NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Soving recurrence relation-Example 1

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Let me state the theorem and paraphrase what the professor just told, let C1 and C2 be real numbers, suppose that X square – C1X – C2 = 0 has 2 distinct roots X1 and X2, then the sequence AN is the solution of the recurrence relation AN = C1 AN-1 + C2 AN-2, if and only if AN = alpha 1, X1 to the N + alpha 2 X2 to the N for N from 0, 1, 2 and so on where these alpha 1's and alpha 2's they are constants.

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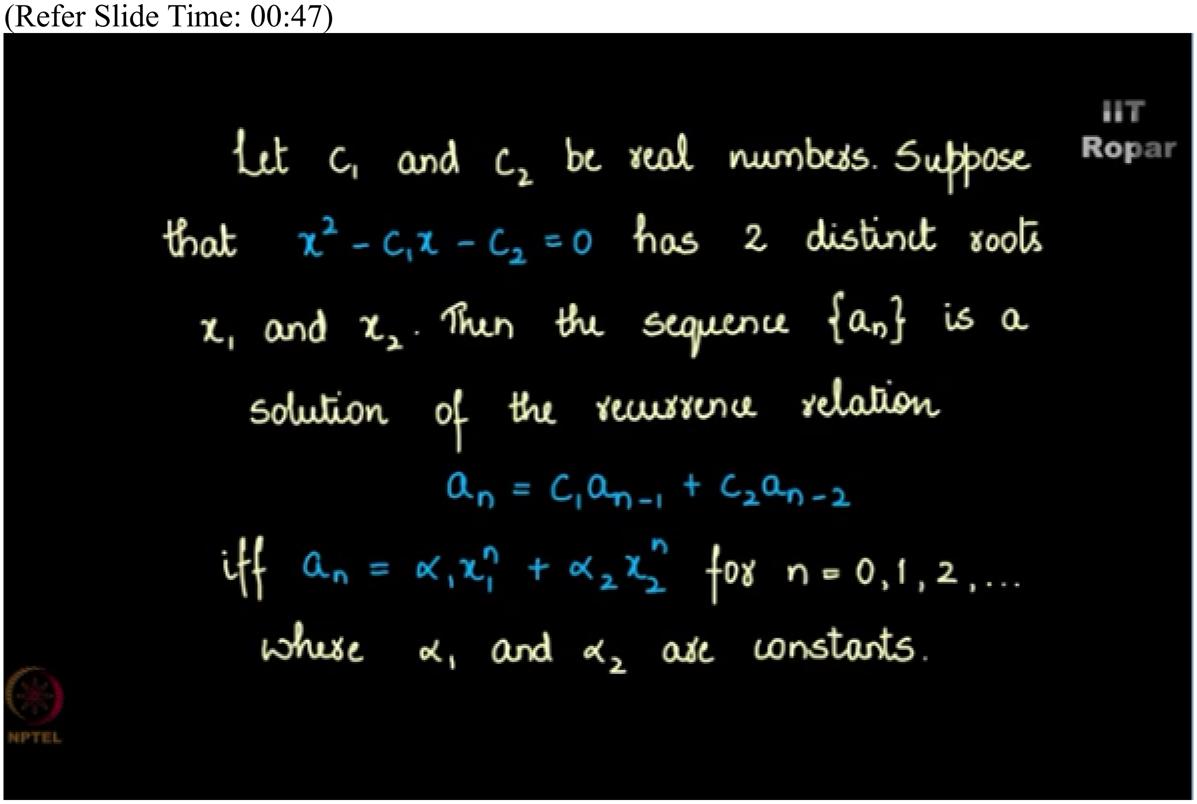

Though the theorem might seem to be very tough and difficult to remember, it is actually very simple.

So given this quadratic equation which has 2 distinct roots X1 and X2, then this sequence AN it will be a solution of the recurrence relation and of what form is this recurrence relation? It is some constant times AN-1 + constant times AN-2, and the solution is of the form alpha 1 X1 to the N + alpha 2 X2 to the N, these X1 and X2 as you remember are the distinct roots of the quadratic equation.

