NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Example 10 - Integer solutions of an equation

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Find the number of integer solutions to the equation X1 + X2 + X3 = 7, well the question doesn't end here there are some conditions given X1 should lie in the range 0 to 2, X2 should lie in the range 0 to 4, and X3 should lie in the range 0 to 3,
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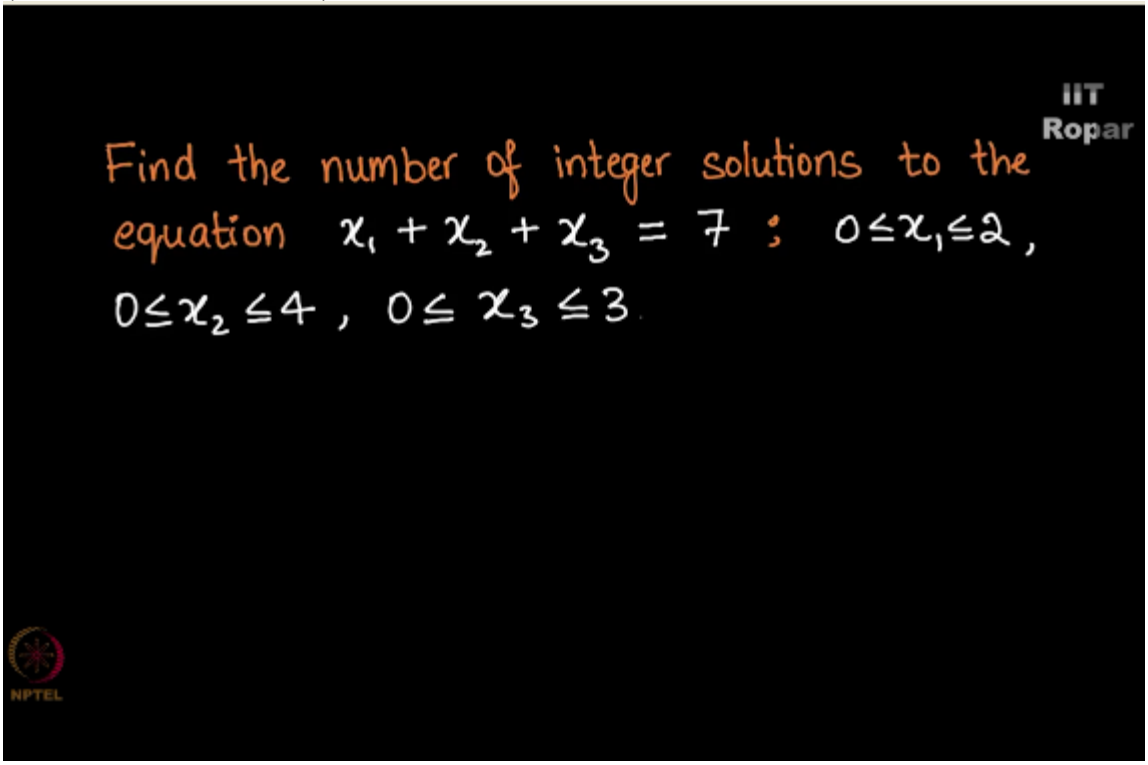

in week 1 we have also seen such questions where rather in counting chapter we have seen questions where we are just asked to find the number of solutions for the equation without any such conditions, but now we have to follow these conditions and find the possibilities which fit this equation.

Now if I consider N to be the set of all possibilities we know that it is going to be 3 + 7 - 1 choose 7, how? By the concept of sticks and cups you know that N is 3 here, R is 7, so by the

formula it is N + R - 1 choose R and hence it is 3 + 7 - 1 choose 7 which is 9 choose 7 which happens to be 36,
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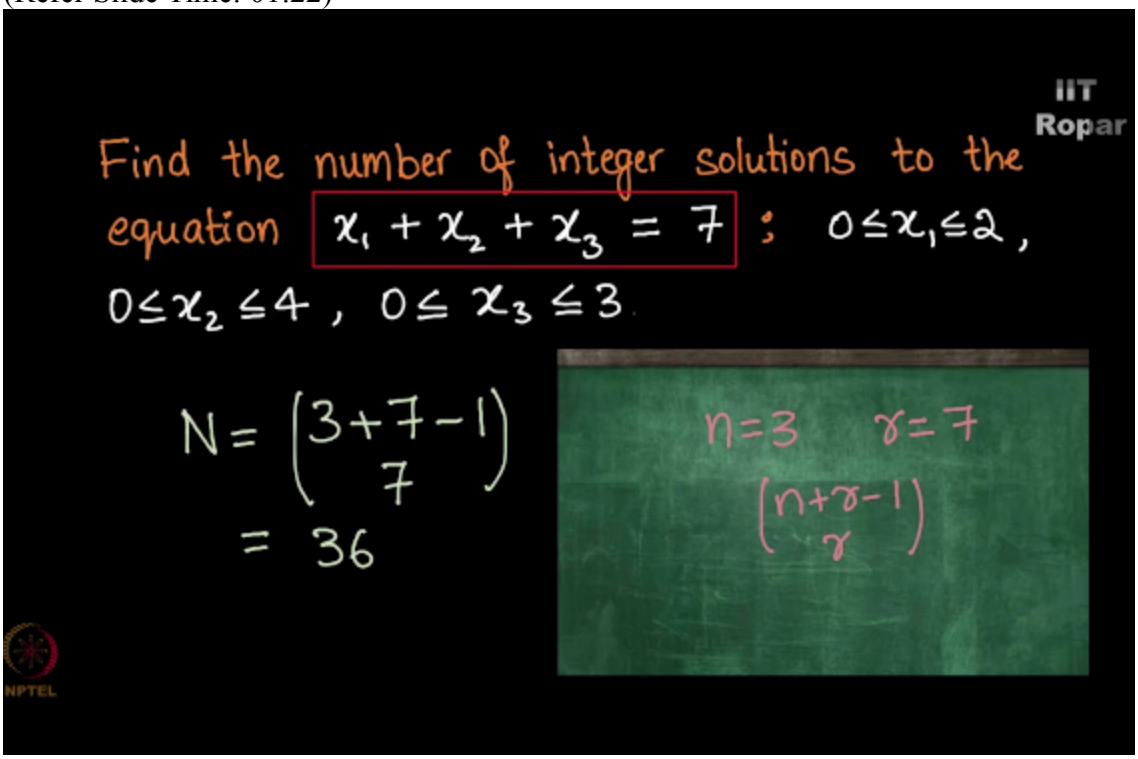

now this is all possibilities, but don't you think we are over counting because here you would have even those possibilities where X1 is at least 3, right it is at least 3 here whereas the given condition is it should lie in the range 0 to 2, so we are over counting some possibilities.

