NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Number of Onto Functions

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Let us now revisit a question that we had to discuss back in the chapter of functions, consider all possible functions from 4 elements to 3 elements, how many possible functions can you think of? We discussed this, it is 3 to the power of 4, all possible functions, right.
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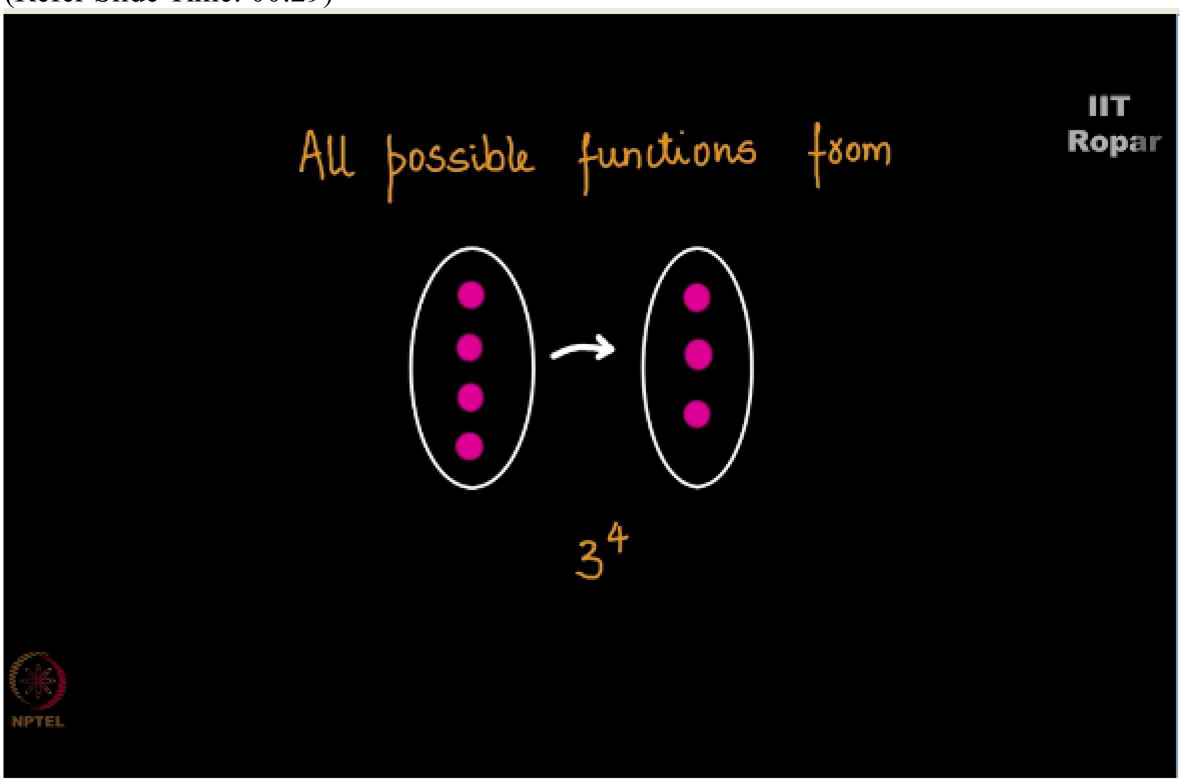