Now let us see how we can apply this theorem in a few problems, if AN is given to be 3 into AN-1 – 2 times AN-2 then what is the solution of this recurrence relation, A naught is given to be 1 and A1 is given to be 2, so you have to find the solution of this recurrence relation.

Now so I am going to write now the quadratic relation of this or the characteristic equation of this recurrence relation, it is R square – 3R + 2 = 0, how did I get this? As you see the constants here C1 and C2 are 3 and -2 respectively, and hence substituting that in the quadratic equation I get R square – 3R + 2 = 0, now how can I solve this? I can solve it using the formula –B + or – B square – 4AC square root / 2A, now solving this I will obtain the 2 roots, so let me substitute for ABC, so I will get 3 + or – root of 9-8/2 and this happens to be 3 + or – 1 /2 which is 2 and 1, so 2 and 1 are the distinct roots of this quadratic equation,
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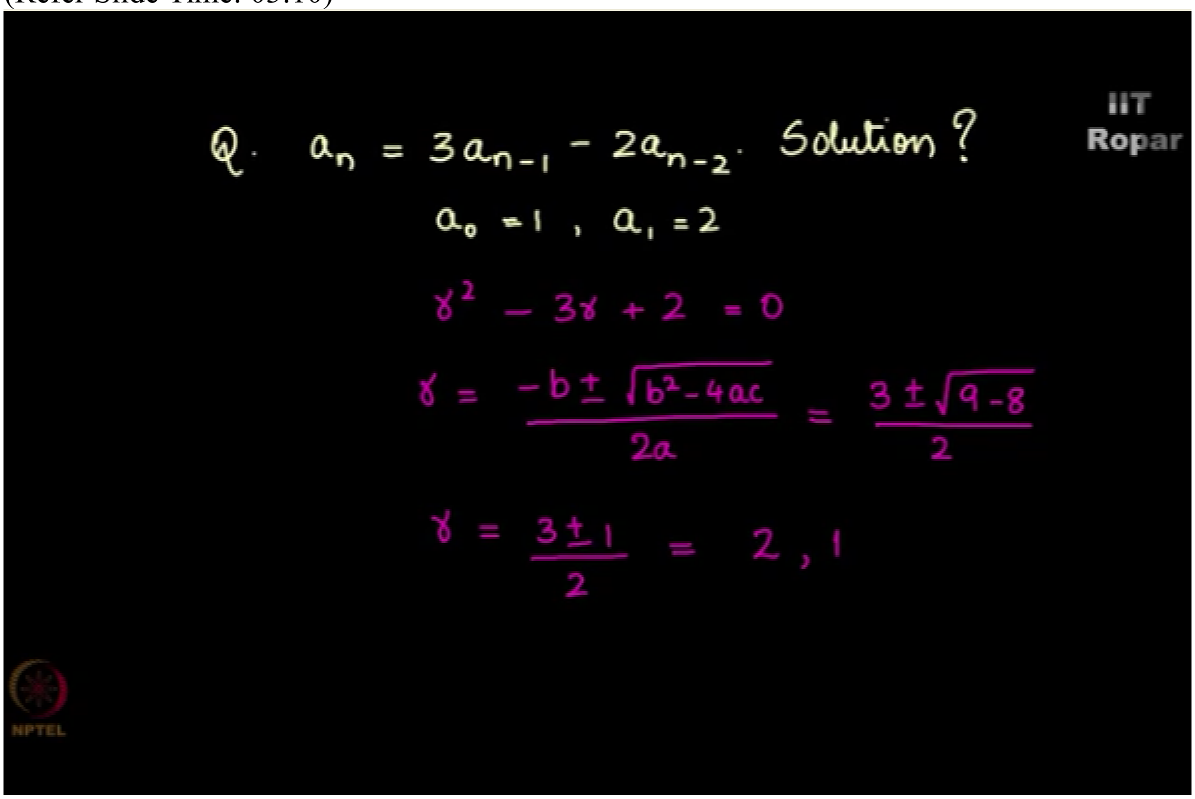

so I can write AN as alpha 1 into 2 to the N + alpha 2 1 to the N, right.

