NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Example 11 - Seating Arrangement - Part 1

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I hope these conditions are clear to you,
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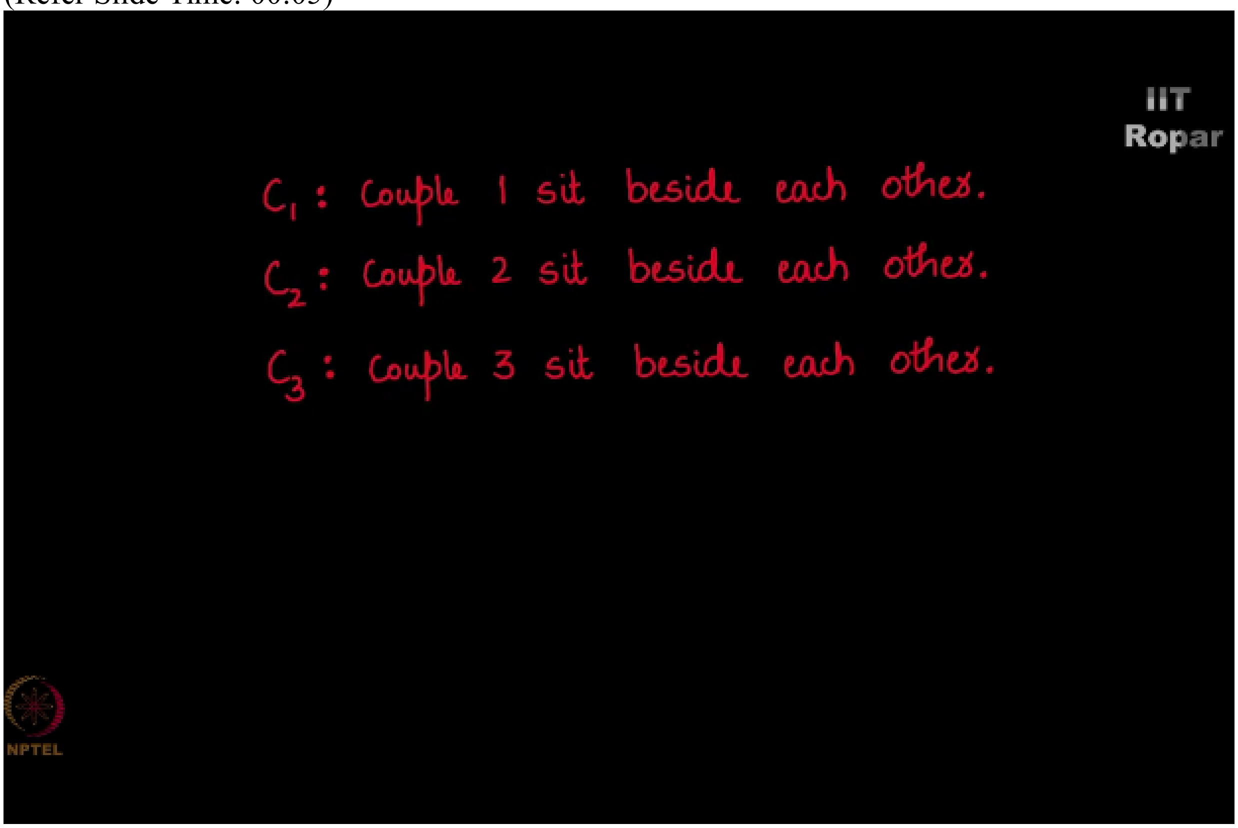

if not please pause the video, look back understand the arrangement, understand the question and then proceed further, because let me warn you at this stage the further you go the more complex it becomes without your understanding, if you understand things become very clear and easy.

Now the conditions are done, now the next question is what is N(C1)? N(C1) are those permutations where couple one are sitting beside each other, we don't know anything about couple 2 and couple 3, right, observe this you have the circular table,

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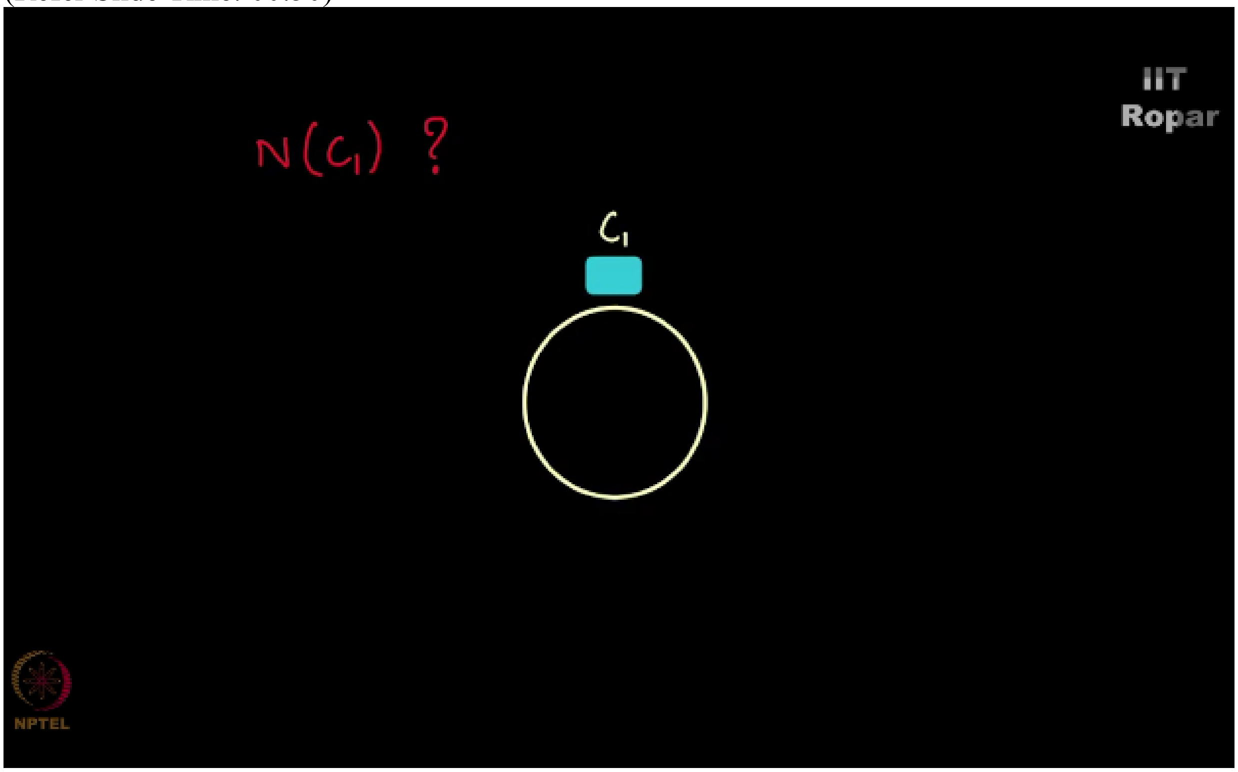

I consider the husband and wife couple 1 as 1 single object, now 2 people out of the 6 people, 6 people I mean 3 couples and hence 6 people, 2 people are considered as 1 unit, 1 object, now how many are remaining? 4 are remaining, I'll name them like this, P1, P2, P3, and P4,