Now the next step would be to subtract all those unwanted possibilities, how do we do that? Let us set C1 to be the condition where it has solutions with X1 greater than or equal to 3, and C2 to be those solutions for the equation with X2 greater than or equal to 5, and C3 to be the condition where X3 is greater than or equal to 4, so these are the three conditions,
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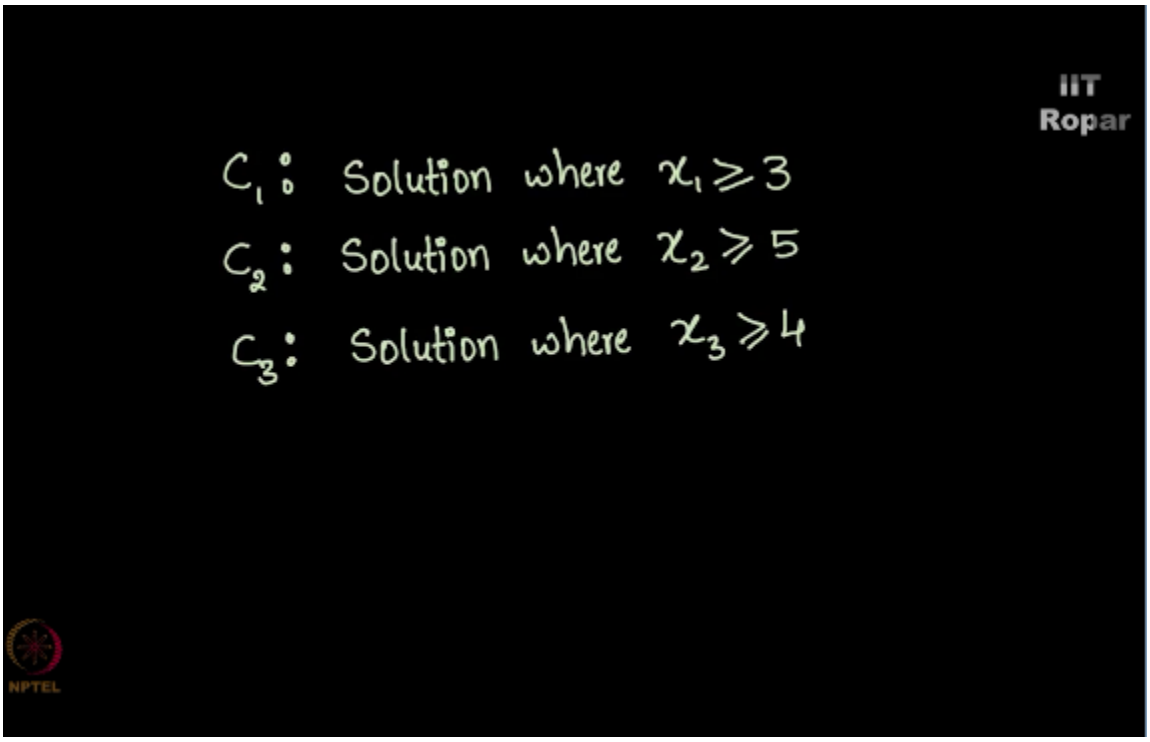

so I am setting the conditions this way, I am going to see what is N(C1)? How do I calculate that? I have to find out all those solutions where X1 is at least 3,

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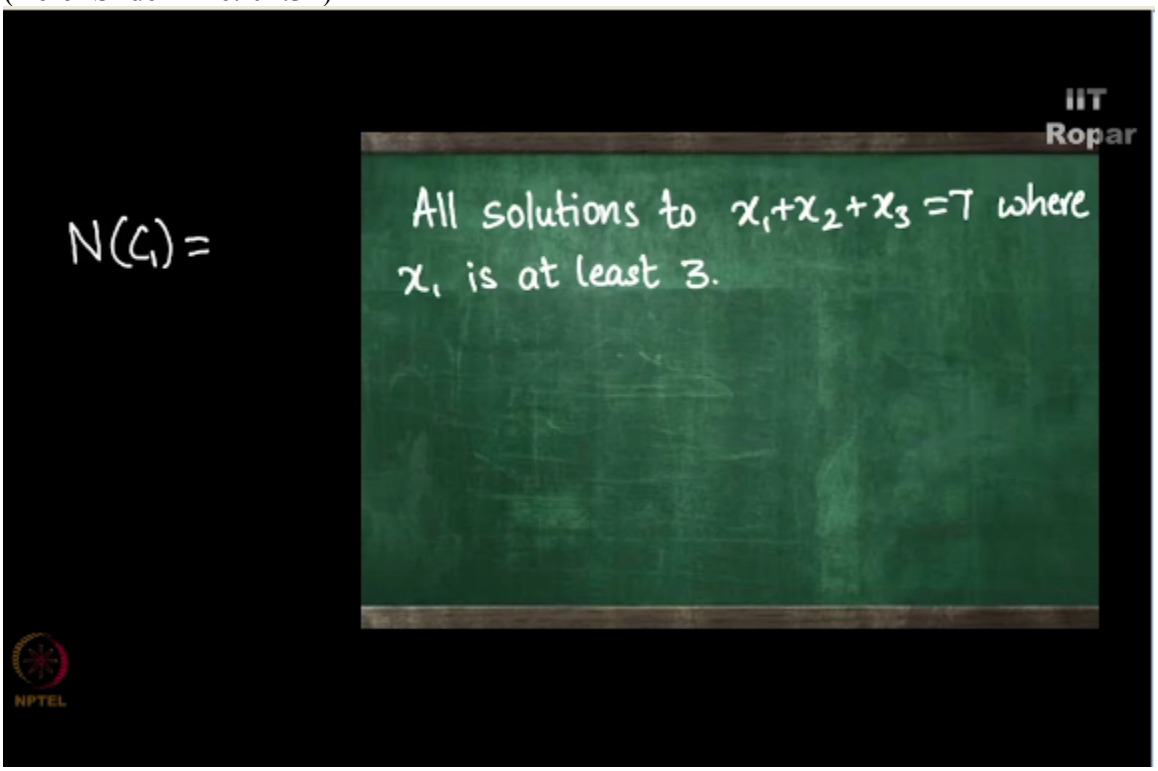

