NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Example-n-bit string without consecutive zero

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How many n-bit strings are possible without consecutive zeroes,
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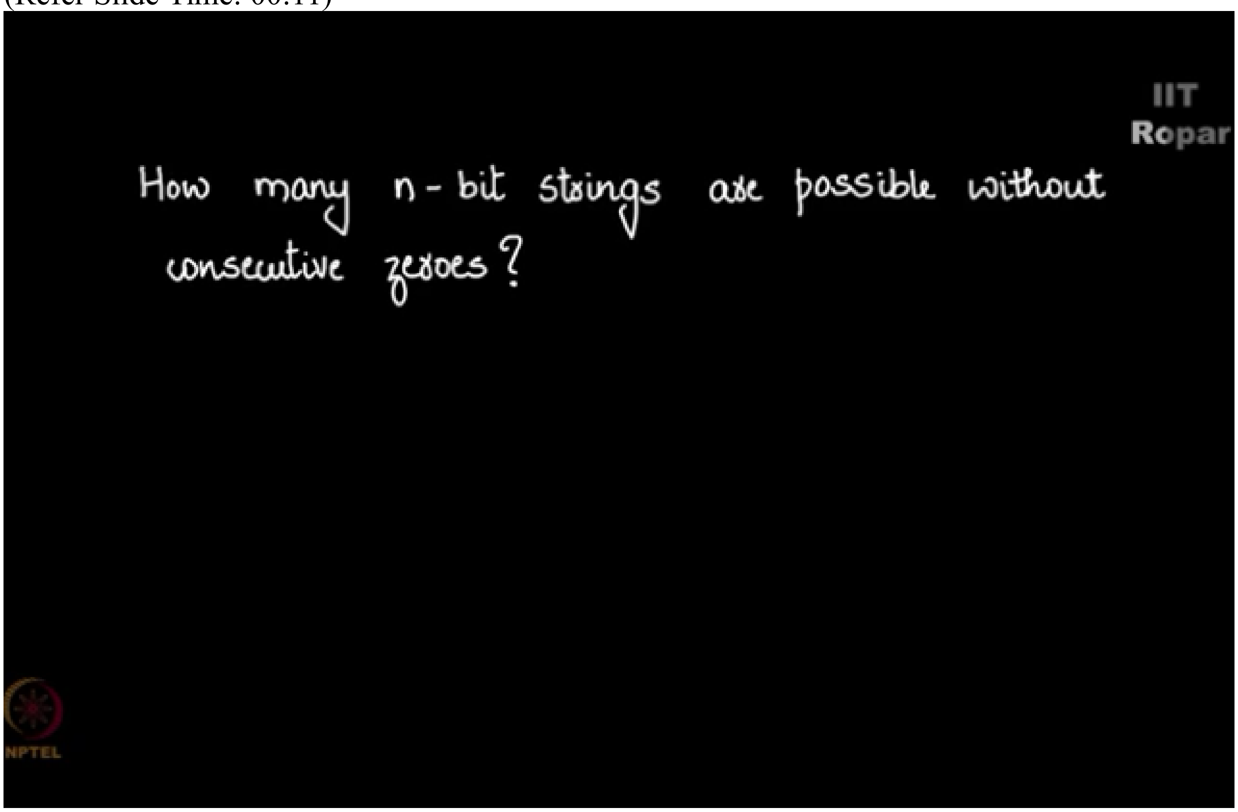

so in this question they're asking you to find out the number of n-bit strings possible, binary digit strings of length N, but you must not consider those cases where there are consecutive zeroes, what do we mean by that? Consider this to be a binary string of length 8, right, but you see there are three zeroes in middle, consecutive zeroes and hence this is not a valid possibility according to the question.
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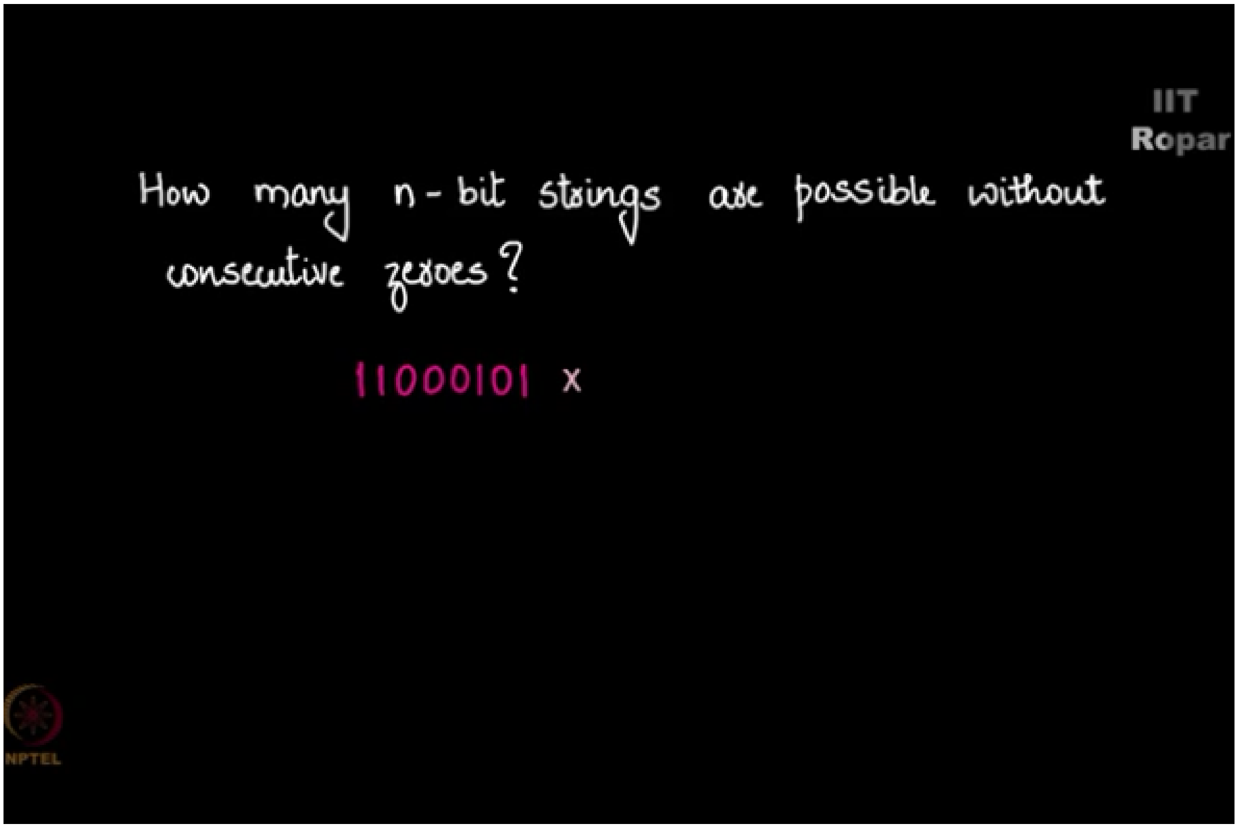

