NPTEL

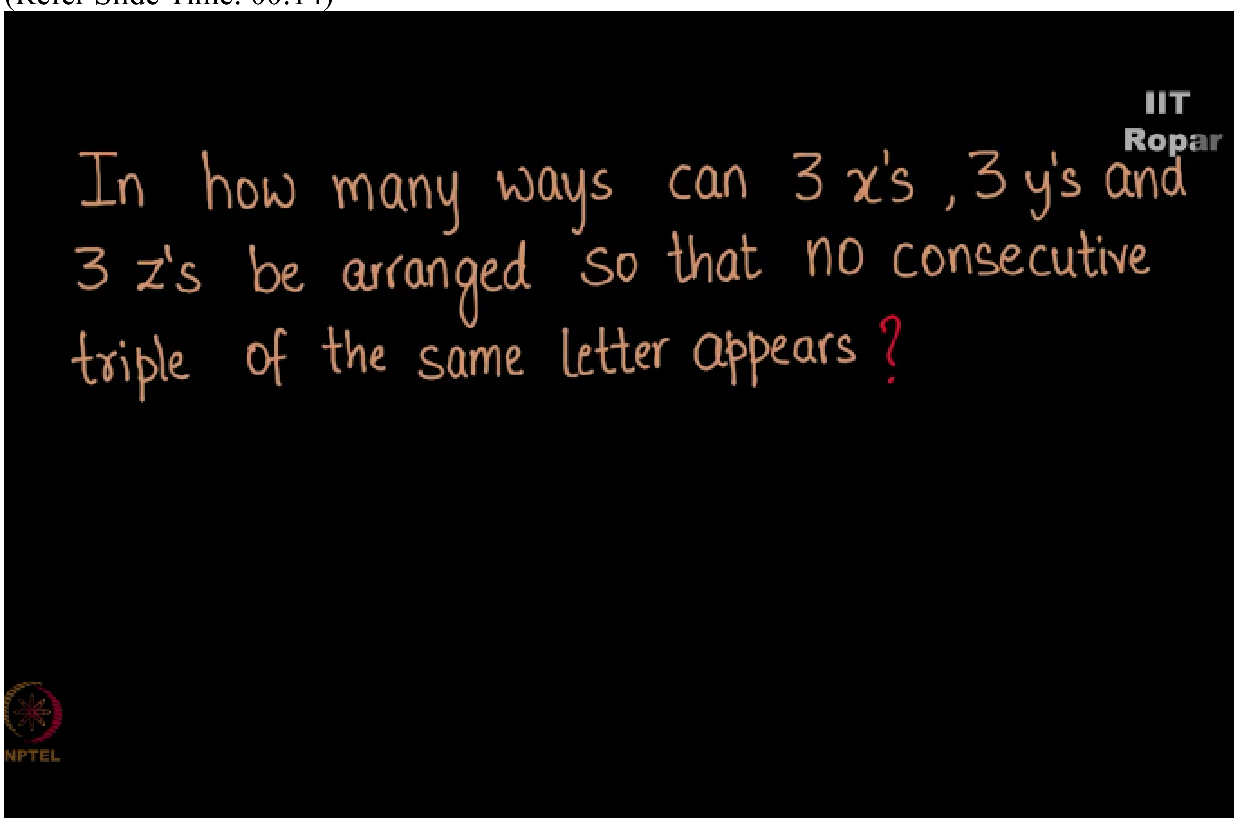
NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

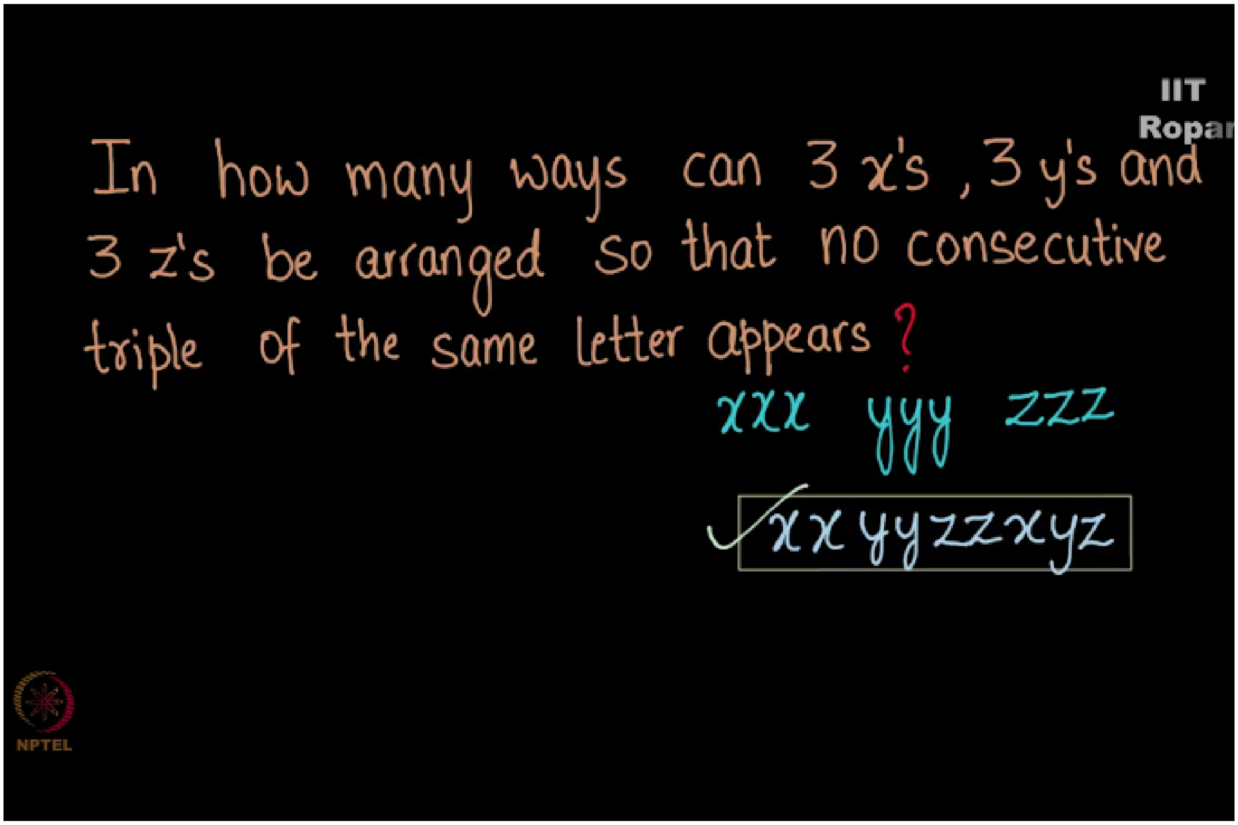
Example 4 - Arranging 3 x's, 3 y's and 3 z's

By
Prof. S.R.S Iyengar
Department of Computer Science
IIT Ropar

In how many ways can 3 X's, 3 Y's, and 3 Z's be arranged so that no consecutive triple of the same letter appears?
(Refer Slide Time: 00:14)

