NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Example 2 - Integer solutions of an equation

By
Prof. S.R.S Iyengar
Department of Computer Science
IIT Ropar

How many integer solutions are there to the equation X1 + X2 + X3 + X4 = 25, where all these exercise that is X1, X2, X3, X4 lie in the range 0 to 10,
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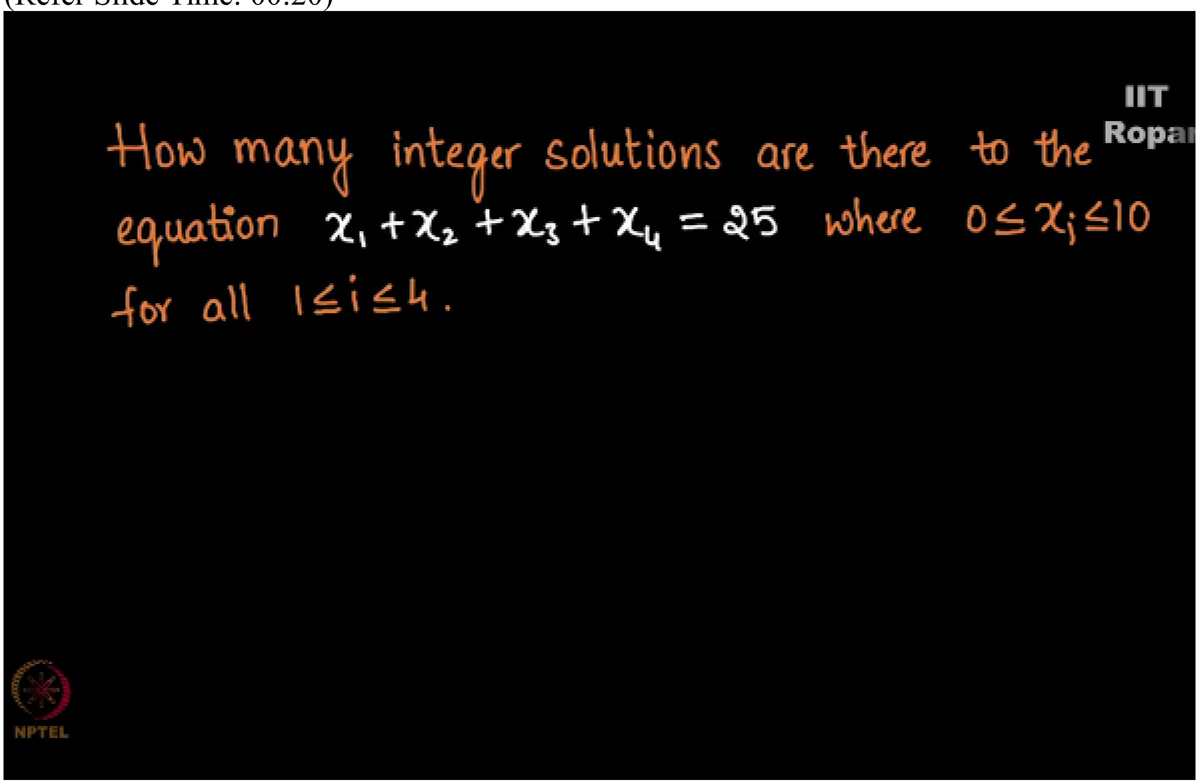

none of them exceed 10, but they are some must be equal to 25, now how many such solutions are possible? I'm going to write C1 as the solutions where X1 is at least 11, and C2 as the solutions where X2 is at least 11, C3 where the solutions where X3 is at least 11, and C4 are the solutions where X4 is at least 11.
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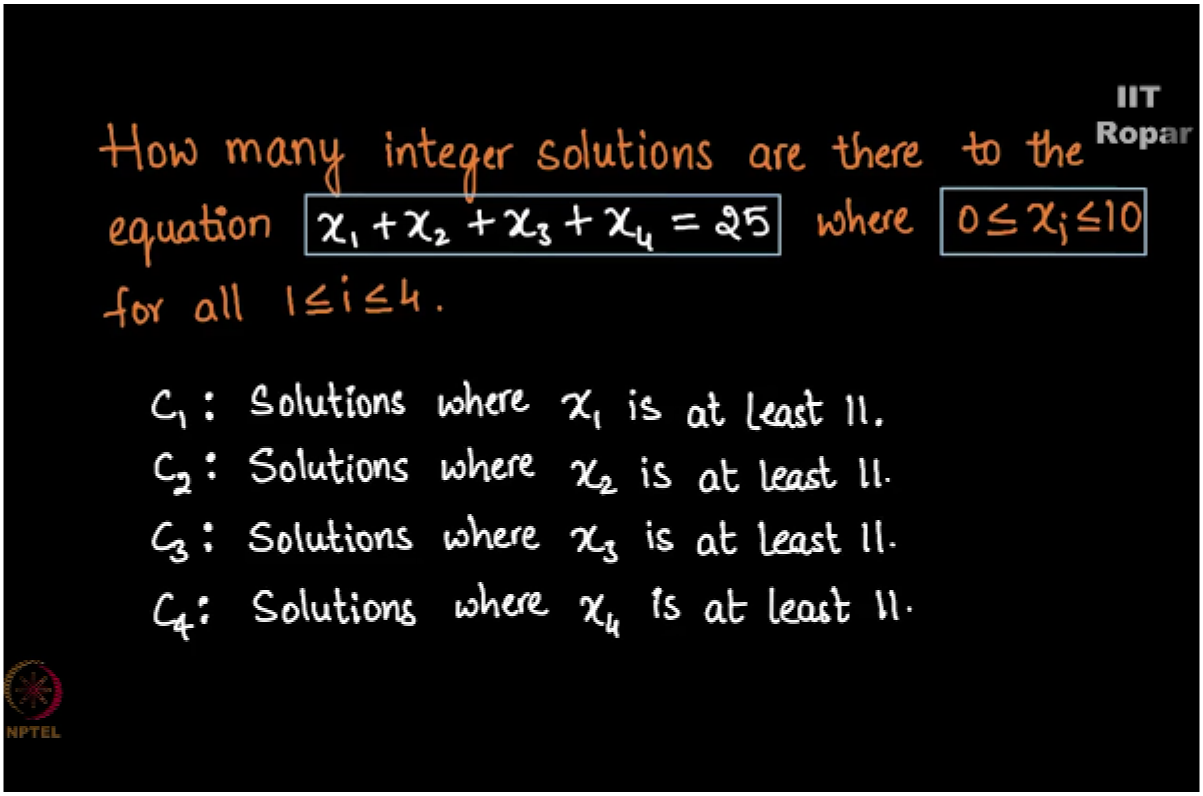

