NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Example 6 - Integers not divisible by 5, 7, or 11

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How many positive integers from 1 to 2000 are not divisible by 5, 7 or 11? So you have to find out those integers which are not divisible by these three numbers,
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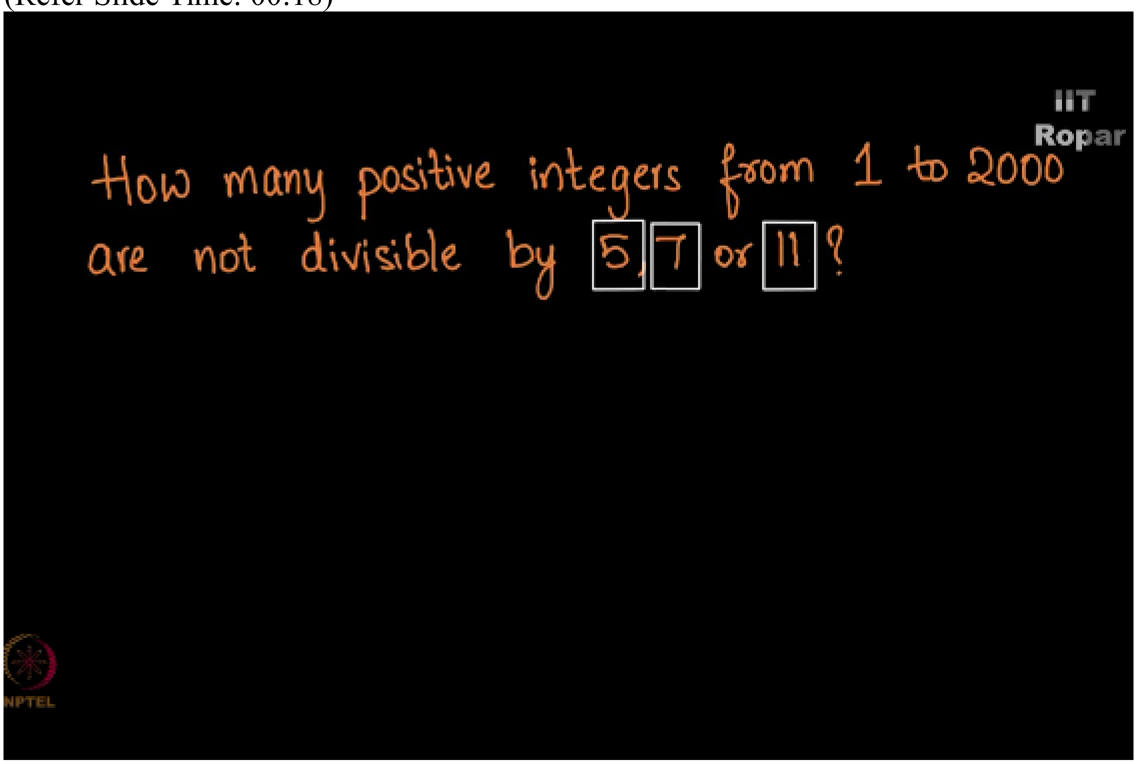

so let me see that a set S is 1, 2, 3, 4, 5, 6 so on up to 2000, the cardinality of S is 2000.

Now what is the condition C1 here? C1 is those integers which are divisible by 5, so integers satisfies this condition if it is divisible by 5, C2 is the condition that an integer is divisible by 7, and C3 is the condition that an integer is divisible by 11,
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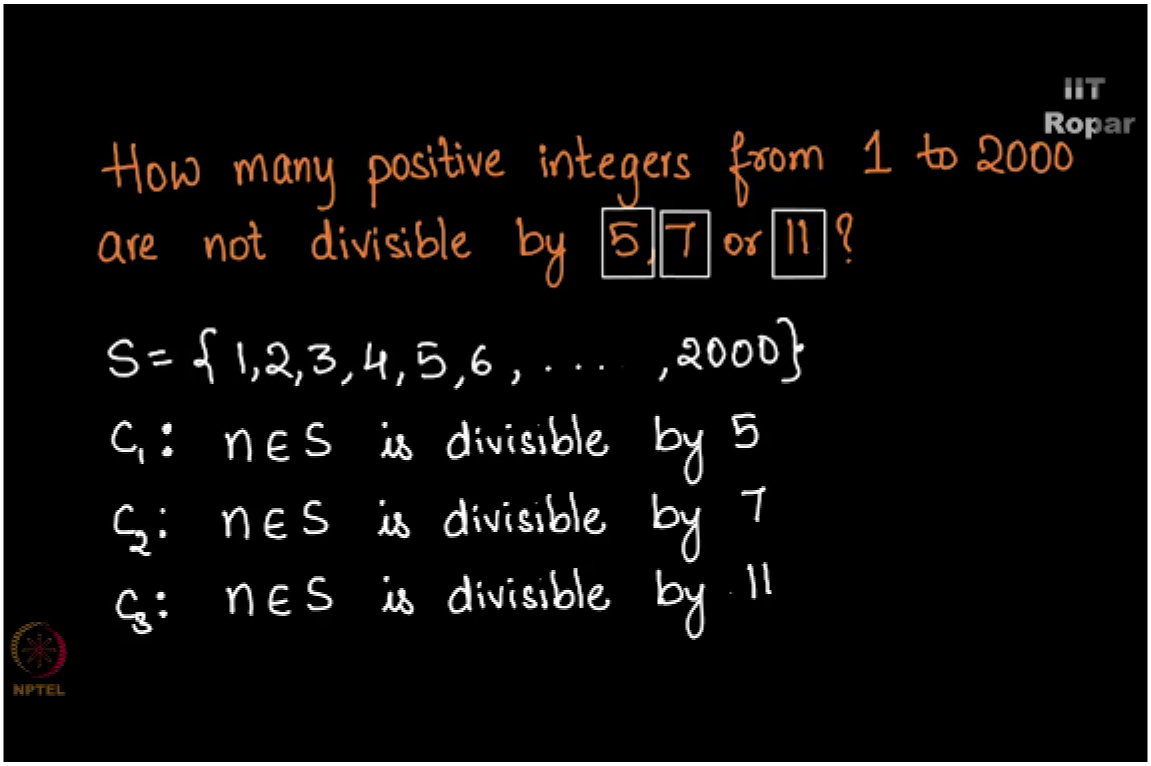

but you have to find out how many integers are not divisible by 5, 7 or 11, so again it is the question of what is N(C1 bar, C2 bar and C3 bar), so in order to find out N(C1 bar, C2 bar, C3 bar) you must know what is S naught, S1, S2, S3, why?