Consider this as another string of length 9, but here there are no consecutive zeroes, right, this is a valid possibility,

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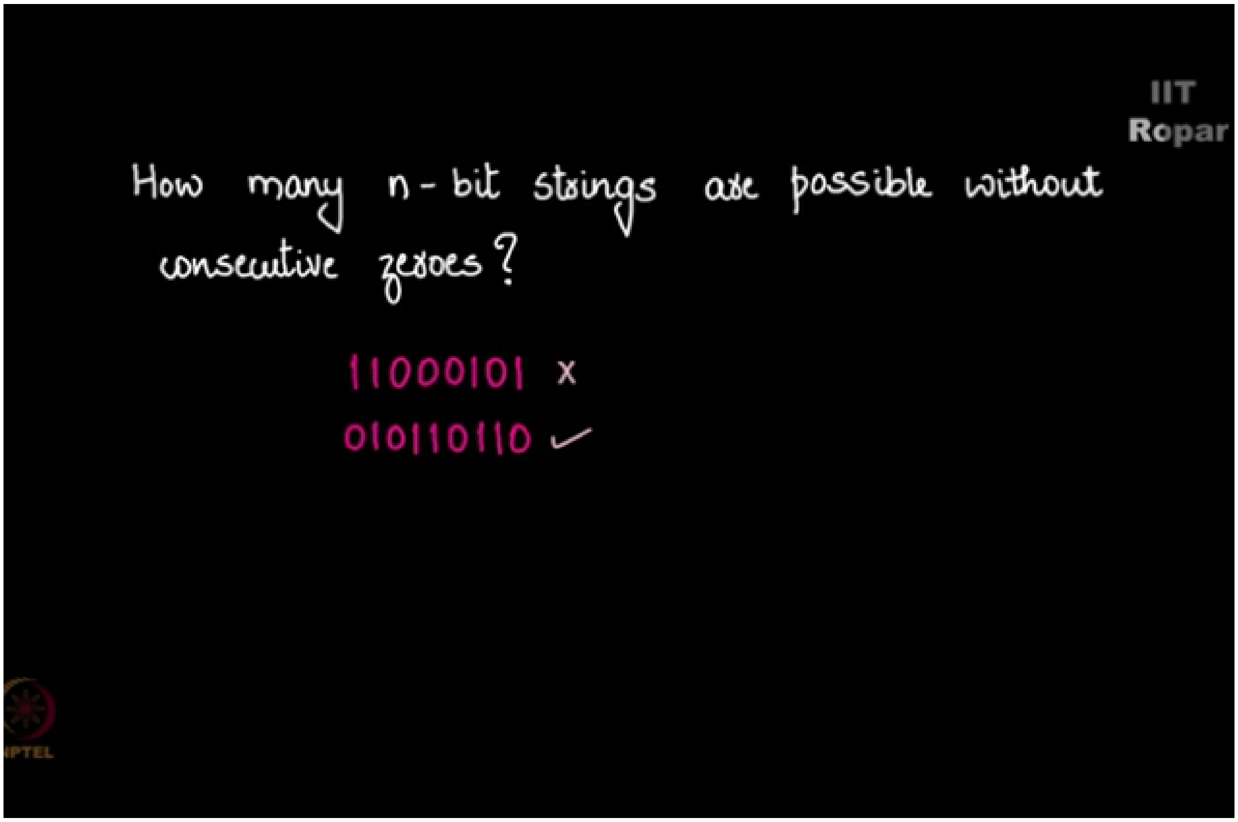

we are not bothered about whether there are consecutive ones or not, we are only bothered about the consecutive zeroes.

