NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Example-n-bit string

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How many binary digit strings are possible of length N?
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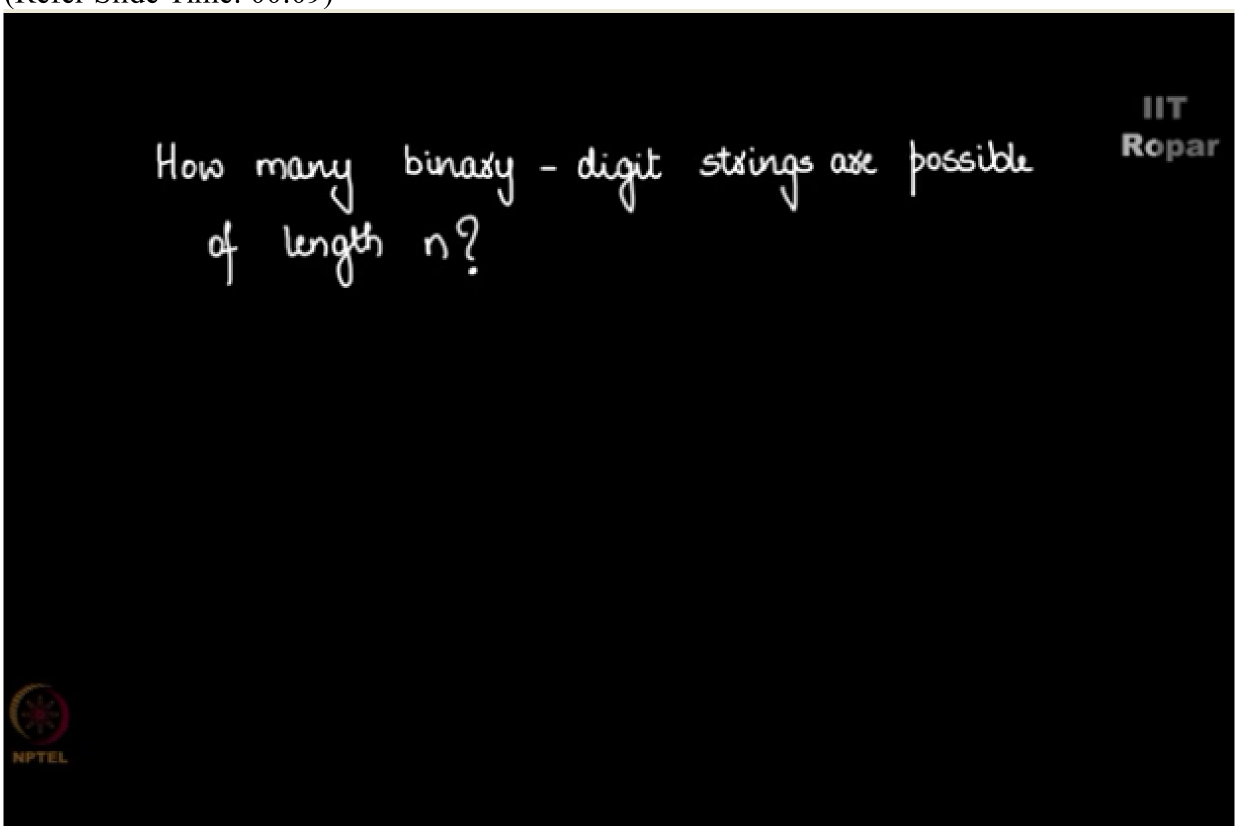

You must have encountered this problem in the earlier videos or of earlier weeks, but let us see how to solve this giving a recurrence relation flavor.

Now the question is how many binary digits strings are possible of length N, so you know that binary digits are only 1 and 0, so how many strings can you form of length N using these two digits?

Now consider this to be a string of length N, so this is an n-bit string,

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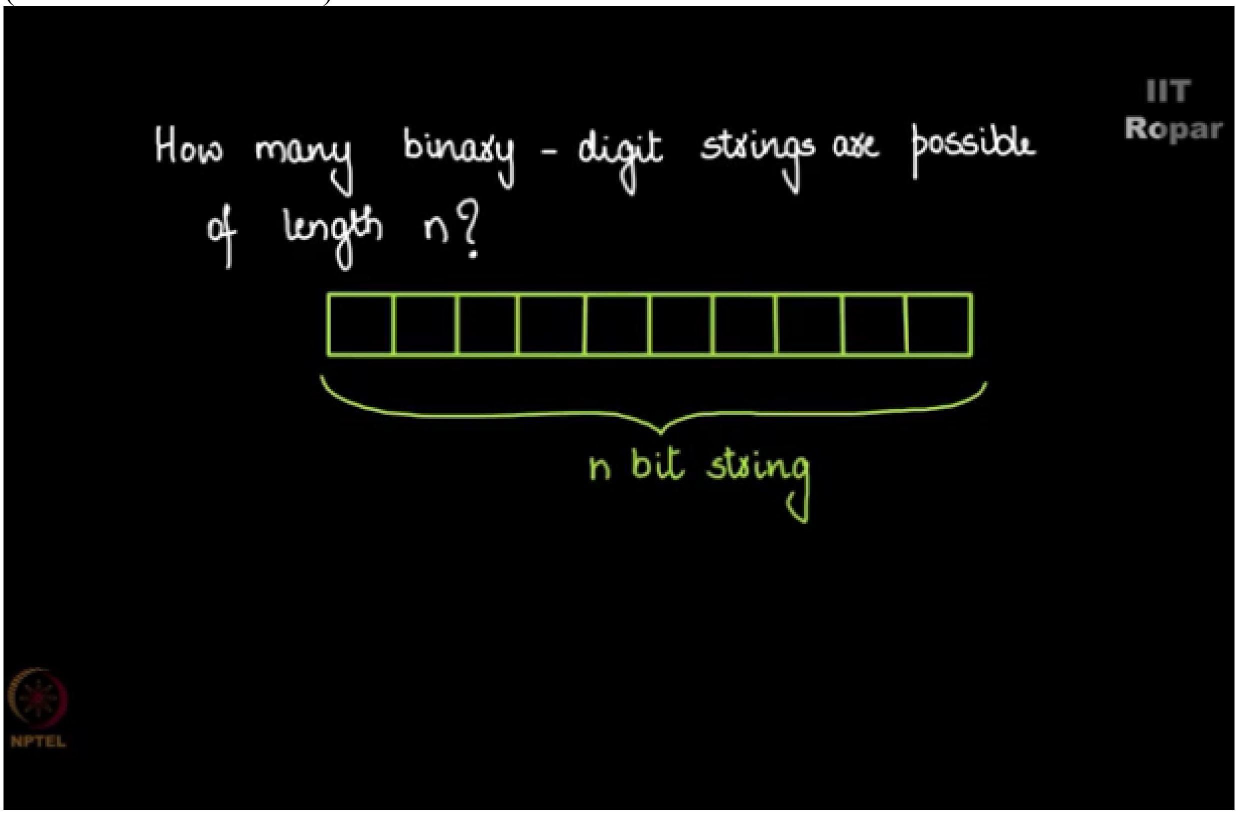