now it is precisely finding out the solution for the equation X1 + X2 + X3 = 4, how did I come to know this? This equation it was X1 + X2 + X3 = 7 – 3 which is 4, now adding 3 on both sides adding 3 to X1 or and to the RHS it gives me the original equation,

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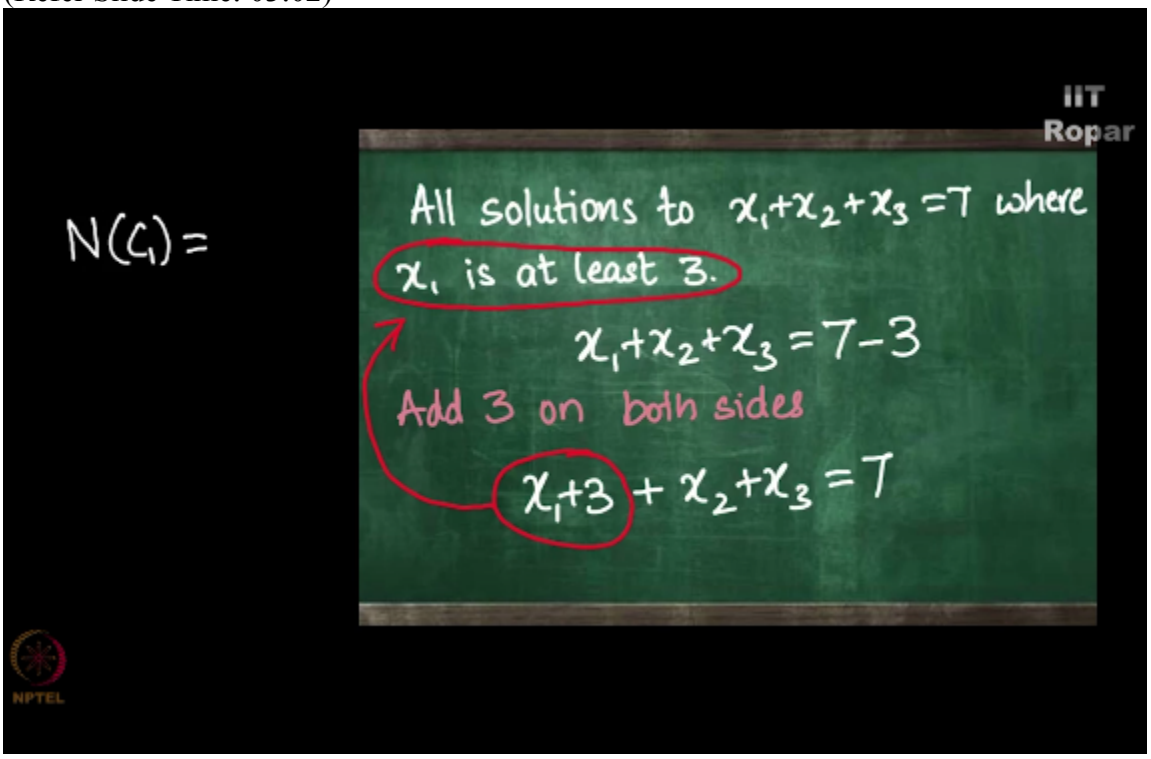

did you observe what I am doing take a moment to think and understand what is happening here, in how many ways can we get solution for this equation, it is nothing but 3 + 4 - 1 choose 4 which happens to be 6 choose 4, which is 15 because it is 6x5/2,

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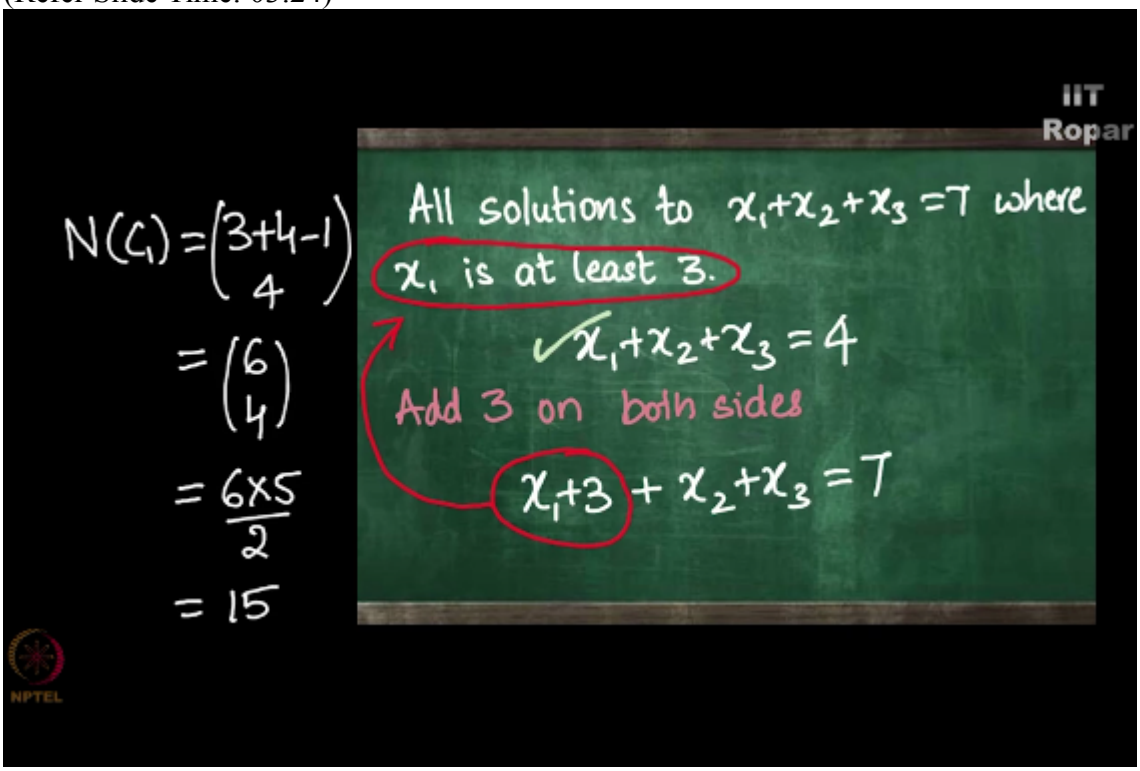