Now consider this one, there is no consecutive zero here and hence it is valid possibility,
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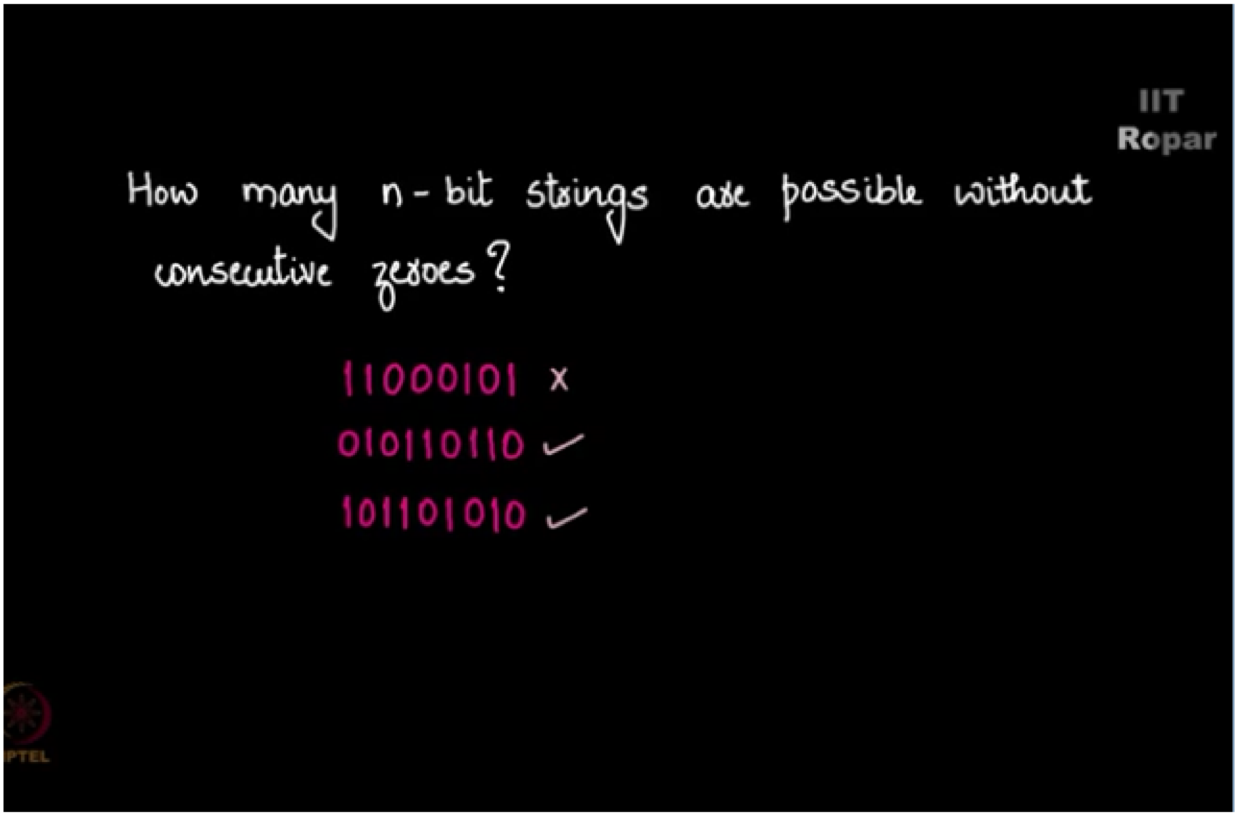

right, these were some of the examples, now supposing N is 2, for N = 2 what are the possibilities? Let me enumerate some of them, this is valid possibility, 1 0, 0 1 is valid, 1 1 is valid, 0 0 is not valid, right, it has consecutive zeroes, so we are not considering this possibility and hence for N = 2 there are 3 enumerations, there are 3 possibilities of writing binary digit strings of length 2, right, without consecutive zeroes, keep this in mind it will be required for our further steps.
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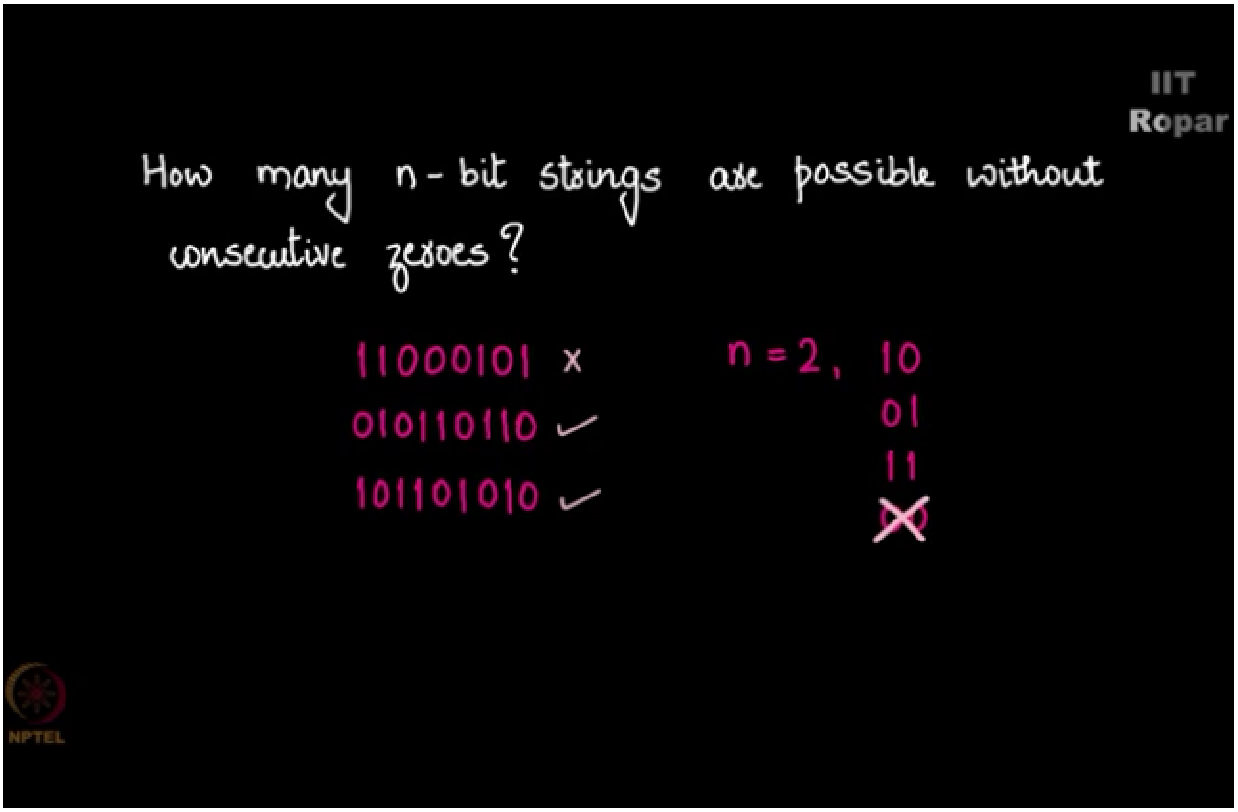