now I will represent the length of this as or rather T(n) I will write it as the number of binary digits, binary digit strings of length N, so if T(n) is the number of binary digit strings of length N, then in case I write as 0 here,

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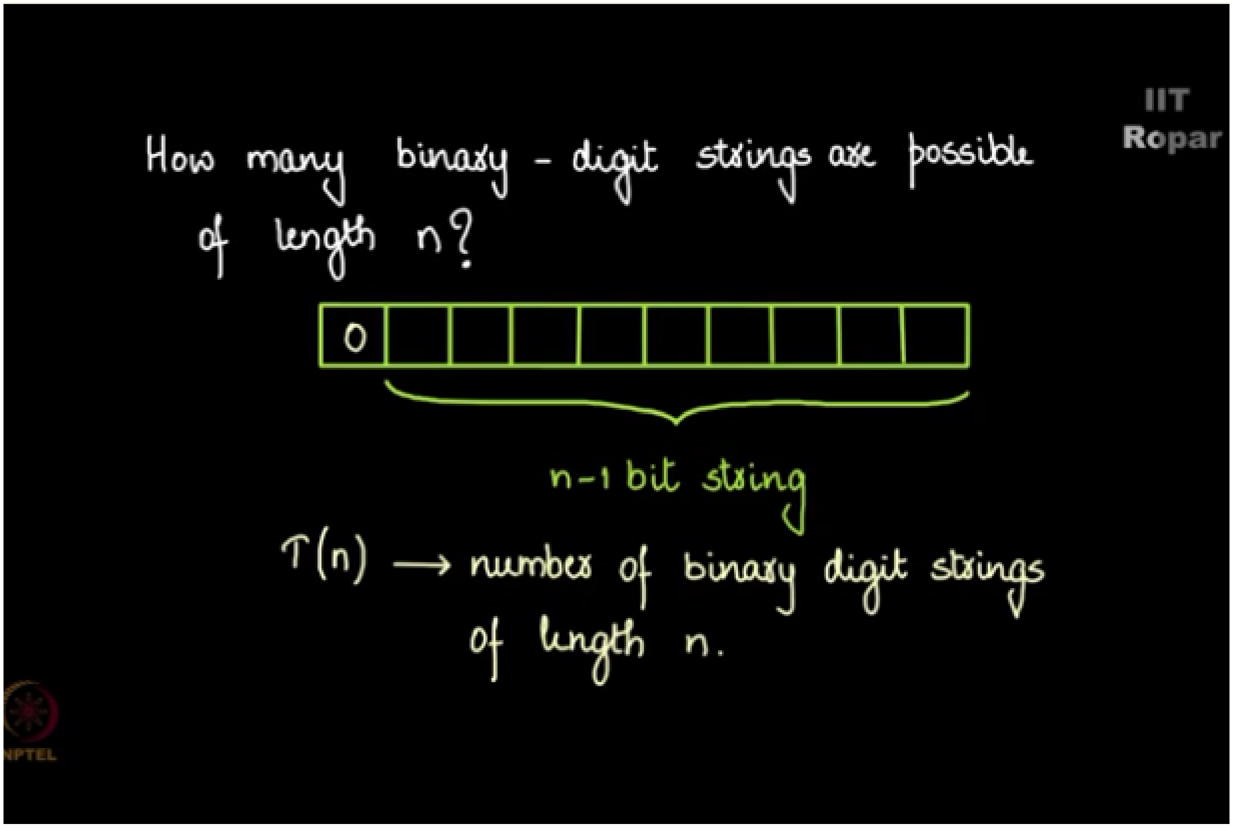

then this would become N-1 string, that is N-1 bit string, right, so in case I write a 0 the rest of it is of length N-1 and then T(n-1) would represent the number of binary digit strings of length N-1, now wait a minute let me explain things more clearly, what did we do initially? We took a string of length N then I represent it T(n) to be the number of binary digit strings of length N, now in case so if I start this with 0 the rest of it as of length N-1, so the number of ways of writing strings of length N-1, I'll represent that as T(n-1), yes this is the progress so far.