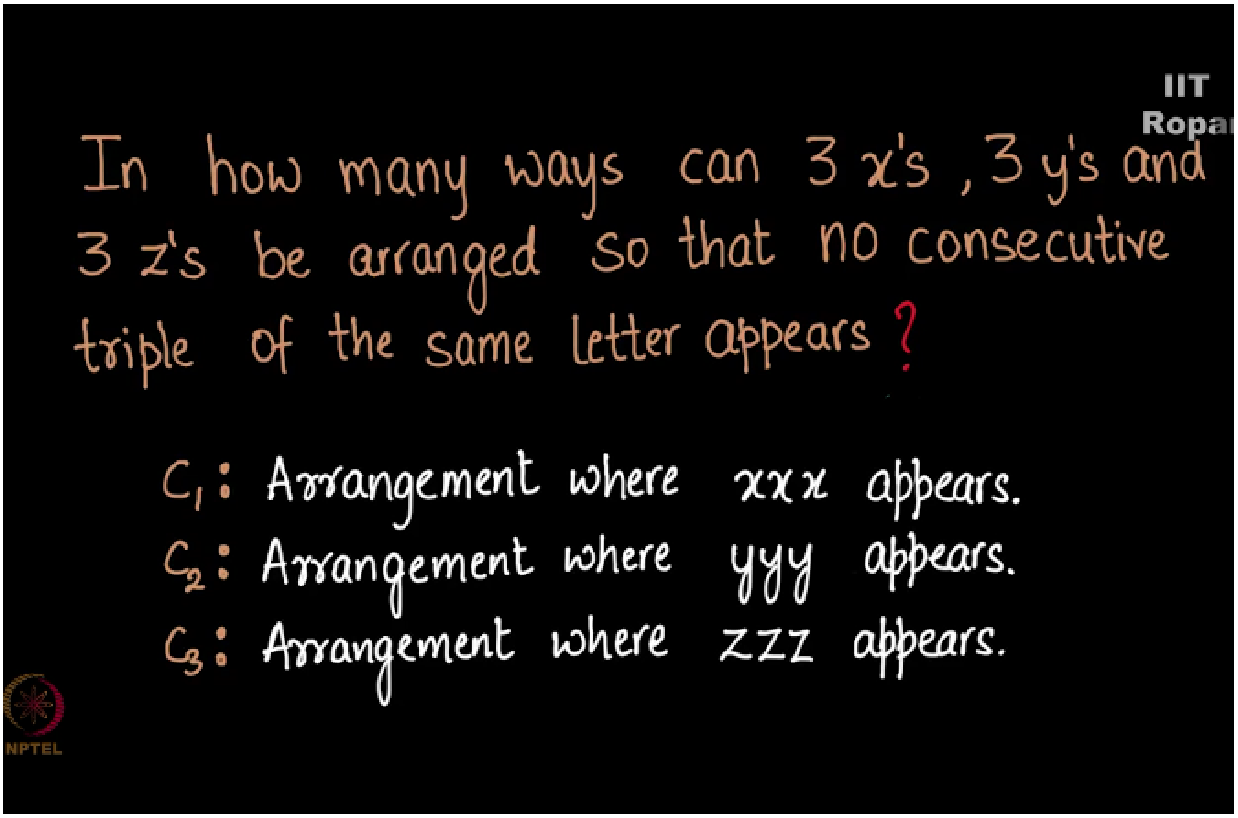


The question is not hard let me explain, so you have 3 X given, 3 Y given, and 3 Z's given, so in how many ways can you arrange them so that you do not have 3 Y together, 3 X together, or 3 Z together,
(Refer Slide Time: 00:31)



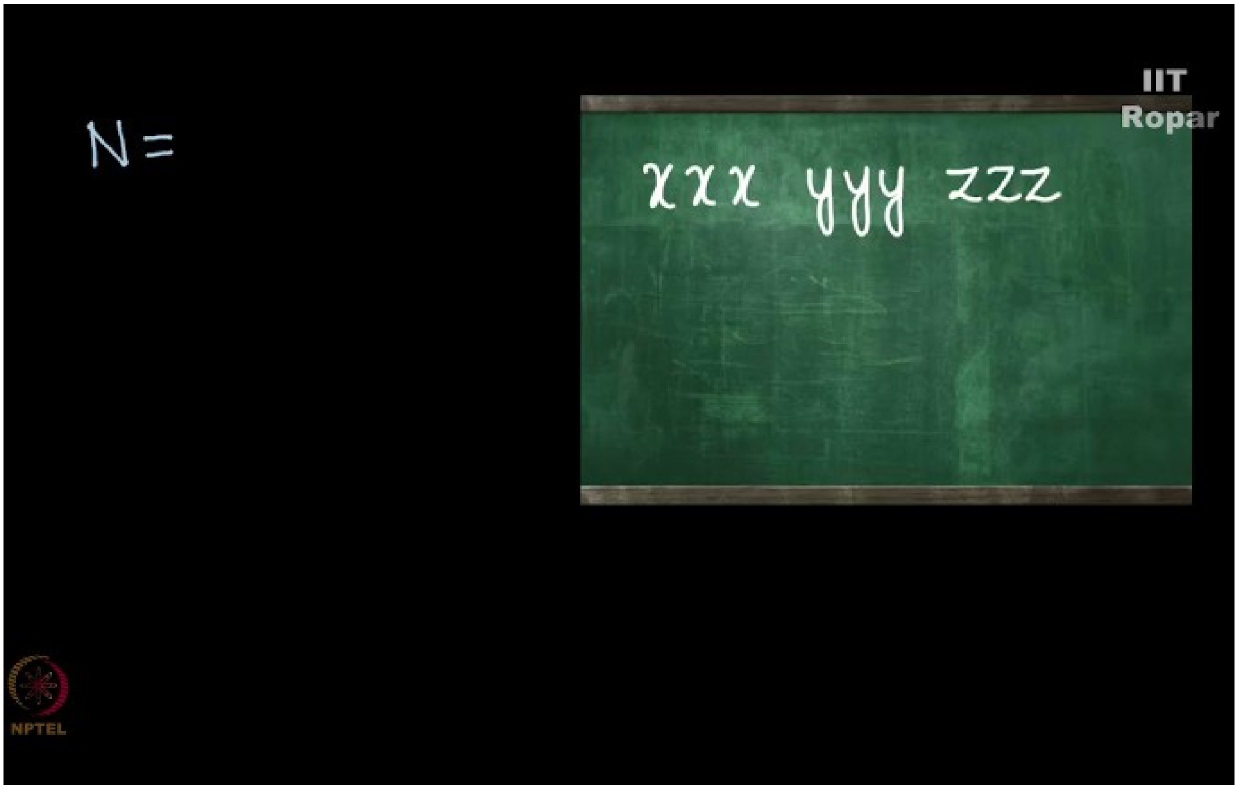
right, it should not appear together.

