NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Example 3 - Words not containing some strings
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How many permutations of 26 letters of English alphabet do not contain the strings, bus, road, and gem?
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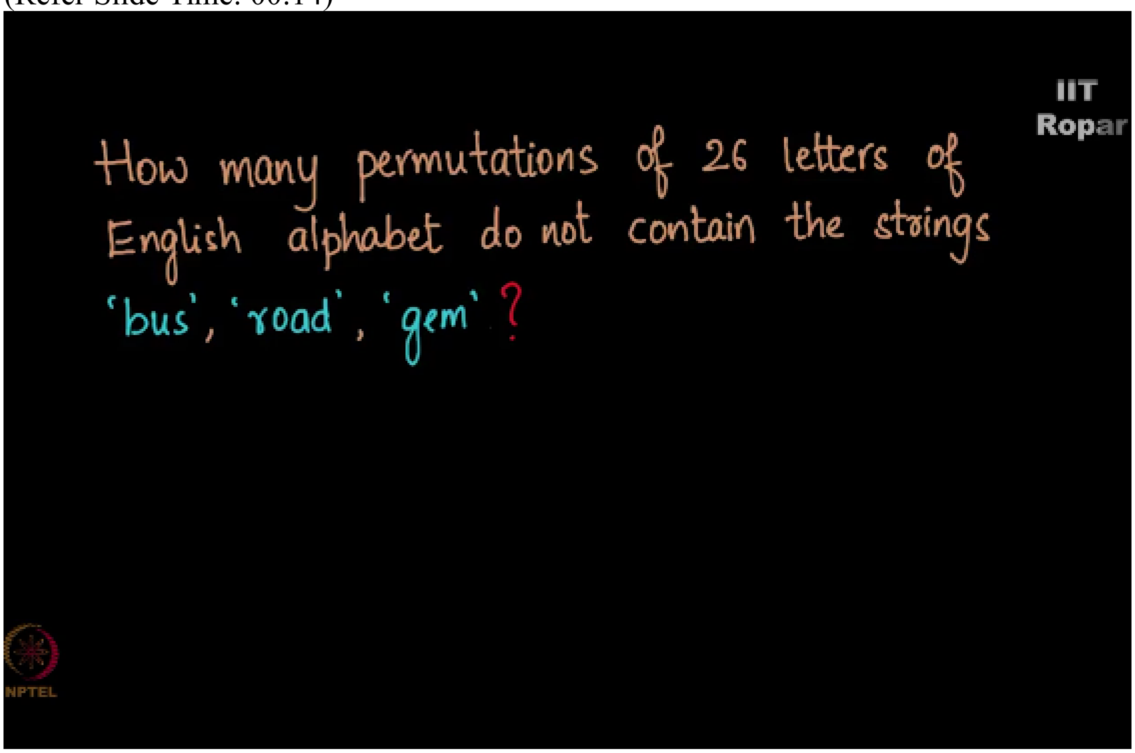

So you see we have to consider all the permutations of English alphabet, and we have to count those, which do not contain these strings as a part of them.