Now in case I write it as, in case I start the N bit string with 1
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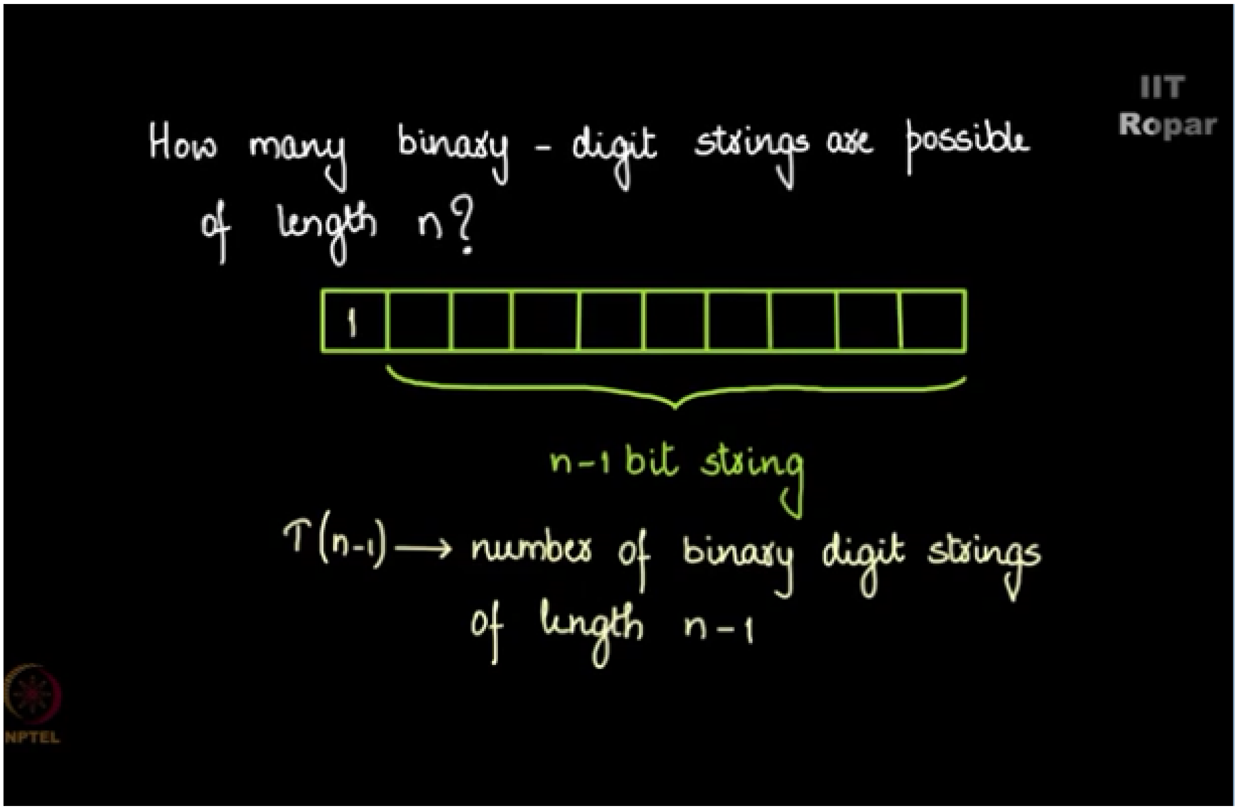

still the rest would remain as T(n-1) itself, that is the N bit string can start with either 0 or 1, that's what it means, now how did I write it as T(n-1)? Of course I can write it because this bit string of length N-1 is independent of what is happening initially, right, it's not dependent on this, and hence it can be written as T(n-1), now so I wrote the n-bit string and then I broke it down into 2 pieces with starting with 0 or 1, and the rest of it I can represent it as T(n-1).

Now writing it in terms of an expression T(n) as T(n-1), you see why I am writing it twice here, one represents the N bit string starting with 0 and the rest of them of length N-1 or the other cases N bit string starting with 1 and rest of them can be written as T(n-1),

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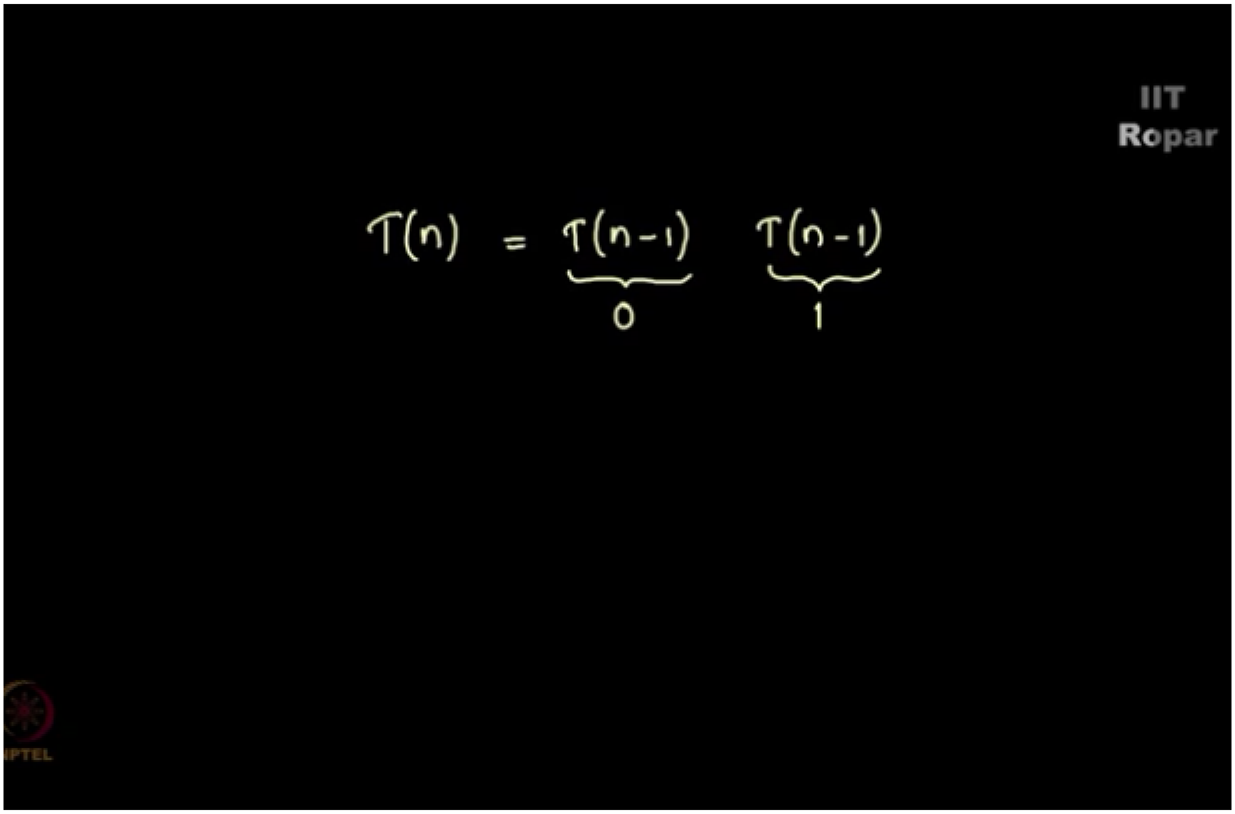