Now let the 3 conditions be like this, let's C1 be the arrangements where XXX appears, right, now let C2 be the condition where arrangements where YYY appears, let's C3 be arrangements where ZZZ appears, right, so out of the 9 alphabets given you have these three as C1, C2, C3, and the corresponding arrangements,
(Refer Slide Time: 01:09)

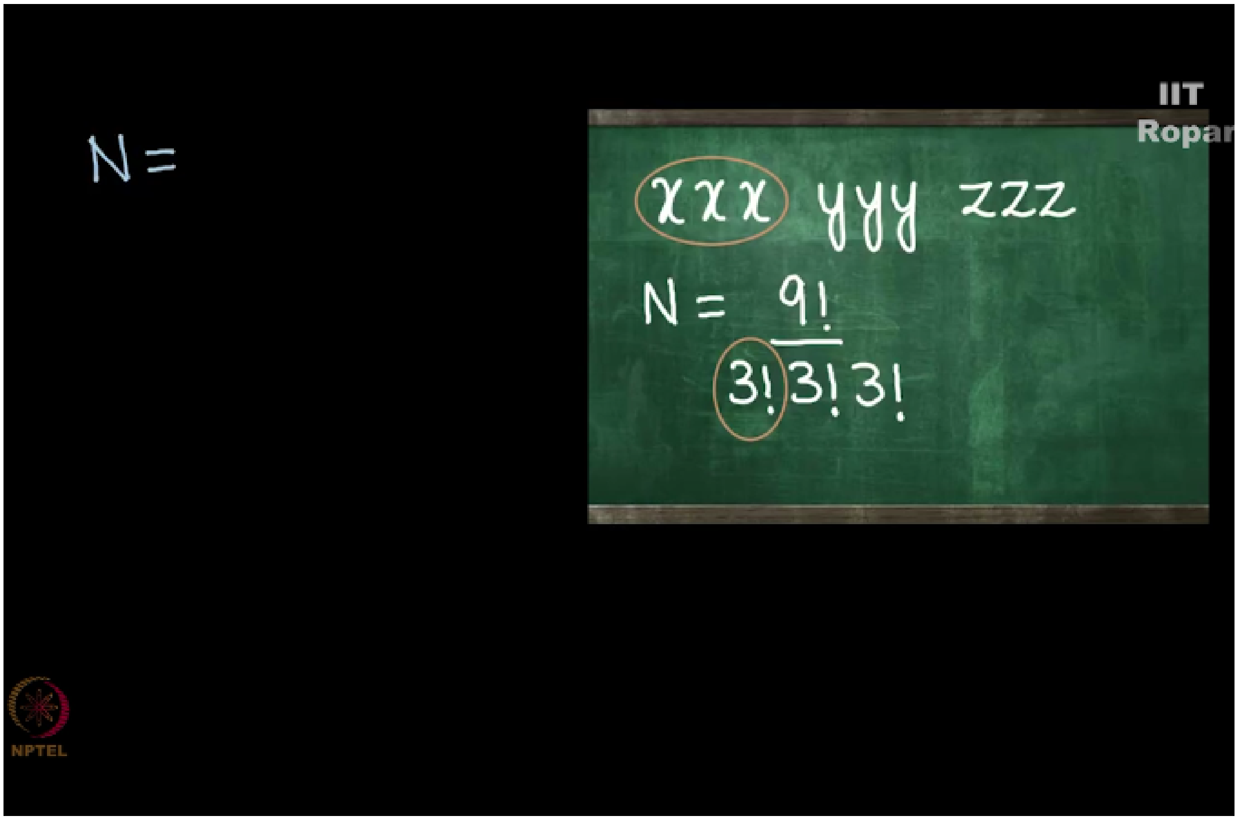


I hope C1, C2, C3 is clear to you, if not pause the video and think now.

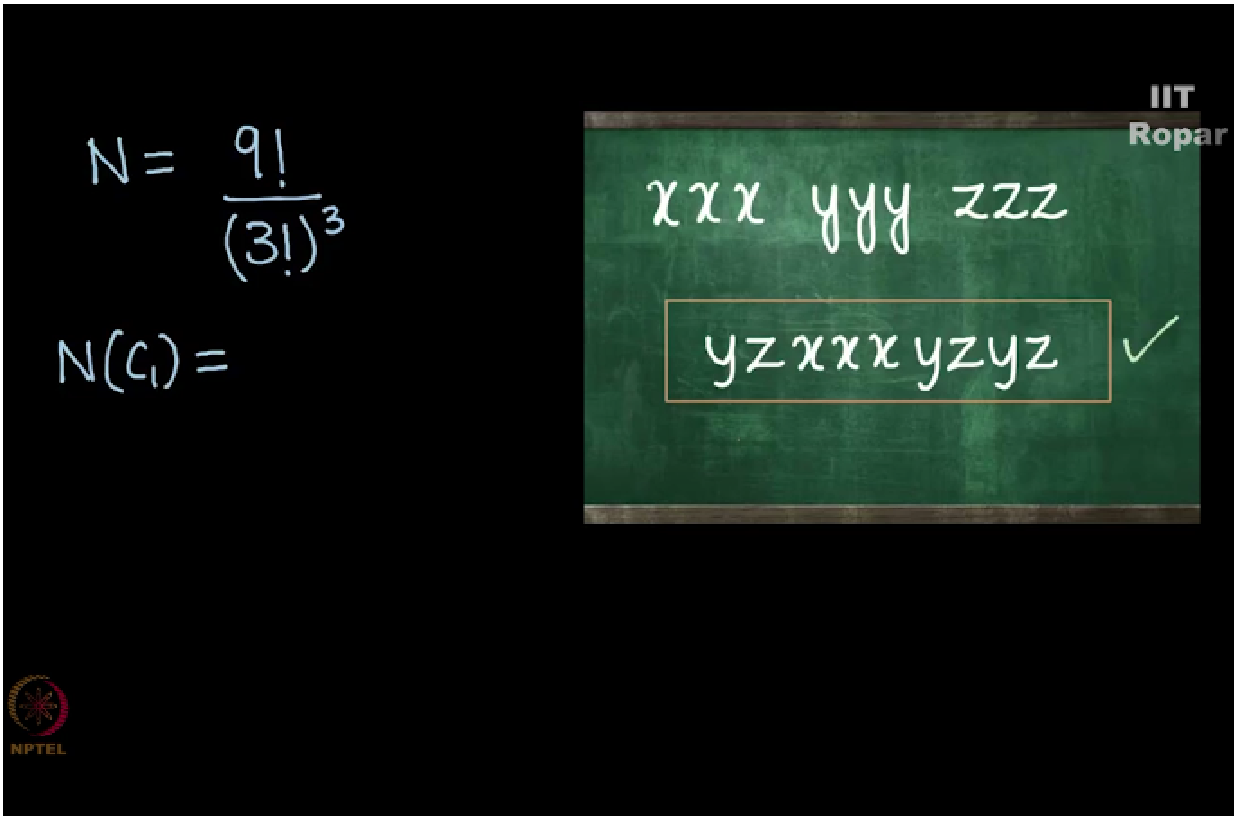
Next what is N here?
(Refer Slide Time: 01:18)