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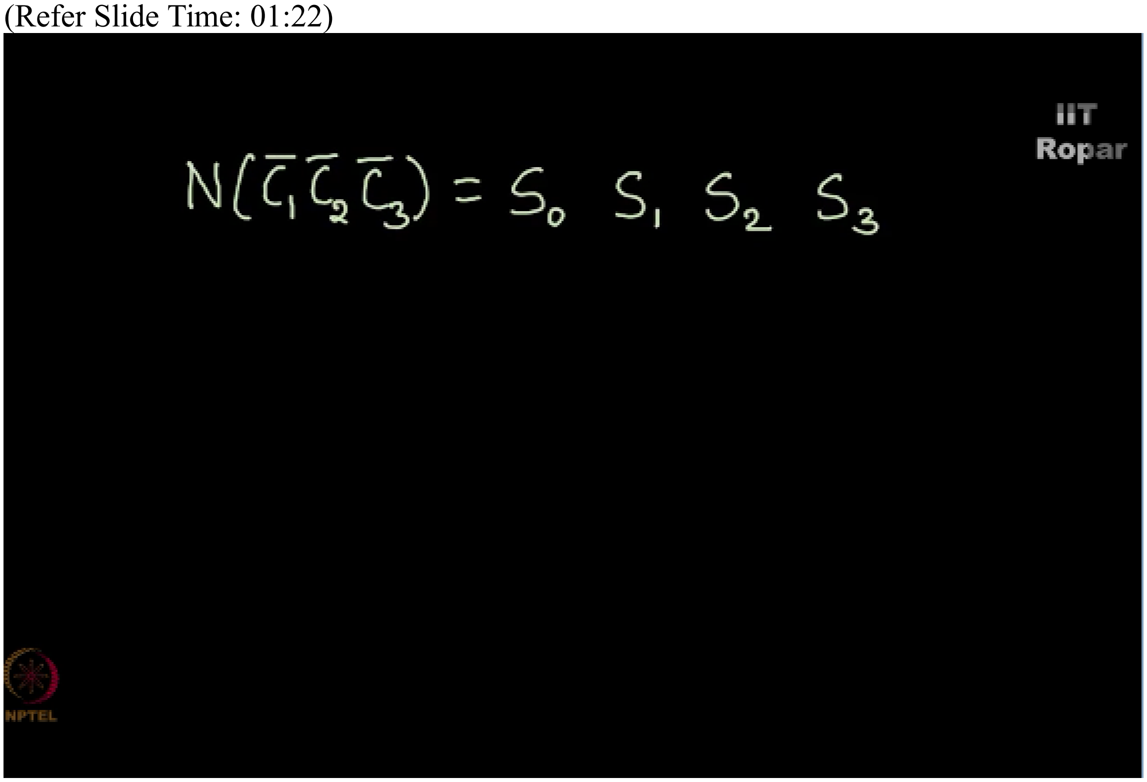

Because N(C1 bar, C2 bar, C3 bar) = S naught - S1 + S2 - S3,

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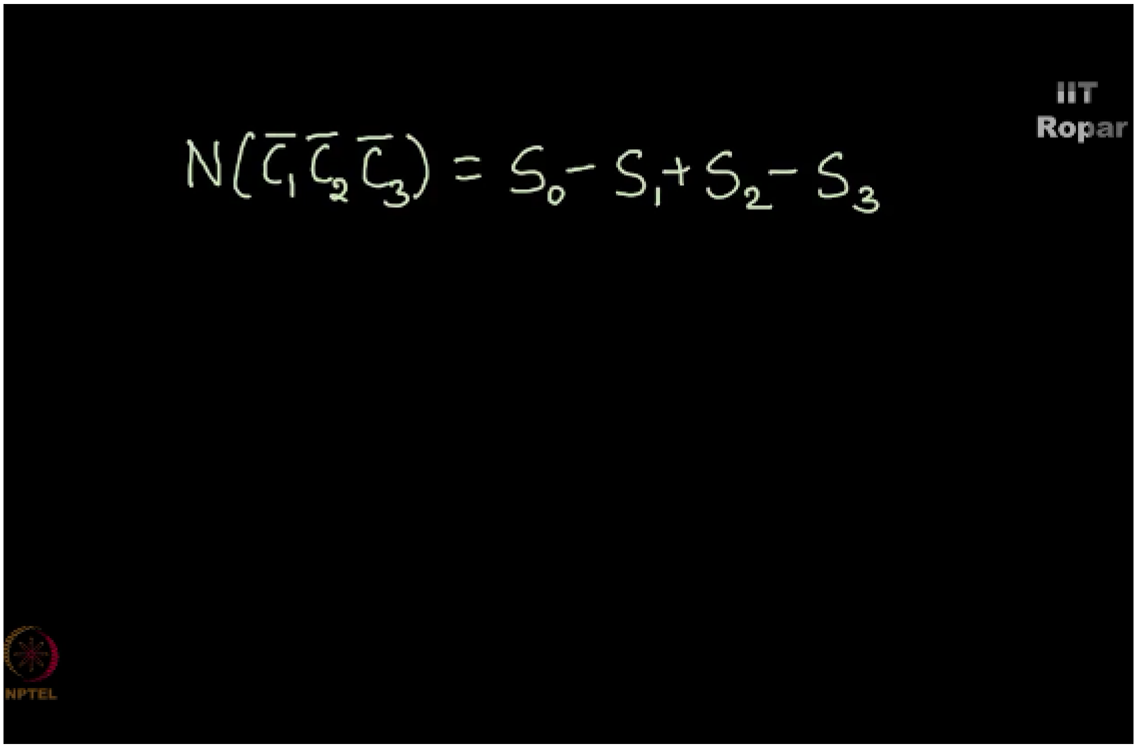

I hope all of you know what is S naught, S1, S2, S3, if not you can follow in the further steps, S naught here happens to be 2000, which is cardinality of S.

