NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Derangements

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So how do we model this problem, the question is write down simply 1, 2, 3, 4 and 5, and below one 1 should be there, of course 2, 3, 4, 5 can be there 1 should not be there, right, this is the equivalent problem that we discussed just now, below 2, 2 shouldn't be there, of course 1, 3, 4, 5 can be there below 3, 3 shouldn't be there, below 4, 4 shouldn't be there, below 5, 5, shouldn't be there, a valid permutation would be 2, 1, 4, 5, and 3 as you can see no number is below itself, when you write 1, 2, 3, 4, 5 and this permutation of 1, 2, 3, 4, 5 namely 2, 1, 4, 5, 3,

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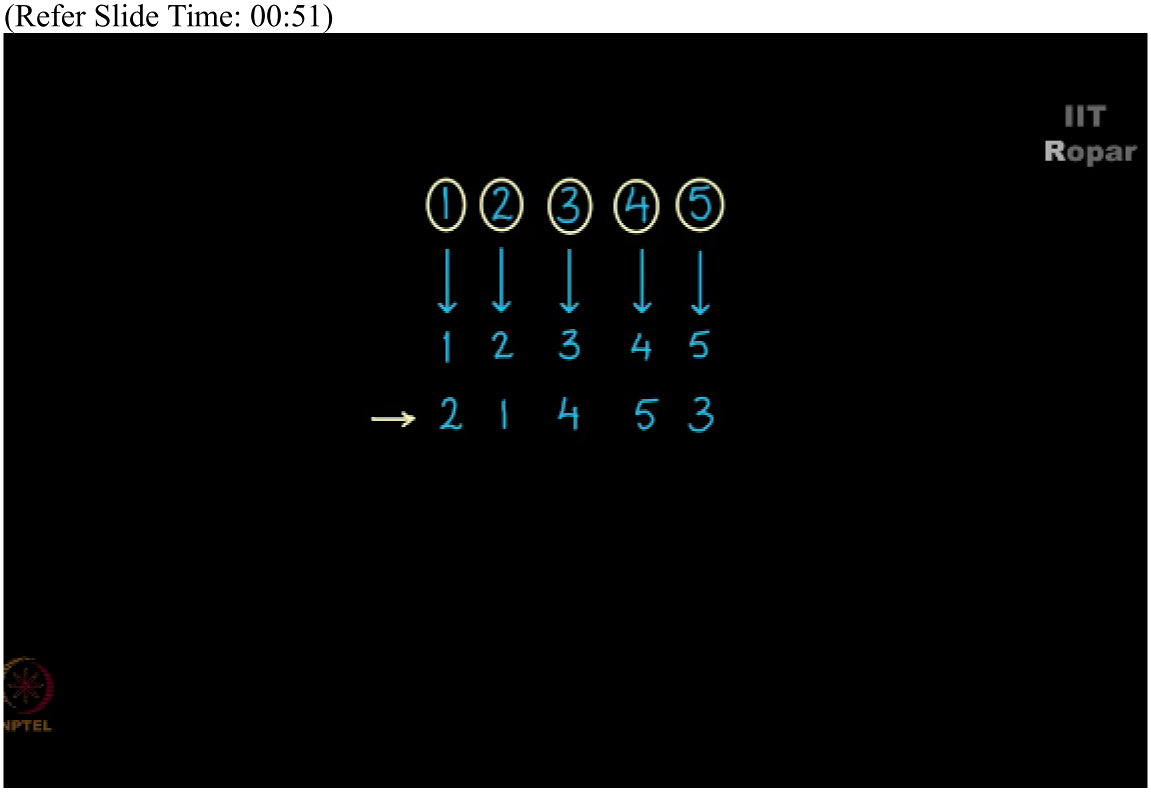

2 is not in the second position, 1 is not in the first position, 4 is not in the fourth position, it's in 3rd position, 5 is in fourth position and hence not in fifth position, and 3 is in the last fifth position.