Now why did I write just the opposite of the given condition? Well, the reason was stated in the first video itself, sometimes when we want the particular thing we should look at what we don't want, and hence since we want to calculate N(C1 bar), C2 bar, C3 bar, C4 bar, I'm going to look at these CI's, N(C1) states the number of solutions where X1 is at least 11,

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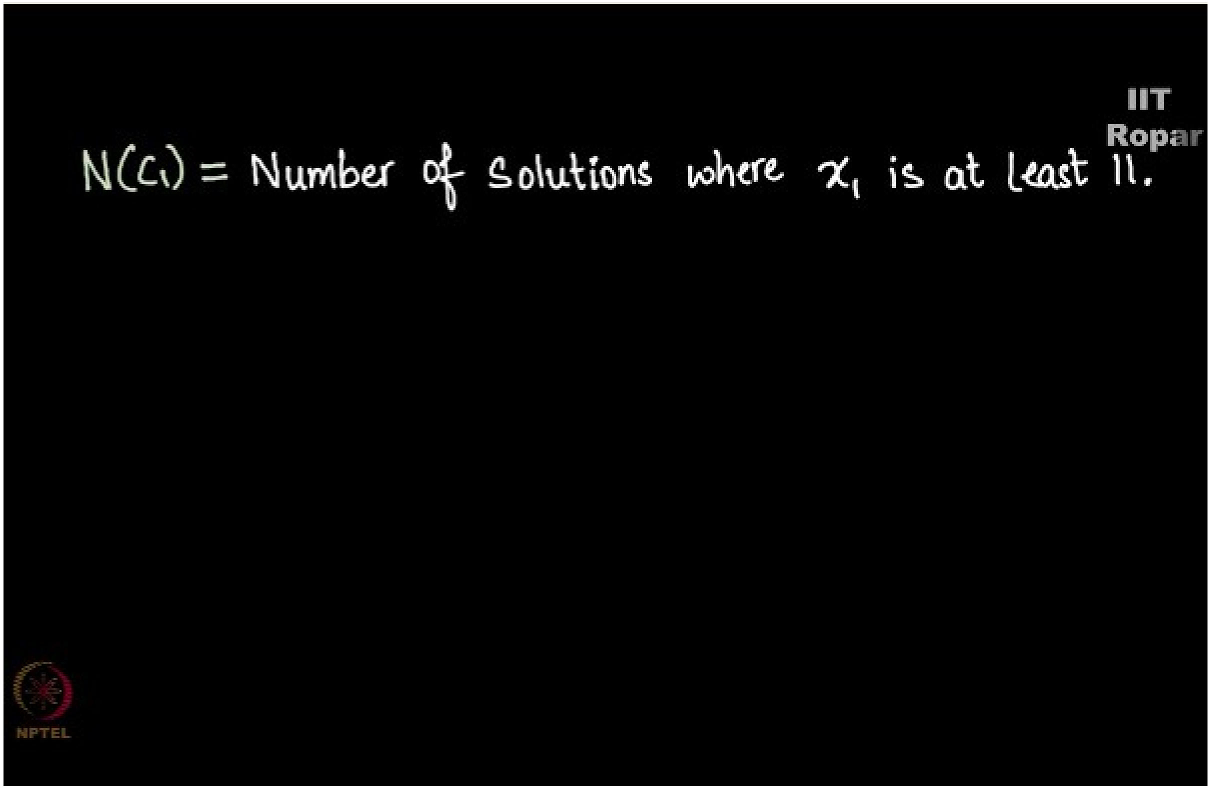

how many such solutions are possible let us see, before going to N(C1) or N(C2) or N(C3) or N(C4) let me tell what are the total possible solutions, the total possible solutions is if you remember the sticks and the cups problem following the same logic here, we see that N is 4 here, there are 4 exercise, and we have R as 25 here, so it is N+R -1 choose R which gives 4 + 25 – 1 choose 25 which is 28 choose 25.