Now consider this to be your n bit string, right,
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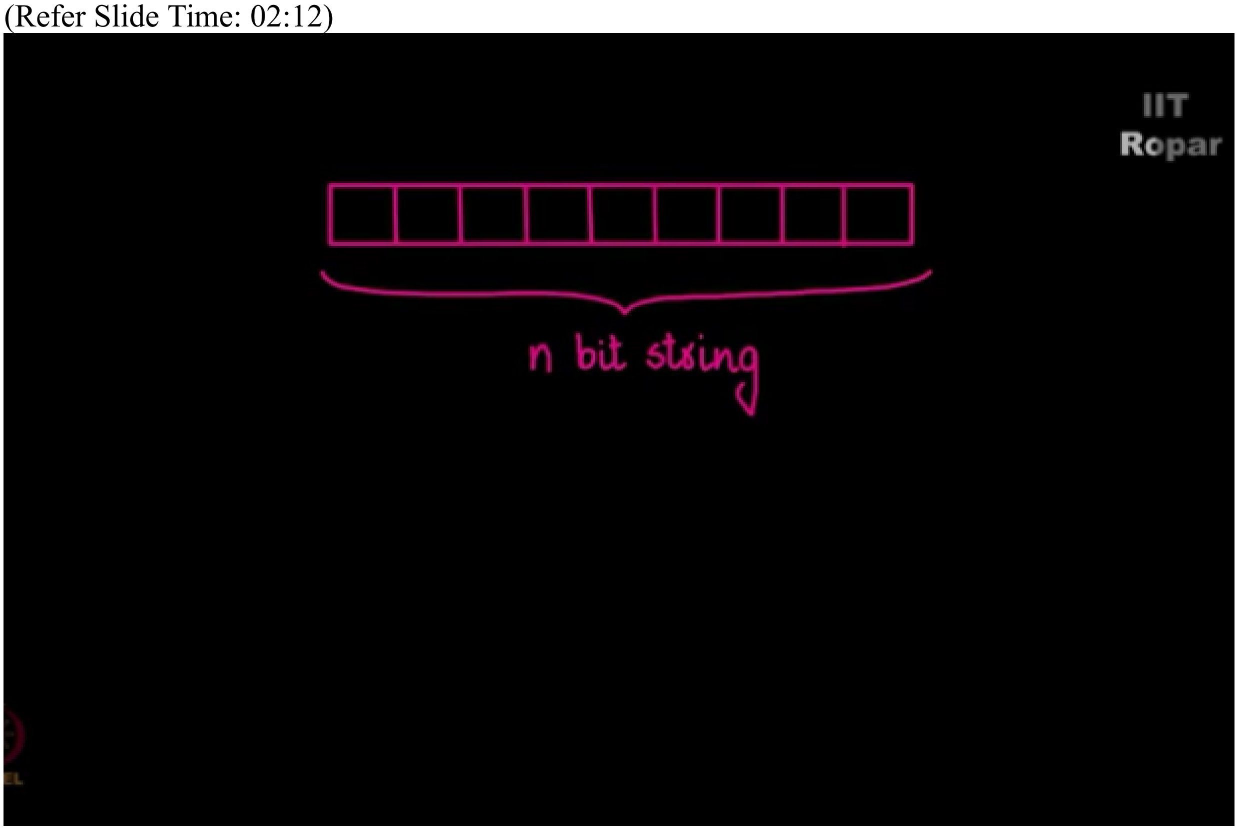

now as in the earlier video where we had seen just writing the possibilities for an n-bit string, I am going to do the same thing here, but what is my T(n) in this case? T(n) is the number of n-bit strings without consecutive zeroes, please do not get confused with a T(n) in the earlier video and this video, here without consecutive zeroes is very important, this is an extra condition in this problem and hence T(n) is always dependent on the question, you cannot consider T(n) to be always the same one and all the problems.
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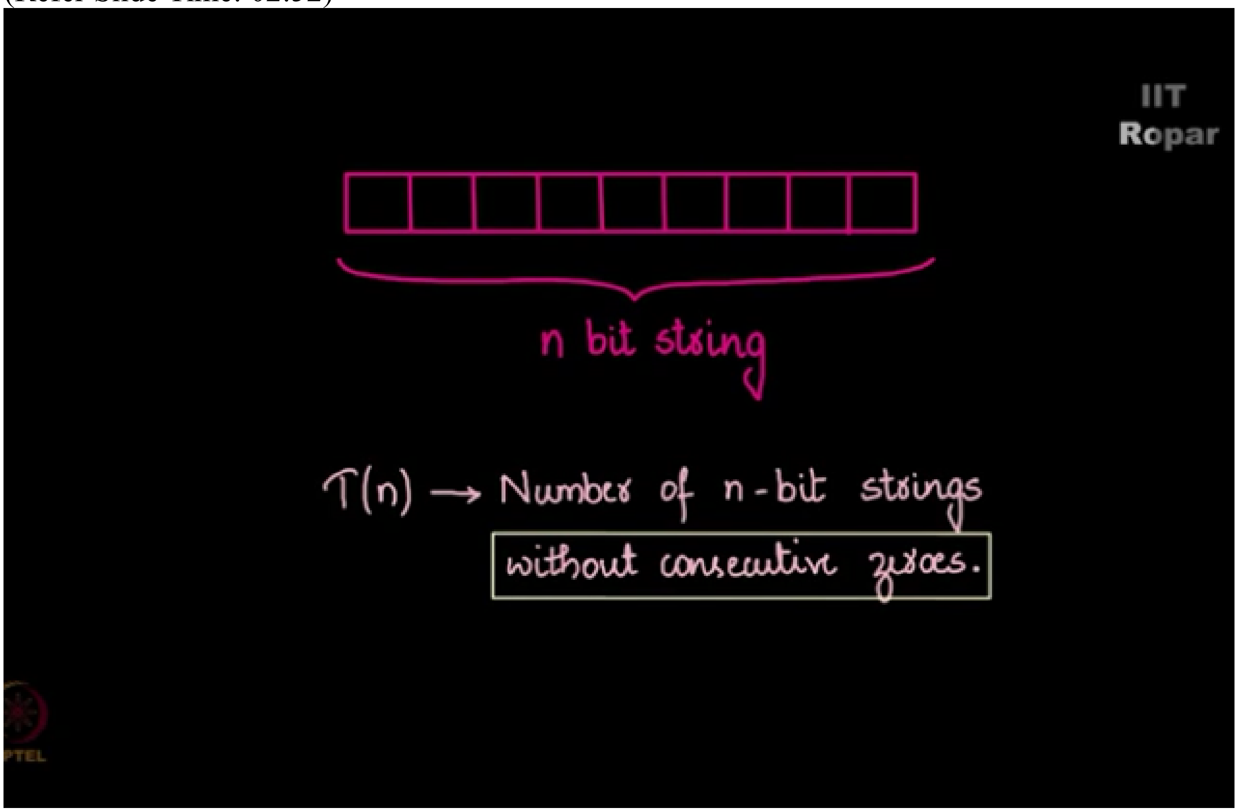