So now count how many of them are onto?
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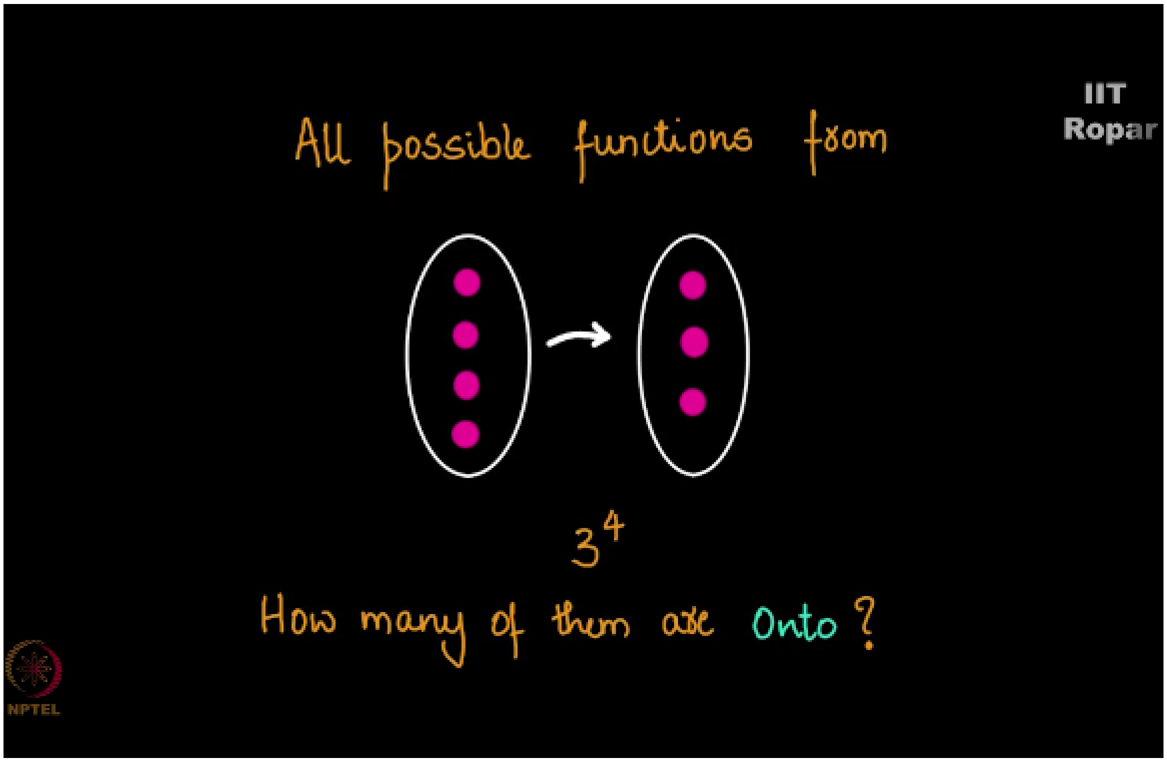

By onto we mean all those functions where no element in the codomain is left out, right, it actually sounds complicated to count this, now look at our principle of inclusion and exclusion coming to a rescue, the best part about this concept is that the formula is so magically applicable that you need not worry about its functionality, simply observe how it can be applied and leave the rest to the formula to give you the answer, you will observe it in action right now look at this.

Now let me consider as and always an application of a principle of inclusion and exclusion depends on how you identify C1, C2, C3?
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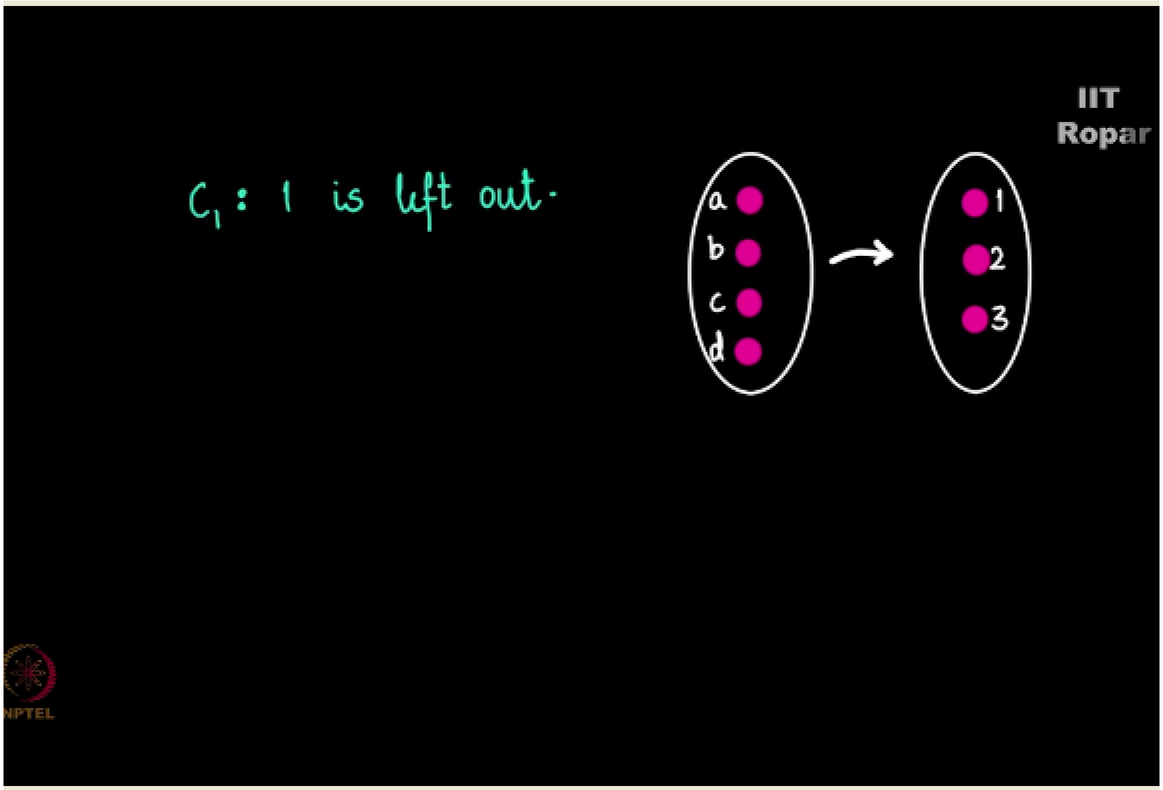