Now let me consider the set S as all the permutations of 26 letters, so cardinality of S will be 26 factorial now I'm not going to explain how it is 26 factorial, if you want more clarification, please once again watch the videos of week 1.
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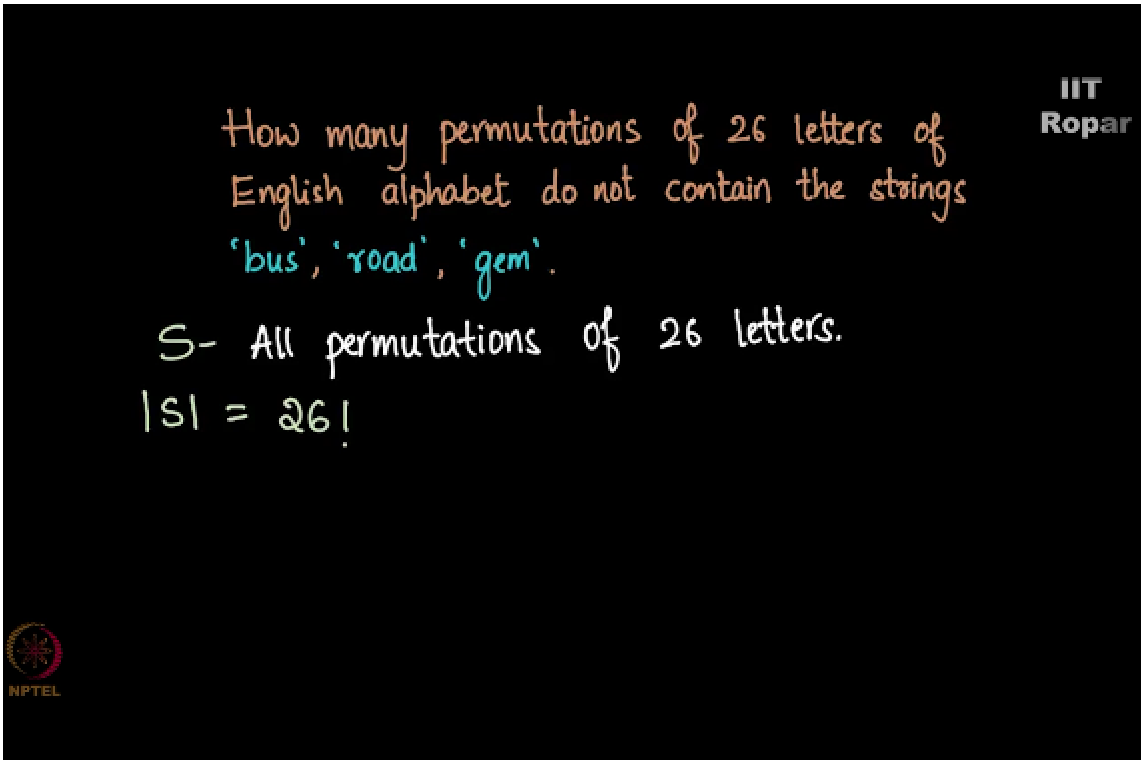

Now how do we use principle of inclusion and exclusion here, what are the conditions? Let us see, C1 is the condition where a permutation satisfies condition 1 if it contains the string bus, C2 is the condition where a permutation satisfies this conditions C2 if it contains road, and C3 is the same if a permutation contains the string, gem.

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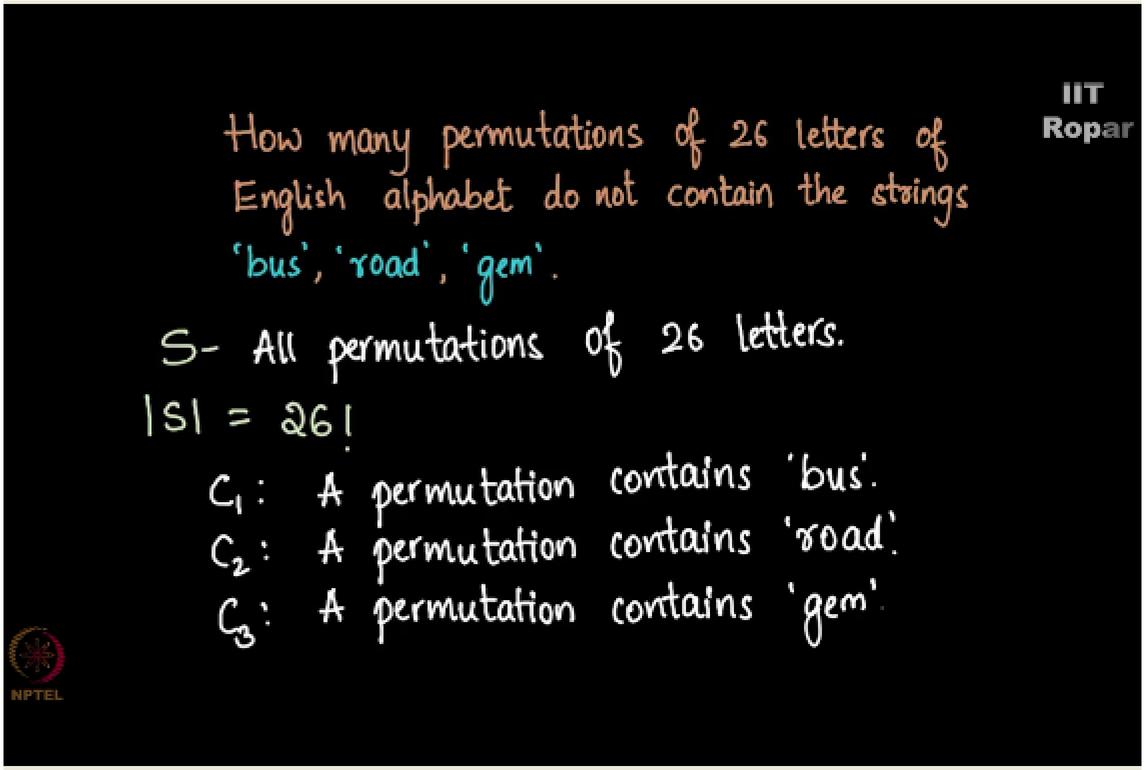