now moving on to number of elements following C2 condition that is N(C2) it happens to be the solutions which are having X2 at least 5, now following on the same lines it is precisely asking the solution to the equation X1 + X2 + X3 = 2, where X2 happens to be at least 5, right.

Now what are the solutions for this equation, it is 3 + 2 - 1 choose 2 which is 4 choose 2, and 4 choose 2 is 4x3/2 which is 6,
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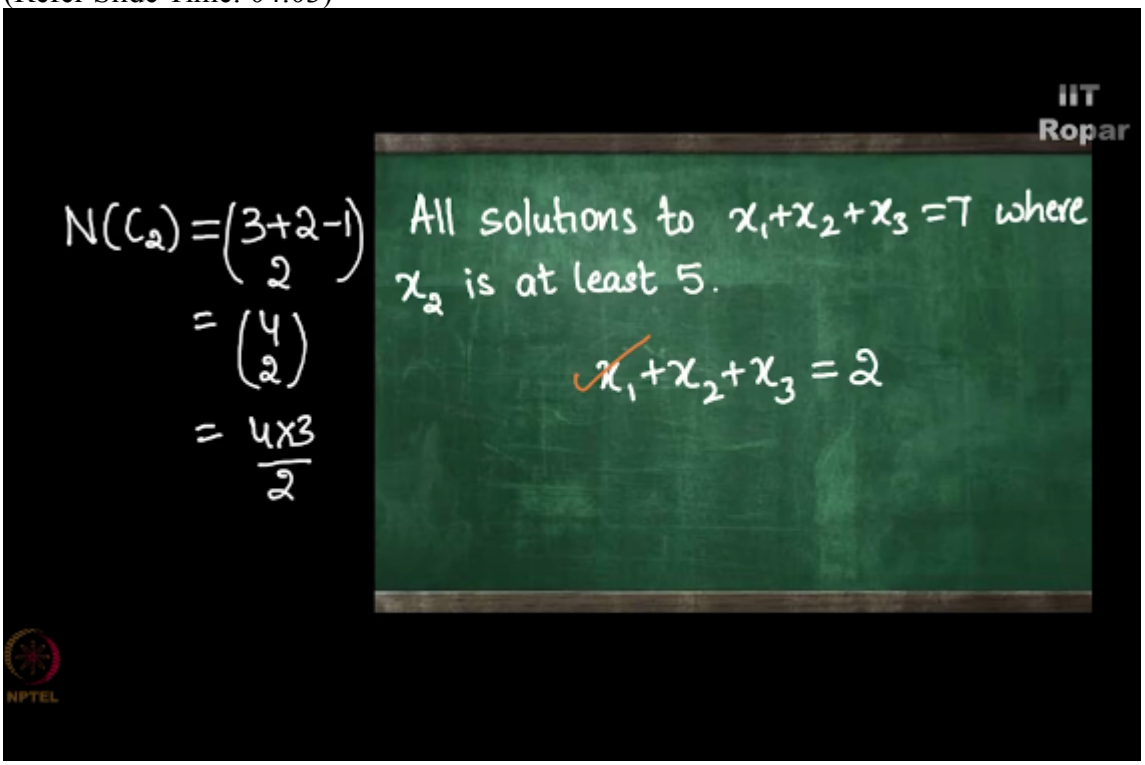

