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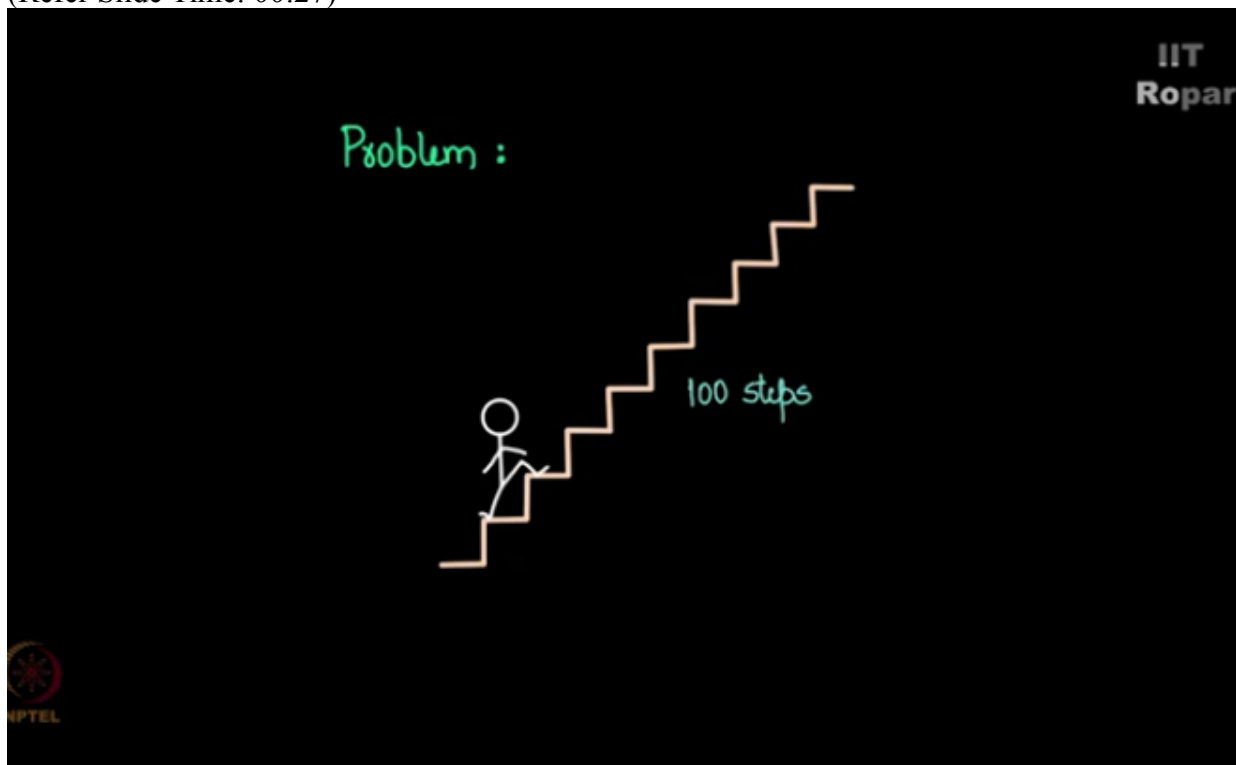
NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