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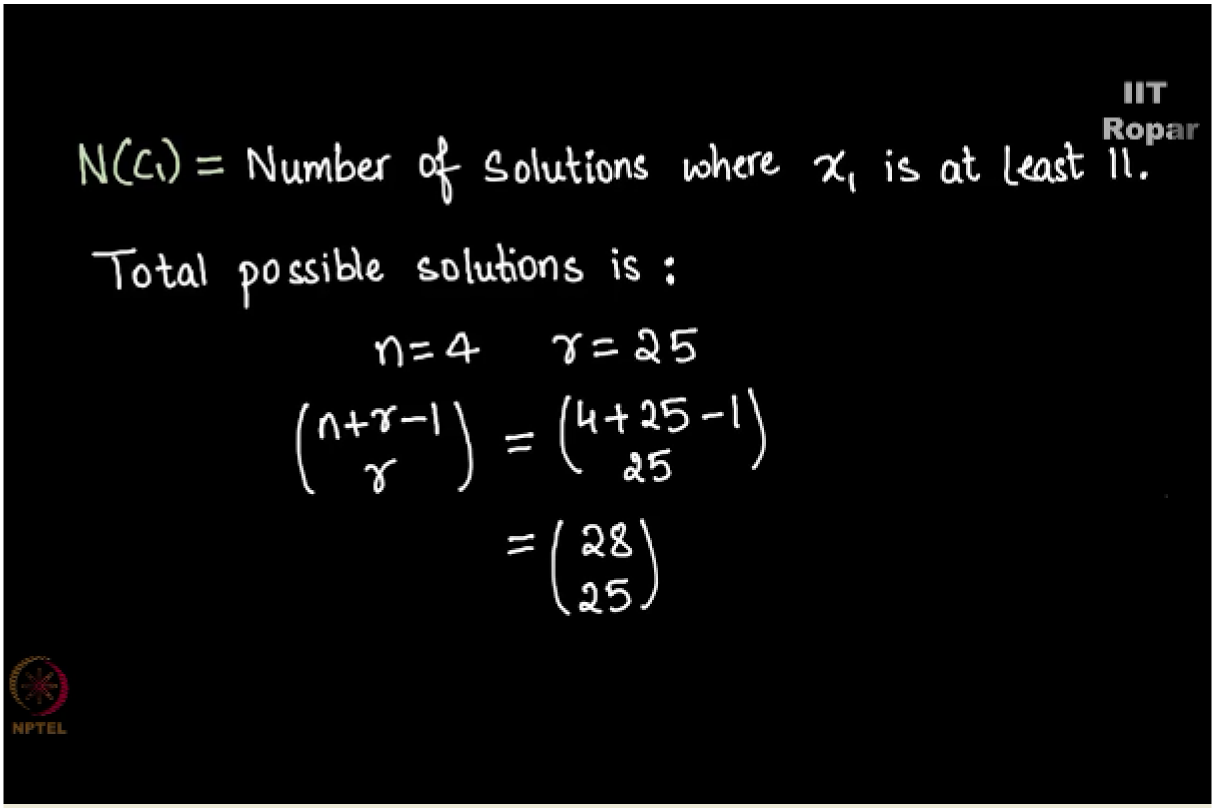

Now N(C1) happens to be the number of solutions where X1 is at least 11, I can turn down this equation as X1 + X2 + X3 + X4 = 14, how? I earlier stated that X1 must be at least 11, now I'm going to just remove 11 from both the sides, what does that become? X1 + X2 + X3 + X4 = 14, earlier X1 had to take the value 11, 12, 13, so on, whatever be the possibility at least 11 it had to take,

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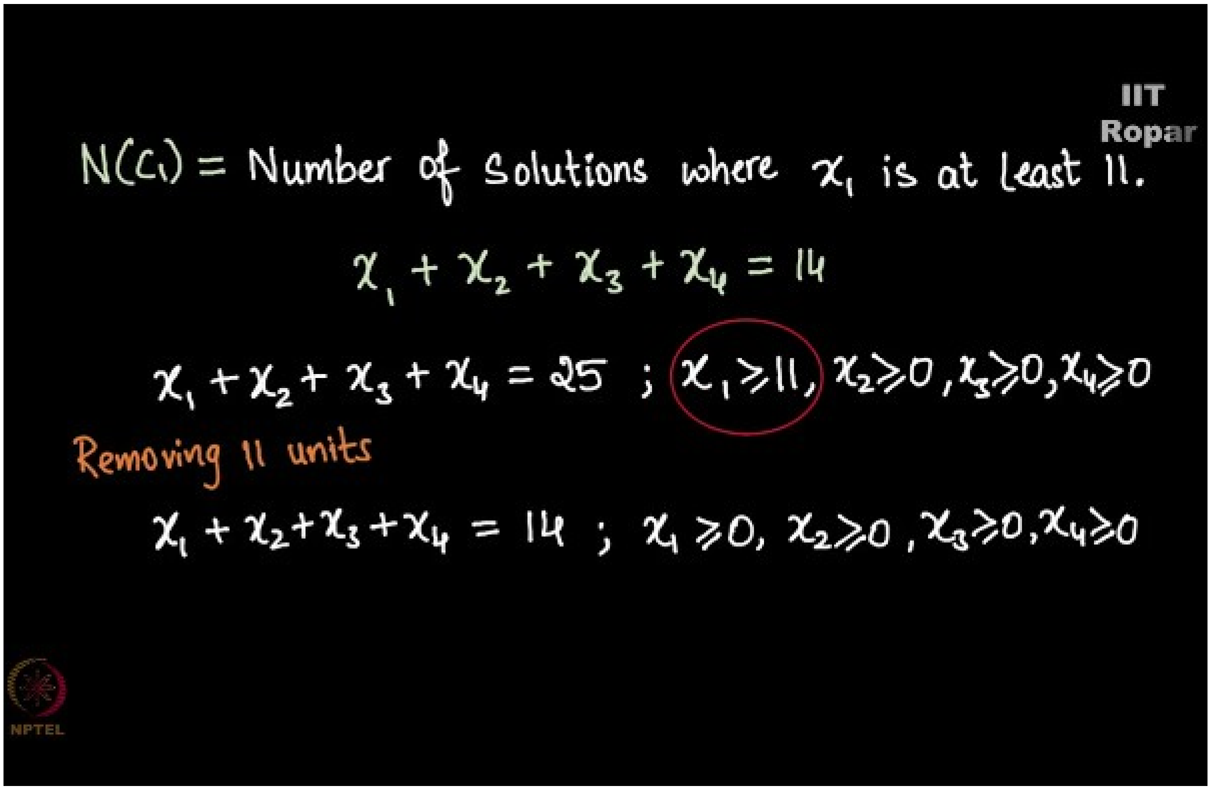