Now we know that N happens to be 26 factorial, now we have to calculate N bar which is N(C1 bar, C2 bar, and C3 bar), before that we must calculate N(C1), N(C2), N(C3) and N(C1,C2), N(C1,C3), N(C2,C3), and N(C1,C2,C3),

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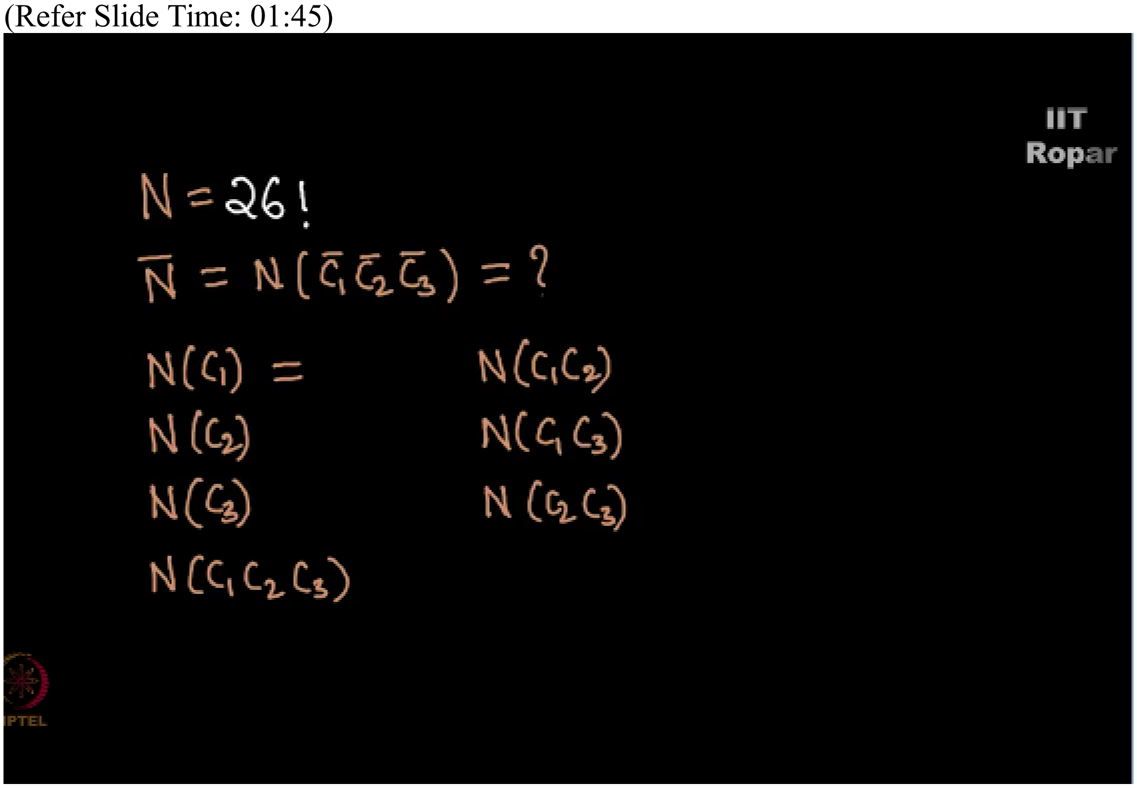

so let us go one by one, so what are those permutations which satisfy condition 1? Condition 1 says it must contain the string bus, so let me write bus here, and then A, I'm not going to write the alphabet B because it is already over here, A, C, D, and so on Q, R, again I'm not going to write S because it is over in the string bus, T, I'm not going to write U as well, because it is being utilized in bus, V, W so on up to Z, permutation of these will give those words which contains bus, now how many of them are there? It is precisely 24 factorial which means 24 factorial permutations will contain the string bus, why 24?