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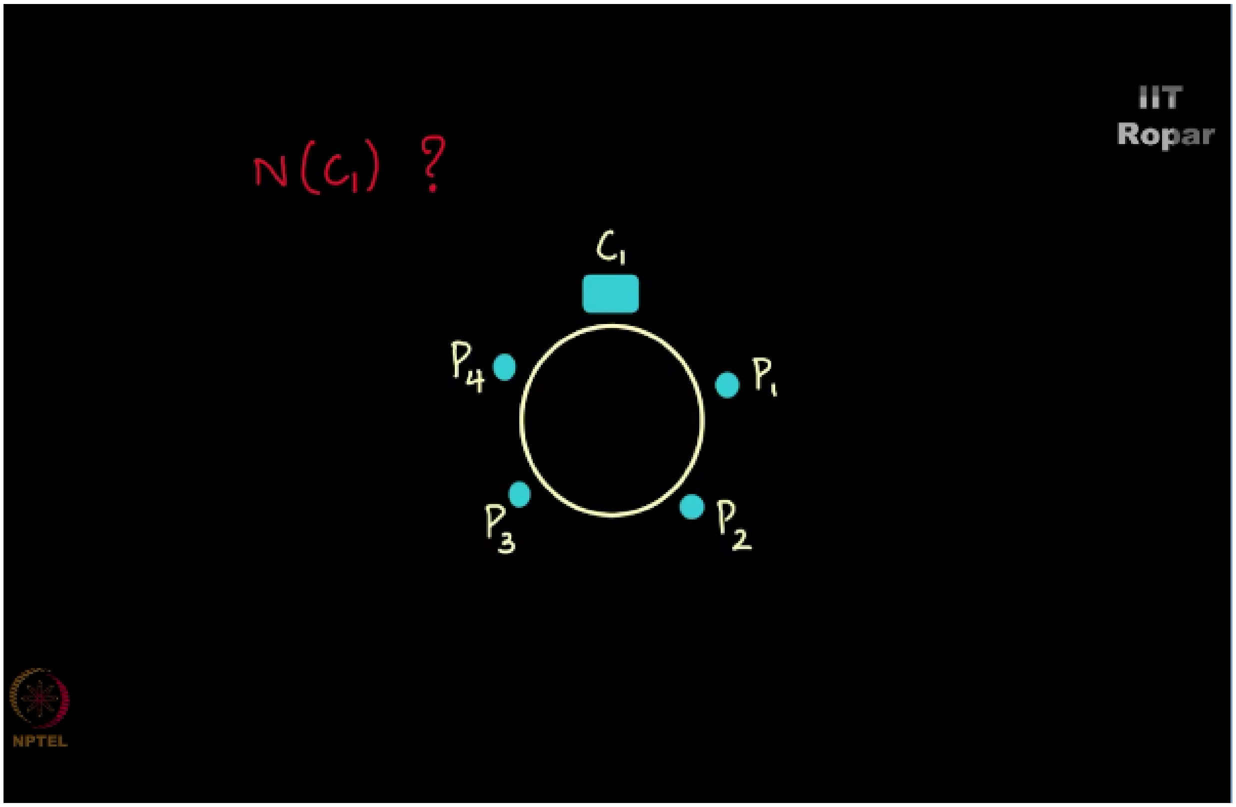

you see we have 5 distinct objects here, earlier we had 6 people now we have turned it down to 5, how? I'm considering couple 1 as 1 unit.

Now in N(C1) you have those permutations where couple 1 sit beside each other, I have locked couple 1 now, the remaining 4 are like this, let me enumerate some of the possible permutations, it is something like this, C1, P1, P2, P3, P4 this is one permutation,
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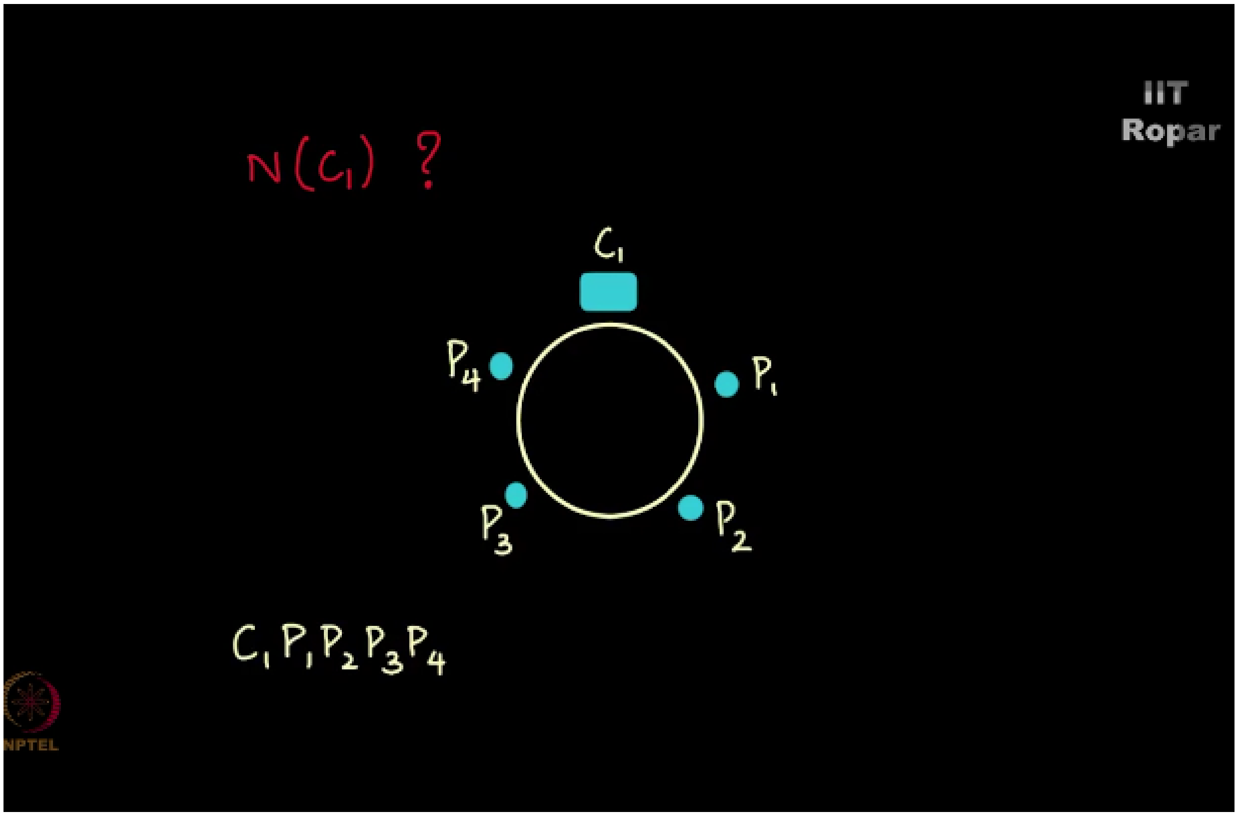

another one could be something like this P1, C1, C2, C3, C4, this is another permutation,
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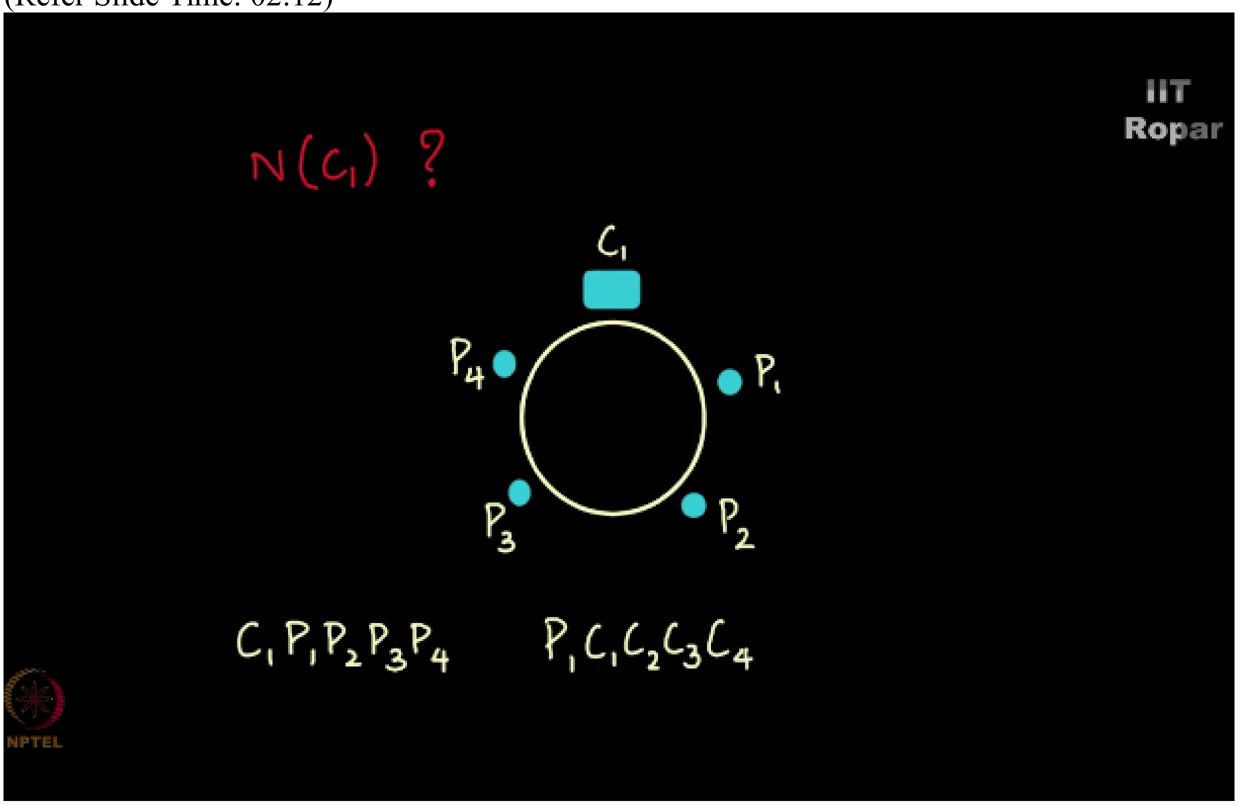