you see by rule of some there is a plus here, why? Because either this is possible or this is possible, you start the string either with 0 or with 1, one of them will hold true, right, and hence T(n) can be written as 2 times T(n-1), now this is the recurrence relation for finding the number of possibilities of n bit string.

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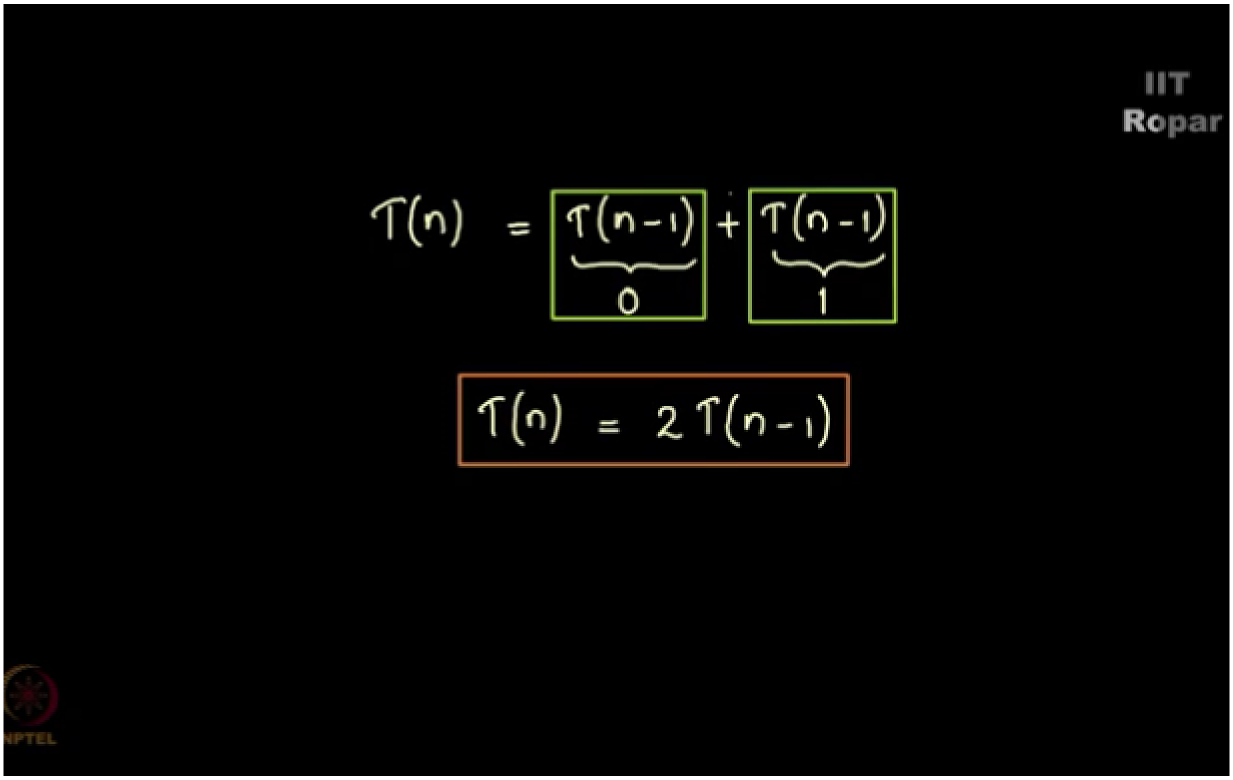

You must be remembering this we have done this even in the functions chapter as well as in the set theory chapter, you might want to revisit that and warm up your mind for it, now how do I calculate this? This is the recurrence relation, we must find a solution to this, so T(n) is 2 times T(n-1), but to calculate T(n-1) I require T(n-2), now to calculate T(n-2) I require T(n-3), for this I require T(n-4) and so on till the end which is T(1), right, T(1) would be 2 times T(0),