C1 will be simply the first element is left out number 1, number 2, number 3 is how we will call the elements of codomain, and the 4 elements in the domain is simply let say A, B, C, D.

Now C1 stands for 1 is left out, as and always we don't say anything about 2 and 3, okay, 1 is left out, what do you think is N(C1) then? N(C1) simply is the function never maps anything from the domain to 1, which means it can map to 2 or 3, which means there are,
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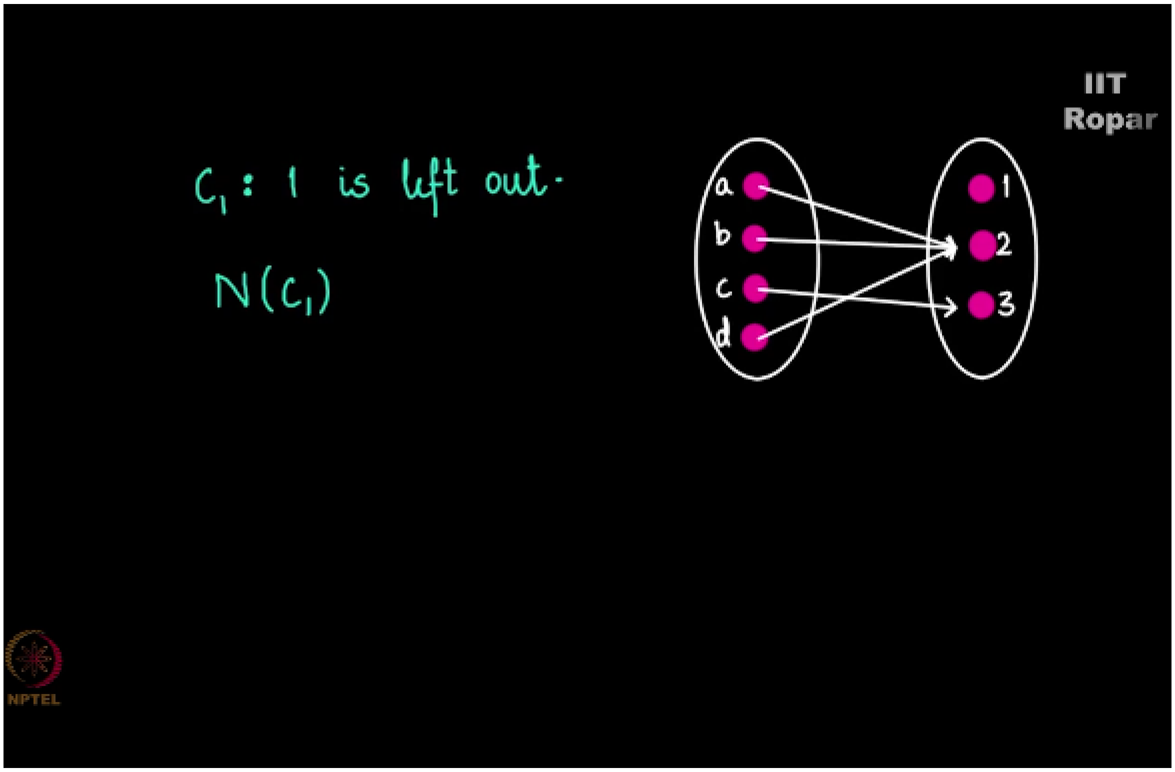

then now N(C1) is same as asking the question in how many ways can you think of a function, how many functions are there from 4 elements to 2 elements, 2 and 3, the answer we know is 2 to the 4,