P1, C1, P2, P3, P4 and this is another one,

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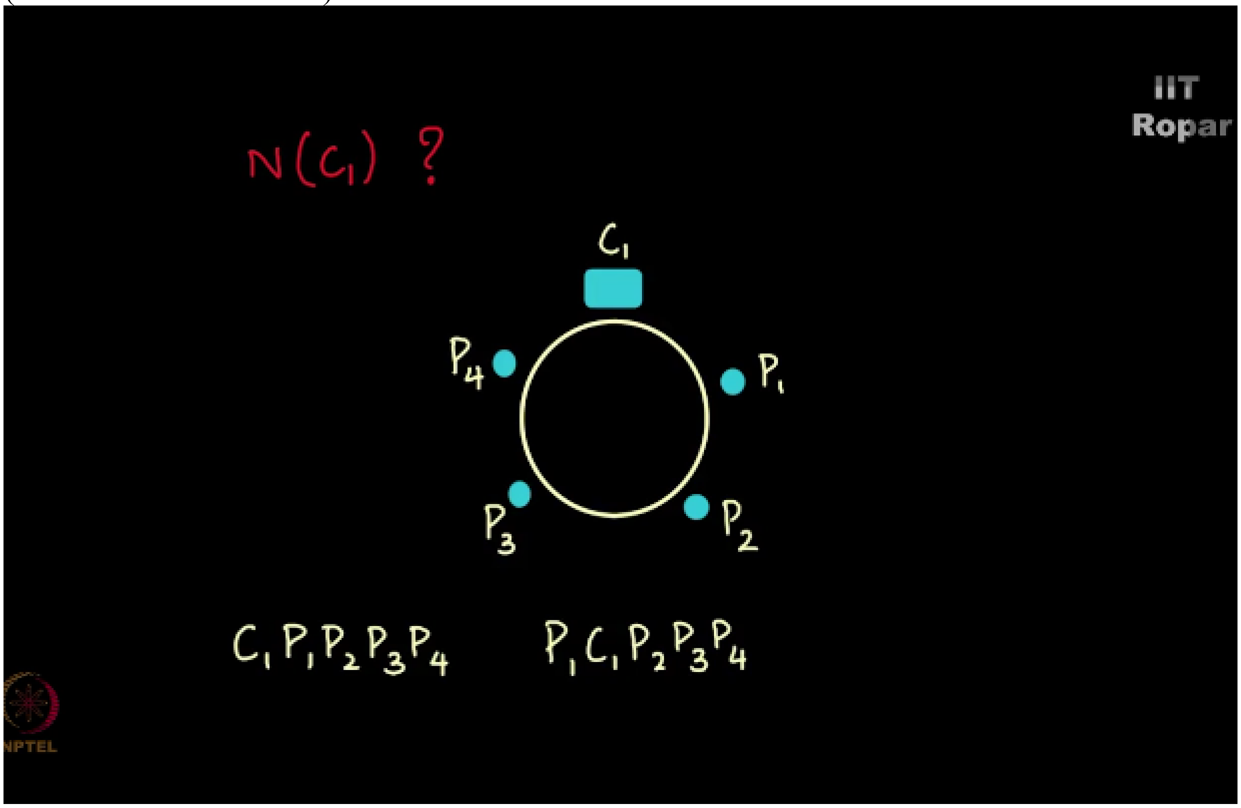

P4, P3, C1, P2, P1 this is another permutation,

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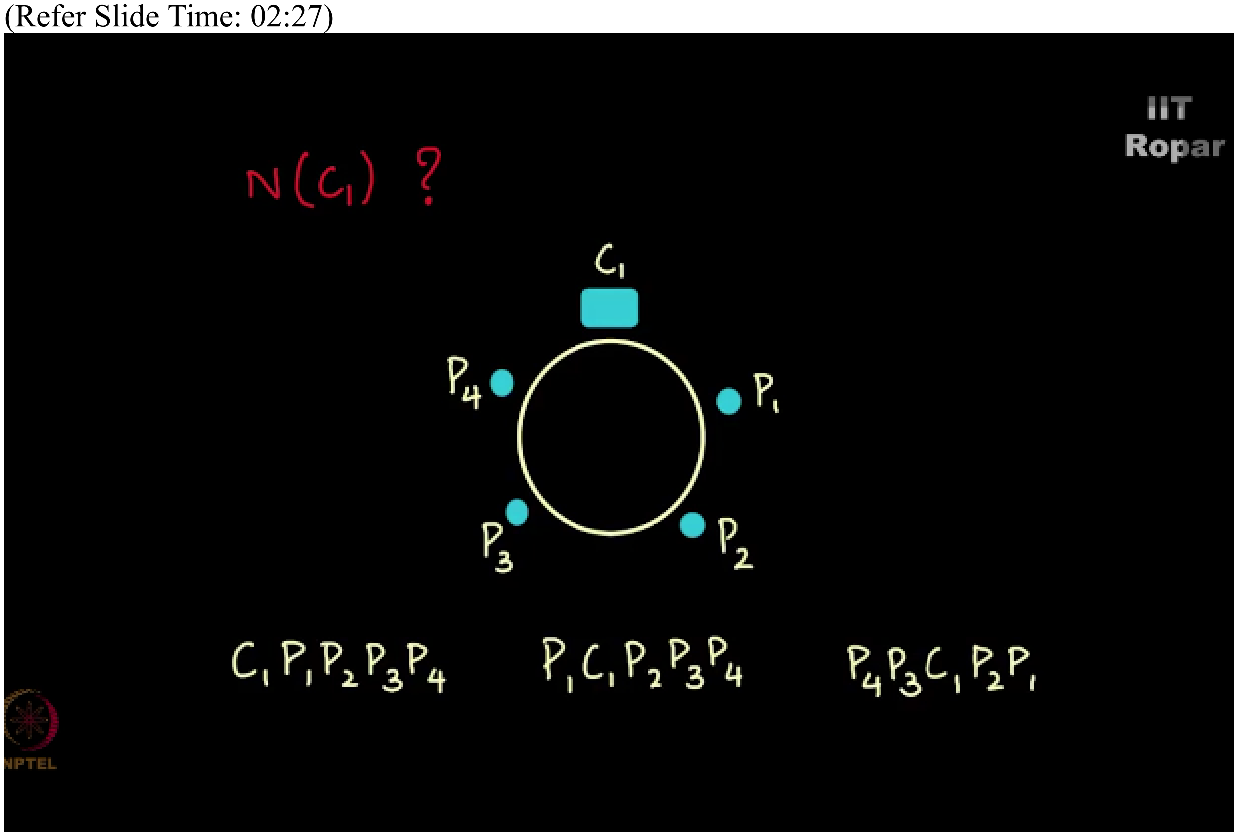

well there are several like this, how many are there let us see? You see all possible such permutations are 5 factorial in number, how? We had 5 distinct objects, C1 as one unit, P1, P3, P2 and P4 these as 4, so 4+1 we had 5 objects and all possible permutations of these 5 objects are 5 factorial, right,

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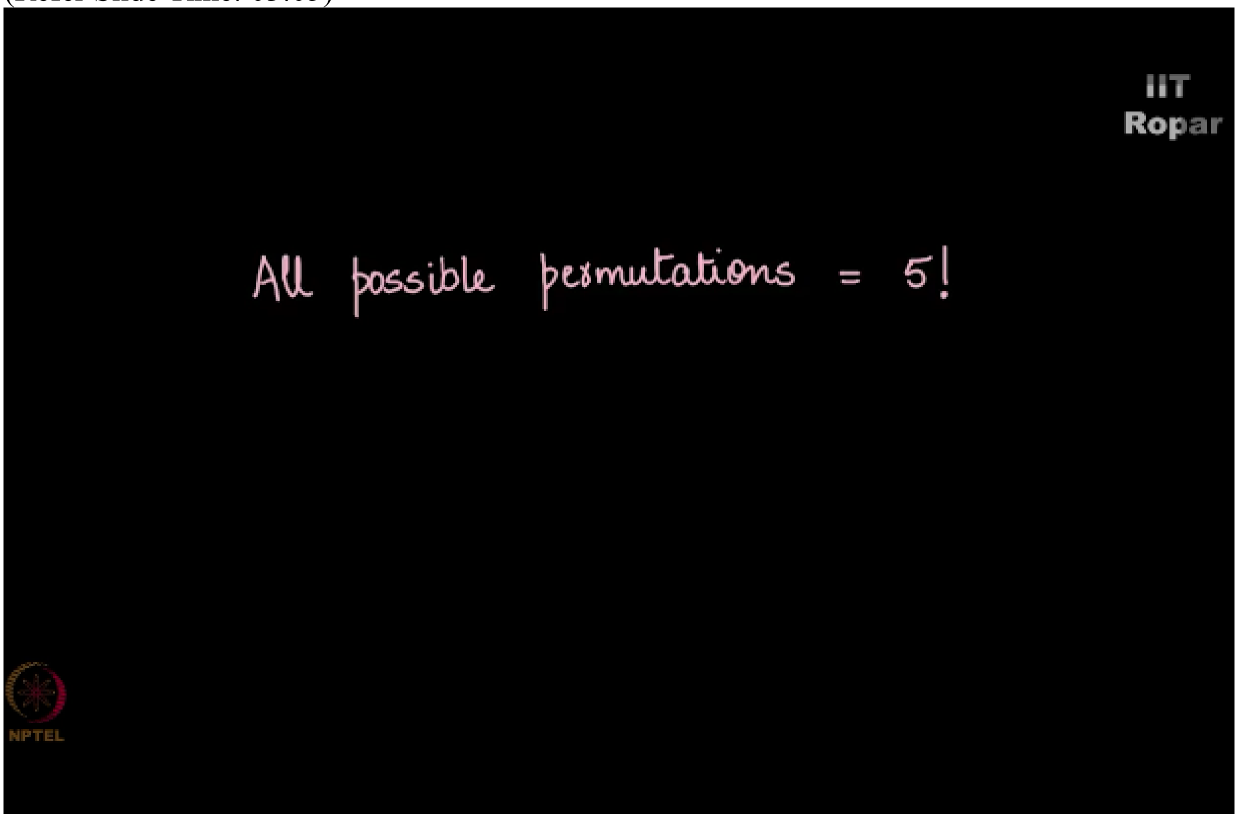