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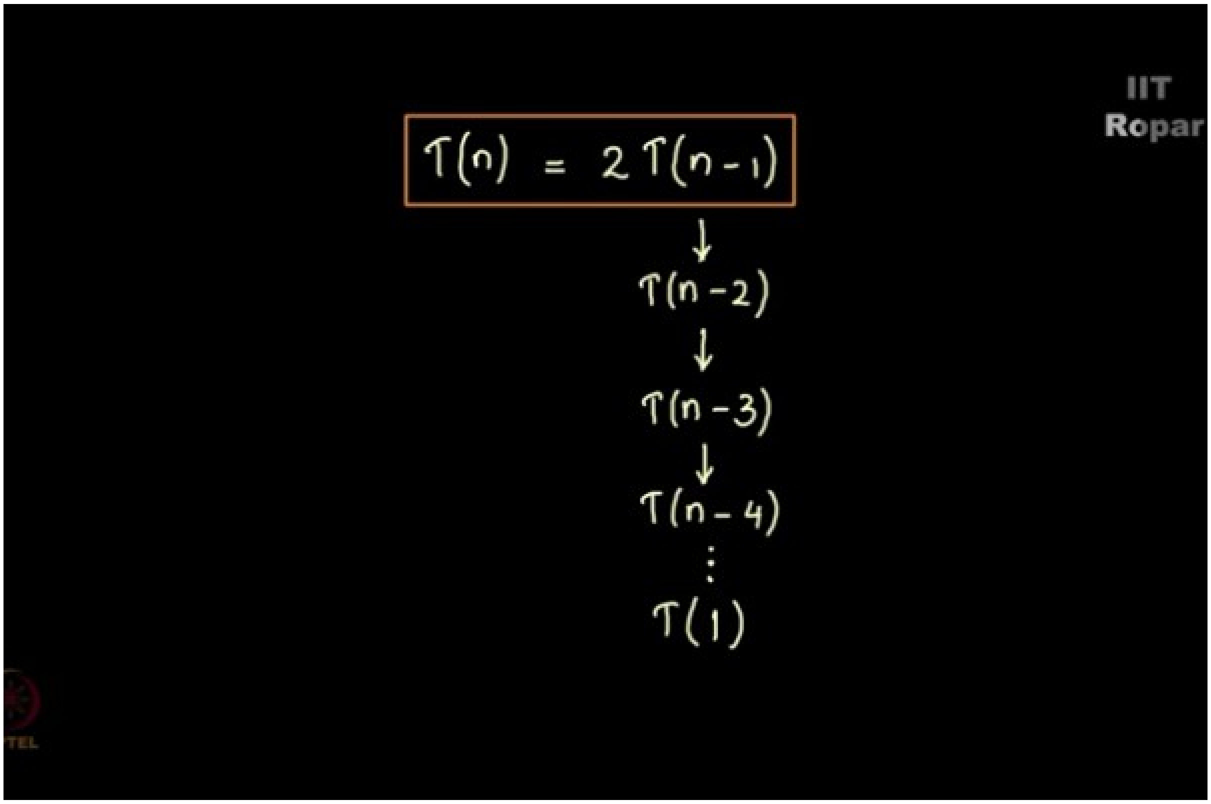

but how do we calculate T(1), till now we have broken T(n) as 2 times T(n-1), now to find T(n-1) we need T(n-2), we need T(n-3) and the process continues till T(1), but we can definitely find out T(1), how can we do that? What does T(1) mean? T(1) means the number of ways of writing 1 bit string, which means the string should be of length 1, if it is of length 1 I can put a 0 here or I can put a 1 here, which means there are only 2 ways, so T(1) happens to be 2, given this information we can see that T(1) is 2.

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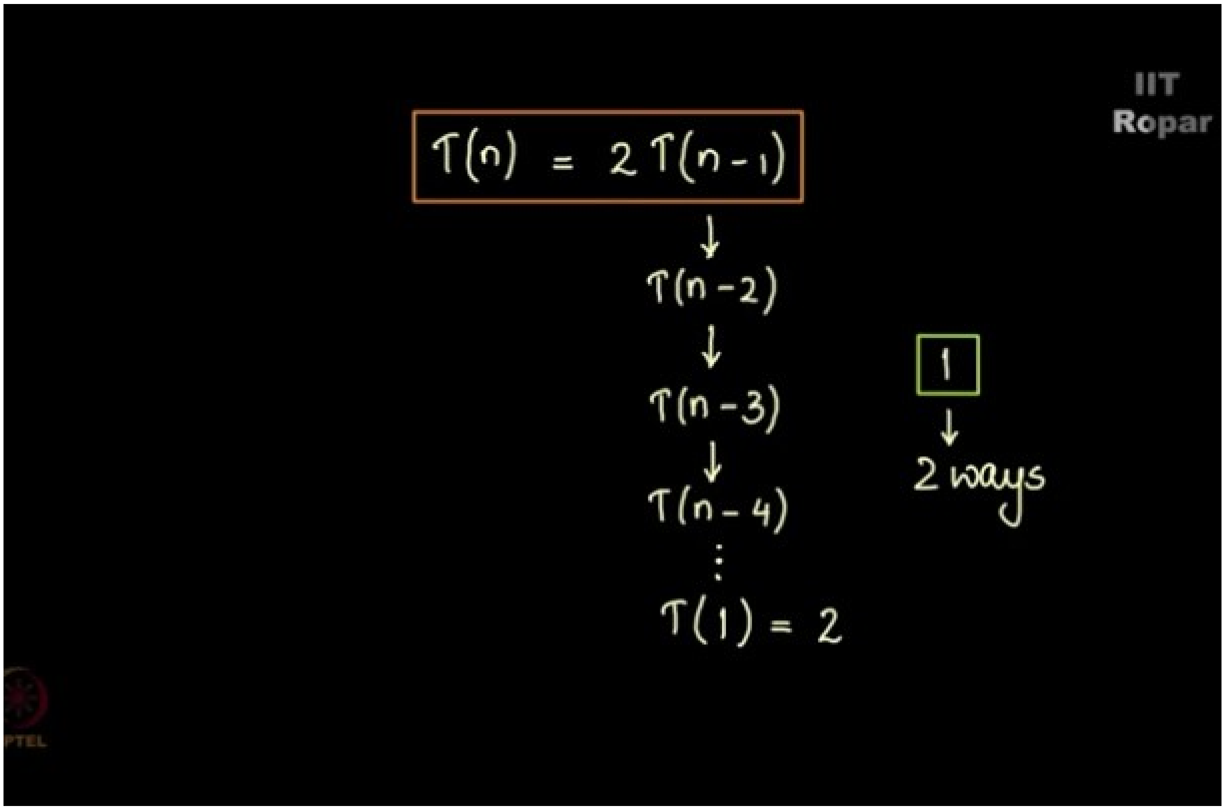

Now T(n) is 2T(n-1) we know this very well, T(n-1) can be further written as 2 times, 2 into T(n-2), how? I can just assume the logic which I have done so far, because I have proved to you that T(n) = 2 into T(n-1) is true, right, we have seen that, again you consider a bit string of length 2, string of length and -1, in the first place you either started with 0 and the rest is N-2 length, you started with 1 again you obtain the same thing that T(n-1) is 2 times T(n-2), now this can be simplified as 2 square T(n-2).