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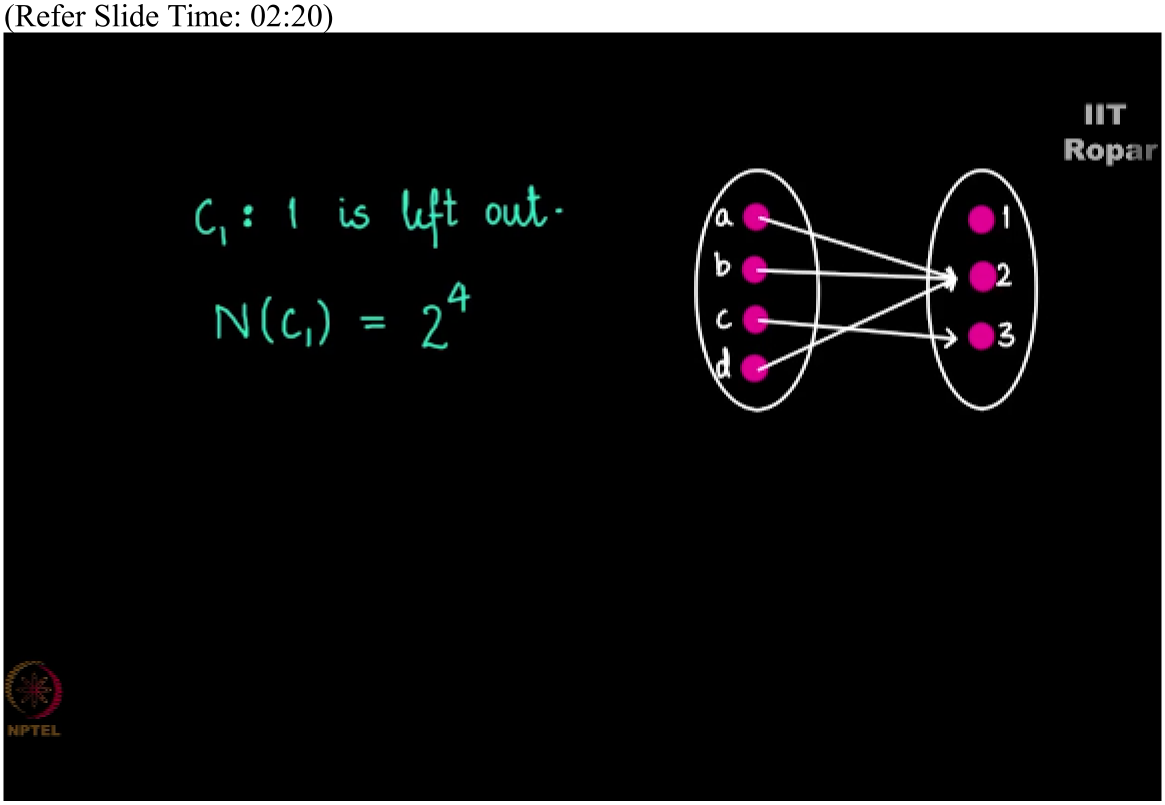