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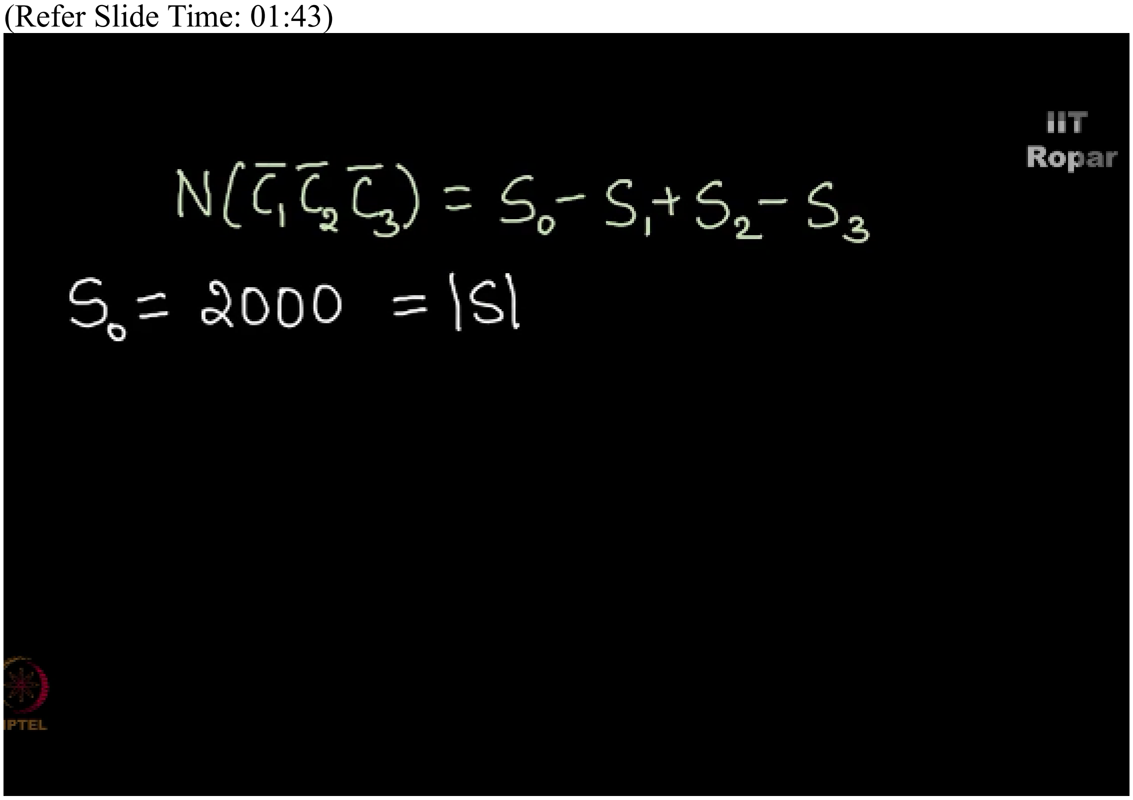

Now S1 is N(C1) + N(C2) + N(C3), N(C1) is the number of integers which are divisible by 5, how many of them are divisible by 5? So 2000/5 will give the answer which is 400,

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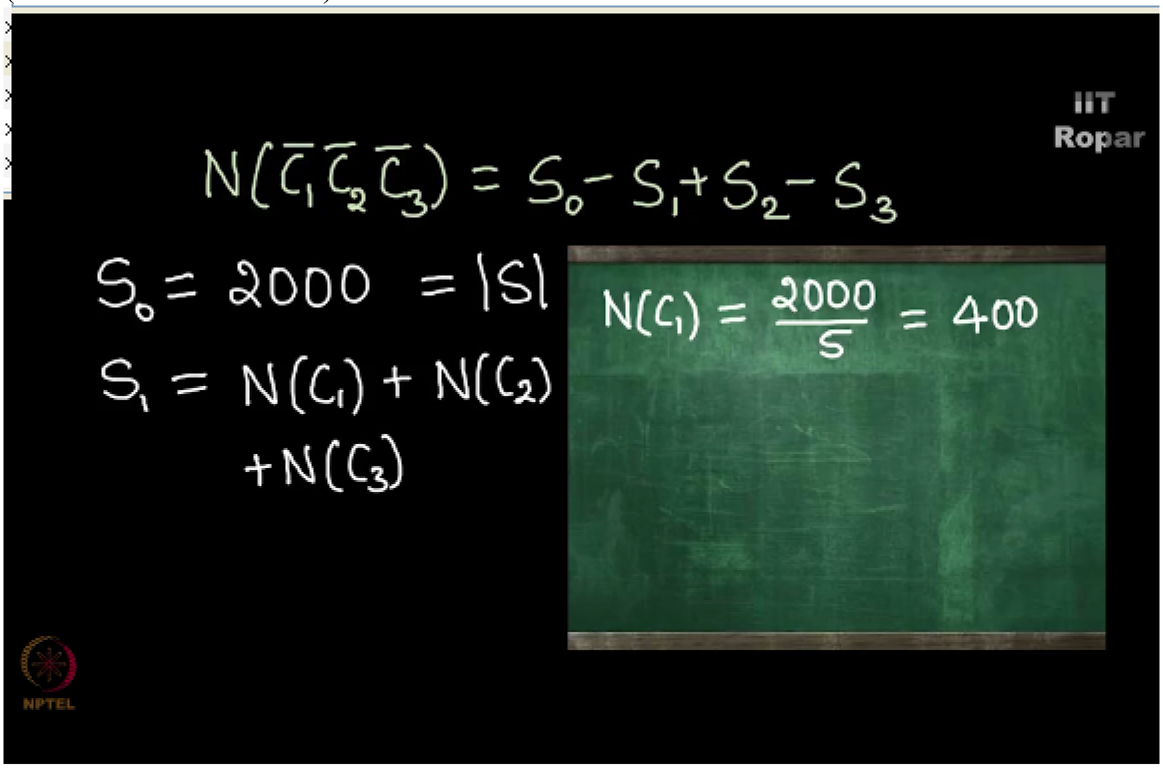

so few integers which are divisible by 0,5, 10, okay, starts from 1 so it is 5, 10, 15, 20, and so on.