now following the same for C3 the solutions where X3 is at least 4 we have to find out N(C3),now it is the same thing as asking the solutions to the equation X1 + X2 + X3 is 3,
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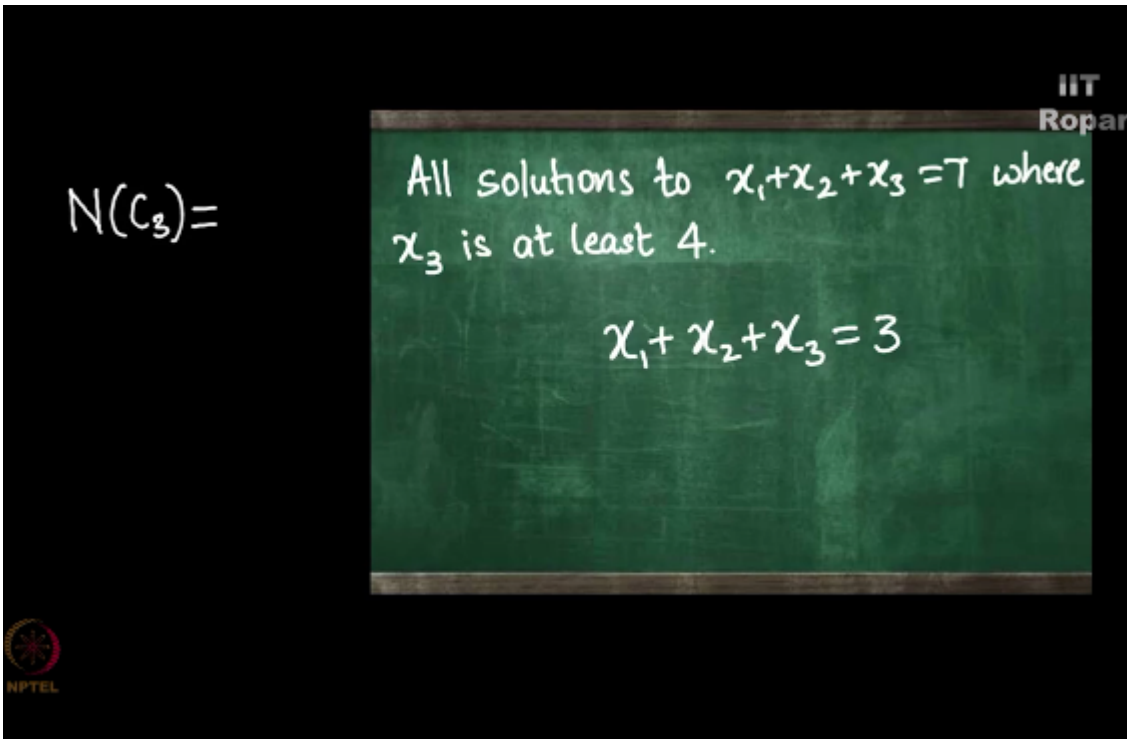

how many solutions do we have which satisfy this equation, it is 3 + 3 - 1 choose 3 which is 5 choose 2, 5 choose 2 happens to be 5x4/2 which is nothing but 10, now that we have calculated N(C1) N(C2) and N(C3)

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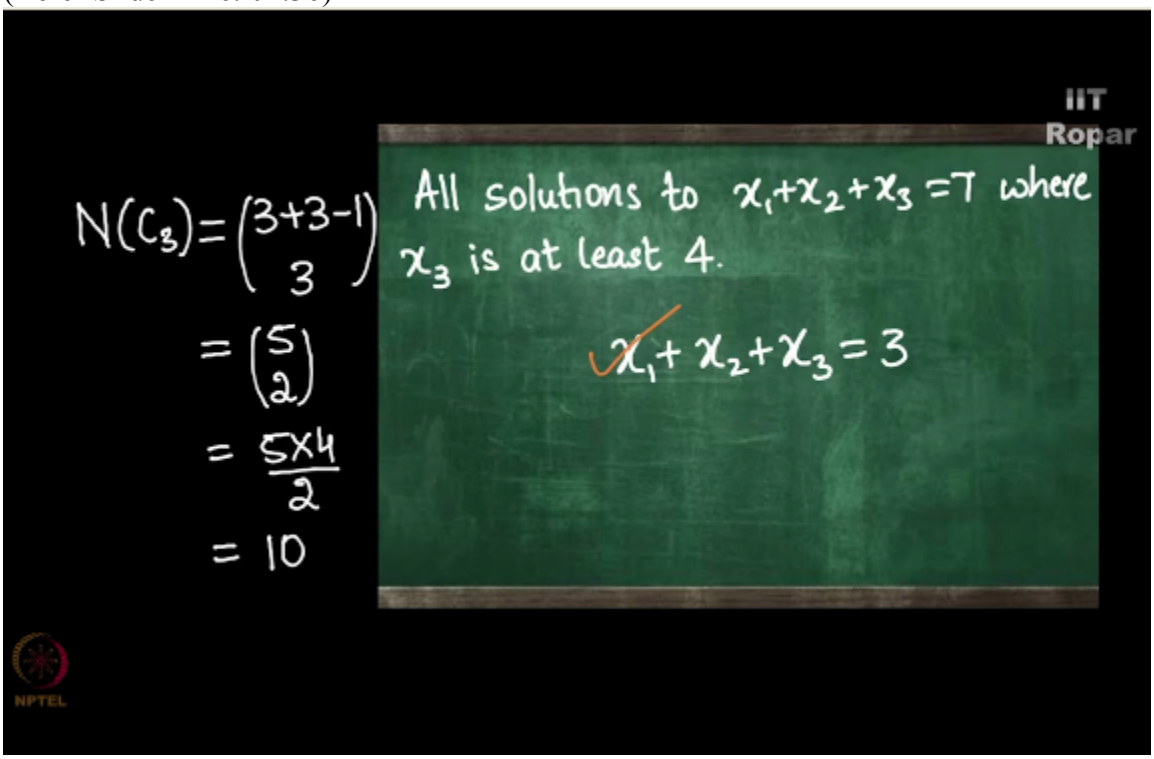