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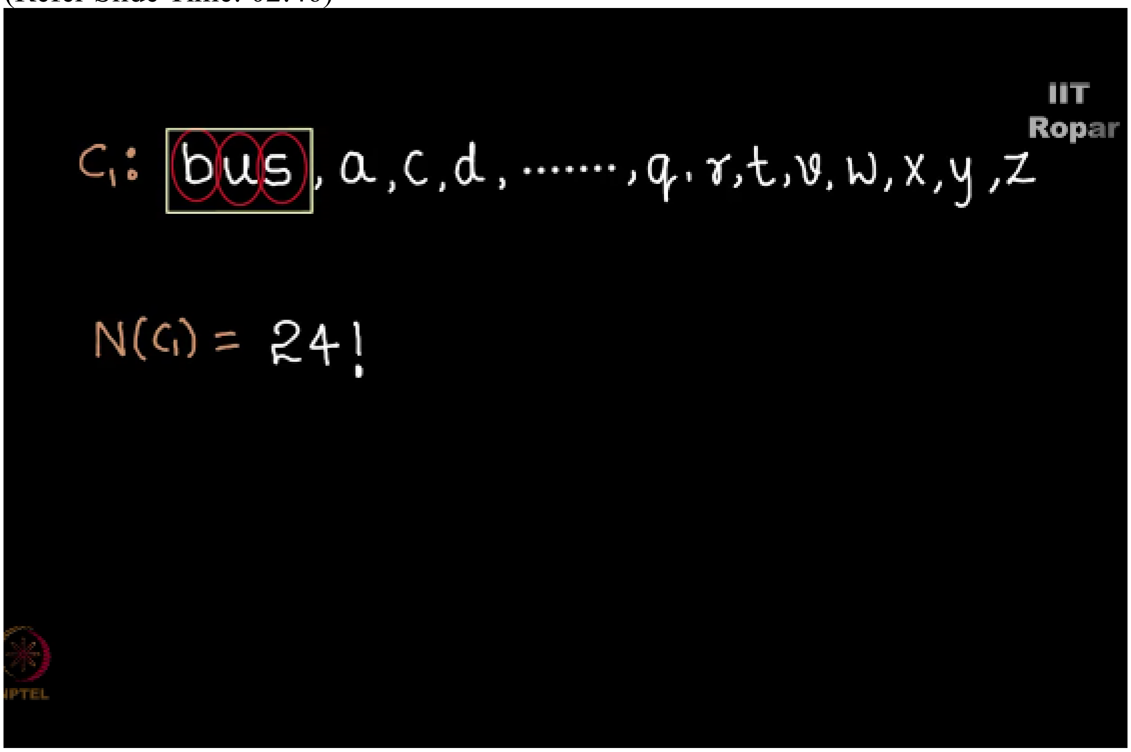

Give a look at it, B U S is one unit, and the rest of the alphabets, how many are there? It is 23, so it is 23 + 1 which is 24 factorial, so in these many ways the words can be permutated which contains bus as a string, so N(C1) happens to be 24.

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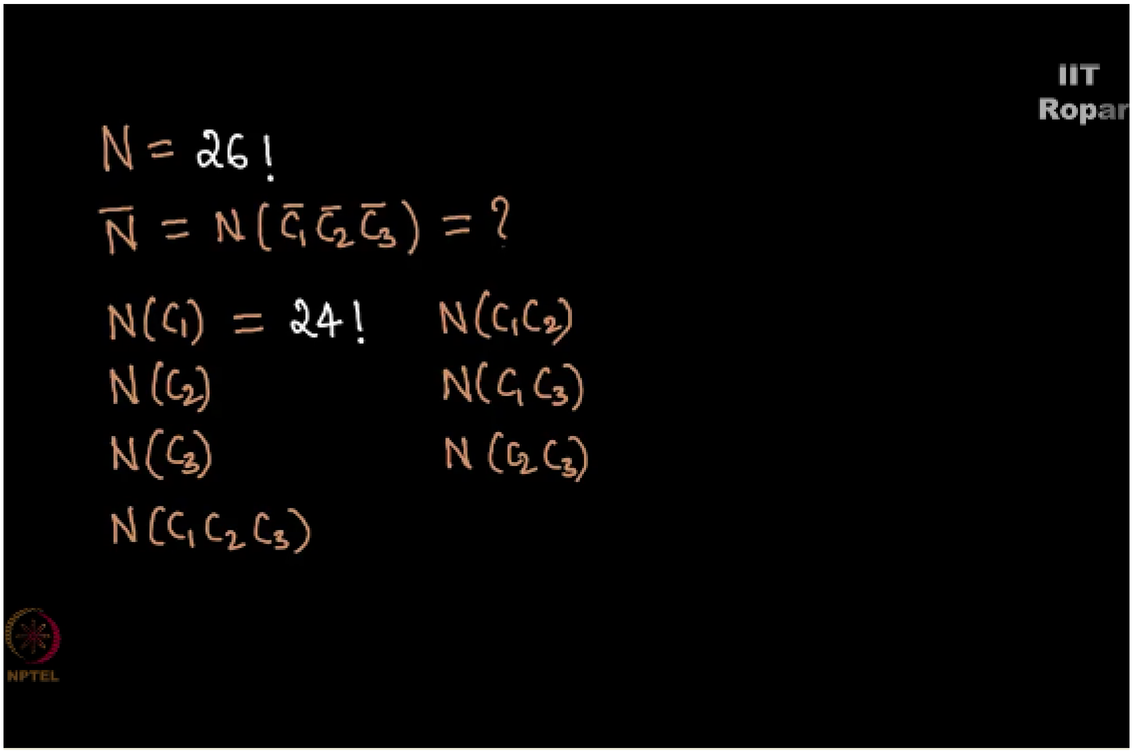