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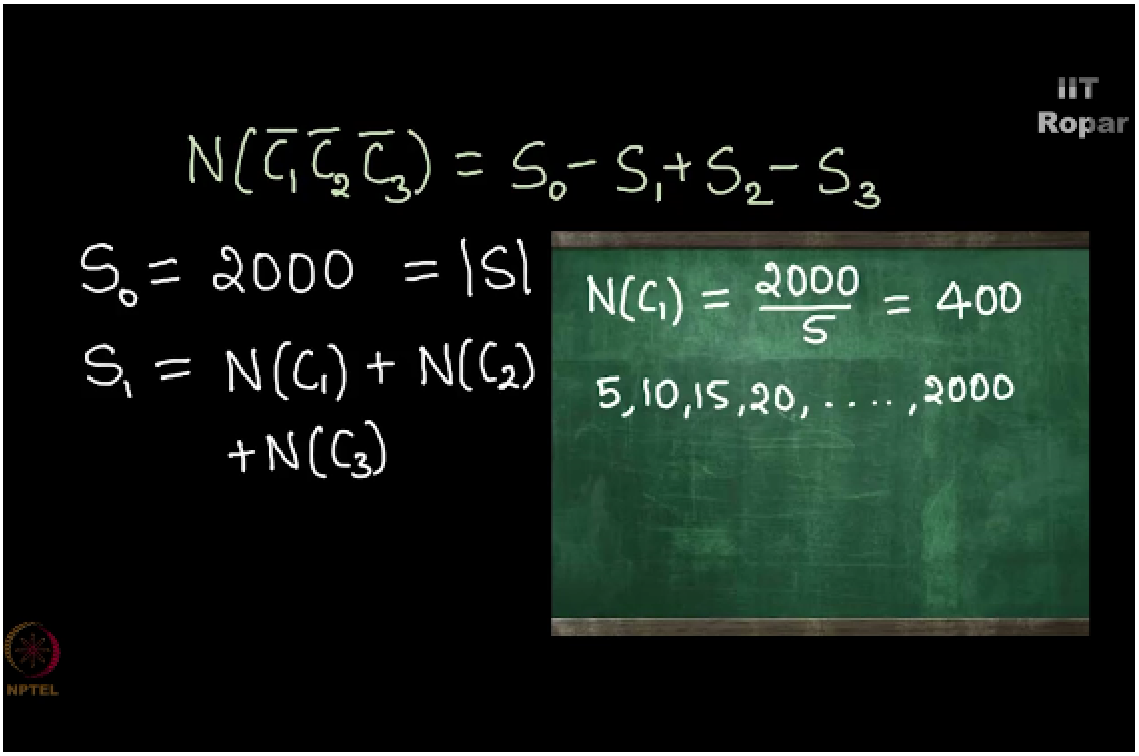

N(C2) will be those integers which are divisible by 7, so 2000/7 and we are going to apply the floor function that is the greatest integer which is less than or equal to 2000 which happens to be 285 here.

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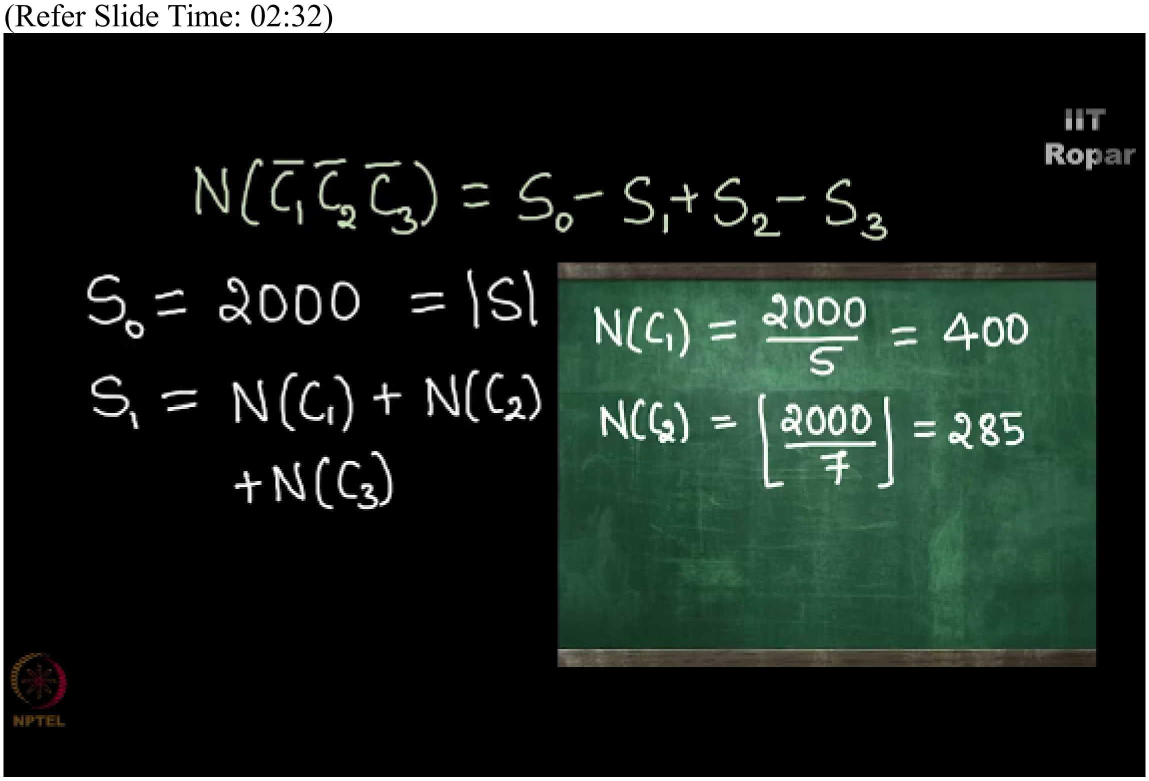

Now N(C3) is those integers which are divisible by 11, and that happens to be 2000 divided by 11 which is 181,