we move ahead to see what is N(C1,C2), what does (C1,C2) mean? X1 should be at least 3, X2 should be at least 5, now the moment I insert X1 to be at least 3 and X2 to be at least 5 the sum becomes 8, boom,
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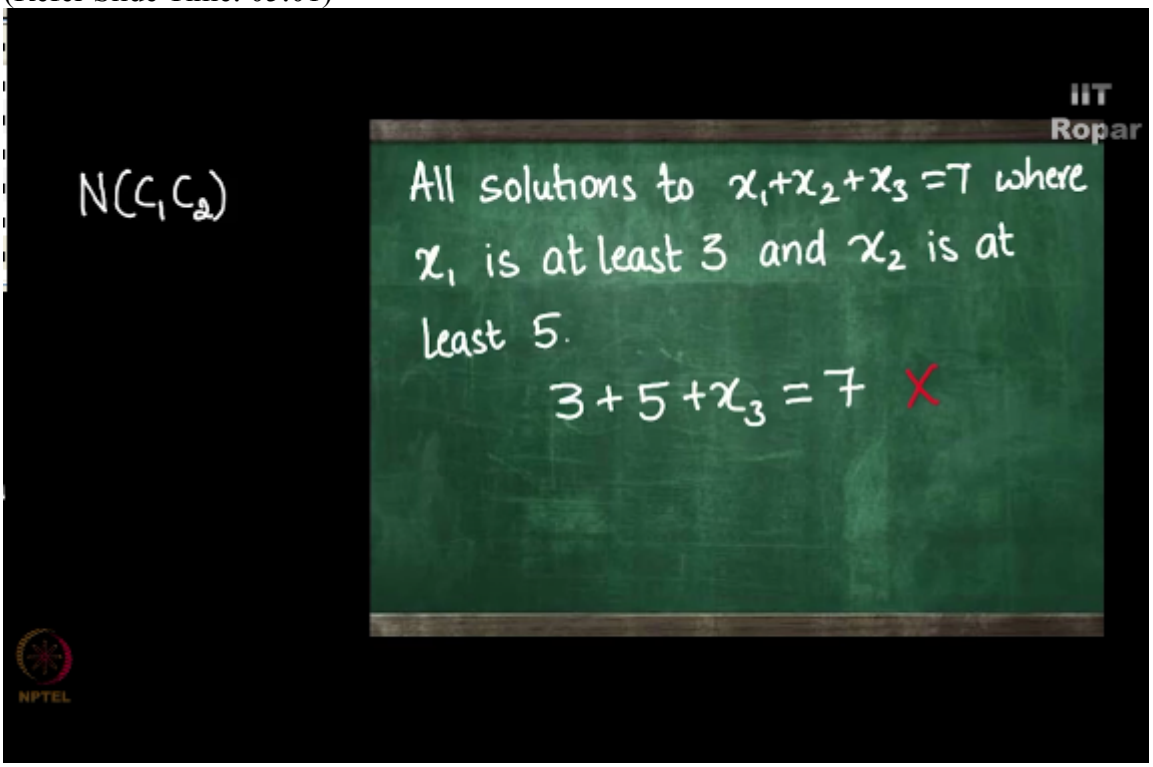

we cannot go ahead because we must get X1 + X2 + X3 to be 7 it is not going to happen here, and hence N(C1,C2) is 0.

Now C2, C3 is X2 should be at least 5, X3 should be at least 4 even if you substitute X2 as 5 and X3 as 4, the sum is 9,
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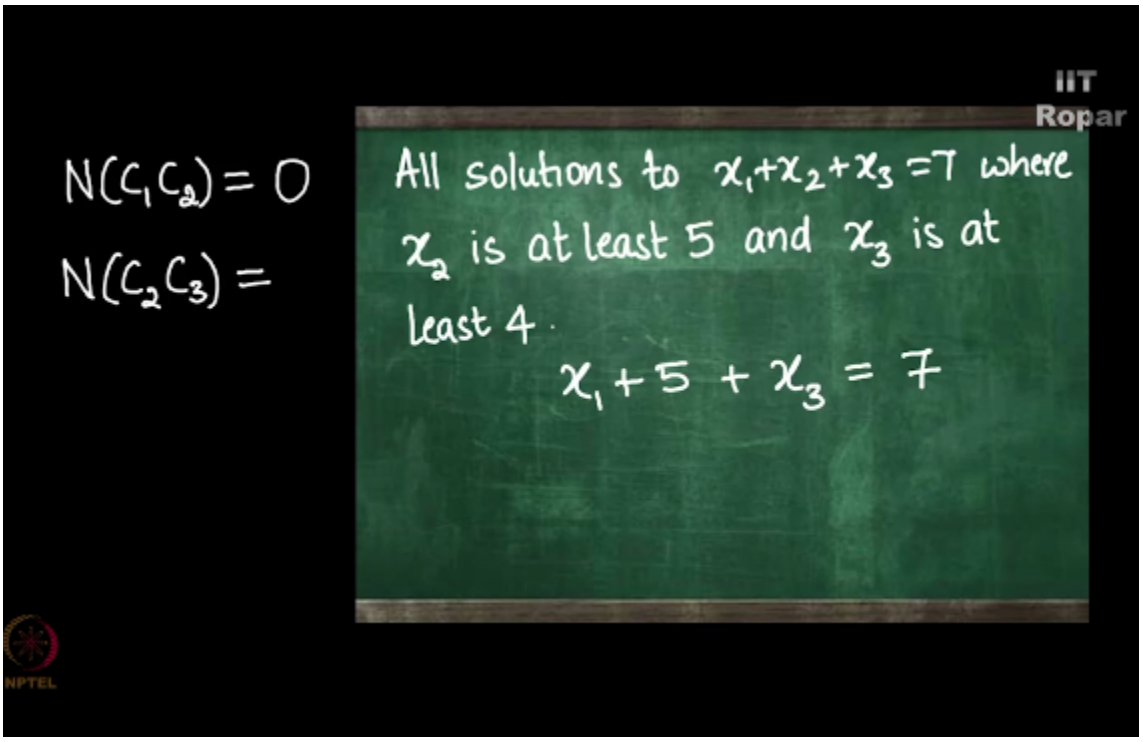

you cannot proceed ahead and hence N(C2, C3) is 0, what happens with N(C1,C3)? N(C1,C3) is those solutions where X1 is at least 3, and X3 is at least 4, if I substitute X1 to be 3 and X3 to be 4 it is satisfying the condition because I can substitute X2 to be 0 and hence the sum is 7,