Now if I substitute N as 0, in this recurrence relation what will I get? A naught is anything to the N is, anything to the 0 is 1 and hence I'll get A naught is alpha 1 + alpha 2, and if I substitute N as 1 and the recurrence relation I'll get A1 as 2 alpha 1 + alpha 2, right, now we have these simultaneous equations with us, but according to the initial condition given we know that A naught is 1 and A1 is 2, right, so 1 = alpha 1 + alpha 2, and 2 = 2 alpha 1 + alpha 2, now when we solve these simultaneous equations we see that alpha 1 is 1, alpha 2 is 0, so I can substitute these values back in the recurrence relation, so what will I get? AN = alpha 1 2 to the N + alpha 2 1 to the N this is the equation or if I substitute for alpha 1 and alpha 2 I'll get it as 1 into 2 to the N + 0 times 1 to the N and hence your final solution will be AN = 2 to the N,

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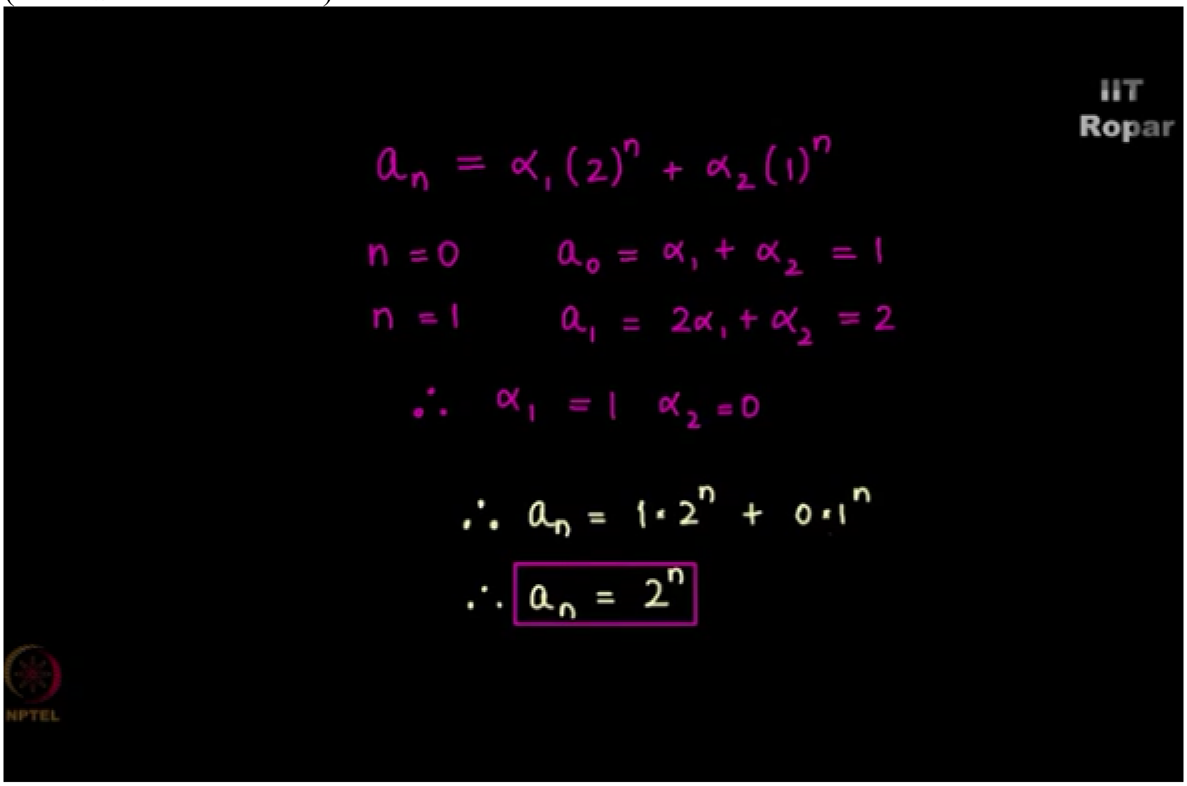

so this is the solution for the recurrence relation that we initially started with.

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