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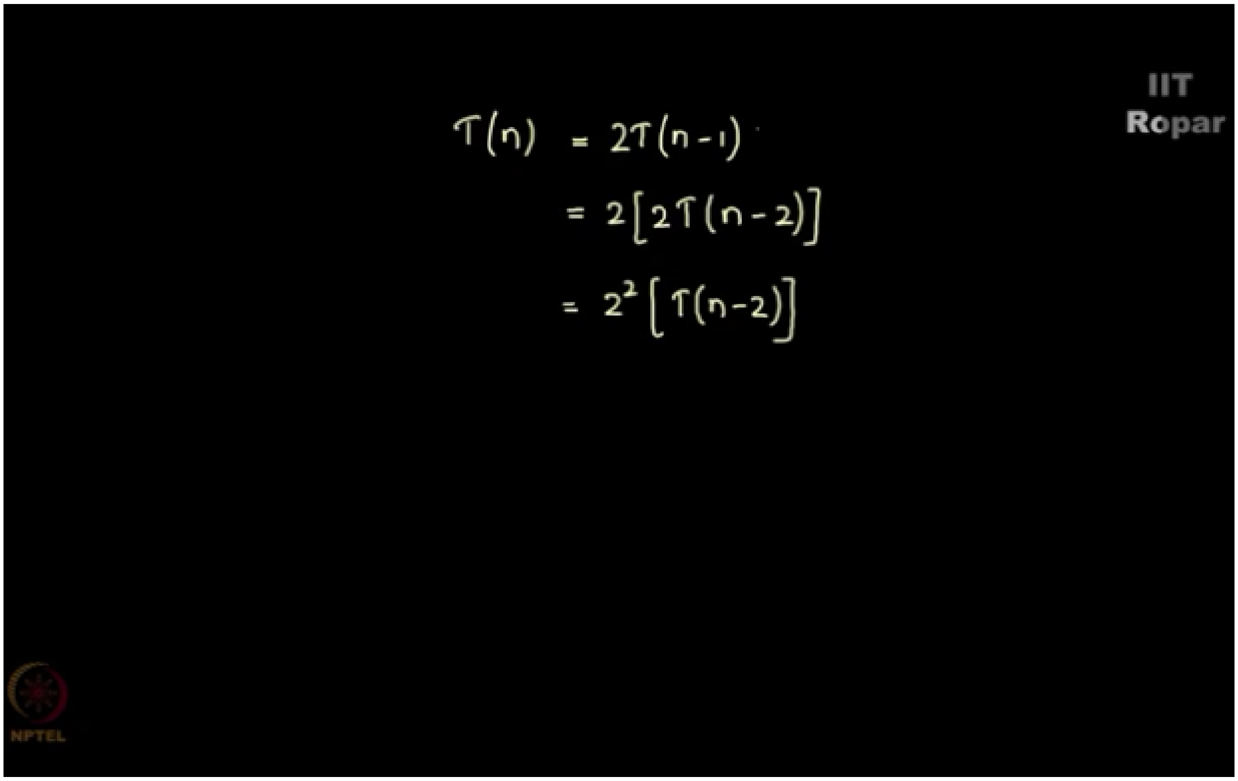

Now T(n-2) can further be written as 2 square into 2 times T(n-3), right. In a simple way this can be written as 2 cube into T(n-3) and so on, this process will continue, but where will be ended? We can write it as observe these 3 steps before we continue 2 into T(n-1), 2 square into T(n-2), 2 cube into T(n-3),

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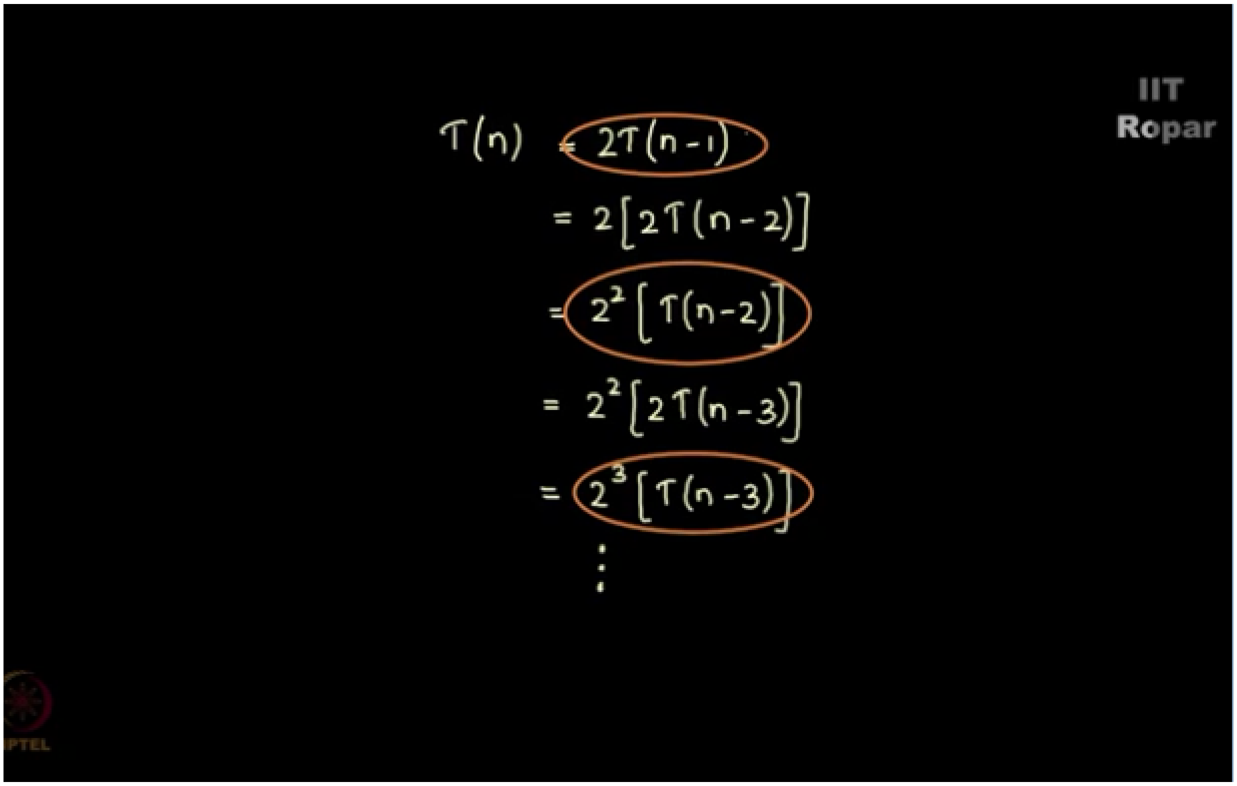