And the question we ask is in how many ways can all these numbers not even 1 is in the same place, in its own place,
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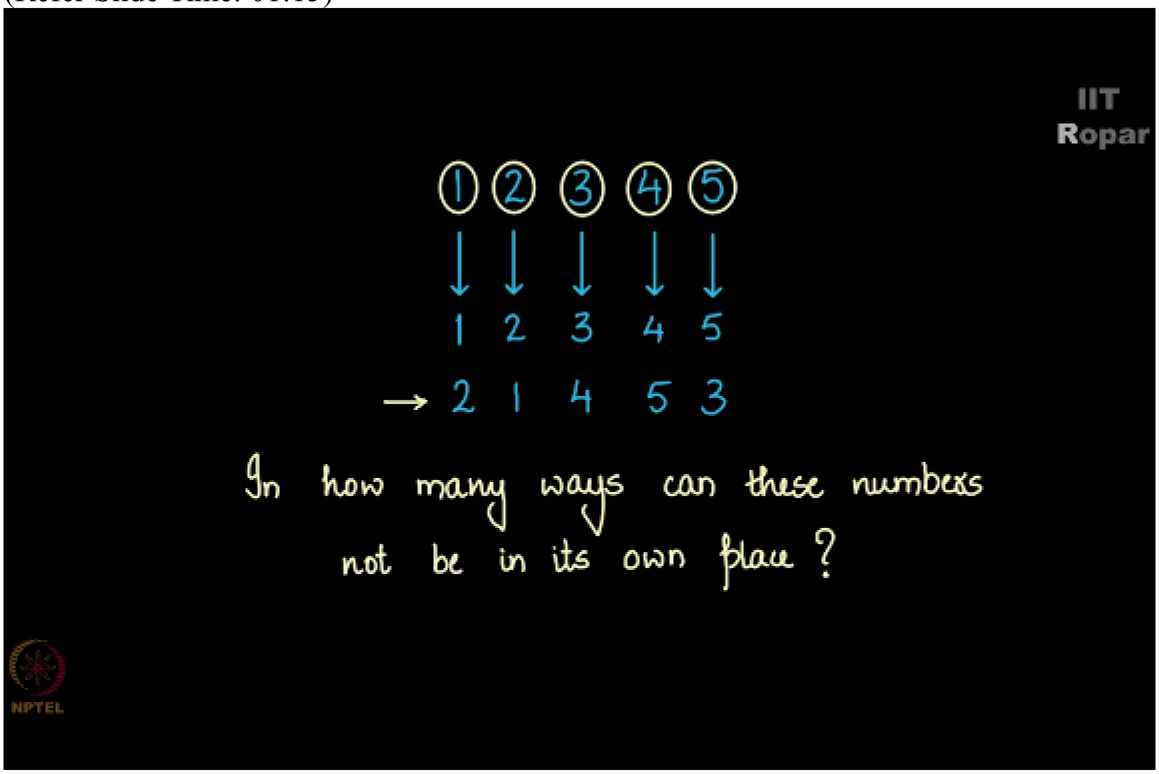

okay let's count this, this sounds a little not so straightforward let us go very slowly, we define C1 as 1 is not, 1 is in the first place, C1 conditions C1 stands for 1 is in the first place, C2 stands for, the number 2 is in the second place, C3 is number 3 is in the third place, C4 4 is in the fourth place, C5 is 5 is in the fifth place,
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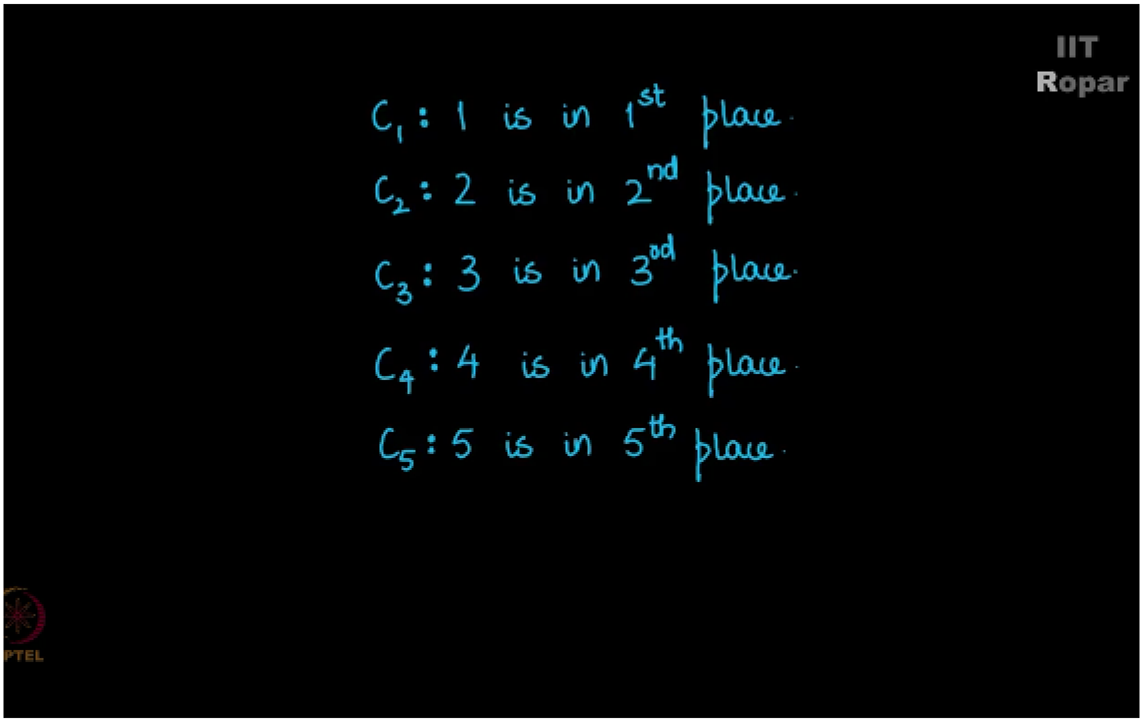

think for a minute all that we ask right now is N bar here, N bar means whatever is outside this C1, C2, C3, C4, C5 possibilities and that is when if it's not in C1 you say 1 is not in the first place, similarly if it's not in C2 you say 2 is not in the second place and so on, 5 is not in the fifth place, correct, and N bar will actually give you the answer.