this is the same answer for N(C2) and N(C3) by symmetry, think about it, right, the total number of ways in which 2 is not mapped is again 2 to the 4, total number of ways in which 3 is not mapped, N(C3) is again 2 to the 4.
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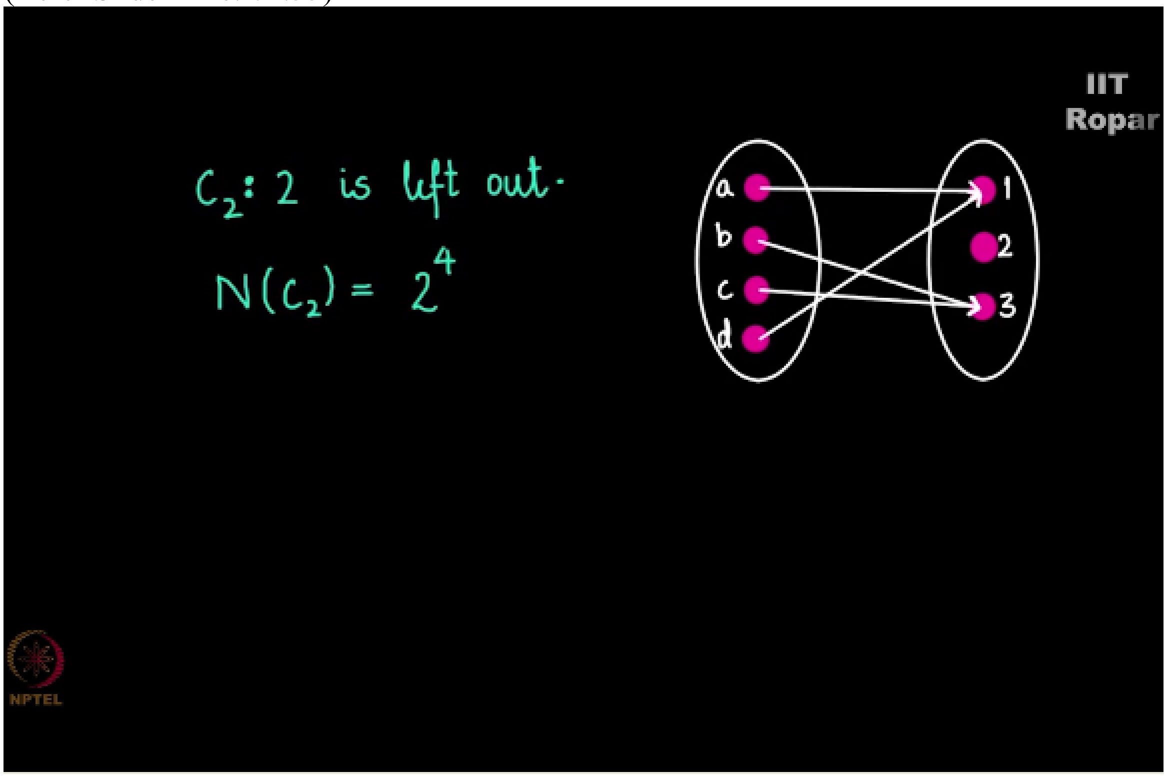