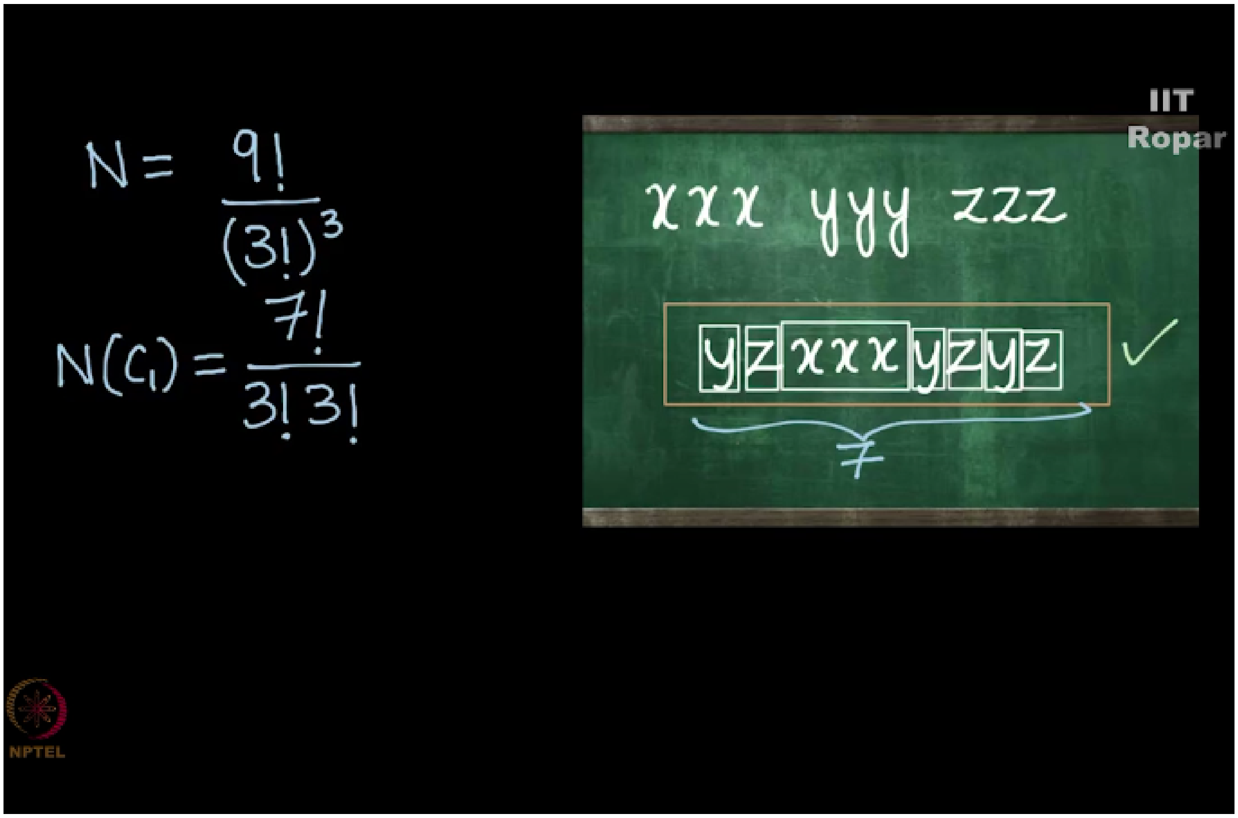
N happens to be all possible arrangements of these 9 alphabets given 3X, 3Y, 3Z and all possible arrangements, how many of them are there? So you have 9 alphabets in all, so it is 9 factorial / 3 factorial x 3 factorial x 3 factorial, why? If you do not catch it here then you must probably revisit multinomial theorem, right, because X is repeating, Y is repeating, Z is repeating each of them thrice, so we divided by 3 factorial thrice.
(Refer Slide Time: 01:56)



Now so 9 factorial/3 factorial whole cube this happens to be N, what is N(C1)? How many arrangements are there where XXX appears, so something like this can happen XXXYZYZYZ or XXXZYZYZY or YZXXXYZYZ,
(Refer Slide Time: 02:27)

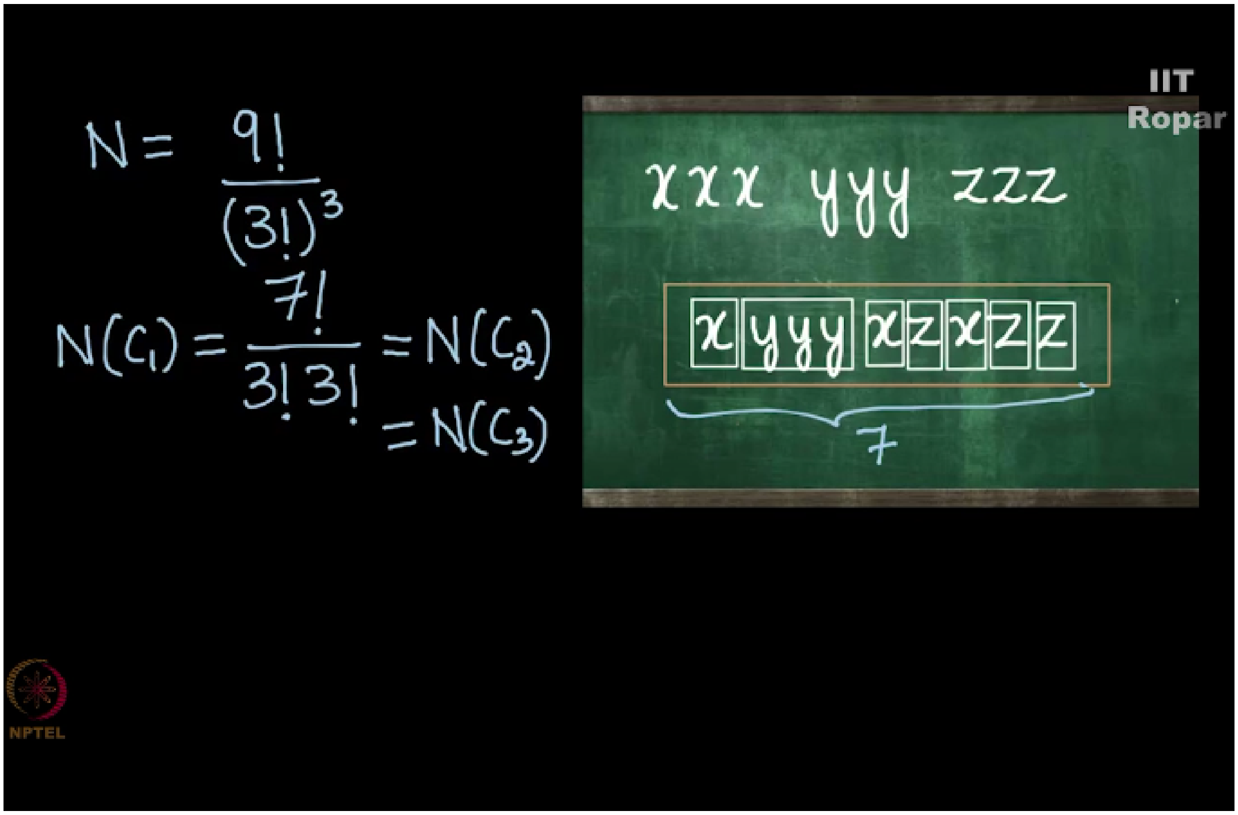


so you see so on, in all these you have XXX together, so how many such possibilities are there? You see I am going to consider XXX as 1 unit and the remaining 6 alphabets as individual 6 alphabets, so it is 6 + 1 unit which happens to be, and the number of arrangements of these happens to be 7 factorial divided by 3 factorial x 3 factorial, why? Because you have 3 Y's and 3 Z's, right, so XXX are considered to be 1 unit, so we are not dividing 3 factorial thrice here, observe here, so N(C1) happens to be 7 factorial by 3 factorial x 3 factorial.