now since I have removed the possibility of 11 it takes the value from 0, right, let me enumerate some of them for you, if X1, X2, X3, X4 their sum is 14 and I say that some of the enumerations go like this, 0, 1, 9, 4 or 1, 2, 10 and 1 and so on, it is equivalent to asking X1 + X2 + X3 + X4 should be 25 with X1 at least 11.

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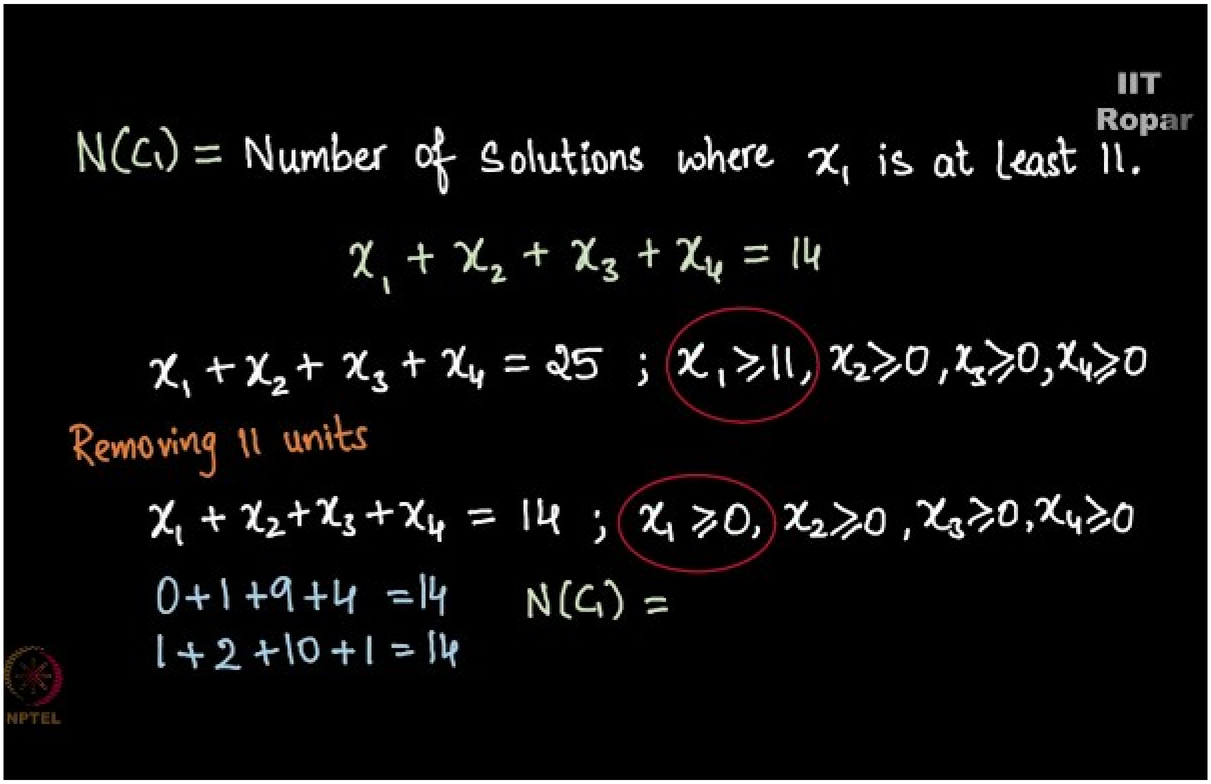

Now how many such possibilities are there? Again following combination with repetitions it is 4 + 14 – 1 choose 14 which is 17 choose 14,