Now in this case T(n) is number of n-bit strings without consecutive zeroes, now let me again do the same thing as the one I did in the earlier problem, in the n-bit string if I start with zero, if the string starts with zero then what can come in the next position, it can be a 1,
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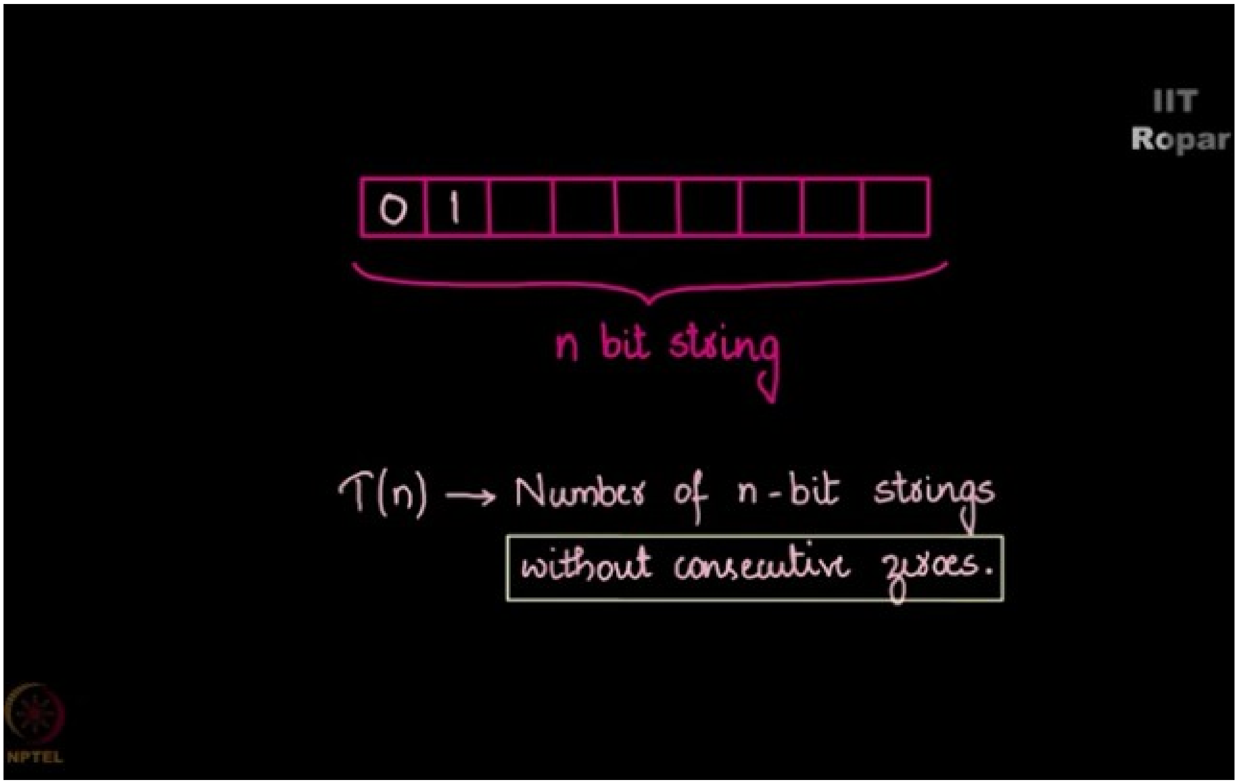

can it be a zero? Definitely no, how? Because we are striking of those possibilities where there are consecutive zeroes and hence only one can occupy the position immediately after zero, and hence this problem reduces to finding the number of possibilities for n-2 bit strings, right, you have locked this positions 1 and 2 where you start with zero, a zero cannot come in the immediate next position, 1 has to come, only 1 can come, right, so now the problem reduces to finding out the number of n-2 bit strings which I can write it as T(n-2),