do you observe the pattern here? I had told you to observe this before we continue, there was a reason for it, when you observe this pattern you can see that T(n) can be written as 2 power K into T(n-k), right, this is the general expression.

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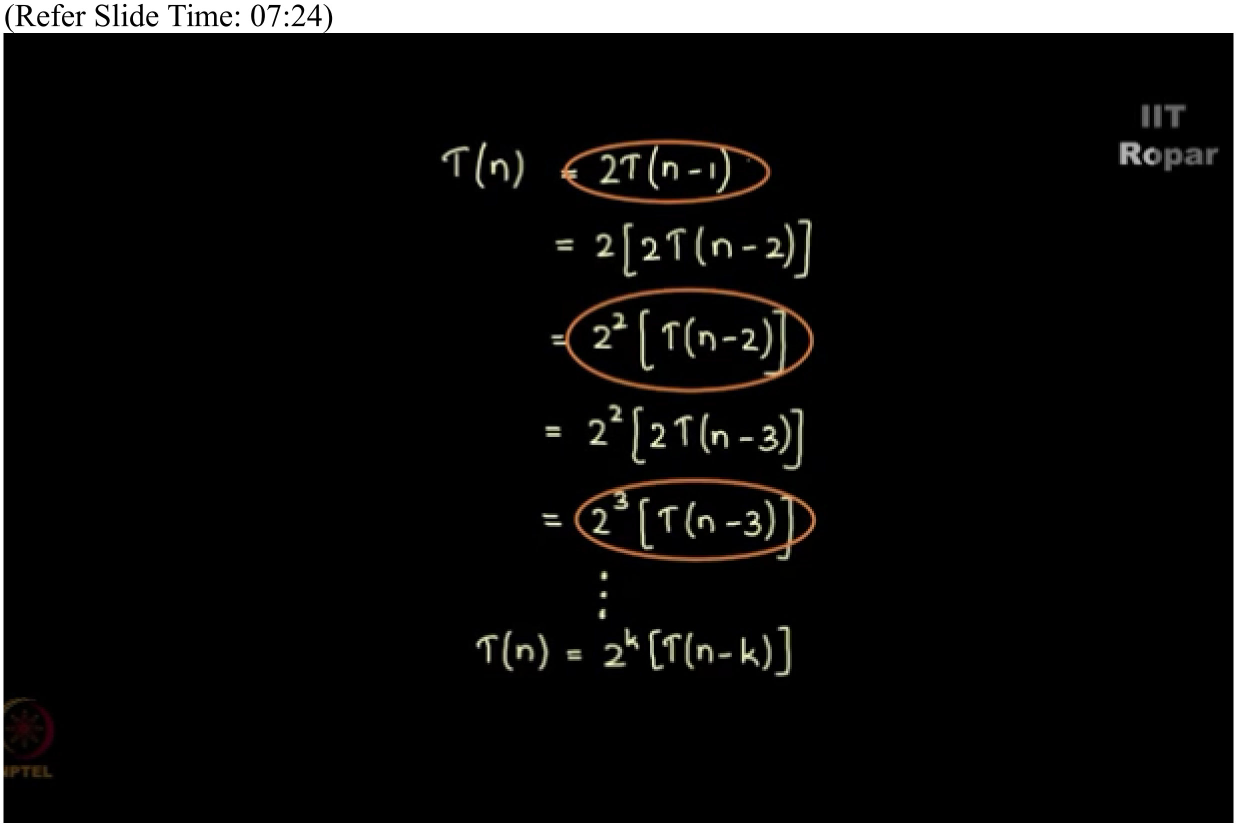

Now T(1) we know that it is 2, right, so if I substitute N-K as 1, why? In the previous step we had seen that T(n) can be written as 2 power K into T(n-k) we don't know what N-K is, we do not have the knowledge of it, but we definitely know that T(1) is 2, now if N-K happens to be 1 then K is N-1, this is simple math, so in place of K if I substitute N-1, so add this point K = N-1, T(n) happens to be 2 to the N-1, because it was 2 to the K earlier you remember, so in place of K, in place of K I have substituted N-1, so T(n) happens to be 2 to the N-1 T(1), and we know what T(1) is, therefore this is equal to T(n-1) into 2, because T(1) is 2, right.