Let us try to find out what is N bar here?
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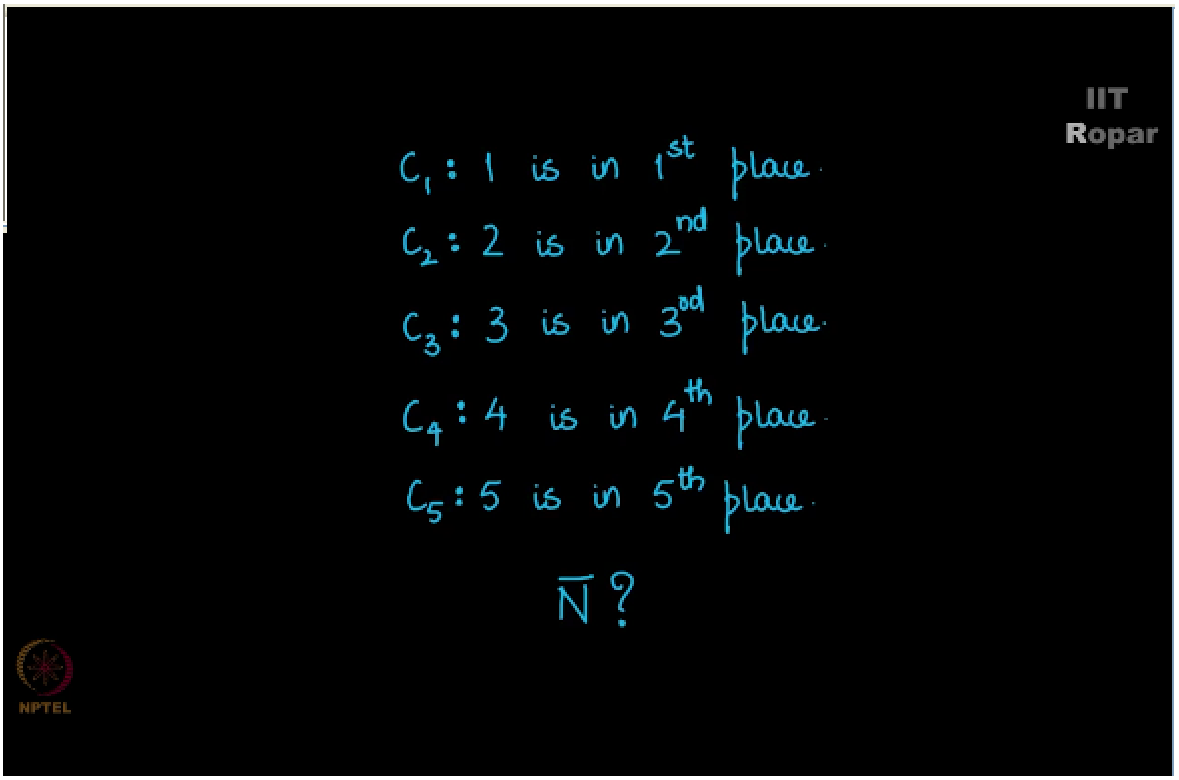

Okay, to begin with let me count what is N(C1), N(C2), N(C3), N(C4), and N(C5) all of them we see will be the same, correct, total number of ways in which 1 is not in the first place will actually be equal to the total number of ways in which 2 is not in the second place, why is that? This is a very symmetric argument you see, it's very easy to observe that, we will not explain that part if you realize we will assume that you understand how N (C1) = N(C2) = N(C3) = up to N(C5), N(C1) 1 is in the first place,