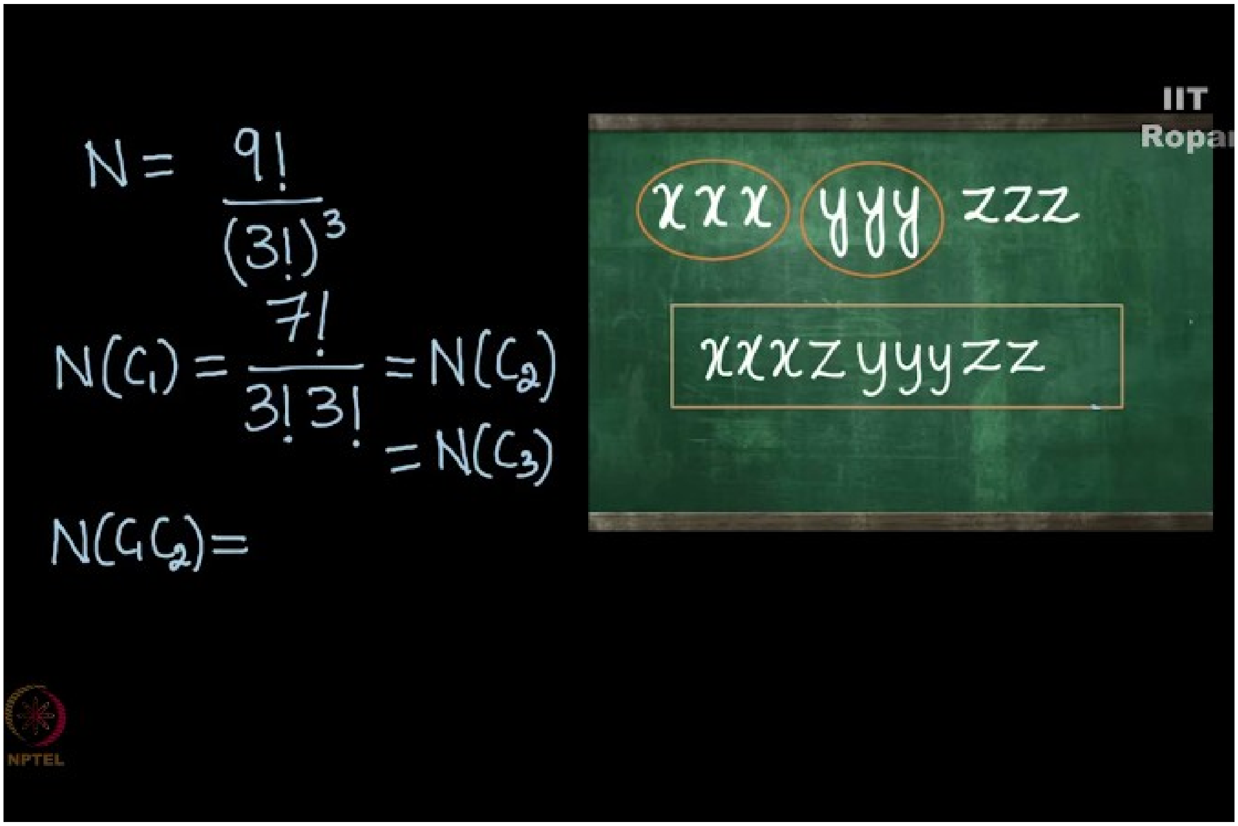
Now don't you think it is the same for N(C2) and N(C3) as well, because I can write it as XYYYXZXZ and X, so I'm again having here YYY as one unit, and the rest 6 of them as individual units which happens to be 7 factorial/3 factorial x 3 factorial, same holds true for C3 as well,

(Refer Slide Time: 03:45)



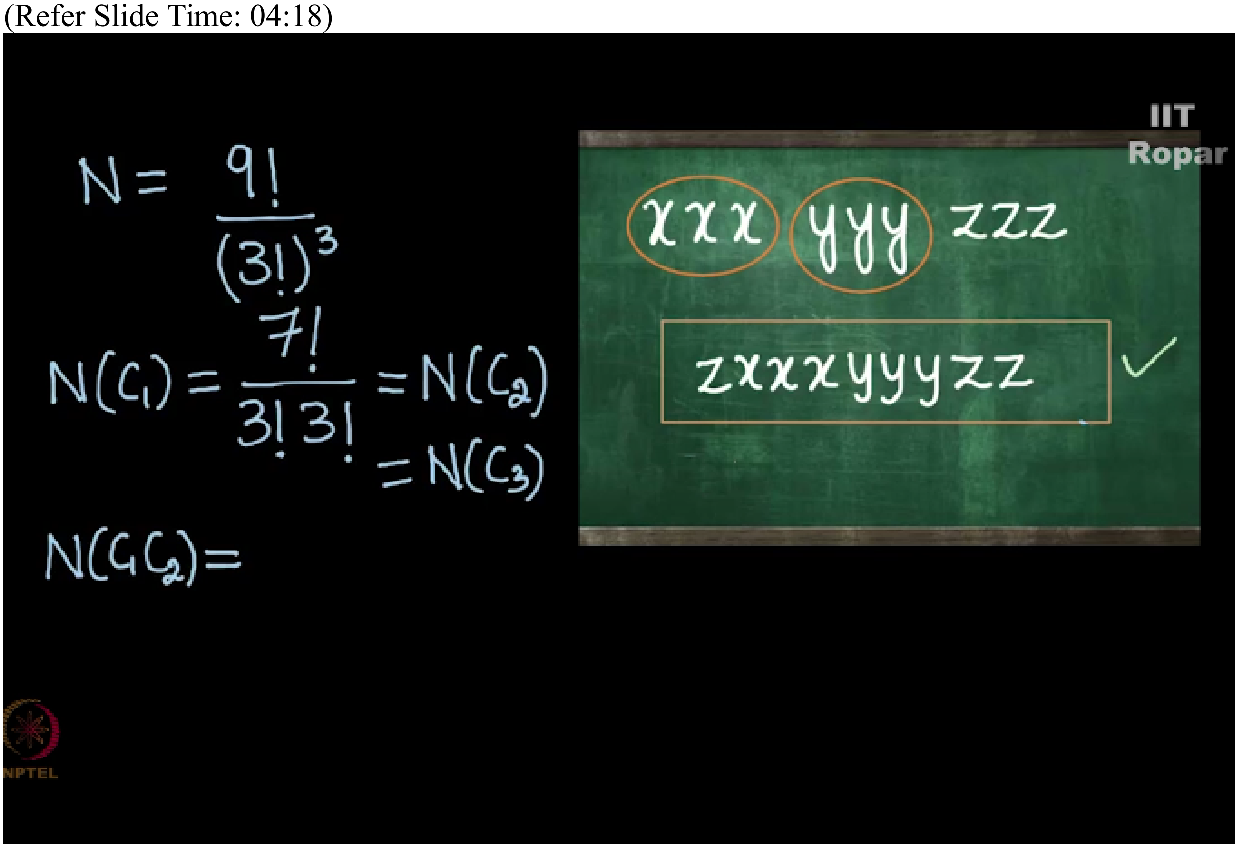
so N(C1), N(C2) and N(C3) are 7 factorial/3 factorial x 3 factorial, well now what is N(C1,C2)? Now we have to consider those arrangements where XXX and YYY will appear, so it is going to look something like this XXXZYYYZZ

(Refer Slide Time: 04:12)