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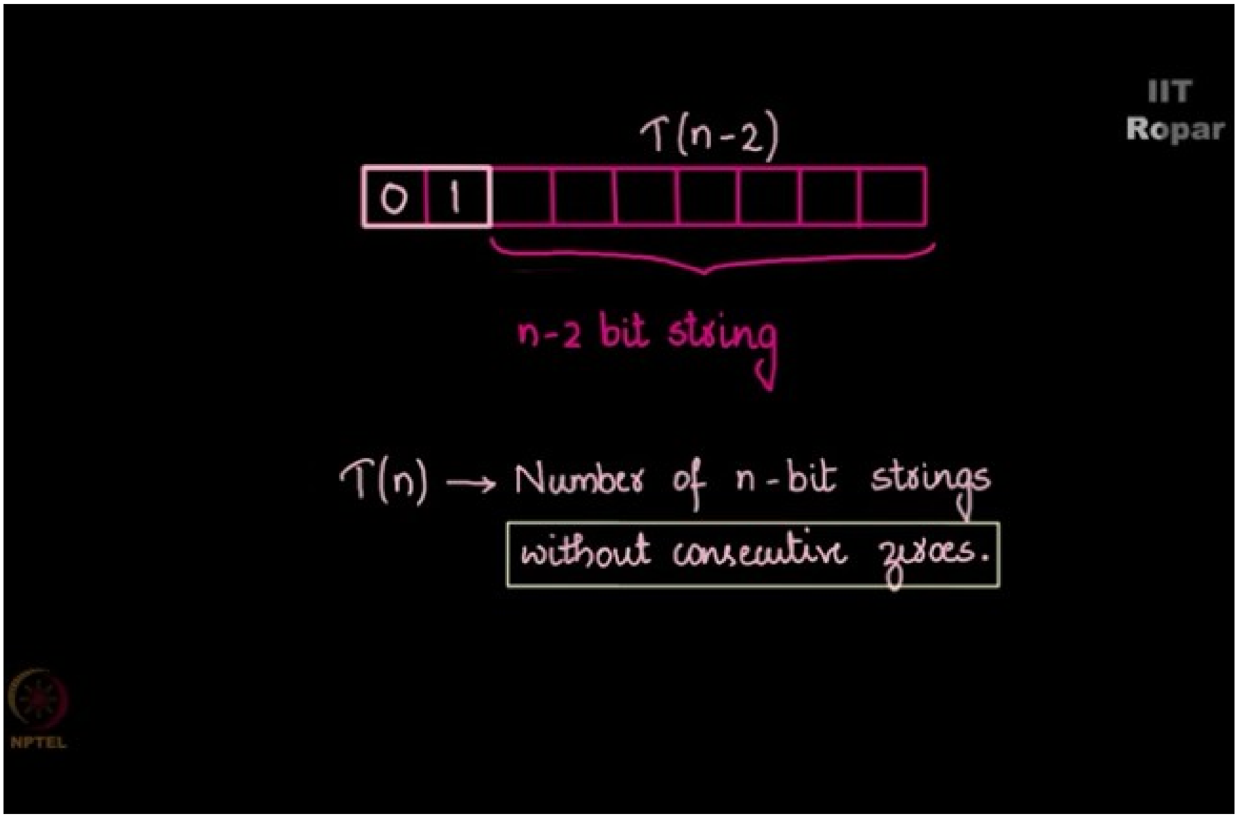

the number of possibilities for bit strings of length n-2, right, now this is the case if I start with zero, what if I start with one, what does it become then?

In case I start with 1, anything can come in the next position, how? There are no restrictions on consecutive ones, right, so you can either have a one in the next place or a zero in the next place, so now this problem reduces to finding your strings of length n-1, right,
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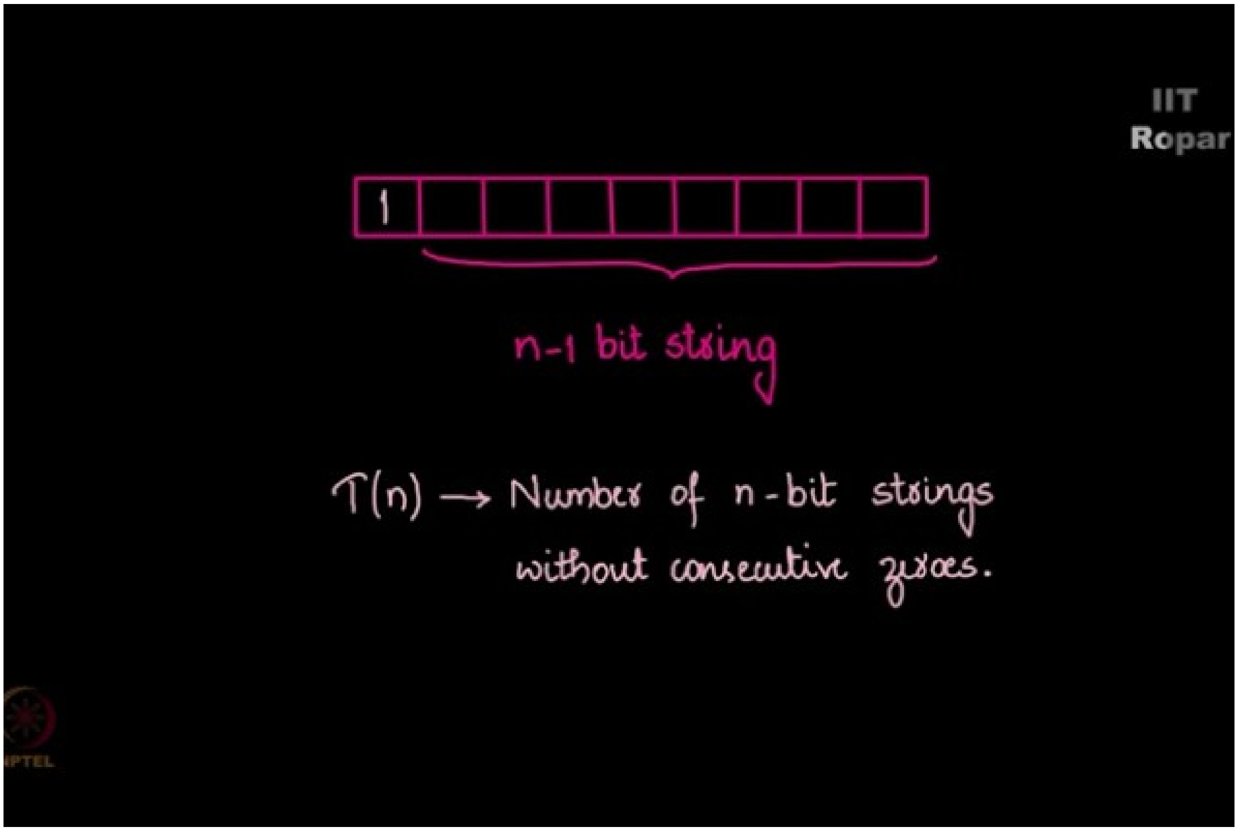

so if you write the one you can find out n-1 length bit strings, right, so the problem now reduces to finding T(n-1) which is the number of n-1 bit strings without consecutive zeroes, clear.