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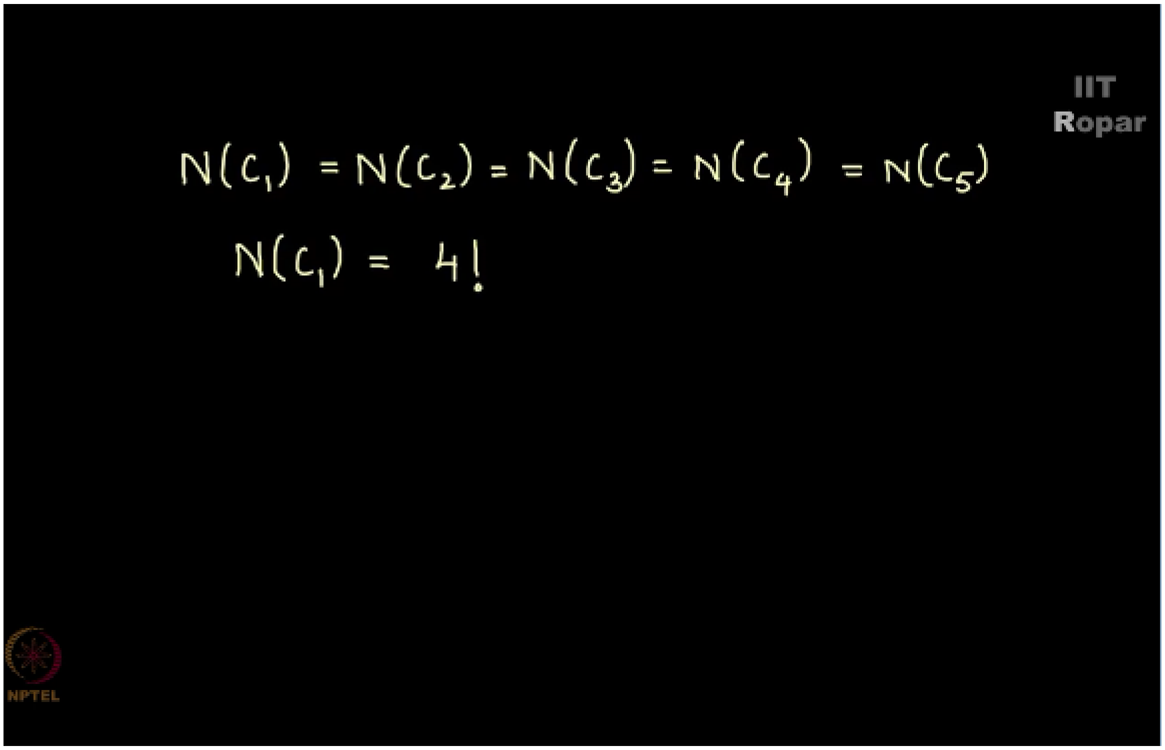

and the other 4 numbers can appear in 4 factorial ways, when I say 1 is the first place I do not, in C1 I do not care where 2 is, 3 is, 4 is, 5 is, it might be in their respective places or not, all I care is 1 is in the first place, so N(C1) happens to be then 4 factorial, so is every other N(CI), for I = 1 to 5.

Now can you think and tell me what is N(C1,C2)? Which means total number of ways in which you can get 1 in the first place and 2 in the second place,
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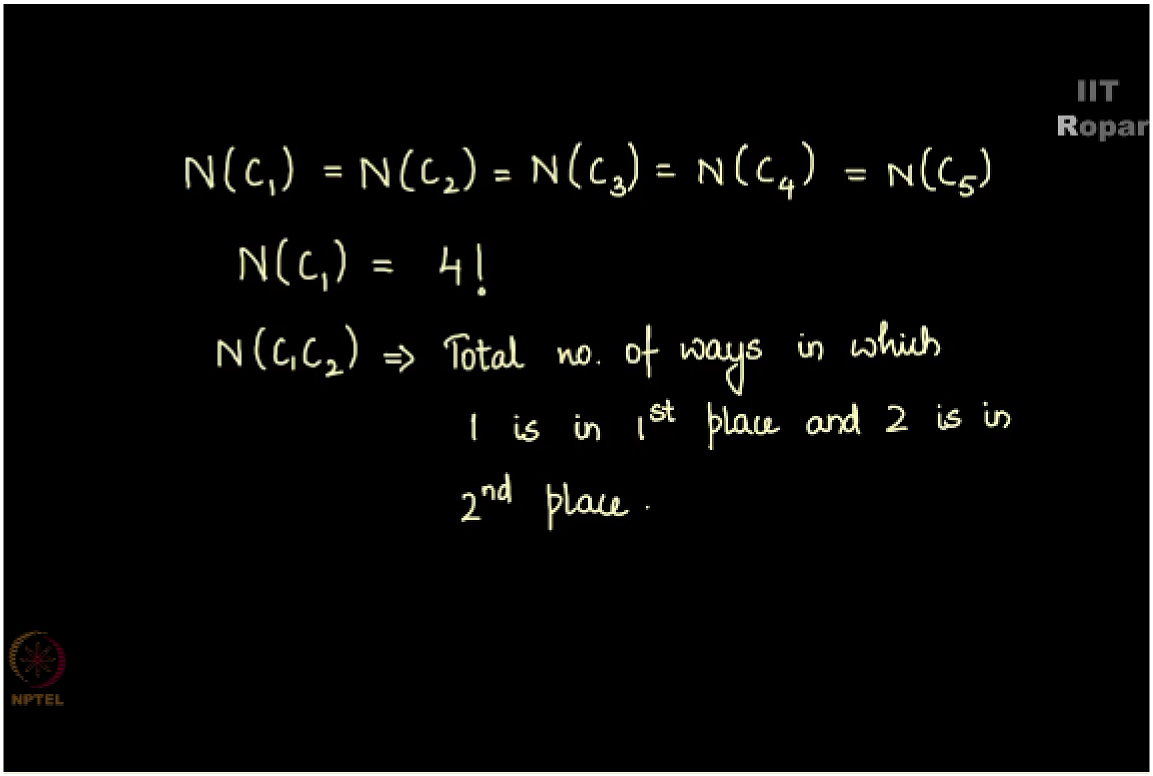

which means you put 1 below 1, 2 below 2, and 3, 4, 5 can appear in 3 factorial ways, so N(C1,C2) is 3 factorial, so is N(C1,C3) (C2,C3) (C4,C5) (C3,C5) and all possible 5 choose 2 combinations all of them will be equal to 3 factorial, N(C1,C2, C3) will then be 2 factorial, N(C1,C2, C3, C4) think will be 1 factorial just for completeness sake I am writing 1 factorial here we all know that is simply 1, N(C1,C2, C3, C4,C5) is 0 factorial, again for convenience and completeness sake we will write it as 0 factorial.