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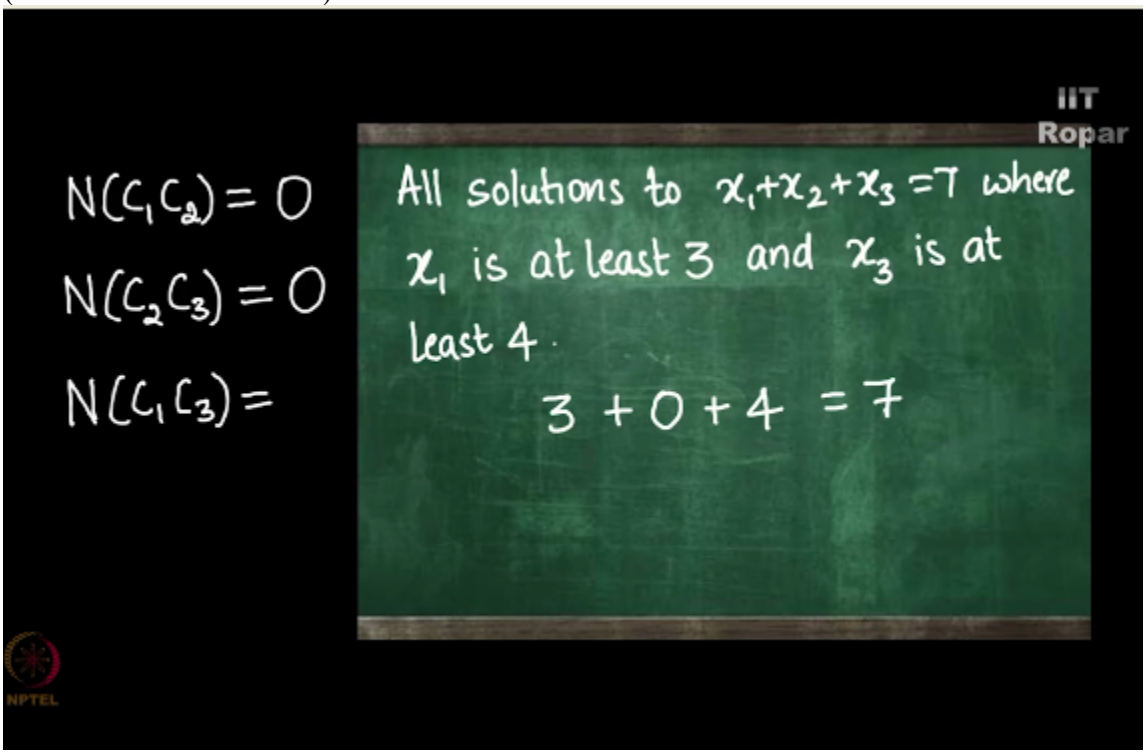

well this is the only possibility here, why? If I swap X1 to be 4 it is possible but X3 cannot be 3 because it must be at least 4, and hence no other solution is possible, the only possibility is 304, X1 3, X2 0, and X3 4, and hence N(C1,C3) is 1.

Now what about N(C1,C2,C3)? If all three conditions must satisfy that X1 must be at least 3, X2 must be at least 5, and X3 must be at least 4, then the sum is going to be at least 3 + 5 + 4 which is 12, which is an invalid solution, and hence N(C1,C2,C3) is 0.
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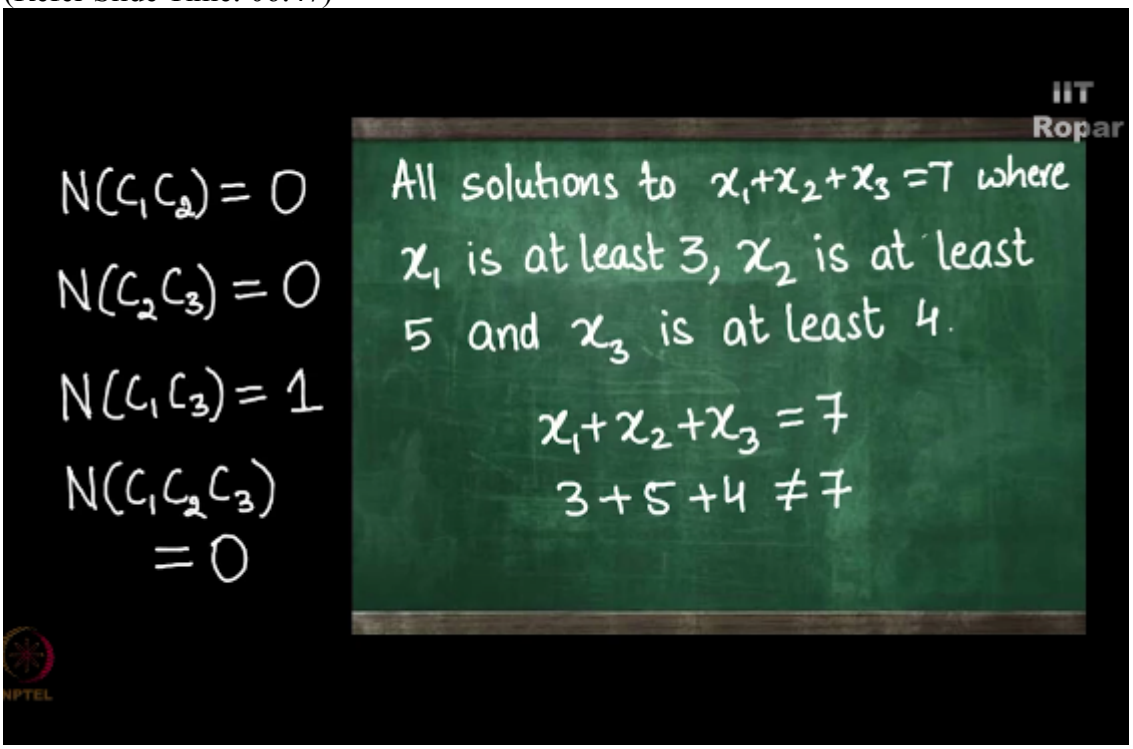