What is N(C1,C2)? In how many ways can you not map 1, 2 at all, there is only one way, what is that? All 4 elements of the domain mapping to 3 alone,
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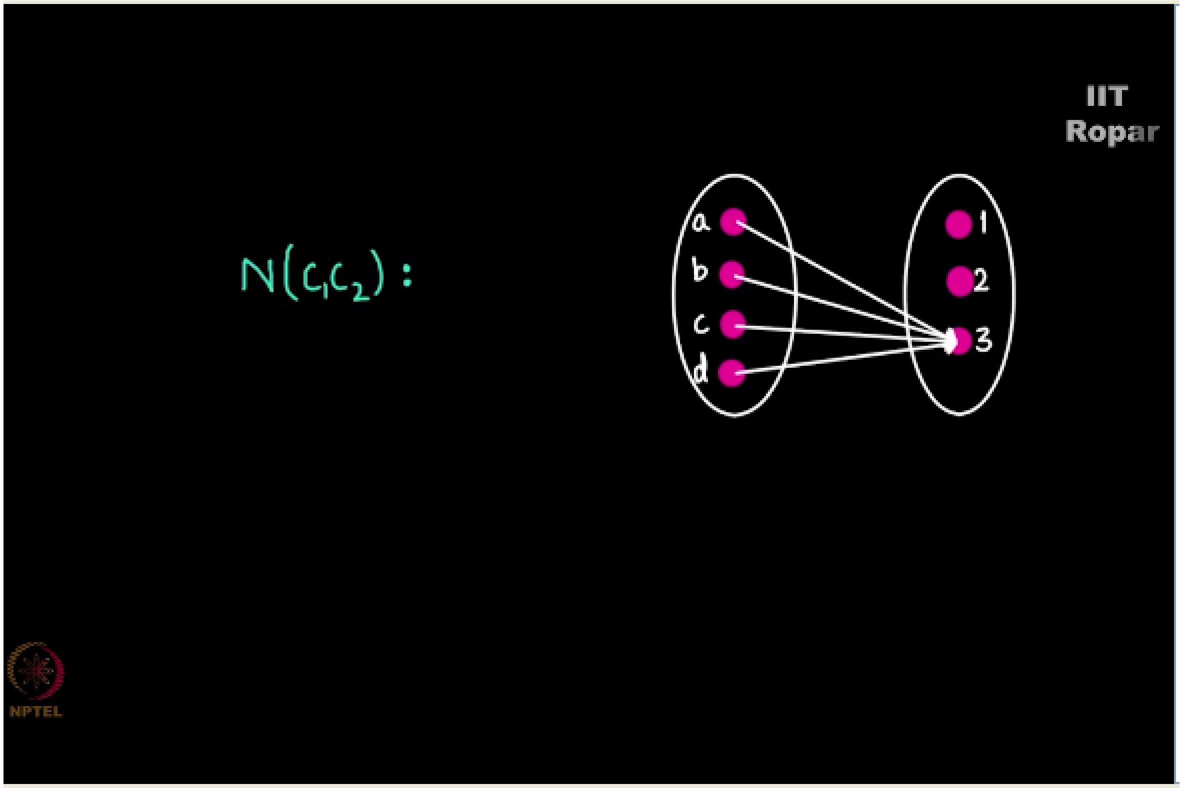

correct, so N(C1,C2) is simply 1, had to be precise let's write 1 to the 4.

Finally let me go ahead and boldly write down what is N bar, because you all know how to use principle of inclusion and exclusion, so N bar will be equal to N which is 3 to the 4, you see why? N has 3 choose 1 times 2 to the 4 + 3 choose 2 times 1 to the 4 – of course is 0 there is no way in which all 1, 2, 3 are not mapped so it's a 0 at the end, and this answer as you can compute is this will be 81 – 48 + 3 which is 36,
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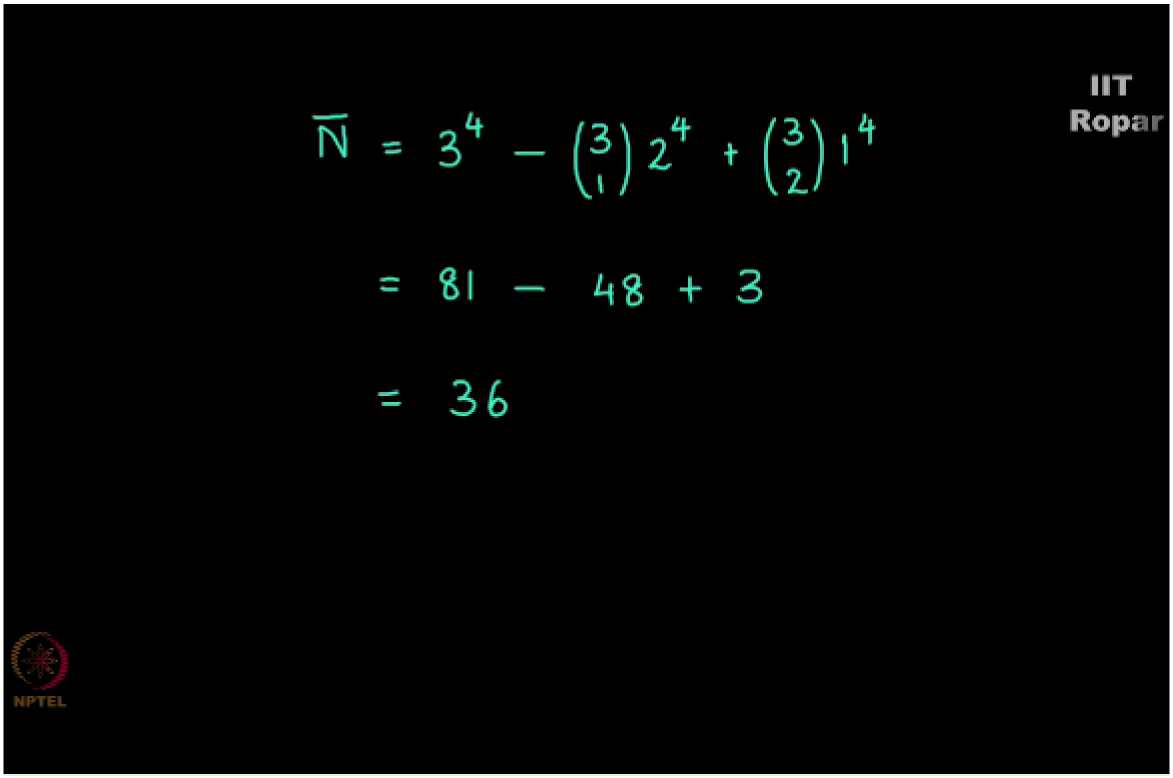