but don't you think we are over counting here, let us understand what we are over counting actually, observe this, C1, P1, P2, P3, P4 don't you think it is the same as P4, C1, P1, P2, P3?

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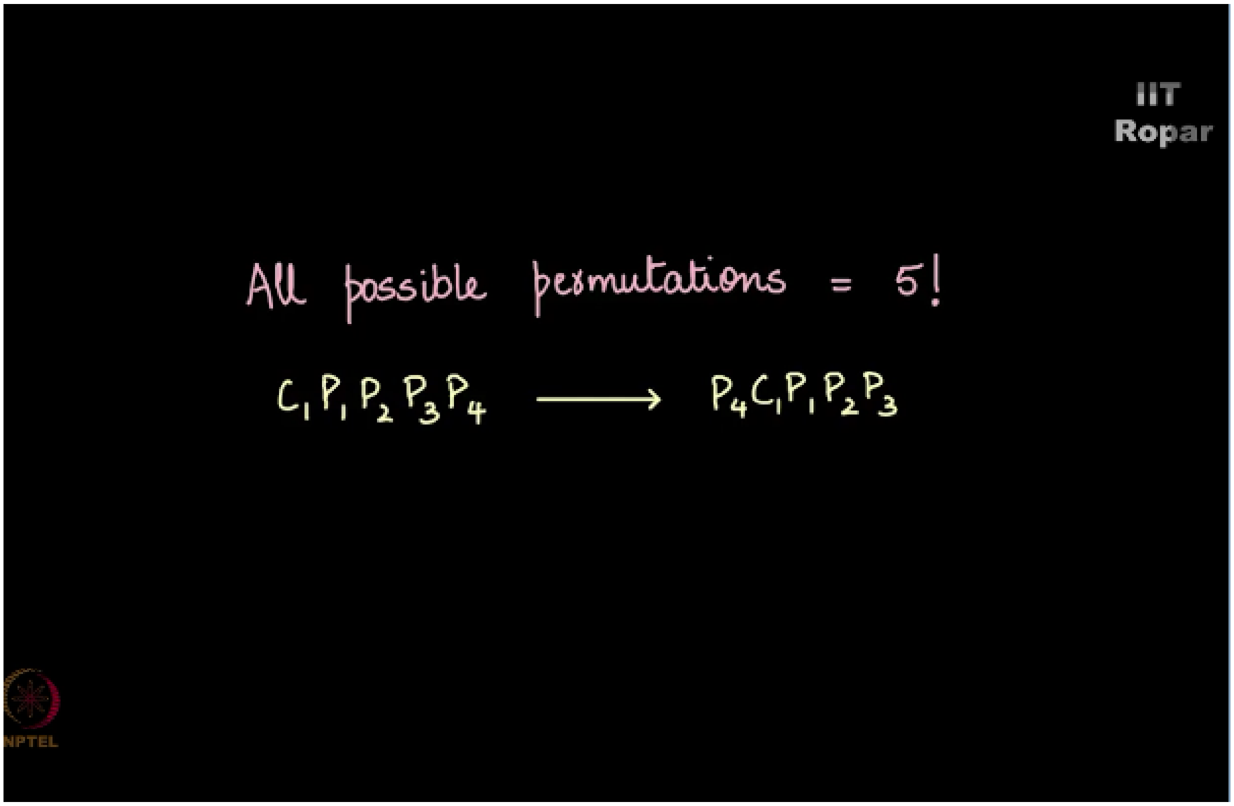

Please observe very carefully, how is it the same, C1 is one unit right, when you rotate it you can write it down on the pen and paper, write down these 5 as in a circular format, give just one rotation don't you think you would get it as P4, C1, P1, P2, P3, it is the same, and what did I mention earlier? I had mentioned that rotations are not considered as different permutations, and hence this is the place where we are over counting, there will be several such rotations.

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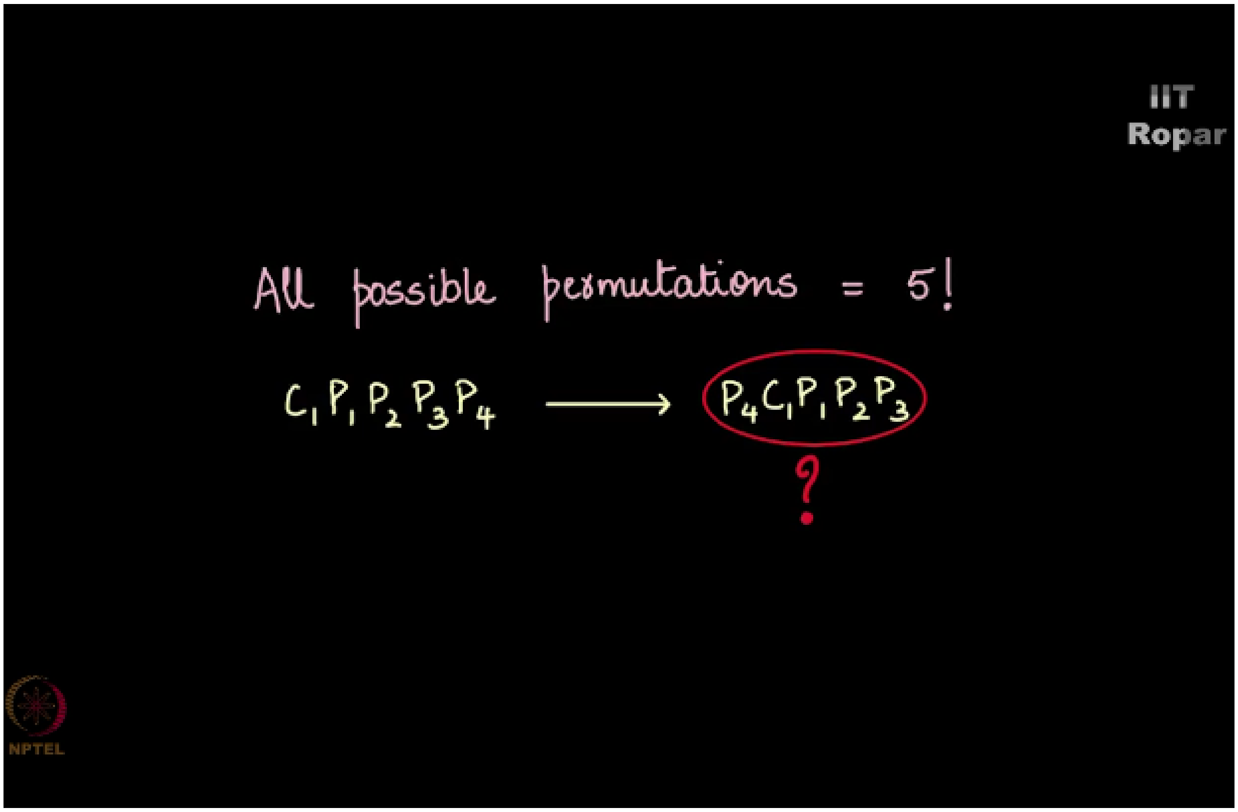

Now the next question is how do we count such extra permutations, rather we do not want such permutations, and how do we come to know how many of these are there let us see. Now this is the place where bijection will come to our rescue, how? Observe very carefully, P1, P2, P3, P4 I'm just taking these 4, leave out C1 for now, observe this P1, P2, P3, P4 I have these 4, I'm going to take it to C1, P1, P2, P3, P4, what did I do? I just appended C1 along with this permutation, I had a permutation I append it C1 to that permutation.

