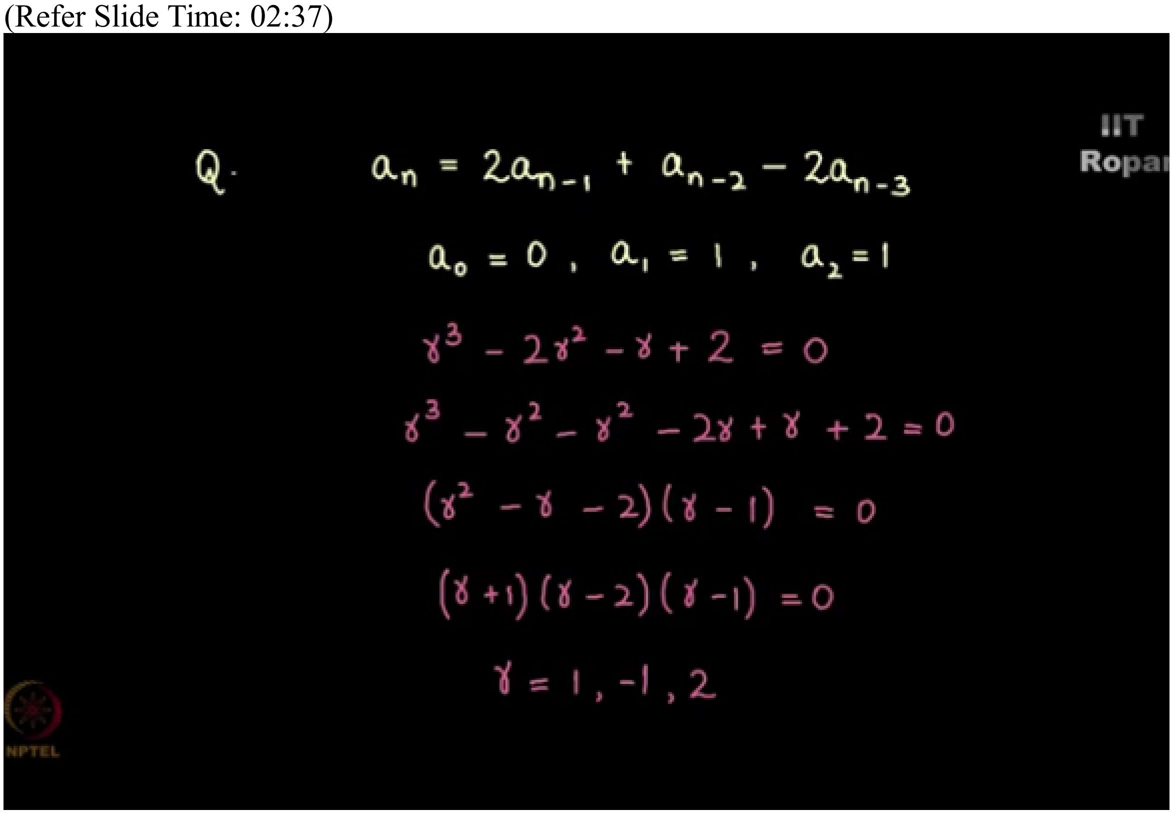

So now that we have found out the root we can find out the solution for the recurrence relation, what will it be? AN = alpha 1 into -1 to the N + alpha 2 into 2 to the N + alpha 3 into 1 to the N, very simple in place of X1, X2, X3, I've just substituted -1, 2 and 1, now this becomes if I substitute N = 0, I get it as A naught = alpha 1 + alpha 2 + alpha 3, and if I substitute N as 1 we get it as A1 = -alpha 1 + 2 alpha 2 + alpha 3, and if I substitute N as 2 I get it as A2 = alpha 1 + 4 alpha 2 + alpha 3, right, just substitute 0, 1, 2 in the above equation and you will get these three simultaneous equations.
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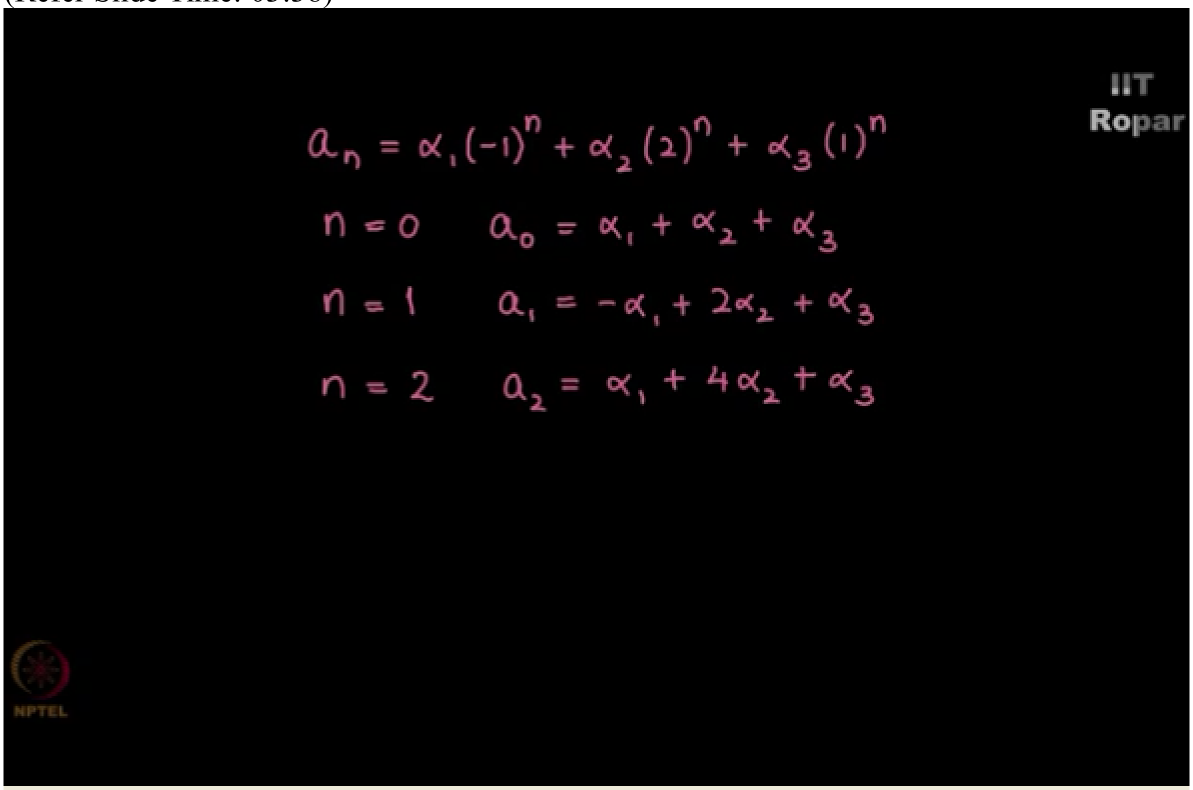

From the given initial conditions we know that A naught is 0, A1 is 1, and A2 is also 1, now solving these 3 simultaneous equations, right, by whichever method you want using the method taught by you in the school you can find out that alpha 1 is 1/6, alpha 2 is 1/3, and alpha 3 is -1/2, now that I have found out what is alpha 1, alpha 2 and alpha 3 I can substitute in this above solution equation and I obtain AN as 1/6 into -1 to the N + 1/3 into 2 to the N -1/2 into 1 to the N, right, so this is the corresponding solution to the recurrence relation.
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$$a_n = \alpha_1(-1)^n + \alpha_2(2)^n + \alpha_3(1)^n$$

$$n = 0 \quad a_0 = \alpha_1 + \alpha_2 + \alpha_3 = 0$$

$$n = 1 \quad a_1 = -\alpha_1 + 2\alpha_2 + \alpha_3 = 1$$

$$n = 2 \quad a_2 = \alpha_1 + 4\alpha_2 + \alpha_3 = 1$$

$$\alpha_1 = 1/6 \quad \alpha_2 = 1/3 \quad \alpha_3 = -1/2$$

$$\therefore a_n = 1/6(-1)^n + 1/3(2)^n - 1/3(1)^n$$

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