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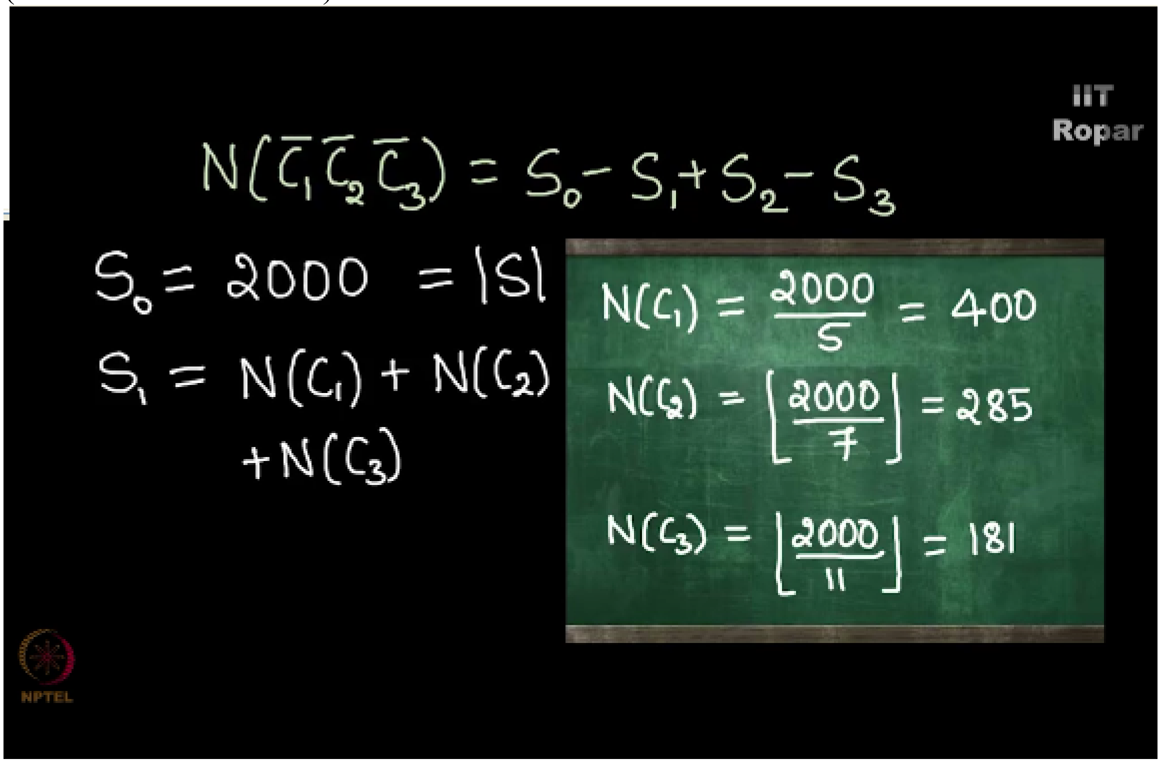

now we have now got N(C1), N(C2) and N(C3), what is N(C1, C2)? Where you have to find out those integers which are divisible by 5 as well as 7, so we take the LCM of 5 and 7 which happens to be 35 so you have to calculate those integers which are divisible by 35, so 2000 divided by 35 happens to be 57, right,

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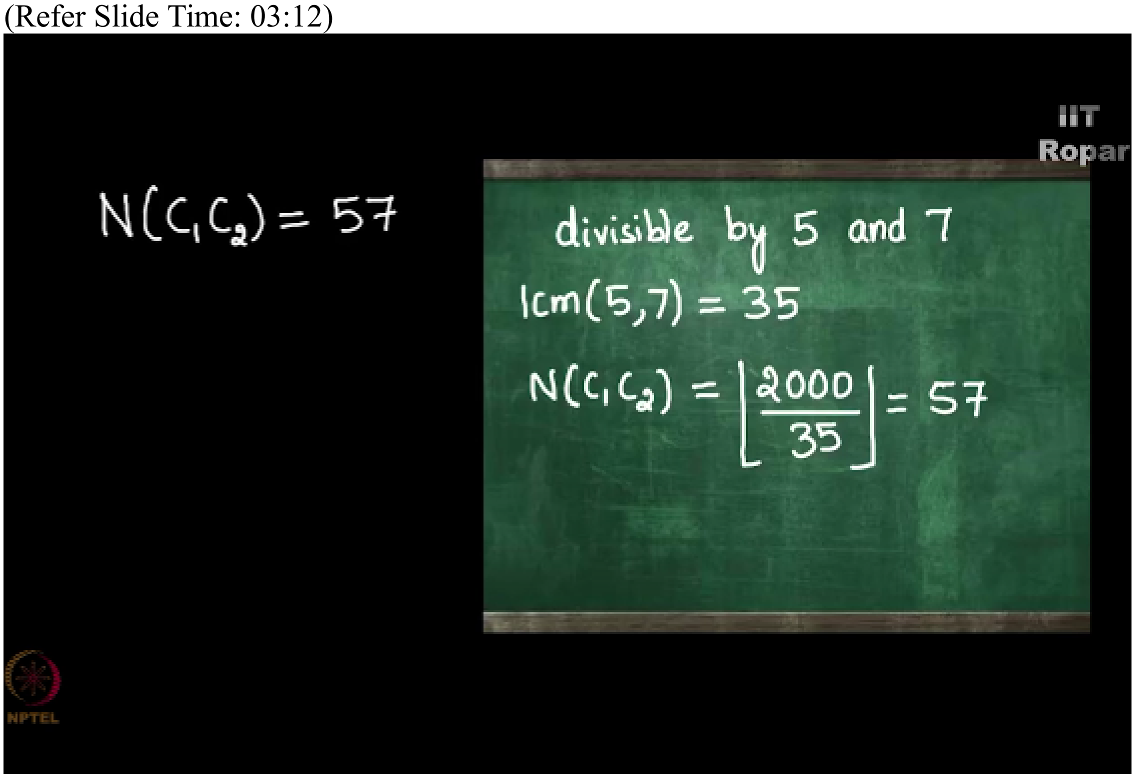

now N(C2, C3) will be the number of integers which are divisible by 7 as well as 11, LCM happens to be 77, so 2000 by 77 is 25, and N(C1,C3) is those integers which are divisible by 5 and 11,