N(C2) are those permutations which contain road as a string, now following the same way you see that road is one unit, and how many alphabets remain? It is 26 – 4 which is 22, 22 of them and this as one unit is 23, so in 23 factorial permutations you can find that road is content, so N(C2) happens to be 23 factorial, N(C3) a permutation which contains gem, such permutations have to be counted, and don't you feel it is the same as bus, it is the 3 lettered word, right, so again it happens to be 24 factorial.

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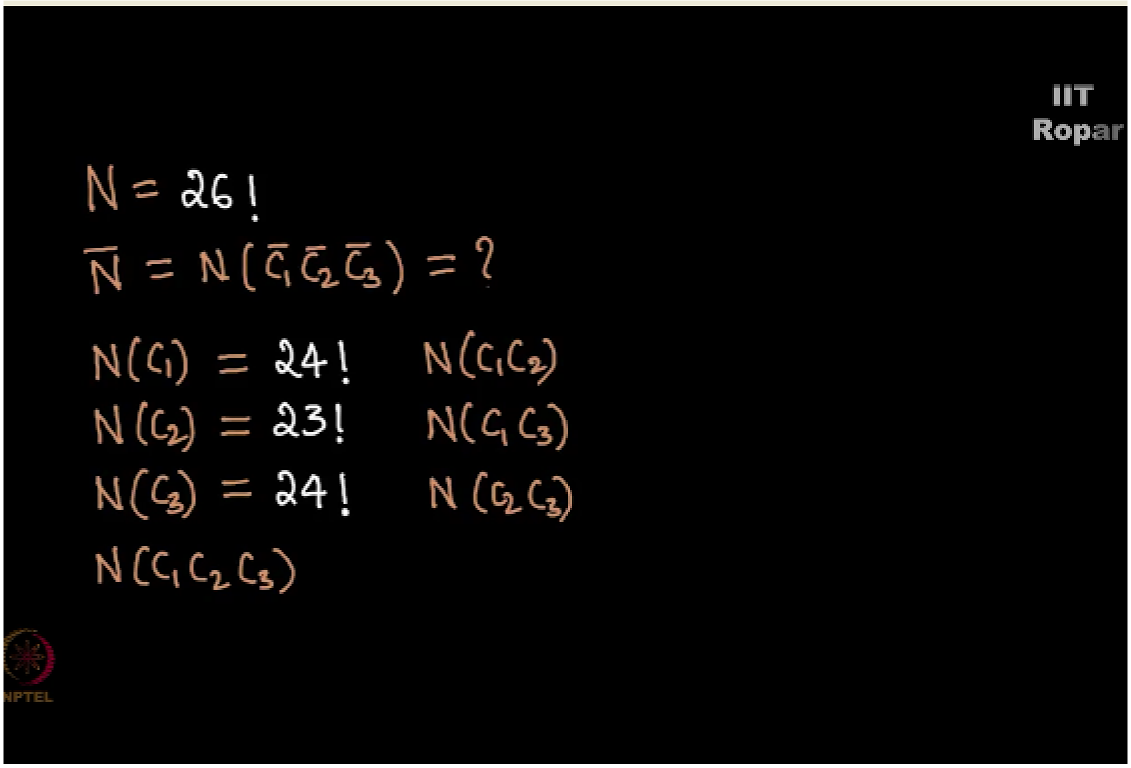

Now N(C1,C2), what is C1, C2? Those permutations which contain bus as well as road, now if I consider bus as one unit, and road as one unit, how many alphabets remain? It is 26 – 7, do you see why? It is 3 + 4, 3 letters in bus, and 4 letters in road, and hence it is 26 – 7, which is 19, 19 + bus as 1 unit and road as 1unit is 21 alphabets, right, and the permutations of these will contain bus as well as road, so it happens to be 21 factorial.