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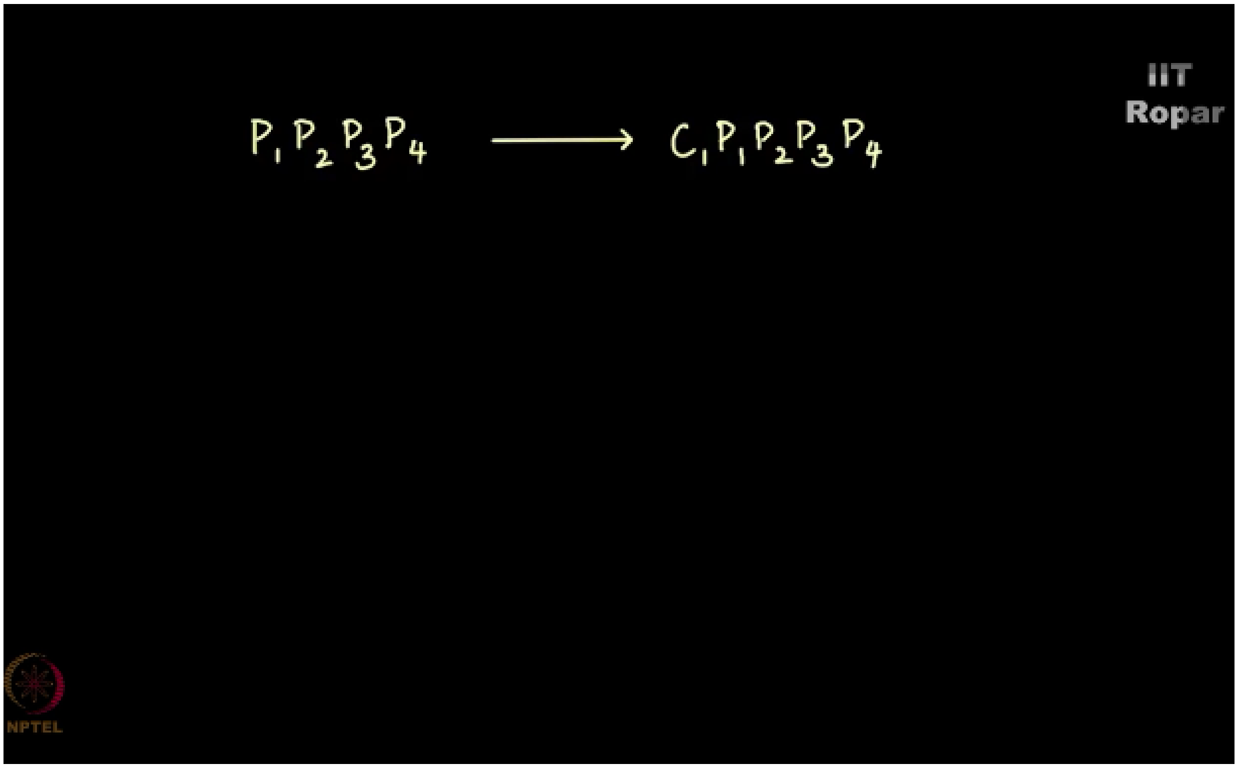

Now consider so C1 here was appended, consider another such permutation P3, P1, P2, P4, this is some random permutation, what is it going to? According to the previous rule or mapping there was a permutation, I just appended C1 along with that permutation, now according to this mapping where will P3, P1, P2, P4 go? It will go to C1, P3, P1, P2, P4, isn't it,

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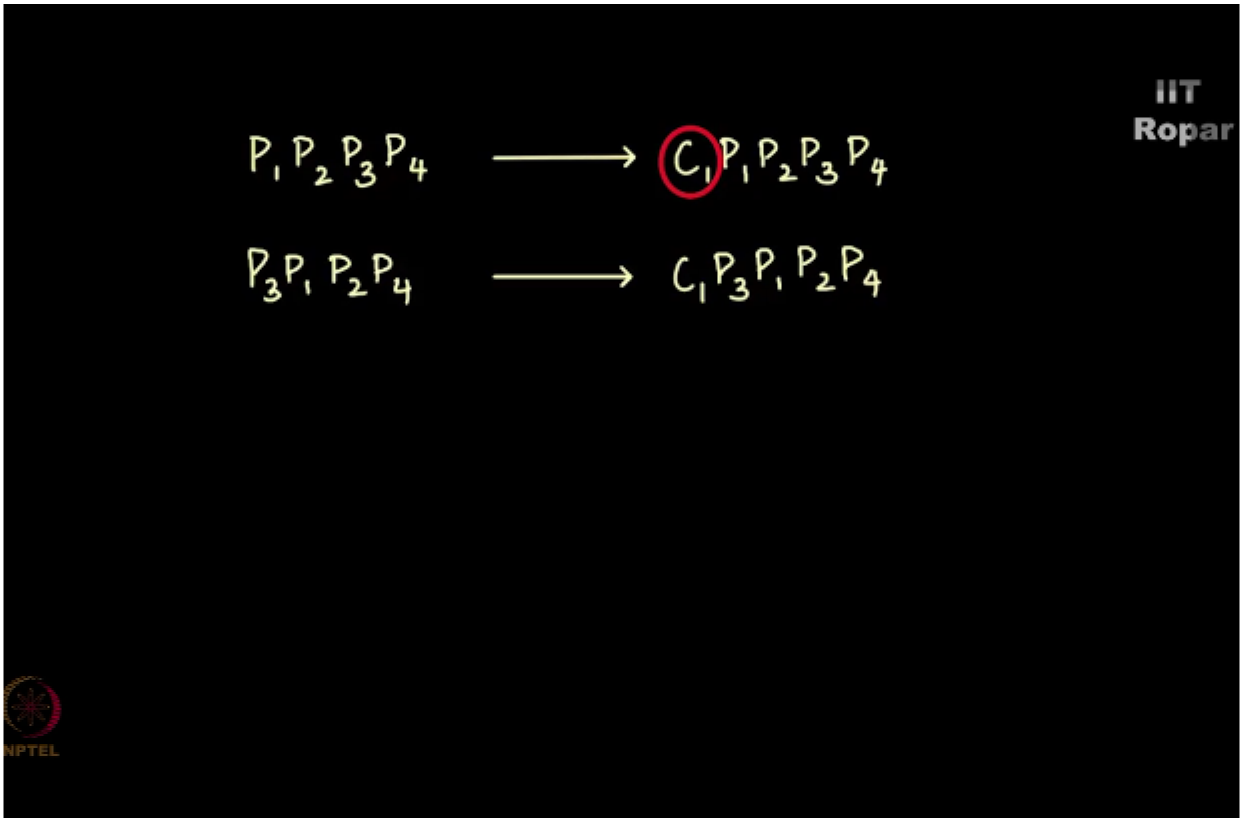

now I can give you any such permutation and you can close your eyes and map it to C1 appended to that permutation, well till now things must be very clear.

Now the next observation we make is how many such permutations do we have, right, we have 4 objects here, 4 people I'm considering into 4 objects, 4 objects all possible permutations of 4 objects, 4 factorial in number, so do you see that the domain has 4 factorial elements, now observe something,

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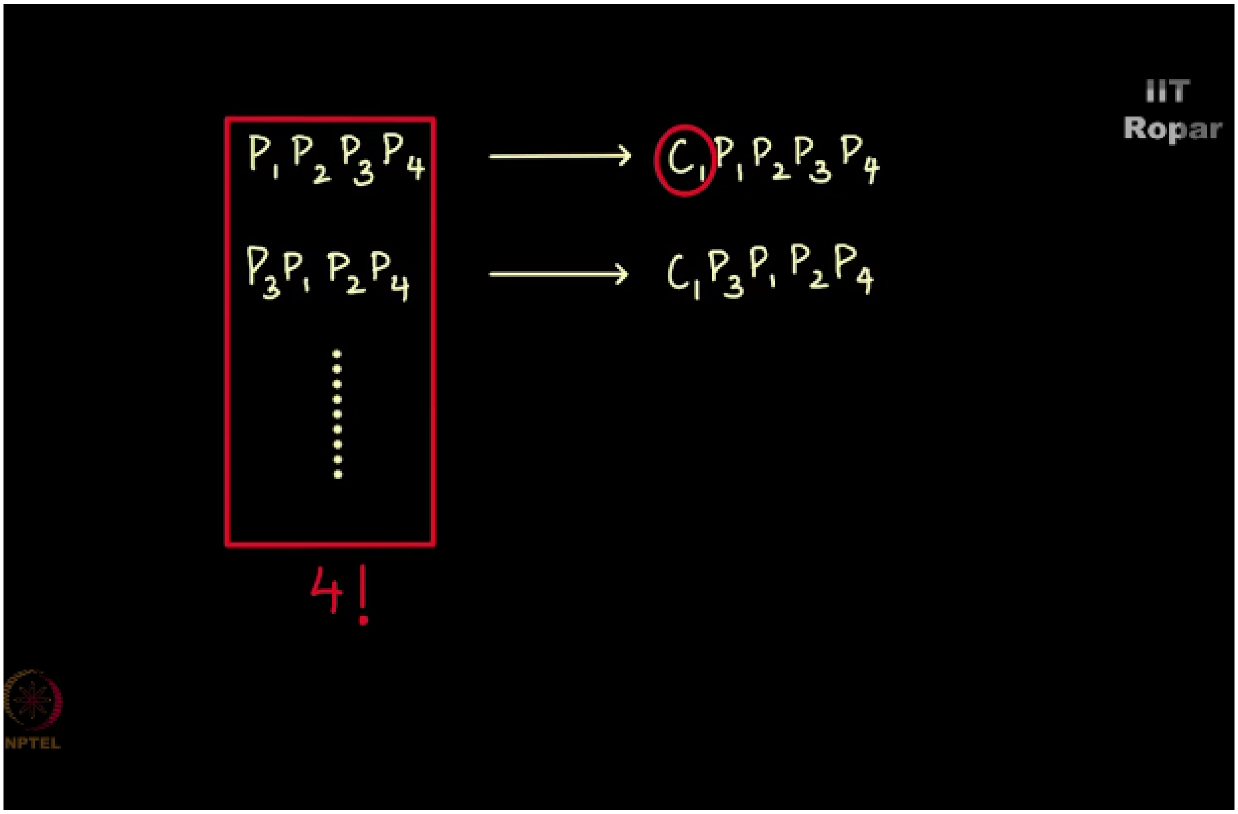