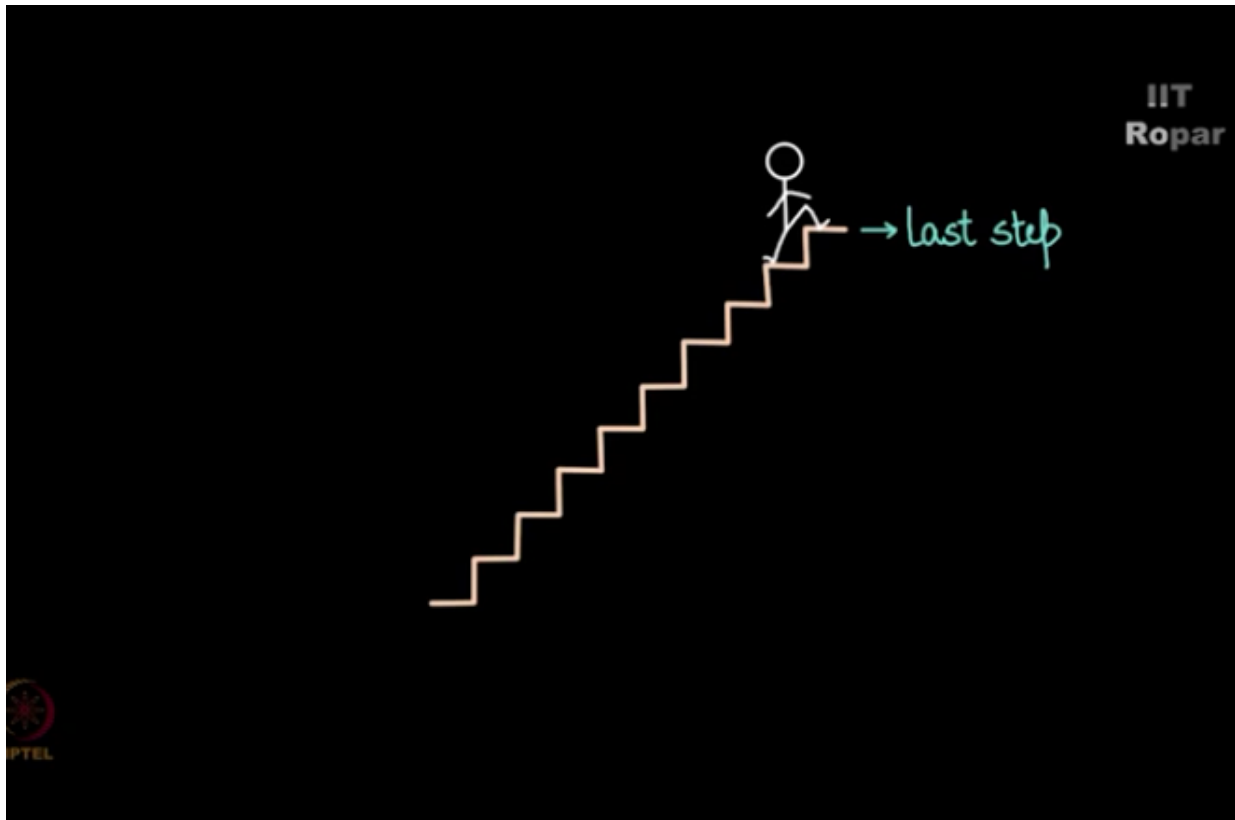
Number of ways of climbing steps:
Recurrence relation

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Here is a typical example of how sometimes to solve a problem you may have to solve the same problem with the smaller instance, there will be sub problems sitting there which when solved you will be solving the bigger problem, for example how do you climb 100 steps,
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a very simple question, forget the previous question, simply you want to climb 100 steps, what do you do? Climb 99 steps and then climb the last step,
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it sounds like a crazy strategy, but you see every, as I told you in the beginning of this chapter every giant mile, every giant journey starts from a simple first step, right, okay.

Now getting back to the problem the question was in how many ways can you go reach the n th step, let the answer be A_n , let A_n denote the total ways in which you can start from the first step and then reach the n th step by taking 1 or 2 steps. Now you see how you will end at the n th step, you are either in $n-1$ th step and you take 1 step jump or you are at $n-2$ step and you are taking a jump of 2 steps,
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In how many ways can you reach
the n^{th} step?

Answer : a_n - All possible ways to
reach the n^{th} step

$(n-1)^{\text{th}}$ step $\xrightarrow{1 \text{ step}}$ n^{th} step

$(n-2)^{\text{th}}$ step $\xrightarrow{2 \text{ steps}}$ n^{th} step



so patiently observe, think, you will realize that the answer for A_N is equal to $A_{N-1} + A_{N-2}$, why is that? The total number of ways in which you can go and reach the N^{th} step is precisely equal to the total number of ways in which you can reach A_{N-1} step and take 1 step + A_{N-2} step and take a 2 step jump,
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$$a_n = a_{n-1} + a_{n-2}$$

$$\begin{array}{l} \text{Total ways to} \\ \text{reach } n^{\text{th}} \text{ step} \end{array} = \begin{array}{l} \text{Total ways to} \\ \text{reach } (n-1)^{\text{th}} \text{ step} \end{array} + \begin{array}{l} \text{Total ways to} \\ \text{reach } (n-2)^{\text{th}} \text{ step} \end{array}$$



these are mutually exclusive events, that are complicated way of saying in math that 2 things a problem can be divided into 2 cases, and then you can call the answer to be case 1 + case 2, the case 1 here is you go up to A_{n-1} and take one step, case 2 is you go up to A_{n-2} th step and take or 2 step link, so $A_n = A_{n-1} + A_{n-2}$, so if you want to compute A_{10} it will be $A_9 + A_8$, but you know A_9 recursively speaking is $A_8 + A_7$, and then your A_8 is $A_7 + A_6$, you can keep breaking this smaller and smaller and you will have all terms A_1 's and you know what is A_1 , A_1 is 1, so adding up you will get the answer for A_{10} .

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$$a_n = a_{n-1} + a_{n-2}$$

$$\text{Total ways to reach } n^{\text{th}} \text{ step} = \boxed{\text{Total ways to reach } (n-1)^{\text{th}} \text{ step}} + \boxed{\text{Total ways to reach } (n-2)^{\text{th}} \text{ step}}$$

$$\begin{aligned} a_{10} &= a_9 + a_8 \\ &= (a_8 + a_7) + (a_7 + a_6) \end{aligned}$$

$$a_1 = 1$$



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