so what is the question for which 36 is the answer? The question is what are the total possible onto functions from 4 elements to 3 elements, the answer is 36, in fact we'll observe that principle inclusion and exclusion can be used to even answer a general case of how many onto functions are there from a domain with M elements to a domain with N elements.

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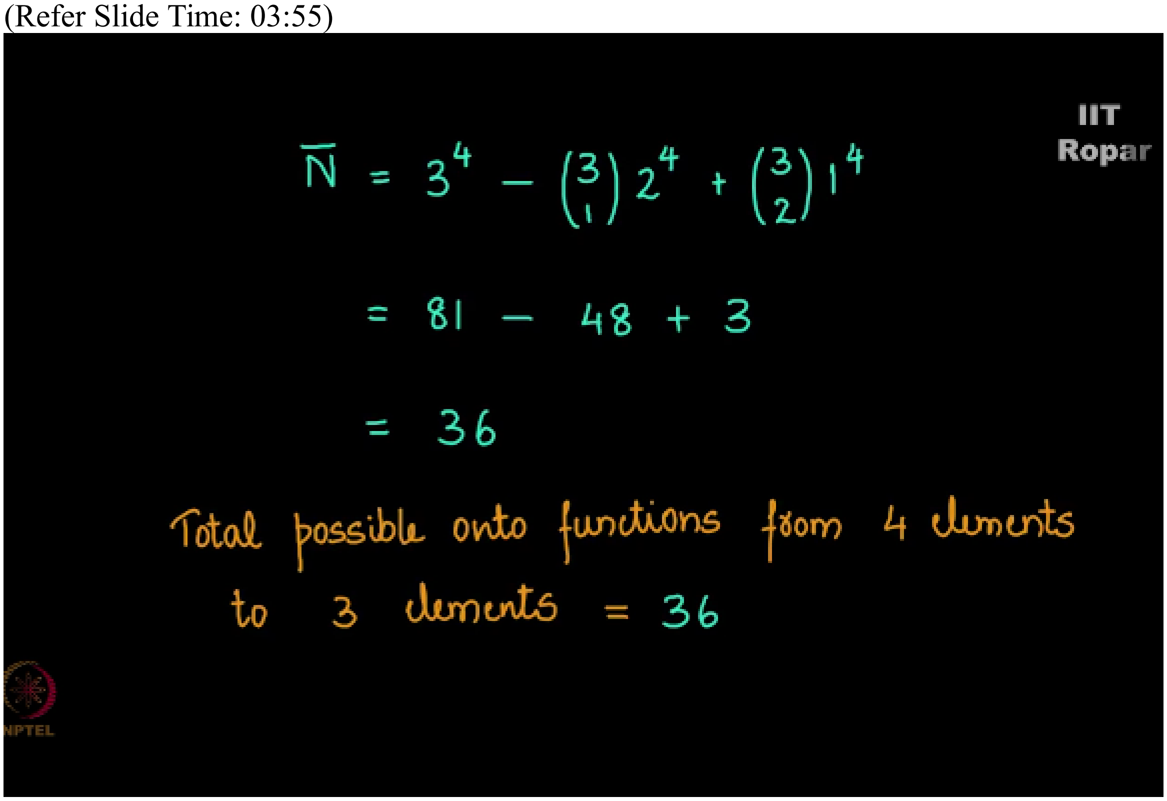

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