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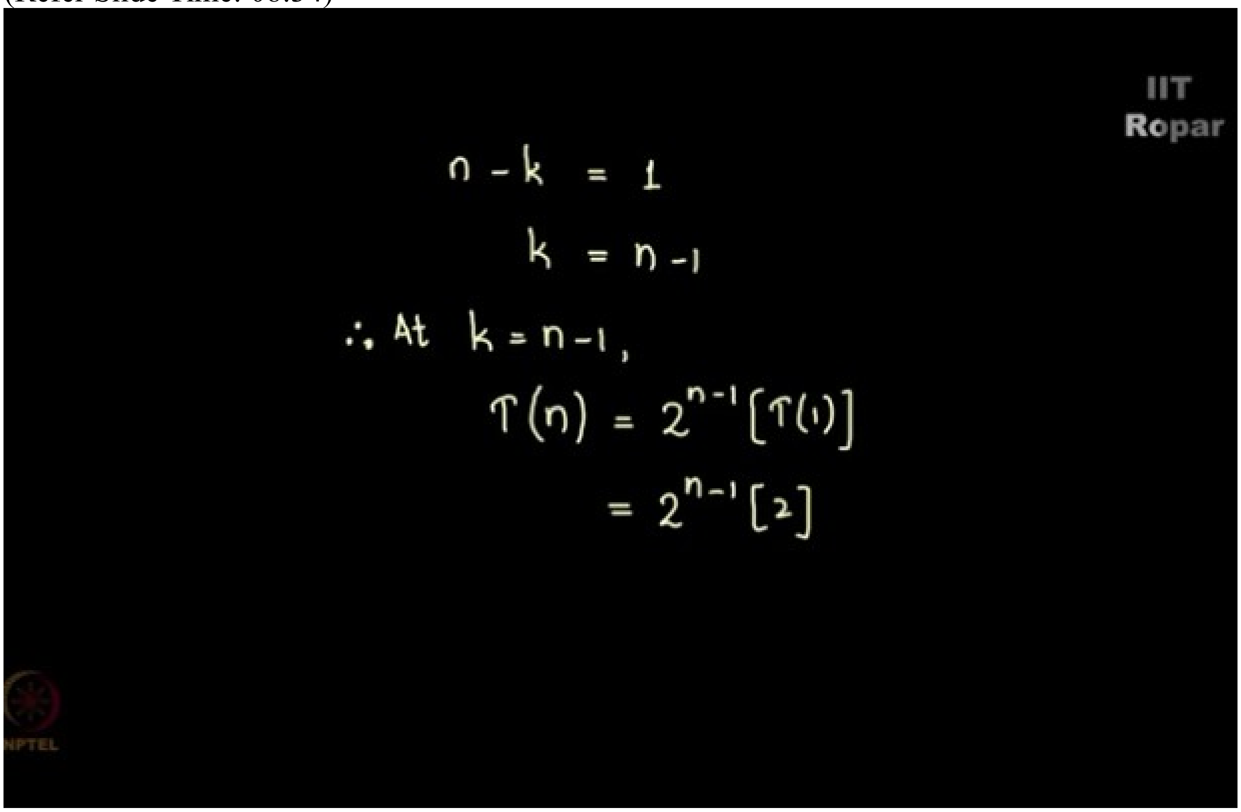

Now simplifying this we get it as T(n) is 2 to the N, now T(n) is 2 to the N, this is the final solution for the recurrence relation where we were finding out the number of ways of writing and N bit string,

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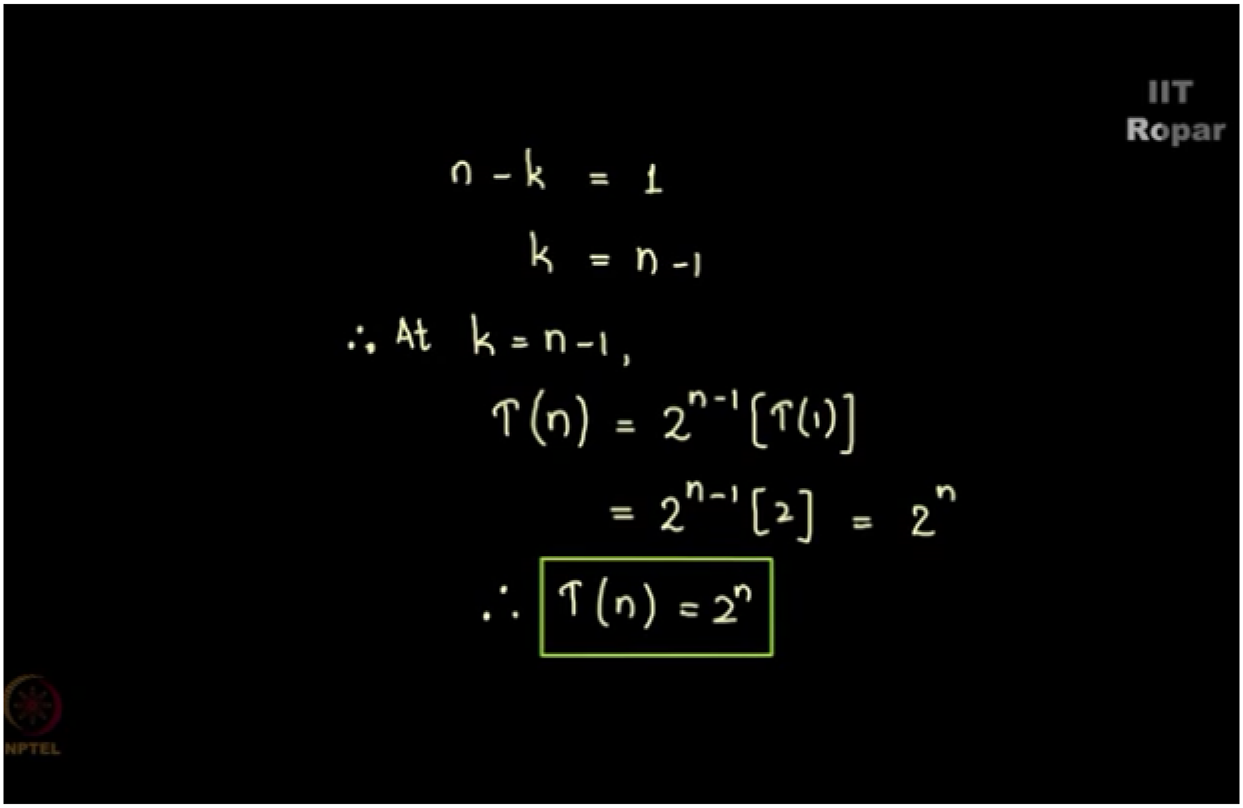

keep this in mind we will further see a variation of this problem, this was to warm up your mind.

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