I'm going to consider one element in the codomain, how? I'm going to consider this element P2, P1, C1, P4, P3, I've considered this permutation of 5 objects, C1 and the other 4, right, P2, P1, C1, P4, P3 don't you think it is the same as C1, P4, P3, P2, P1?

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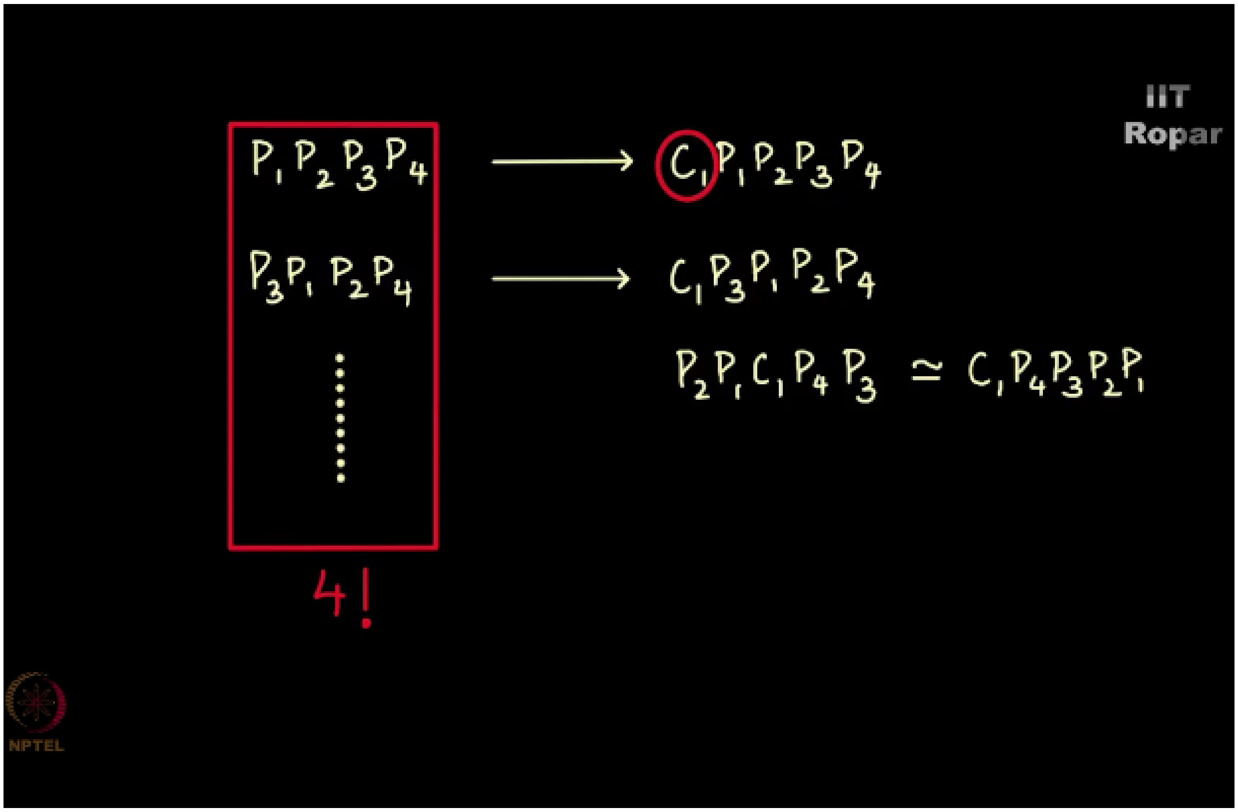

Please observe very minutely here, we haven't change anything, it is rotation again, if you observe carefully I have just rotated and got it as C1, P4, P3, P2, P1, you can probably write it down as a circle and check it.

Now if this element is there in the codomain from where did it come, I can take it back to this element, P4, P3, P2, P1,

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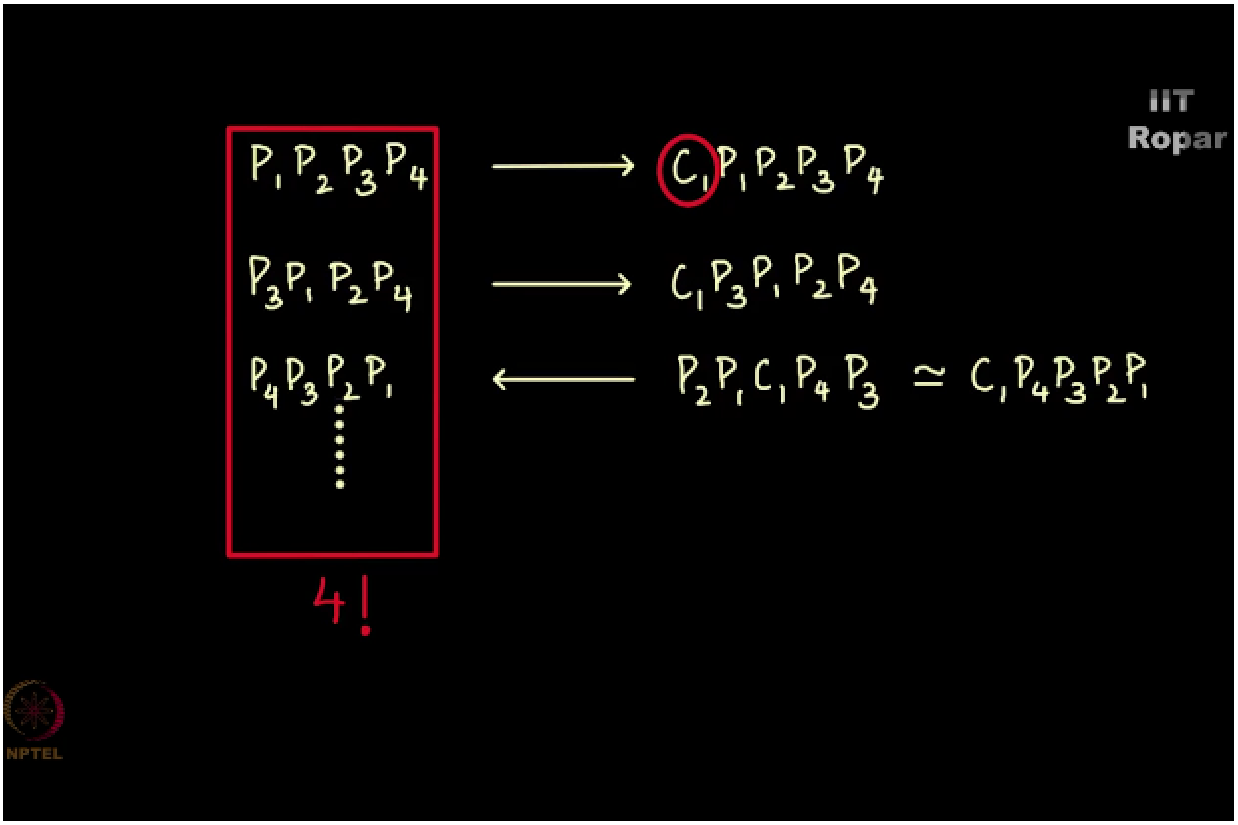

how, how? How did I do this? See this element here in the codomain has C1 appended to this permutation, right, now given any such permutation in the codomain don't you think we can obtain a corresponding element in the domain? Yes we can, and hence how many such elements are there here as well, it must be 4 factorial, why?