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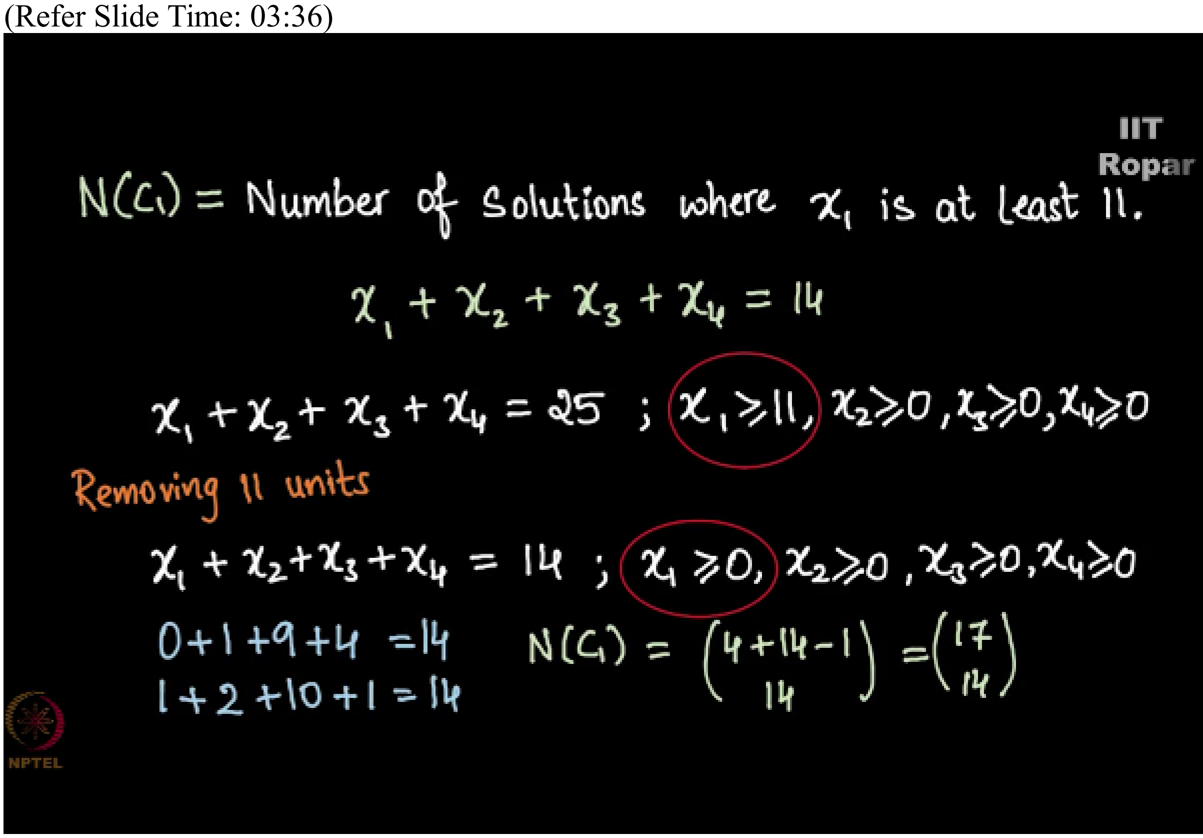

N(C1) happens to be 17 choose 14, don't you think it is the same for N(C2) as well, because X1 + X2 + X3 + X4 = 25 with X2 at least 11, is the same as removing 11 from both the sides, right, and making the equation as X1 + X2 + X3 + X4 = 14, I have turn down earlier question to this question and hence it happens to be 17 choose 14 the answer, the same for N(C3) and N(C4) as well.

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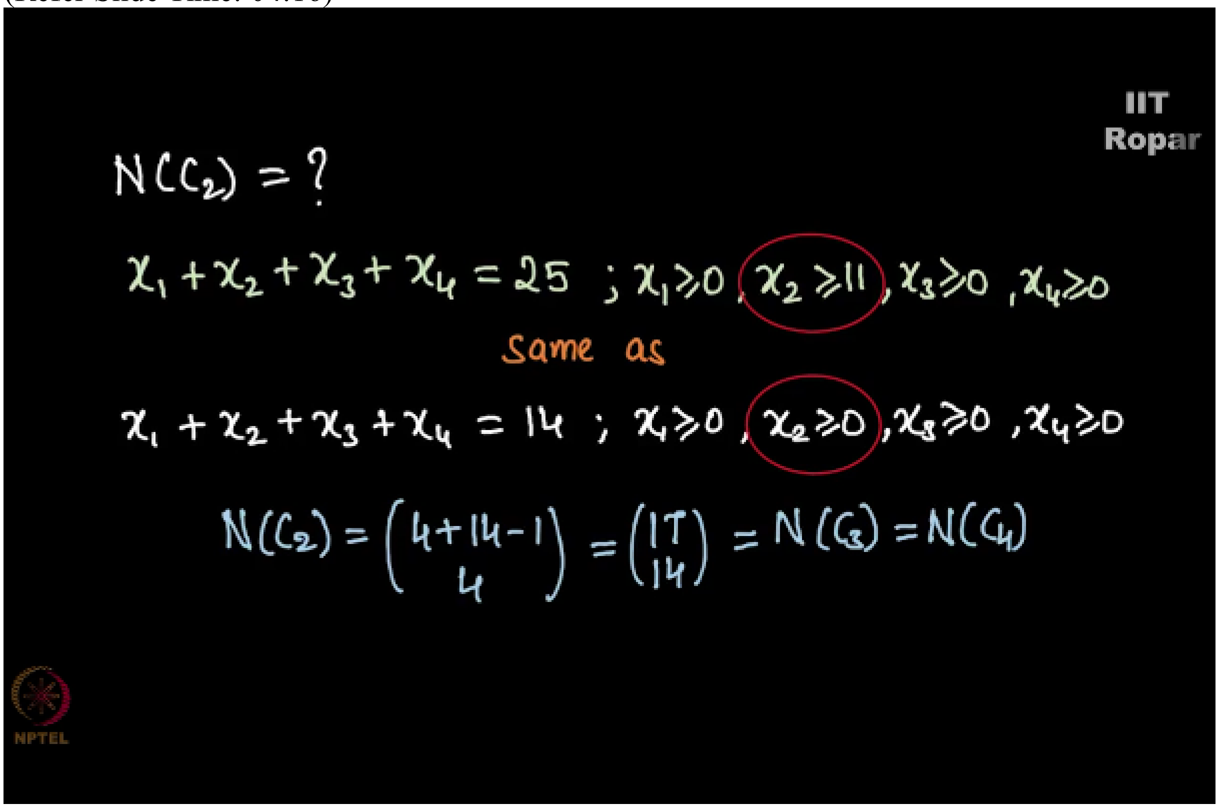