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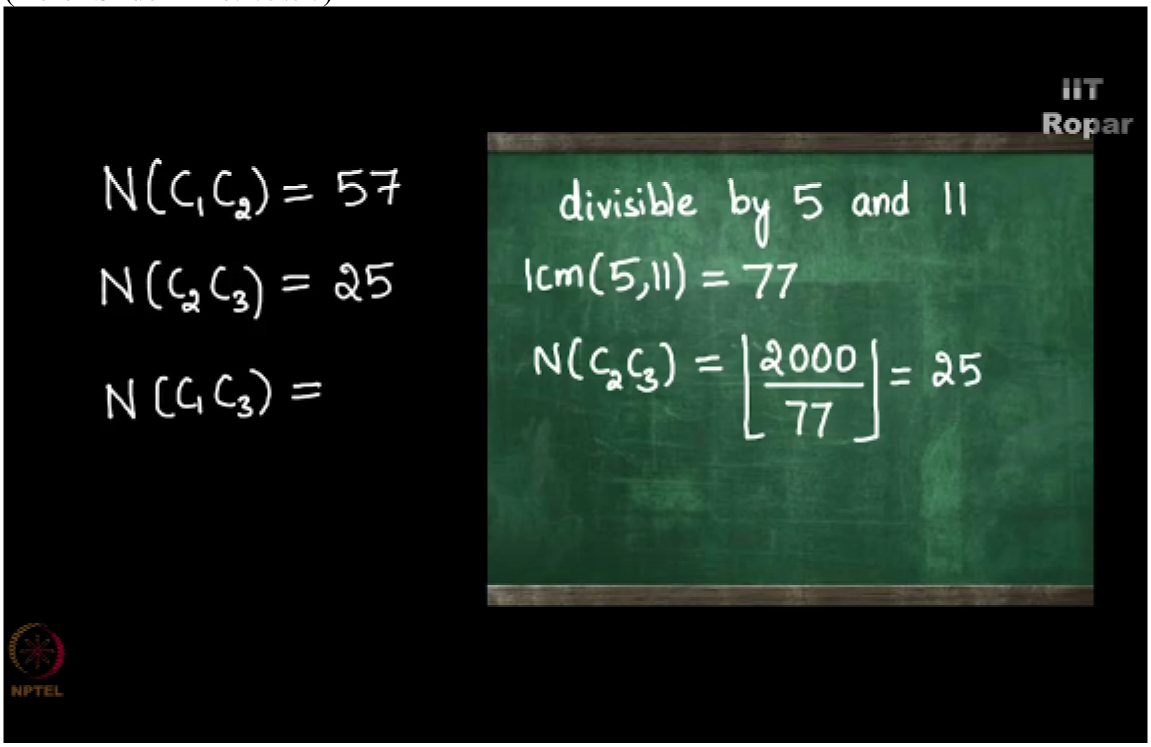

how do we do that? LCM of 5 and 11 happens to be 55, so 2000 divided by 55 is 36,

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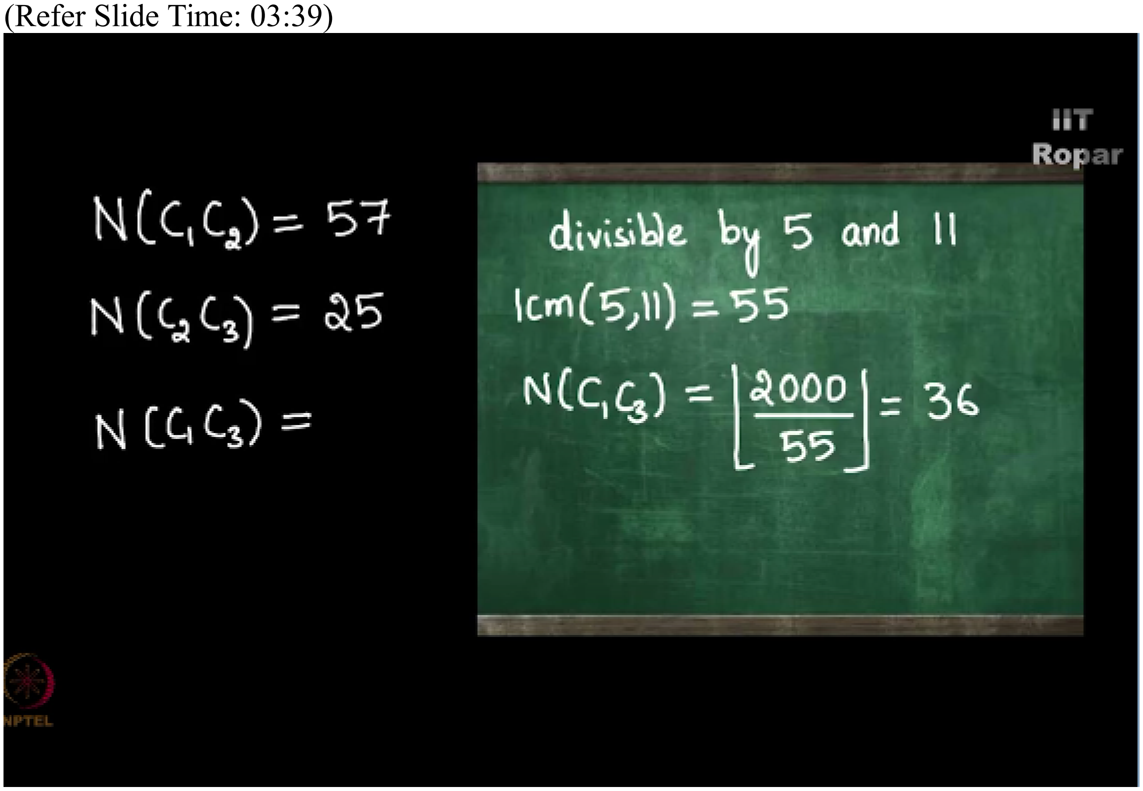