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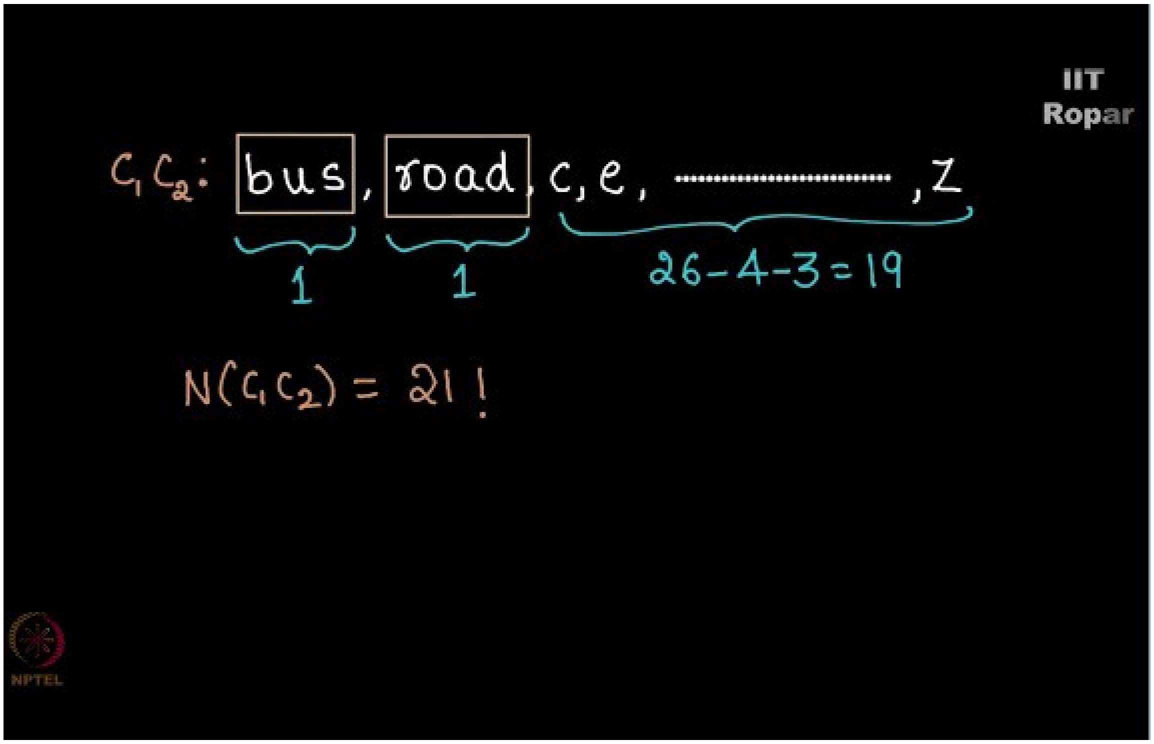

Now C2, C3 will be those permutations which contain road as well as gem, again it is 7 of them, 26 – 7 which is 19 + 2 of these as individual units and we get 21 factorial, so in 21 factorial ways you can arrange the words which contain these strings road and gem.