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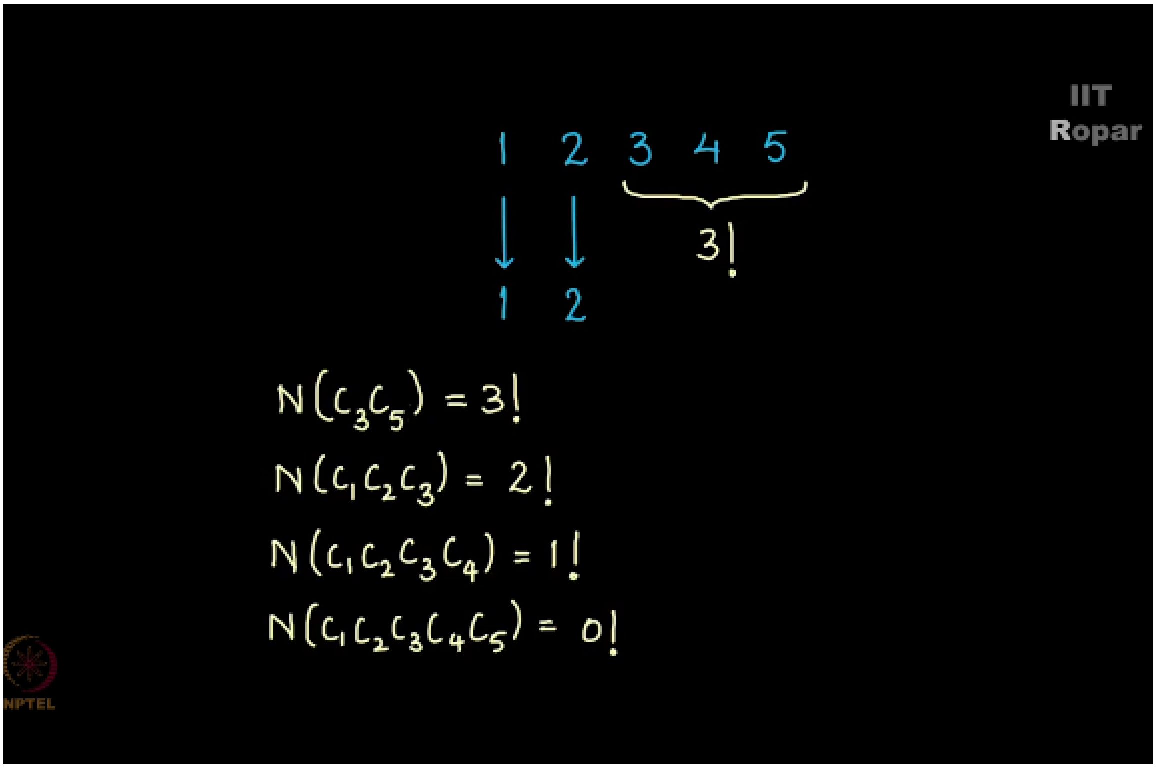

What is N bar now? N bar = N which is all possible ways in which you can write 5 numbers which is 5 factorial - 4 factorial 5 times, N(C1) N(C2) up to N(C5), so I'll write this as 5 for some convenience, for my convenience I'll write this as 5 choose 1, by factorial – 5 choose 1 times 4 factorial and then alternating signs that's the formula for your principle of inclusion and exclusion which we explained and gave a proof as well, + 5 choose 2 times 3 factorial you see why, correct, -5 choose 3 times 2 factorial + 5 choose 4 times 1 factorial - 5 choose 5 times 0 factorial.