so N(C1,C2) is 57, N(C2,C3) is 25, and N(C1,C3) is 36.
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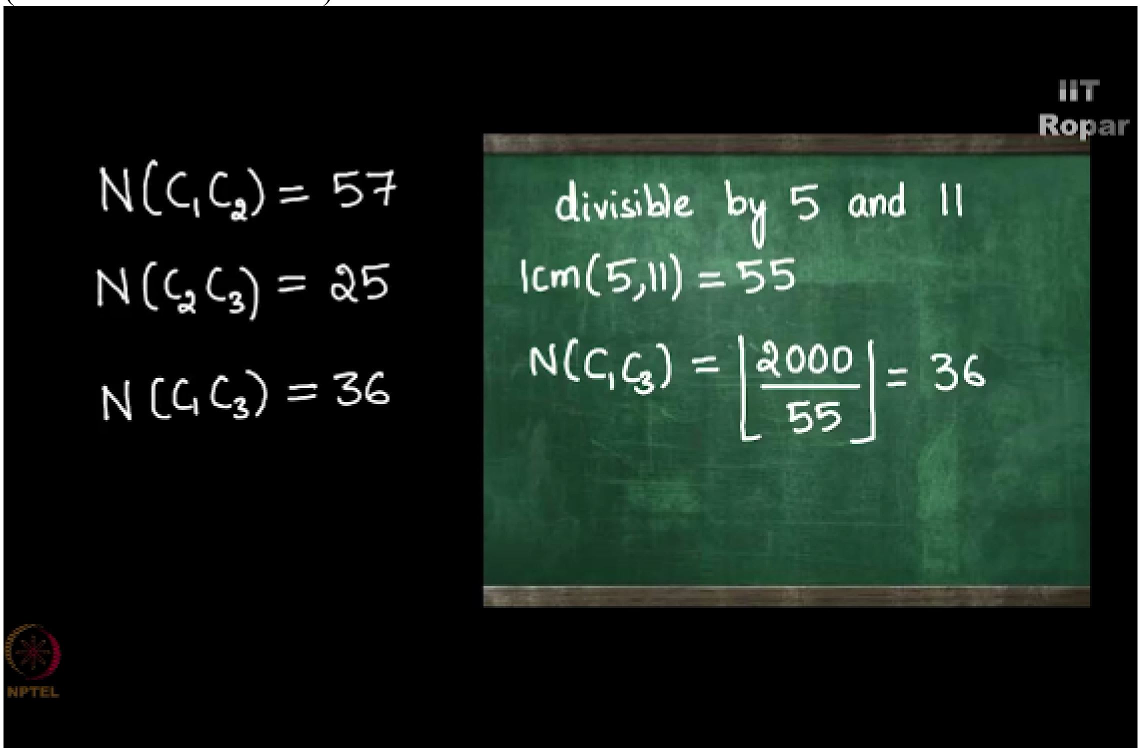

Now N(C1,C2,C3) is the number of integers which are divisible by 5, 7 and 11, now the LCM of 5, 7, and 11 happens to be 5 x 7 x 11 which is 385, and 2000 divided by 385 is 5, so2,000 divided by 385 will give you 5 on applying the floor function,
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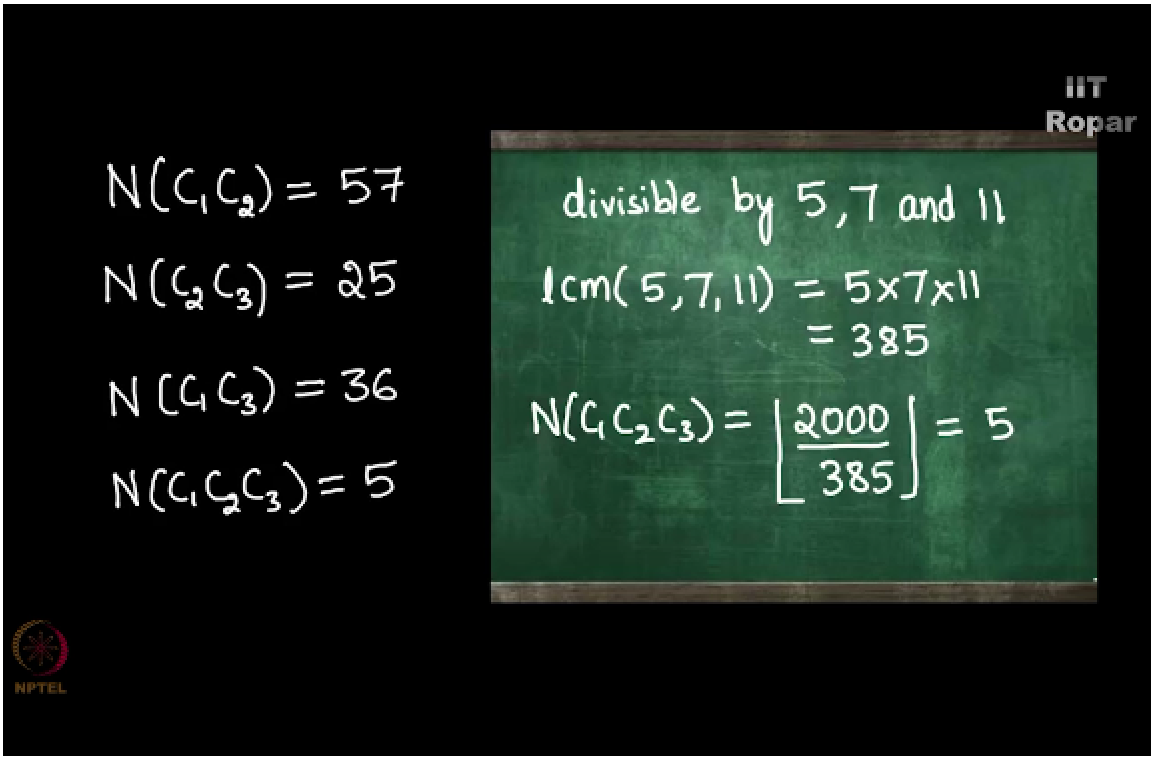

now N(C1 bar, C2 bar, C3 bar) is S naught - S1 + S2 – S3, we know that S naught is 2000, S1 happens to be 400 + 25 + 181, and S2 is 57 + 25 + 36, and S3 is 5,