Now we move ahead and write what is T(n)? T(n) we saw that if you start with zeroes there is one condition, if you start with one there is another condition, right, so T(n-1), T(n-2) I have written both the possibilities T(n-1) is the case when you start with one, T(n-2) you have to calculate this when you start with zero, right, so T(n) depends on T(n-1) or T(n-2) whether you start with one or zero, so either of them can occur and hence by rule of thumb there is a plus in middle, right, you might want to watch the previous video for more clarity,
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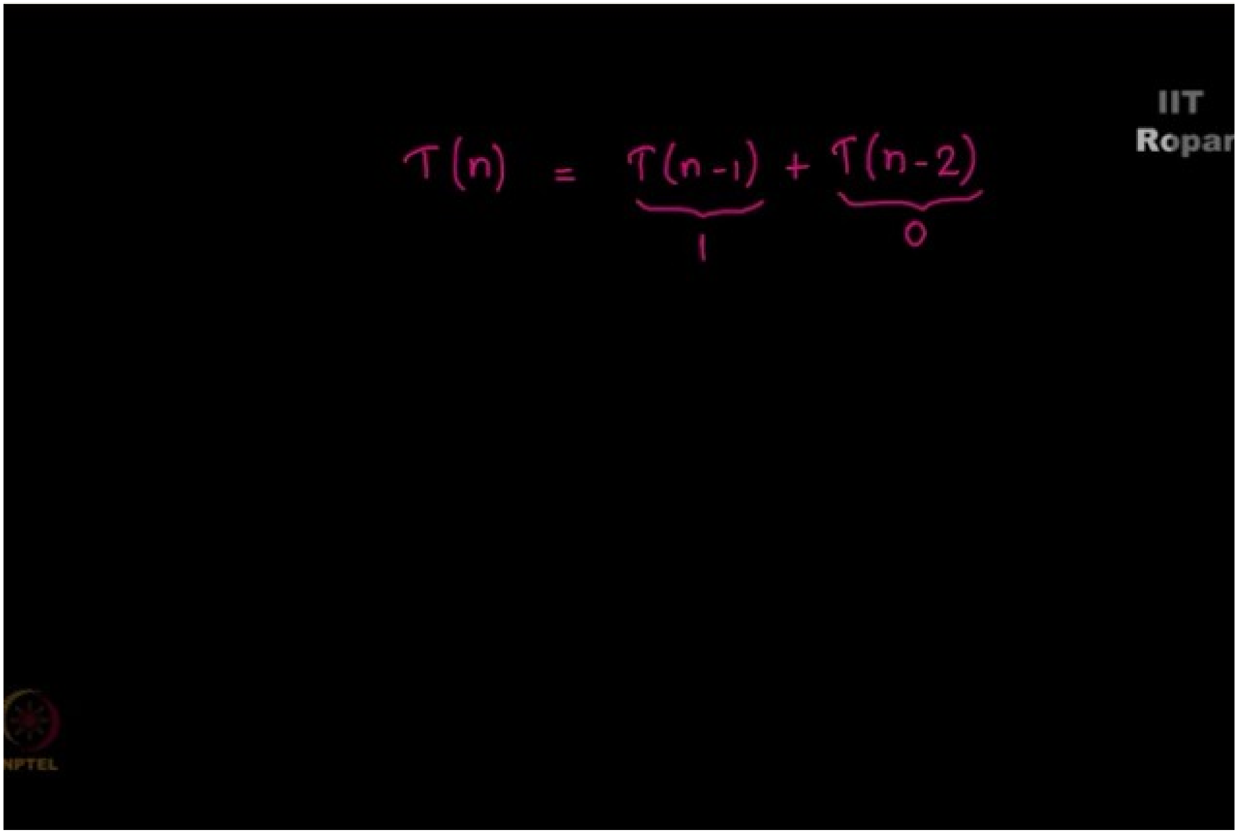

so T(n) is given to be T(n-1) + T(n-2) and this is the recurrence relation, but can I stop here?
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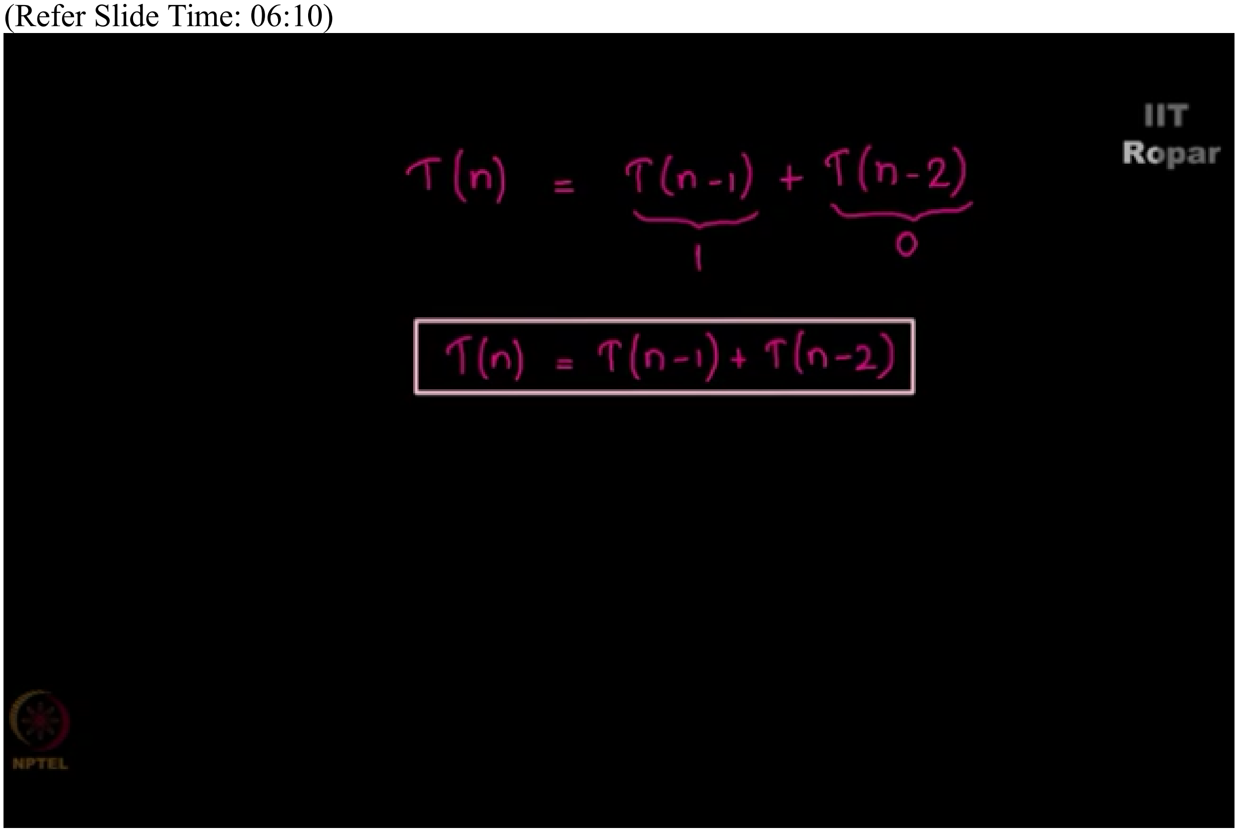