Now when it comes to N(C1)(C2), how do we do it? The solution must have X1 at least 11, and X2 at least 11, we want such solutions, right, now X1 + X2 + X3 + X4 = 25 this can be turn down to the equation, X1 + X2 + X3 + X4 removing 22 from RHS, it becomes 3 here, right, how do we, we need solutions for this equation,

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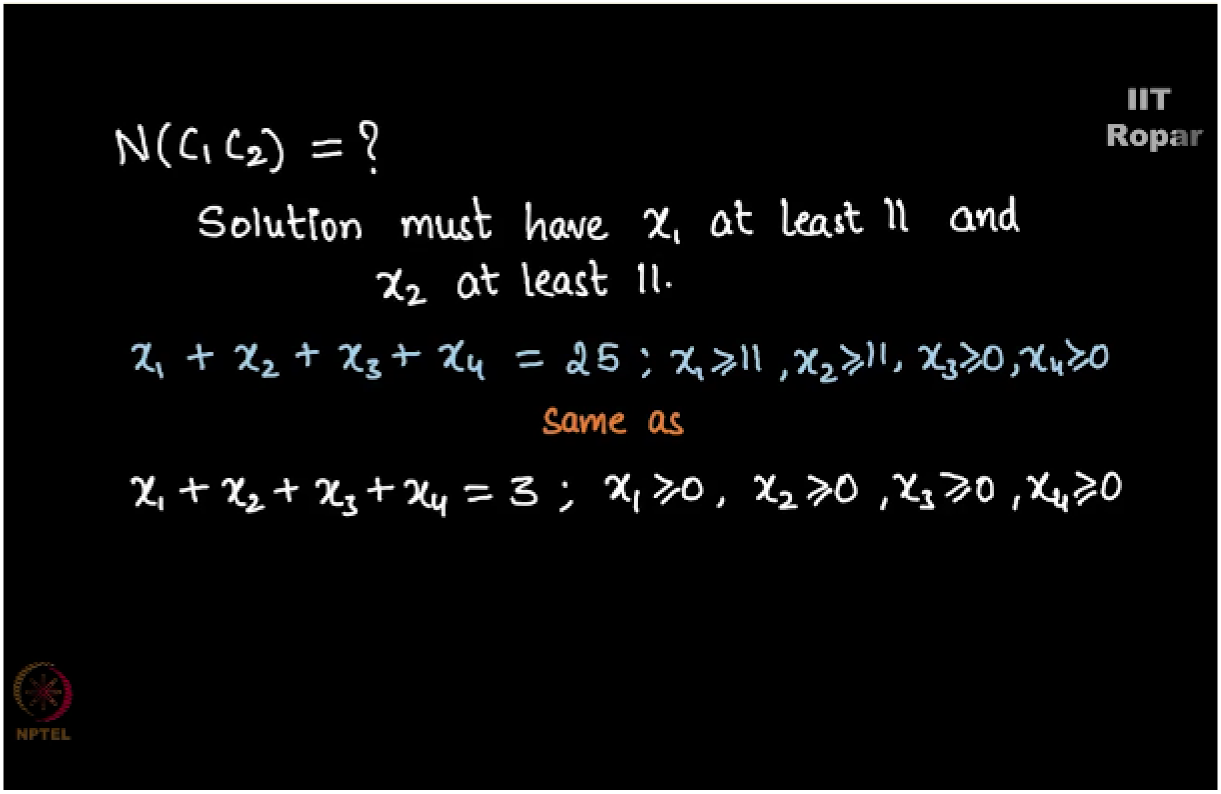

now if I enumerate the possibilities of solutions for this equation, it happens to be something like this, 1 1 1 0, 1 1 0 1, 0 1 1 1, 1 0 1 1, 2 1 0 0, 1 2 0 0, and so on,