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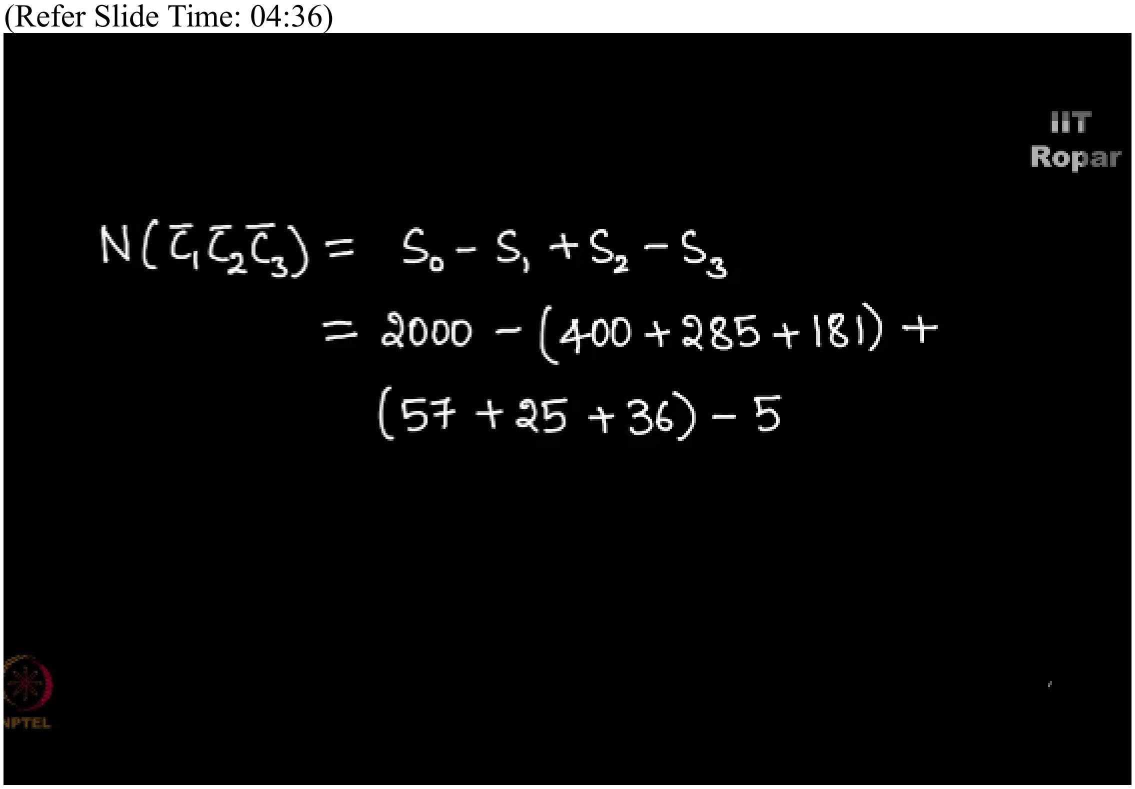

so we see that N(C1 bar, C2 bar, C3 bar) is 2000 - 400 + 25 + 181 happens to be 866, and 57 + 25 + 36 happens to be 118 – 5, so on calculation the final answer is 1247, so 1247

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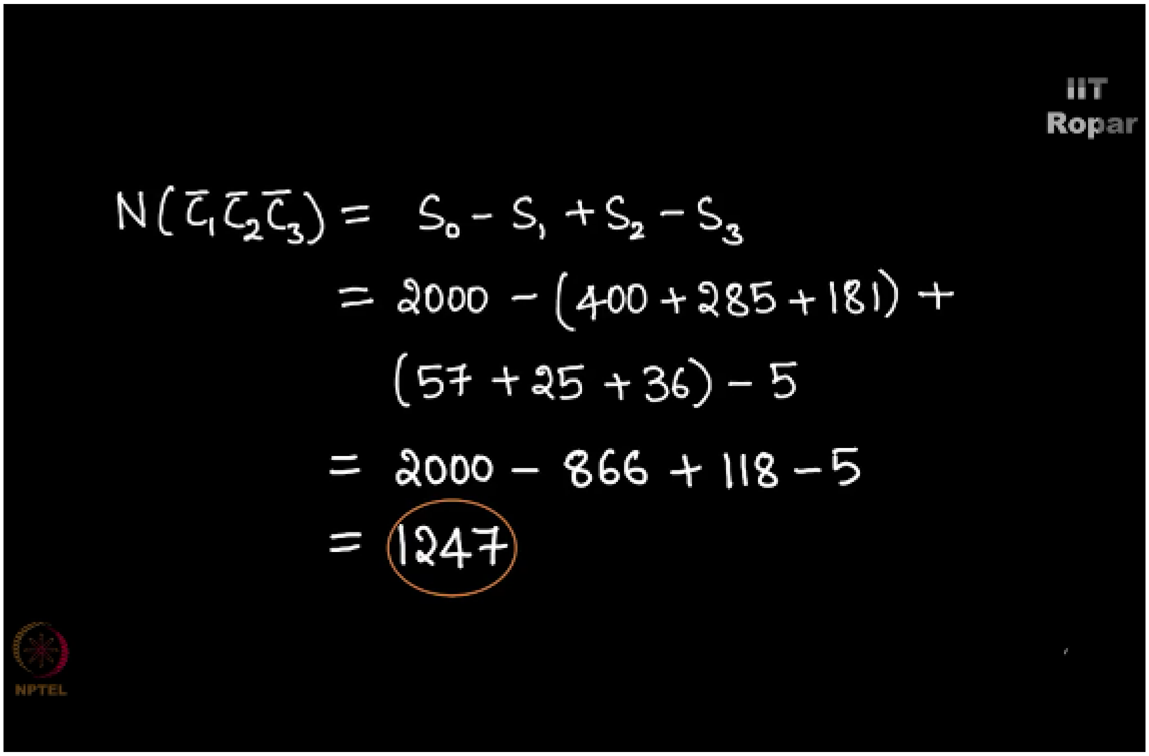

numbers are there between 1 and 2000 which are not divisible by 5, 7 or 11.

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