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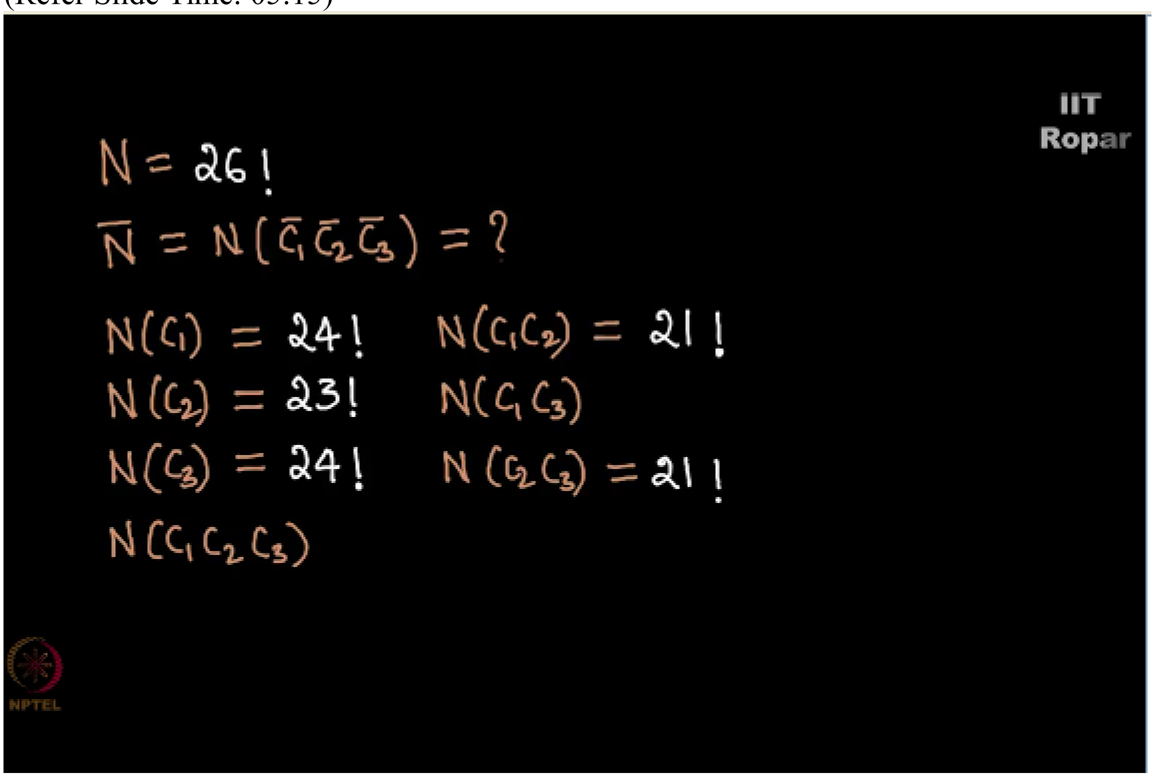

C1, C3 you saw that N(C1,C2) is 21 factorial, and N(C2, C3) is 21 factorial, now N(C1,C3) will be those permutations which contain bus and gem, right, bus has 3 alphabets one unit, gem has 3 alphabets another unit, now how many remain? It is 26 alphabets – 6 alphabets which is 20, right, 20 alphabets and bus has one unit, gem has one unit which happens to be 22, so permutations of these 22 units will give you words that contain bus and gem as strings, so N(C1,C3) is 22 factorial.

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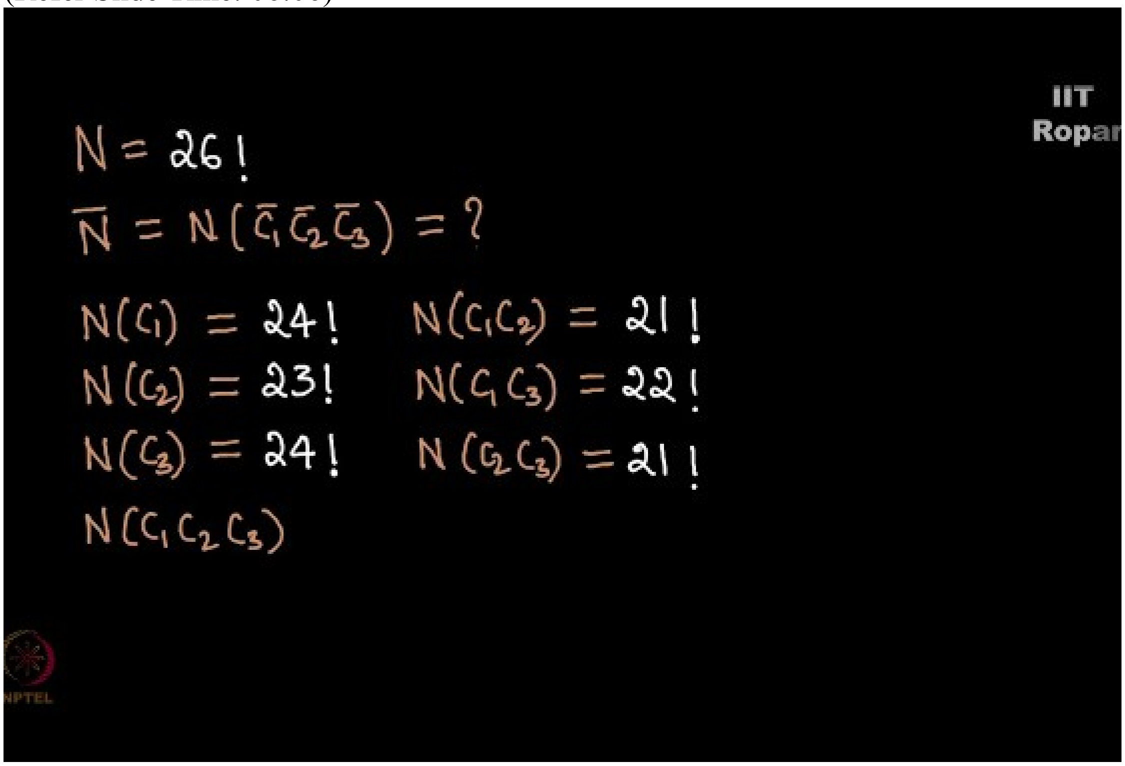