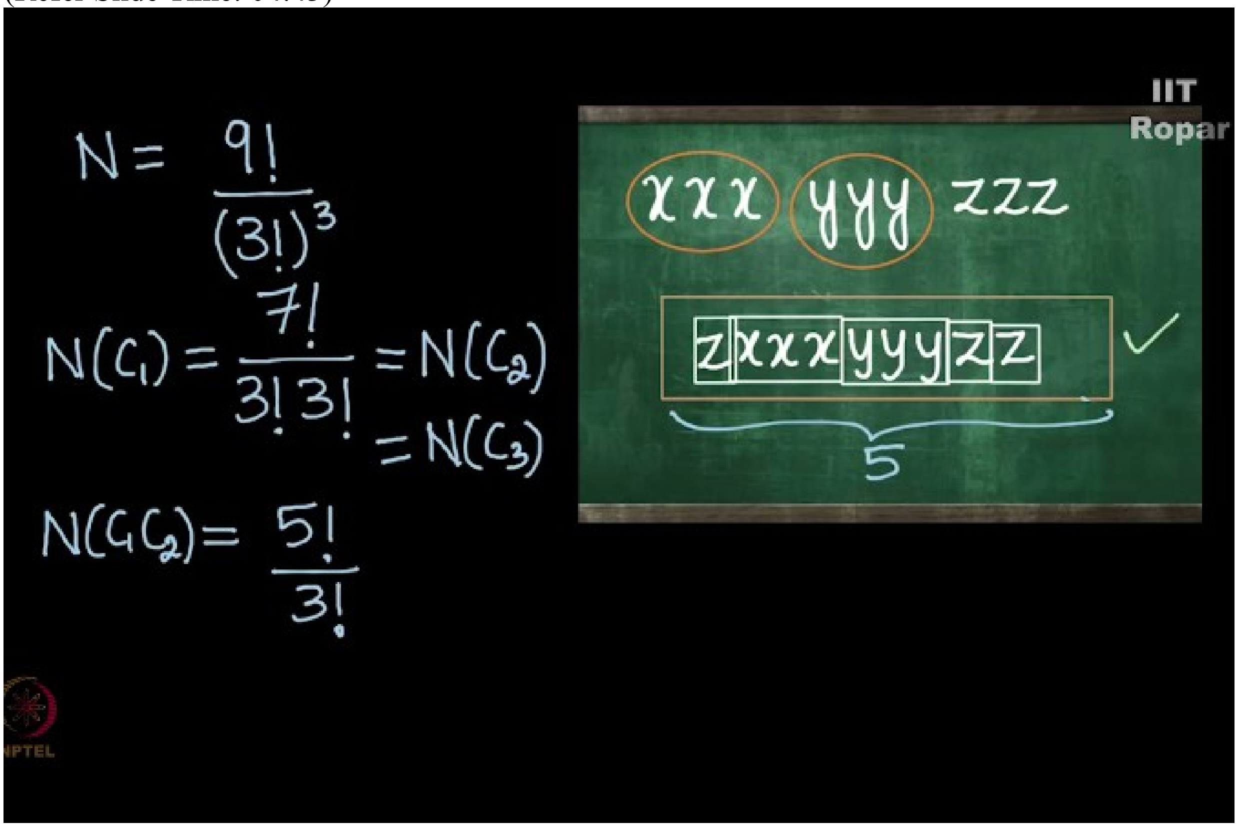


or

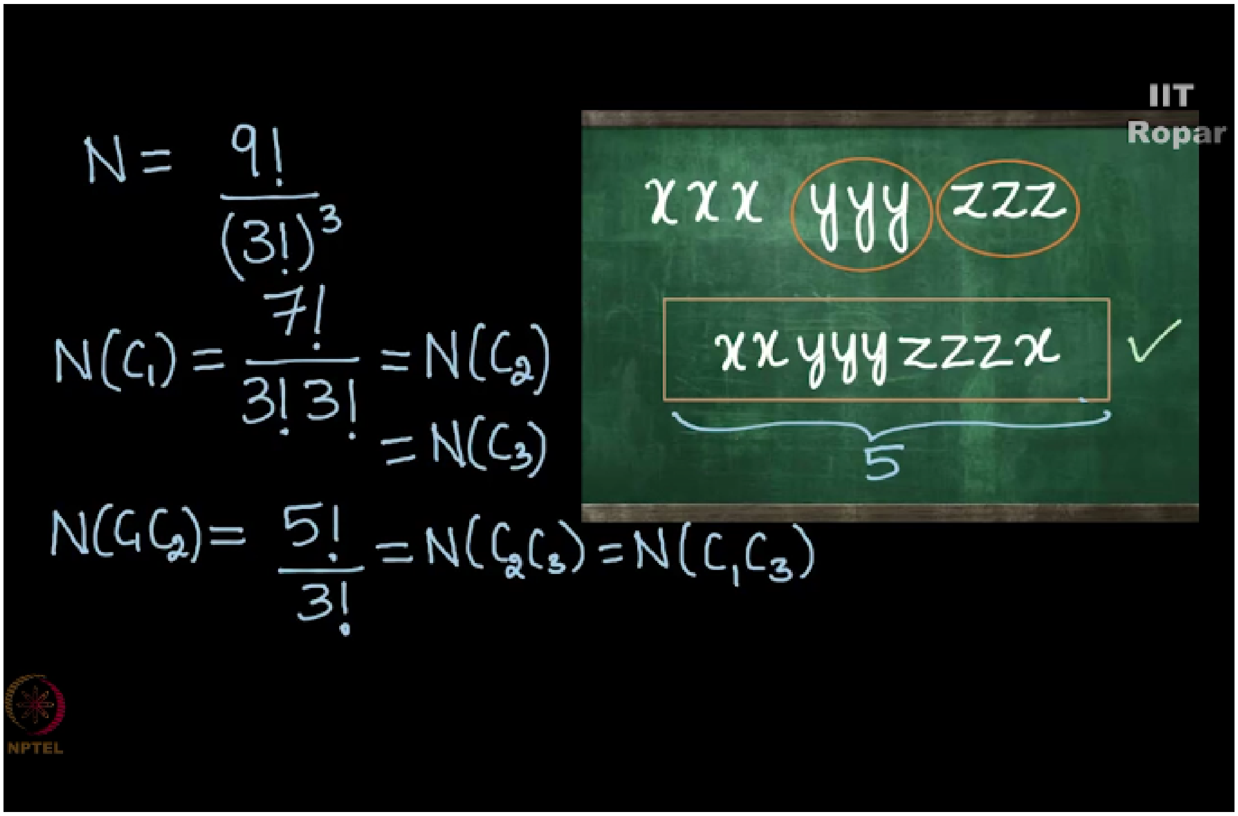
(Refer Slide Time: 04:18)