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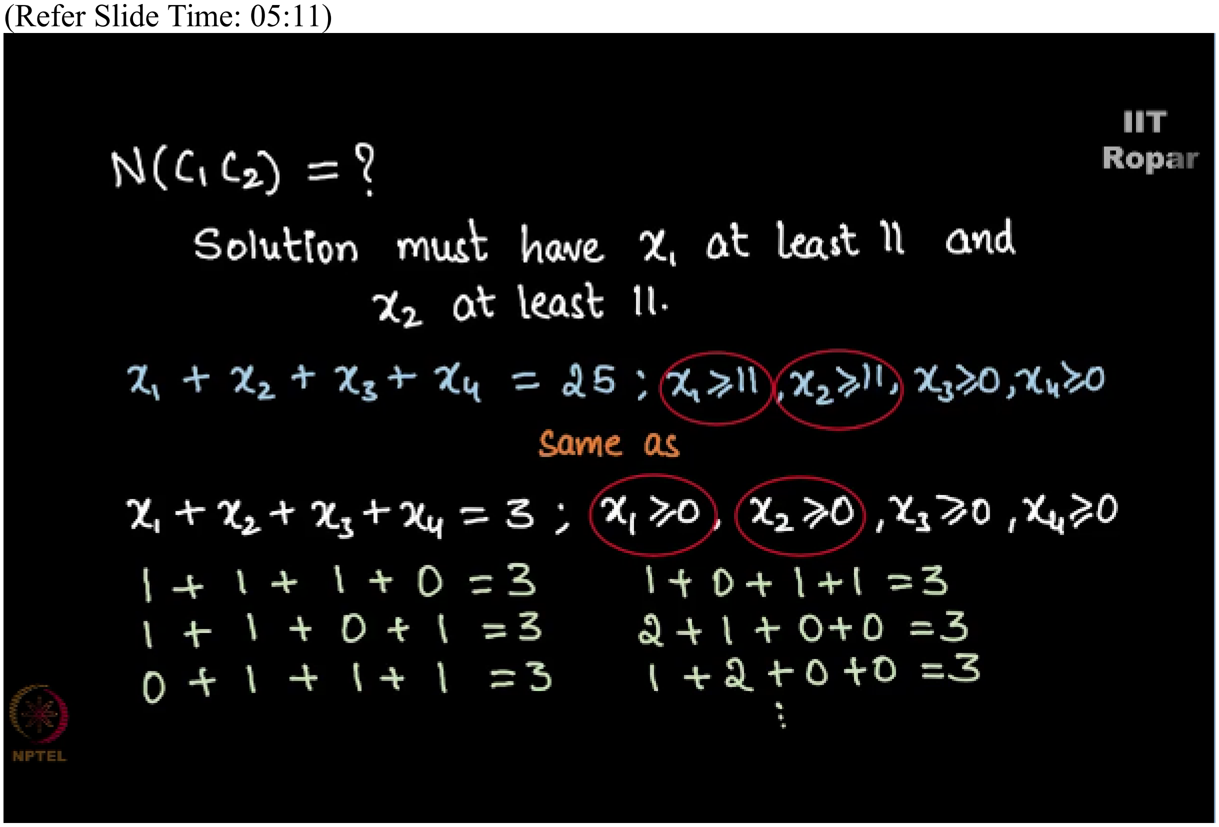

now the possibilities for X1 + X2 + X3 + X4 = 25 with X1 and X2 at least 11, if I write the possibilities for this, some of them are like this, 11 + 11 + 3 + 0, 11 + 11 + 0 + 3, 12 + 11 + 2 + 0, 12 + 11 + 0 + 2, and so on, now there is a proper bijection between the enumerations of this equation and this equation, let me show you a few of them, 11 will go to 0, so 11 + 11 + 3 + 0 will be mapped to 0 + 0 + 3 + 0.

Now 12 + 11 + 2 + 0 will be mapped to 1 + 0 + 2 + 0 and so on, and hence between all the enumerations of this equation and this equation there is a proper bijection, if you are interested you can take some pause of like say 10 to 15 minutes, write down all the enumerations of both the equations and you can yourself find out the proper bijection,