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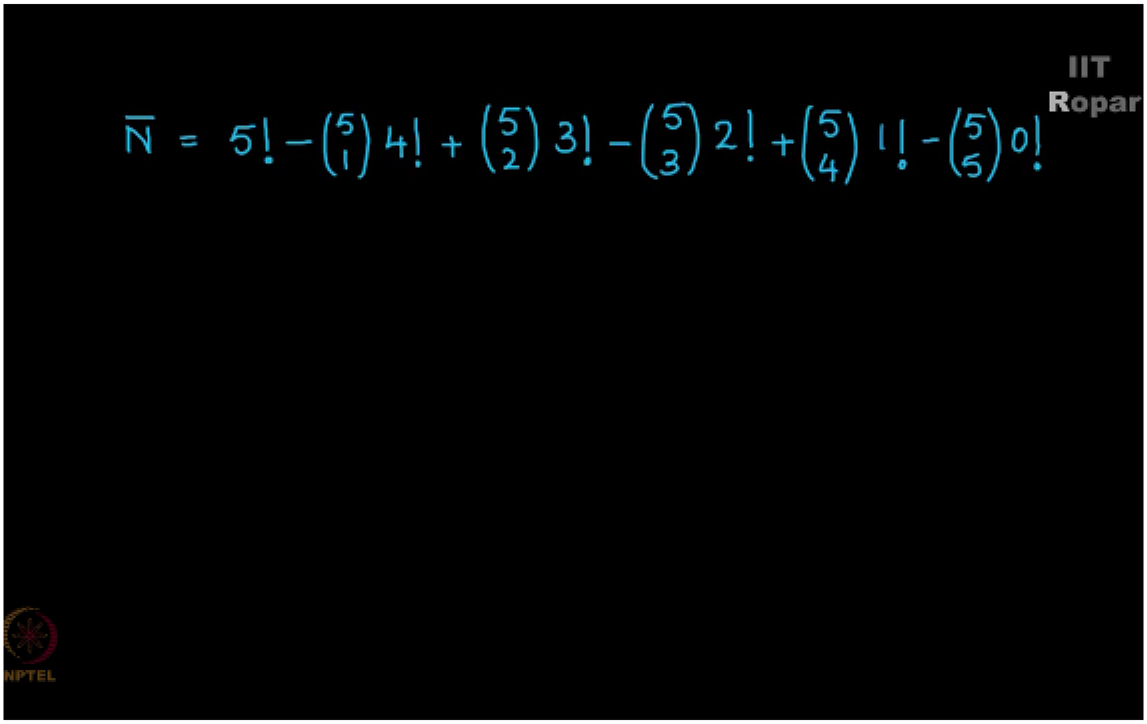

Now let me write this down as 5 factorial - let me expand 5 choose 1, which becomes 5 factorial divided by 1 factorial x 4 factorial times 4 factorial, correct, this 4 factorial comes and sits here + 5 factorial divided by 2 factorial x 3 factorial times 3 factorial minus so on and so forth, I'll expand and write everything you will observe 4 factorial 4 factorial gets cancelled, 3 factorial 3 factorial gets cancelled, 2 factorial 2 factorial gets cancelled and so on,

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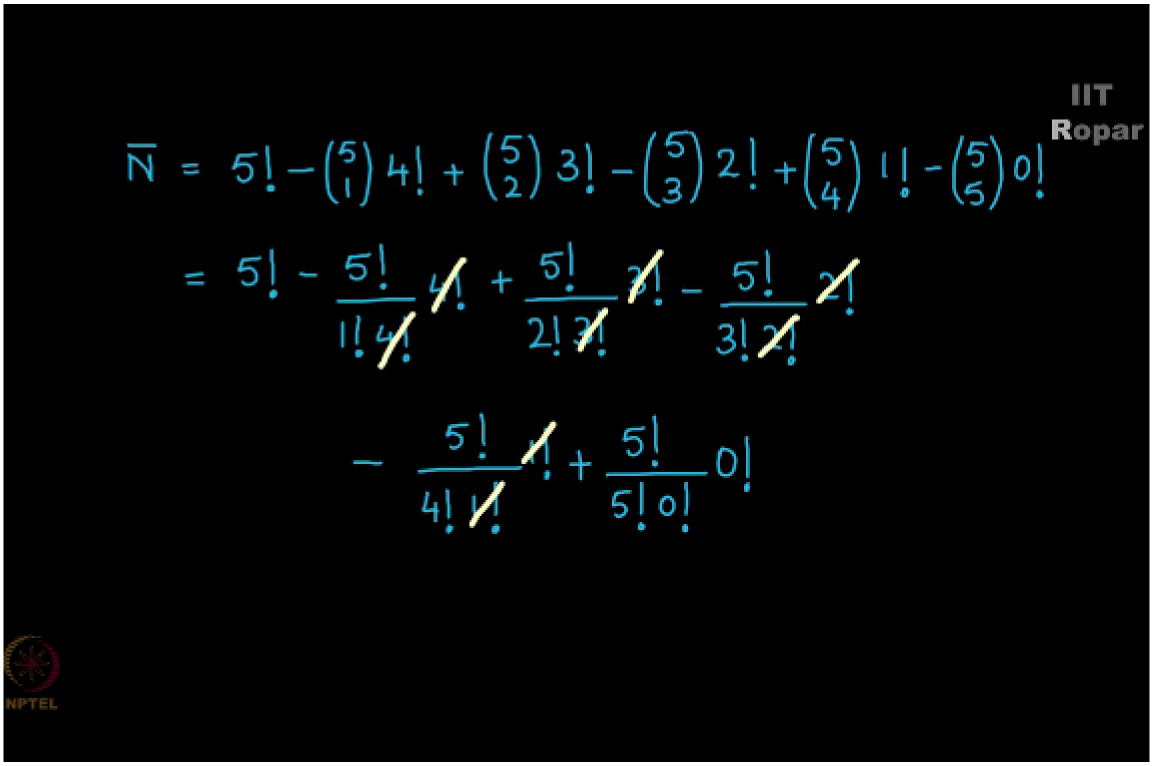

ultimately what you will get is 5 factorial I'll take it outside and it will be 5 factorial times 1 - 1 over 1 factorial + 1 over 2 factorial - 1 over 3 factorial + 1 over 4 factorial - 1 over 5 factorial right, this number if you go on up to infinity is 1/E,