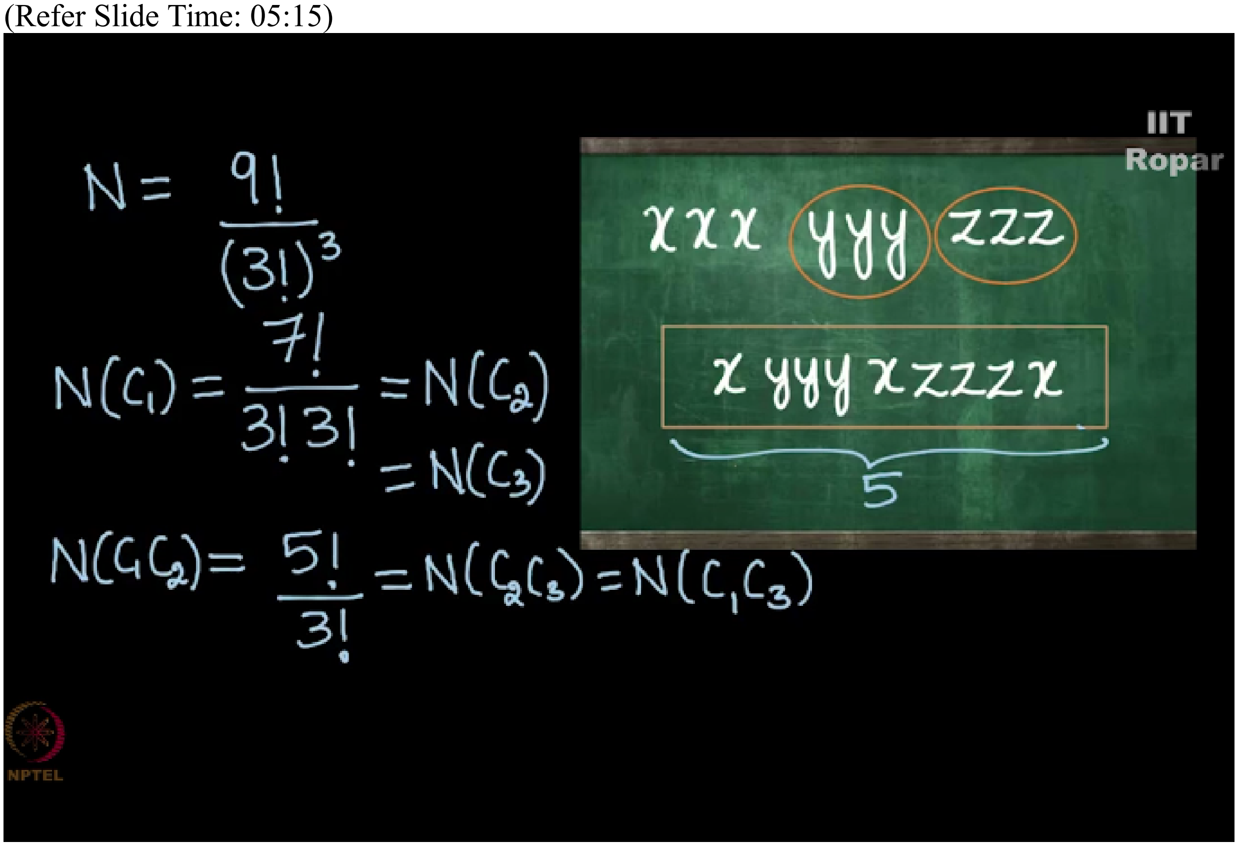
ZXXXYYYZZ do you see something like this should happen, XXX and YYY both should come together, so now I'm going to consider XXX as 1 unit YYY as another unit, and these Z's as separate units, so how many entities do we have? We have 5 in all, so it happens to be 5 factorial / 3 factorial.
(Refer Slide Time: 04:43)



Now N(C2, C3) and N1(C1,C3) will be on the same lines, what is N(C2,C3)? N(C2,C3) will be those arrangements where YYY and ZZZ will appear, right, so it's going to be something like this XXYYYZZZ and then X,
(Refer Slide Time: 05:09)

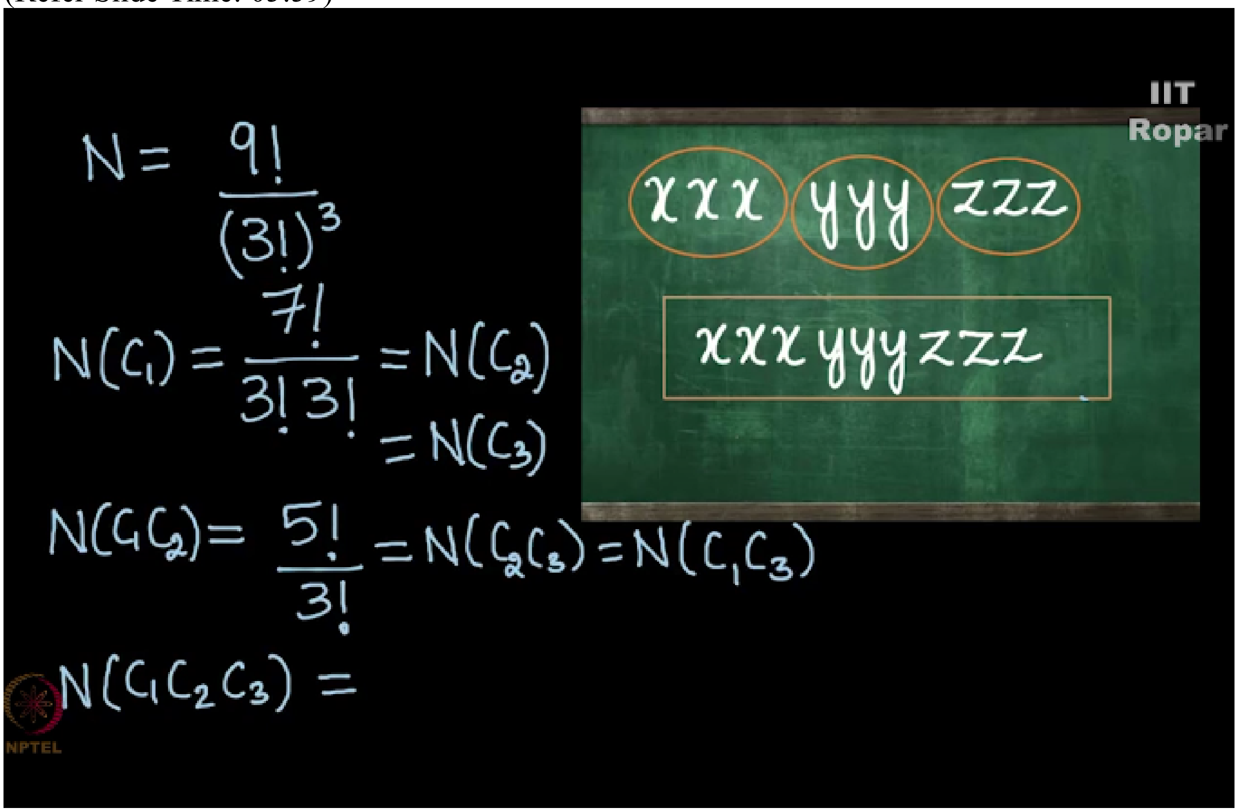


or XYYYXZZZ and another X,
(Refer Slide Time: 05:15)