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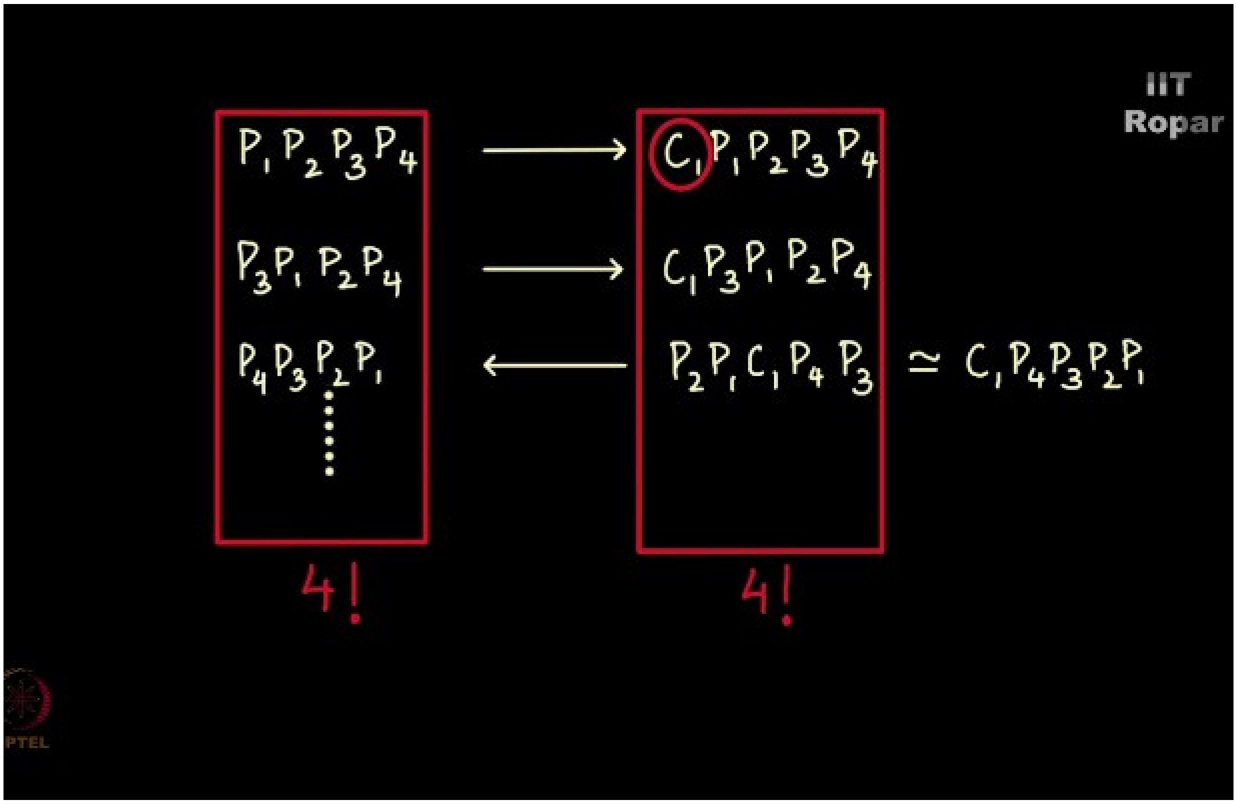

I have just proved to you that this is a bijection, it is 1 1, as well as onto, it is a proper bijection, right, did you see how ingeniously bijection came to our rescue here, so I had earlier asked you how many such over counts did we do? How many such elements did we count more than once? 4 factorial is the number, so 5 distinct objects can be arranged around a circular table in 5 – 1 factorial which is 4 factorial ways.

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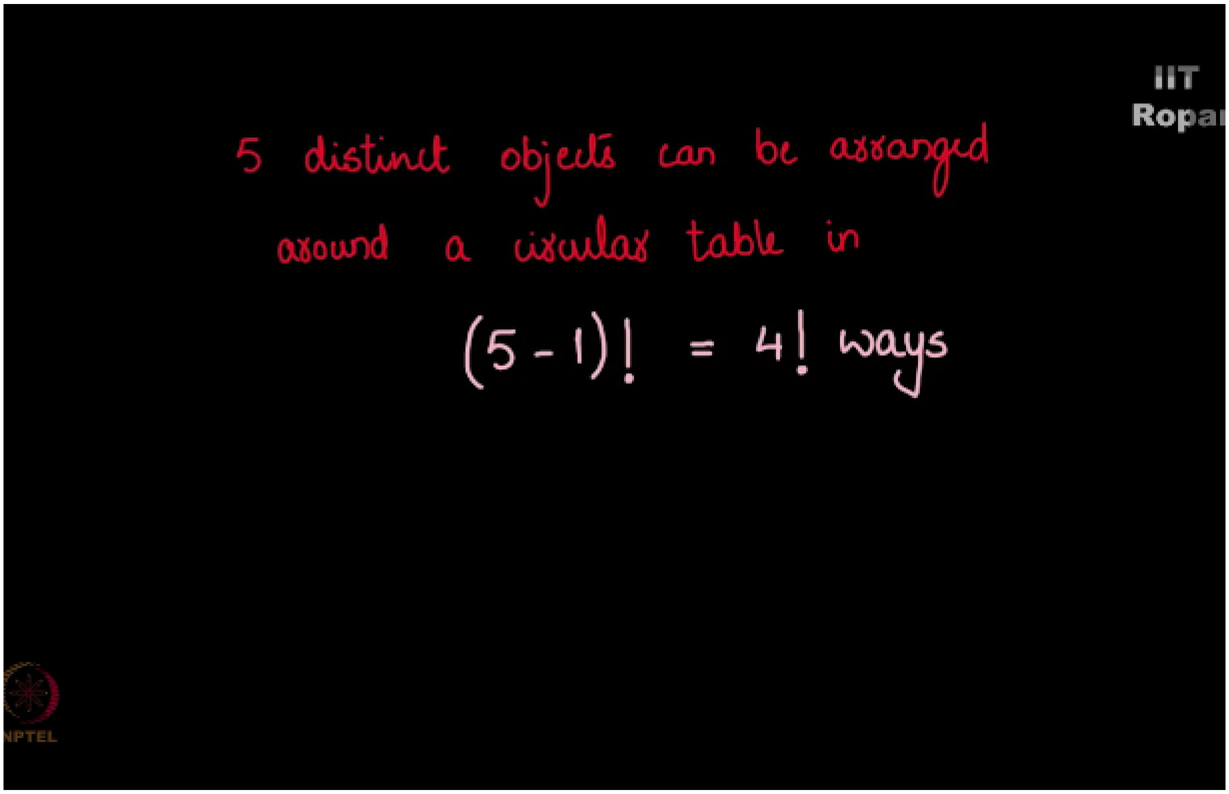

Now I can write it as in general N distinct objects can be arranged around the circular table in N-1 factorial ways,

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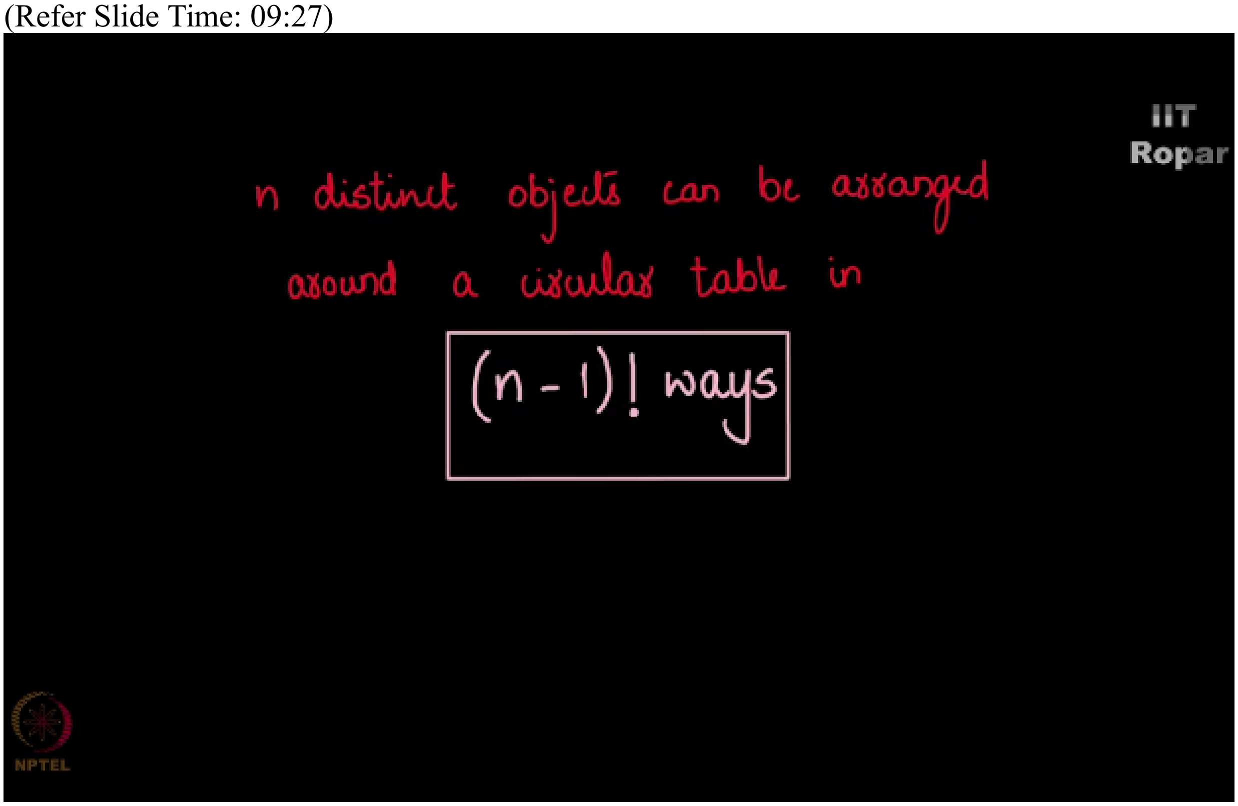