Now N(C1, C2, C3) this remains, right, so those permutations which contain bus, road, and gem all the three we must calculate how many of them are there, now bus, road, and gem, how many alphabets are covered here? It is 3 + 4 + 3 which happens to be 10, and 26 – 10 is 16, so 16 alphabets are remaining, 16 + 1 unit of bus, 1 unit of road, and 1 unit of gem, so it is 19 units remaining, and permutations of this 19 units will give words which contain bus, road, and gem as strings, so N(C1,C2, C3) will be 19 factorial.

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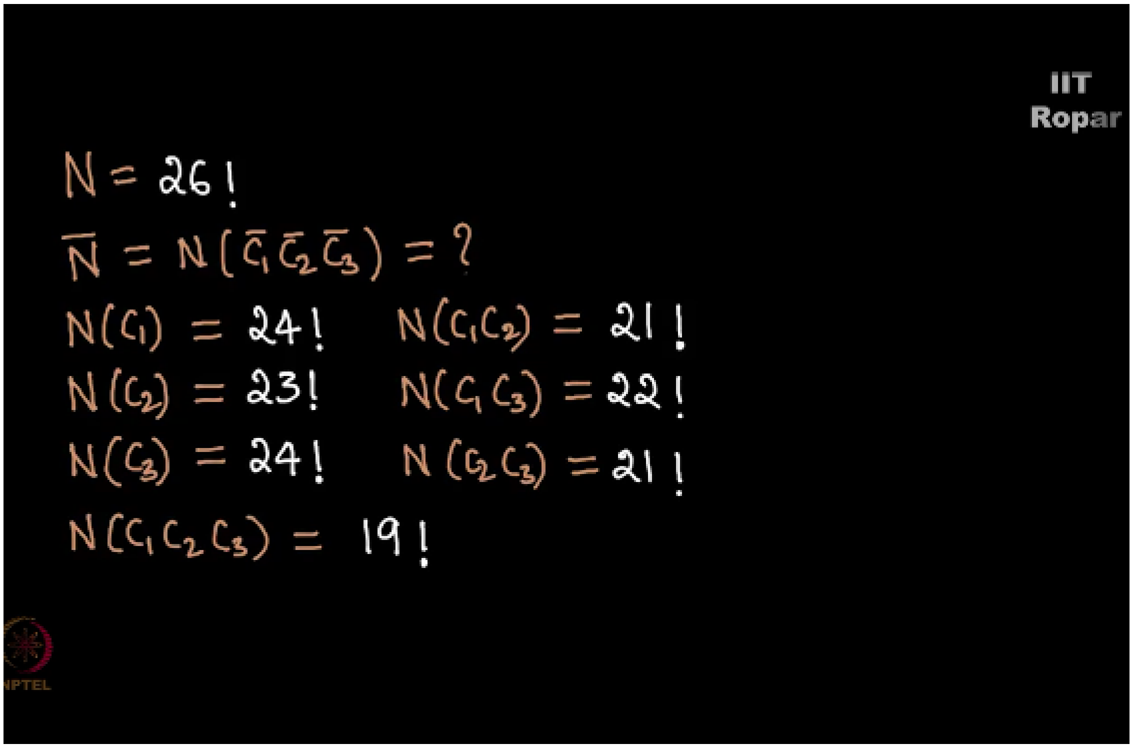

Now applying the formula of inclusion and exclusion we see that N(C1 bar, C2 bar, and C3 bar) is 26 factorial – 2 x 24 factorial + 23 factorial + 2 x 21 factorial + 22 factorial – 19 factorial, so you need not expand these huge numbers, these many permutations of English alphabet will not contain the strings bus, road and gem.

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