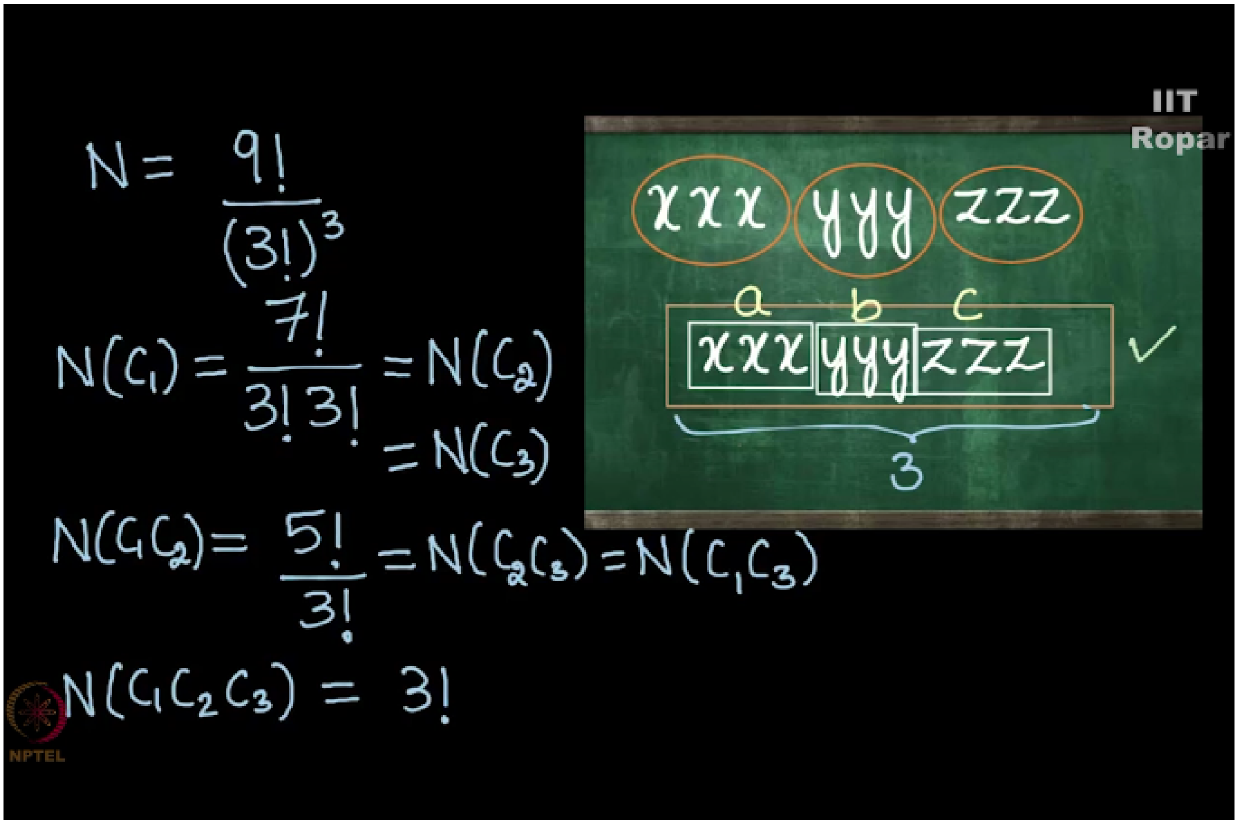


so this can be done in 5 factorial/3 factorial ways, because I've considered YYY and ZZZ to be 2 units, and the rest 3 of them for X's, the same holds true for N(C3) as well, you can probably try enumerating 1 or 2 of them.

What is N(C1,C2,C3)? Well those arrangements where XXXYYY and ZZZ will appear, we want those arrangements, let me try showing you how it is going to look like, well XXX, YYY, ZZZ and this is one such possibility,
(Refer Slide Time: 05:59)

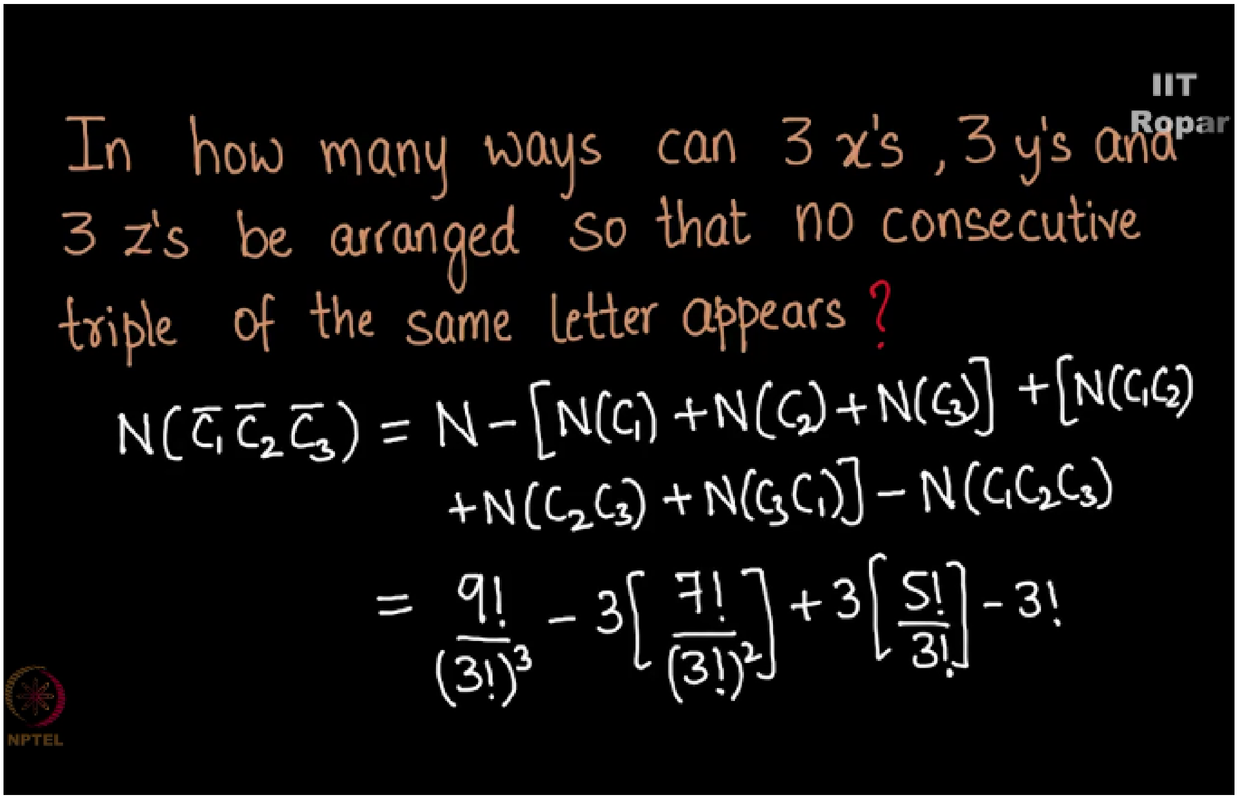


now I am going to consider XXX as 1 unit YYY as one unit and ZZZ as another unit, so I have 3 units in all, now in how many ways can I rearrange them? It is very obvious to see that it is 3 factorial ways, right, so N(C1,C2,C3) happens to be 3 factorial, you can probably understand it like this if I represent XXX as say A, YYY as B, and ZZZ as C, in how many ways can I arrange ABC then? The answer is 3 factorial right.
(Refer Slide Time: 06:39)



Now so what is the answer for the question, the question was in how many ways can these 9 alphabets be arranged, so that no consecutive triple of the same letter appears, so basically we were trying to ask what is N(C1 bar, C2 bar and C3 bar) which is N – N(C1) + N(C2) + N(C3) + N(C1,C2) + N(C2,C3) + N(C1,C3) – N(C1,C2,C3), so the answer goes like this, N(C1 bar, C2 bar, C3 bar) happens to be 9 factorial/3 factorial whole cube - 3 times 7 factorial/3 factorial whole square + 3 times 5 factorial/3 factorial - 3 factorial,

(Refer Slide Time: 07:42)



again you need not expand it and leave this as the final answer.

IIT MADRAS PRODUCTION

Founded by
Department of Higher Education
Ministry of Human Resources Development
Government of India

www.nptel.iitm.ac.in