you can probably box this and keep, it will be useful for our further steps, so N(C1) happens to be 4 factorial, but can I stop at this position? I cannot still conclude that 4 factorial is the answer, how? You will appreciate it after this step, you observe couple 1 can either sit as husband wife, or wife and husband and hence it is 2 x 4 factorial, because every permutation this is possible, you consider C1, P1, P2, P3, P4 there itself you have 2, either WH or HW,

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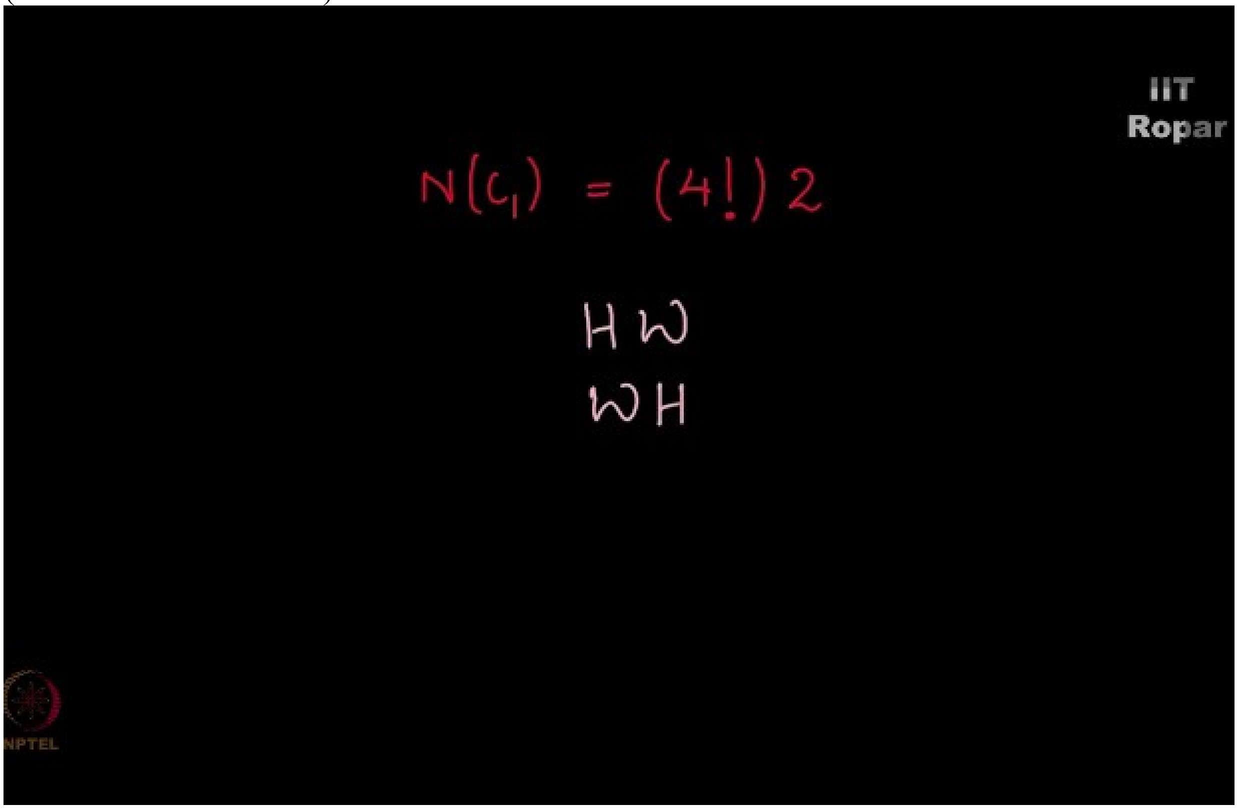

for every such permutation 2 sub permutations are possible and hence it is 2 x 4 factorial, this entire story was for couple 1.

Now the story continues for couple 2 as well, what does it mean? N(C2) happens to be 2 x 4 factorial, N(C3) happens to be 2 x 4 factorial as well, well we have observed that N(C1), N(C2), N(C3) happen to be 2 x 4 factorial.

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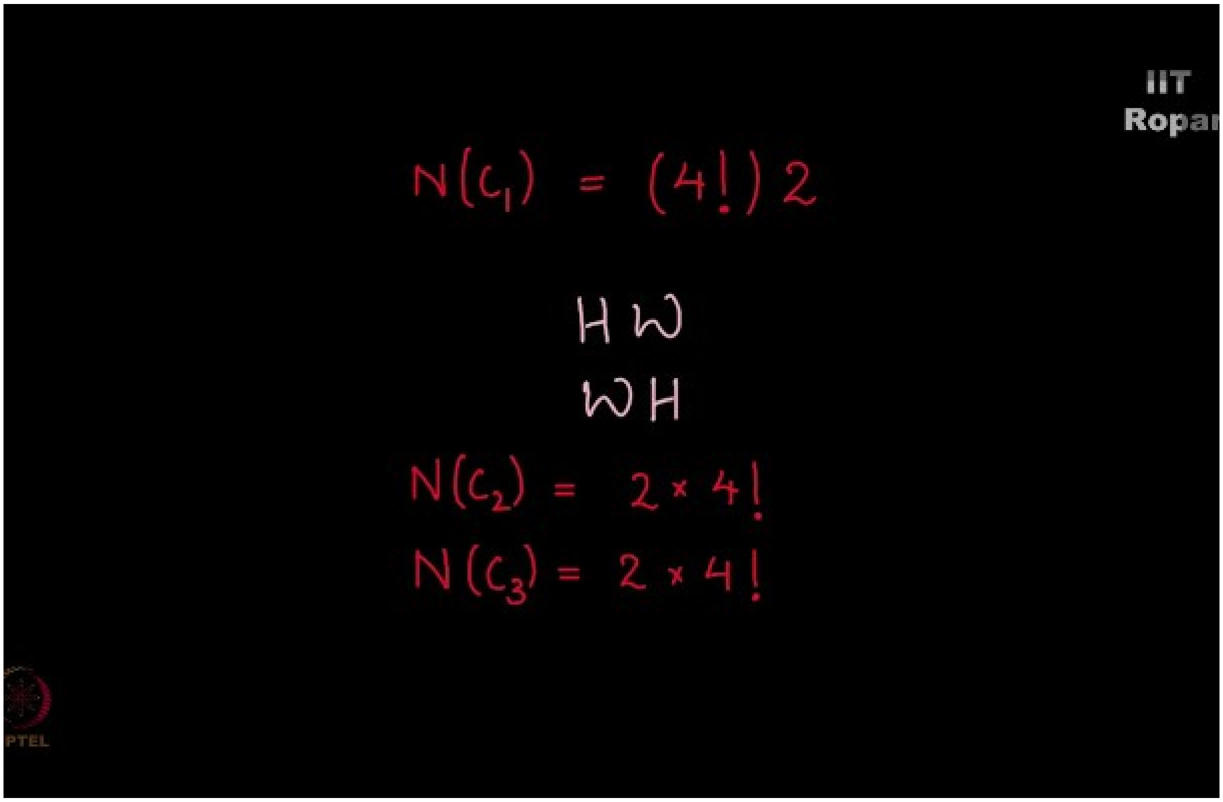

Now let us see what is N(C1,C2) (C2, C3) (C3, C1) in the next video, you must probably really want to write down things, watch the video several times here and there to understand the example clearly.

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