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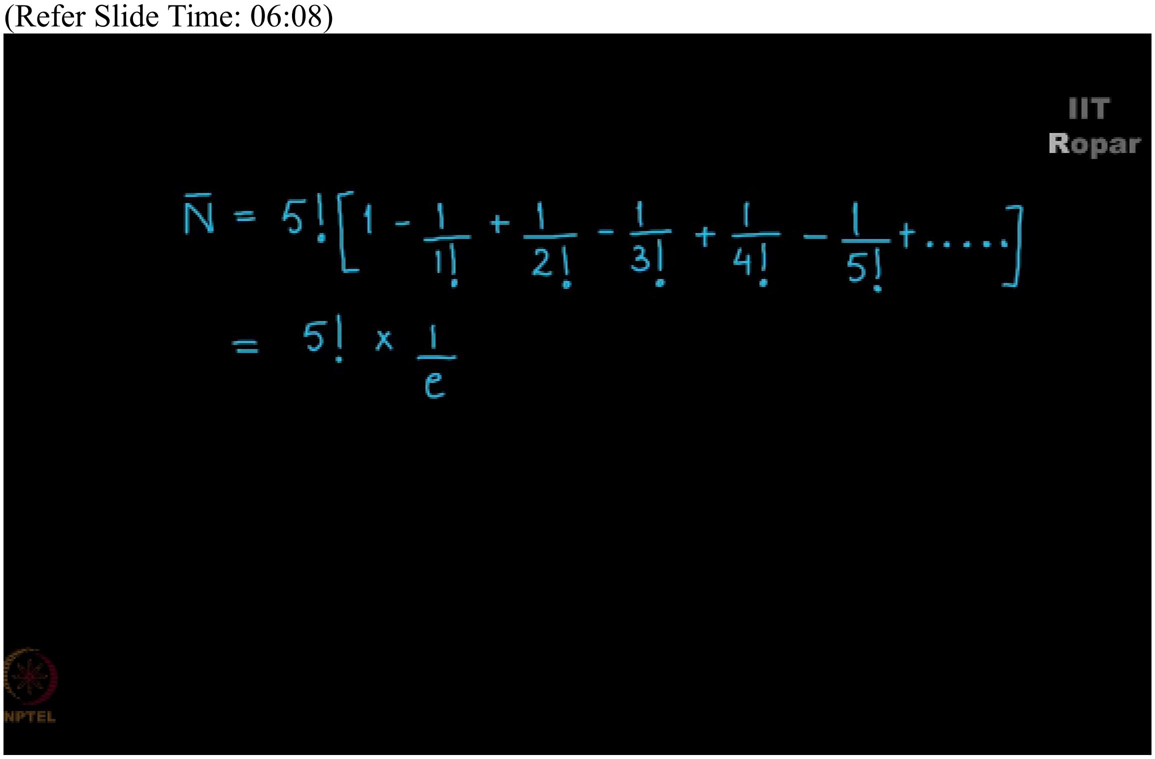

do you know this? If not look it up it's a very interesting number, number is actually 1/E this infinite series is 1/E we are stopping it at 1/5 factorial and the rest of the numbers in fact ahead are actually smaller numbers which again comes very close to 1/E, so I can replace this by 1/E when number is sufficiently large, you see I did this for 5 you can in fact do it for 10 also, 100 as well, or 200 as well, for N sufficiently large this will simply be N factorial into 1/E, N factorial into 1/E,