Please note we must always mention what is the base case, what do I mean by base case? You know that T(n) is T(n-1) + T(n-2), but if you go on like this, right for further finding out T(n-1) you need T(n-2) and T(n-3), right, where will you stop? Since it's a finite step process you must stop at a point, so you know that when there is only one string, one length, the string is of length one we had seen that there are two possibilities either it is a zero or it is a one, there is no question of consecutive zeroes in this case, where the string is of length one, right.

And now T(1) is 2 in that case, when you have N = 2, that is there are 2 positions, the string is of length 2 you had seen a few minutes back you had seen that the number of possibilities are 3 in that case, right, so now these 2 are the base cases,

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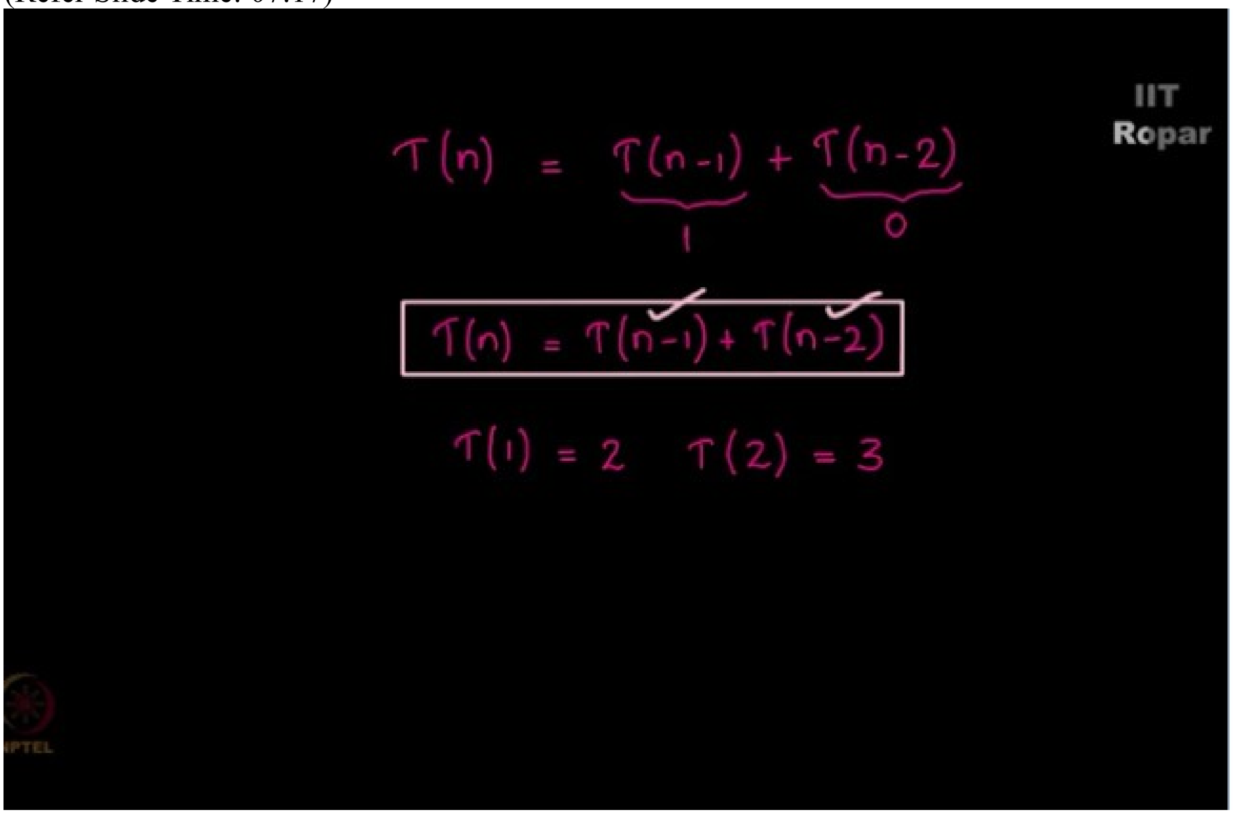

so T(n) = T(n-1) + T(n-2) is the recurrence relation for the number of possibilities of n-bit strings without consecutive zeroes and the base case for this recurrence relation is T(1) is 2, and T(2) is 3.

Well, till now we have seen the recurrence relation, the solution of this is left as a challenge for you people, you please do it yourself, try it yourself and if you do not succeed it is quite okay, you will be seeing the solution in the further videos.

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