Now the question is how many solutions exist for this equation with these conditions, right, so what do we have to calculate N(C1 bar,C2 bar, and C3 bar)? N(C1 bar,C2 bar, and C3 bar) here is equal to S naught - S1 + S2 - S3 which is nothing but 36 which is S naught, S1 is 15 + 6 + 10 which is 31, so 36 - 31 + S1 which is N(C1,C2) + (C2,C3) + N(C1,C3) which is 1, so 36 - 31 + 1, and last one is -S3 which is 0 here, so we need not worry about that, so the final answer is going to be 36 - 31 + 1 which is 6,
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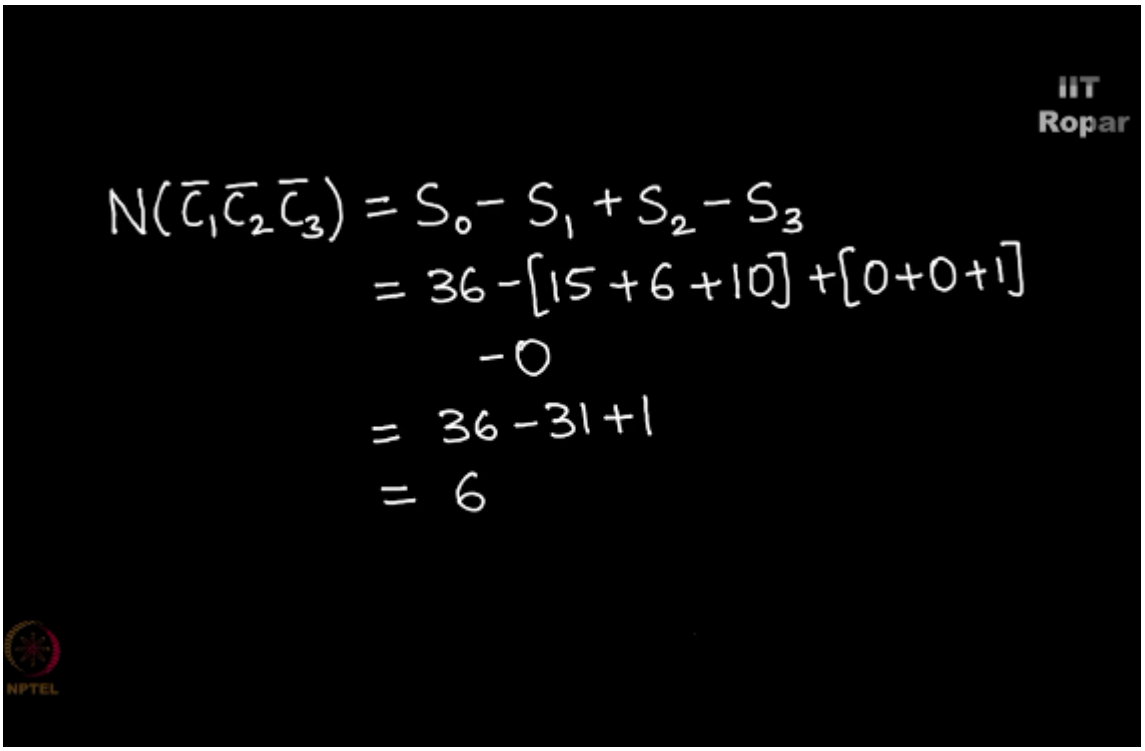

so the number of integer solutions to the equation X1 + X2 + X3 = 7 satisfying these conditions is 6.

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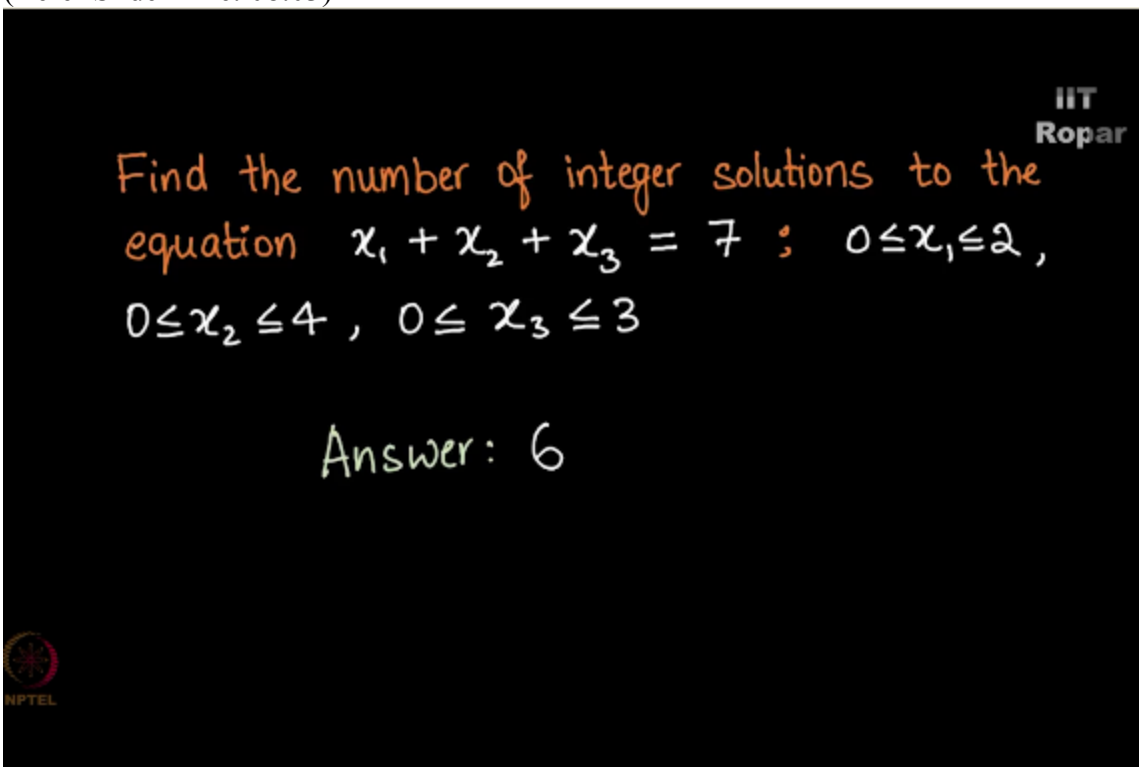

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