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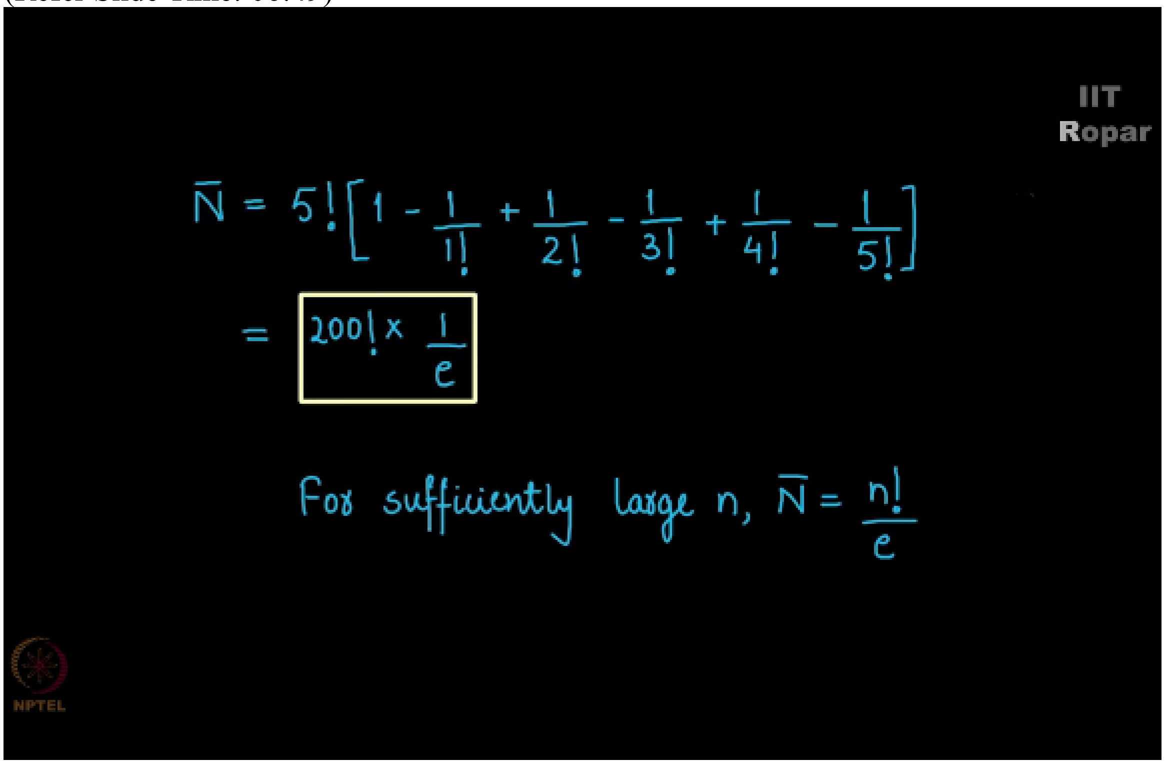

which means 1/E you see is roughly 1 over 2, E is 2.71 right, so which is just more than 2 which means 1 over E is just less than 1/2, correct, so 5 factorial times 1 over E is the amount of permutations where nothing is in the right place that is called a derangement.

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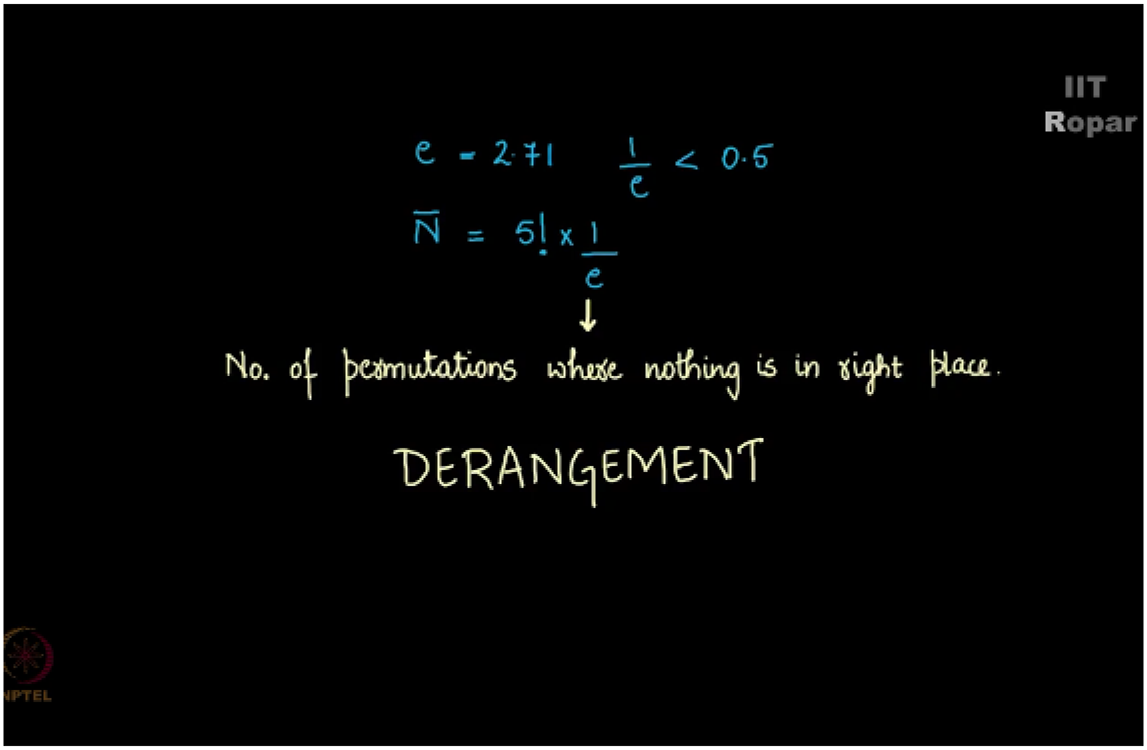

So roughly little less than 1/2 the permutations will have this property, in fact which means the moral of the story is good number of permutations less than 1/2 though, but still it's a good number, it doesn't feel like it but a good number of permutations will satisfy a derangement condition which means nothing is in the right place, right, so go through the entire thing once more you will realize what we are saying this keeps appearing in many situations, it is good to make note of it and that is why you study this topic as part of your discrete math curriculam.

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