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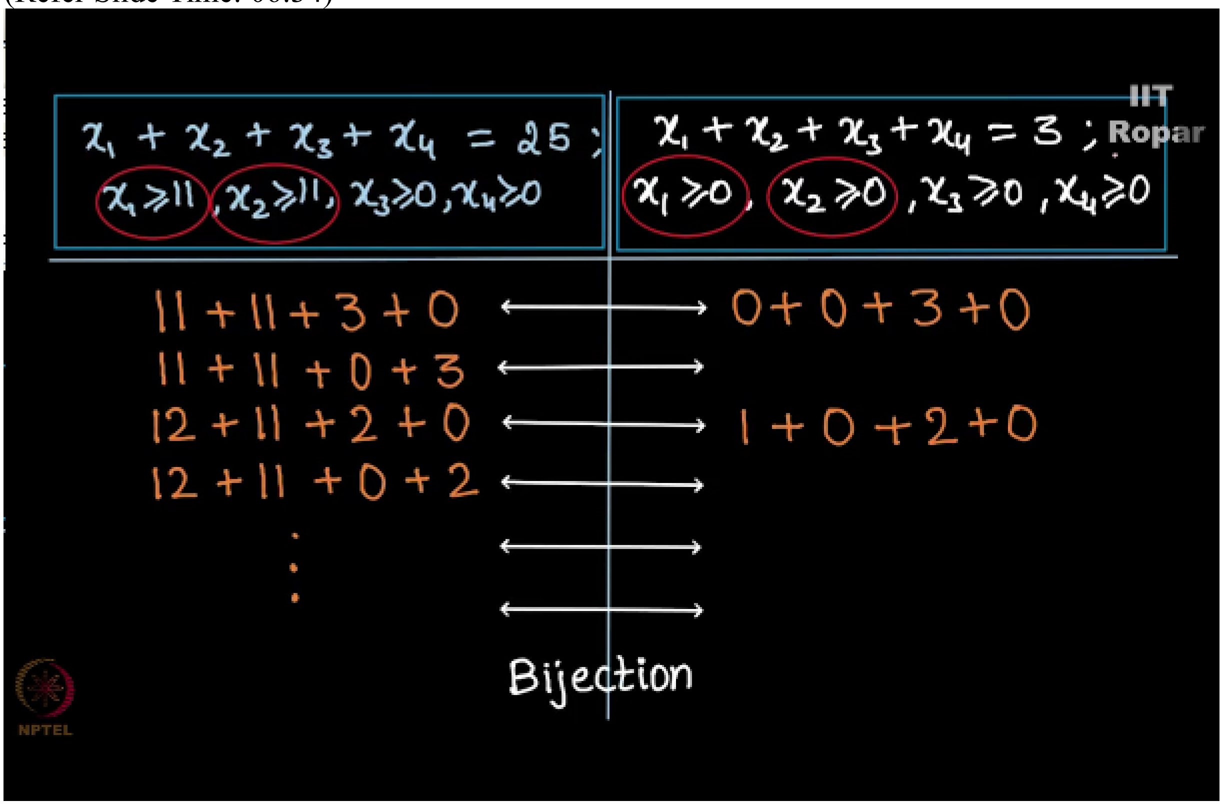

this can be followed for N(C1) and N(C2) as well, like for N(CI's) when I turn down the given equation to another equation I can always give you a bijection between the 2 equations, the possibilities, there is always a bijection,

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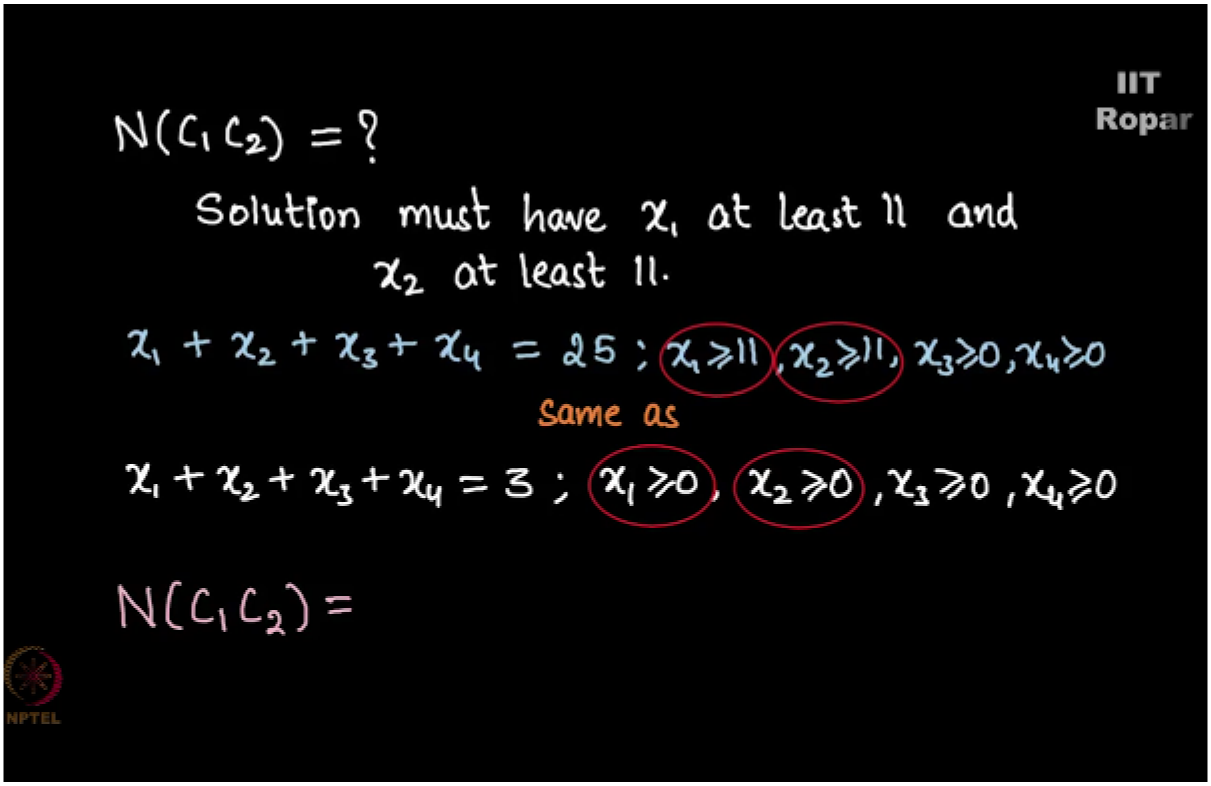

like how I stated just now, you can try it out yourself, so the number of possibilities for X1 + X2 + X3 + X4 = 3 happens to be 4 + 3 – 1 choose 3, which is 6 choose 3 that is 6 x 5 x 4 / 6 which is 20, right.

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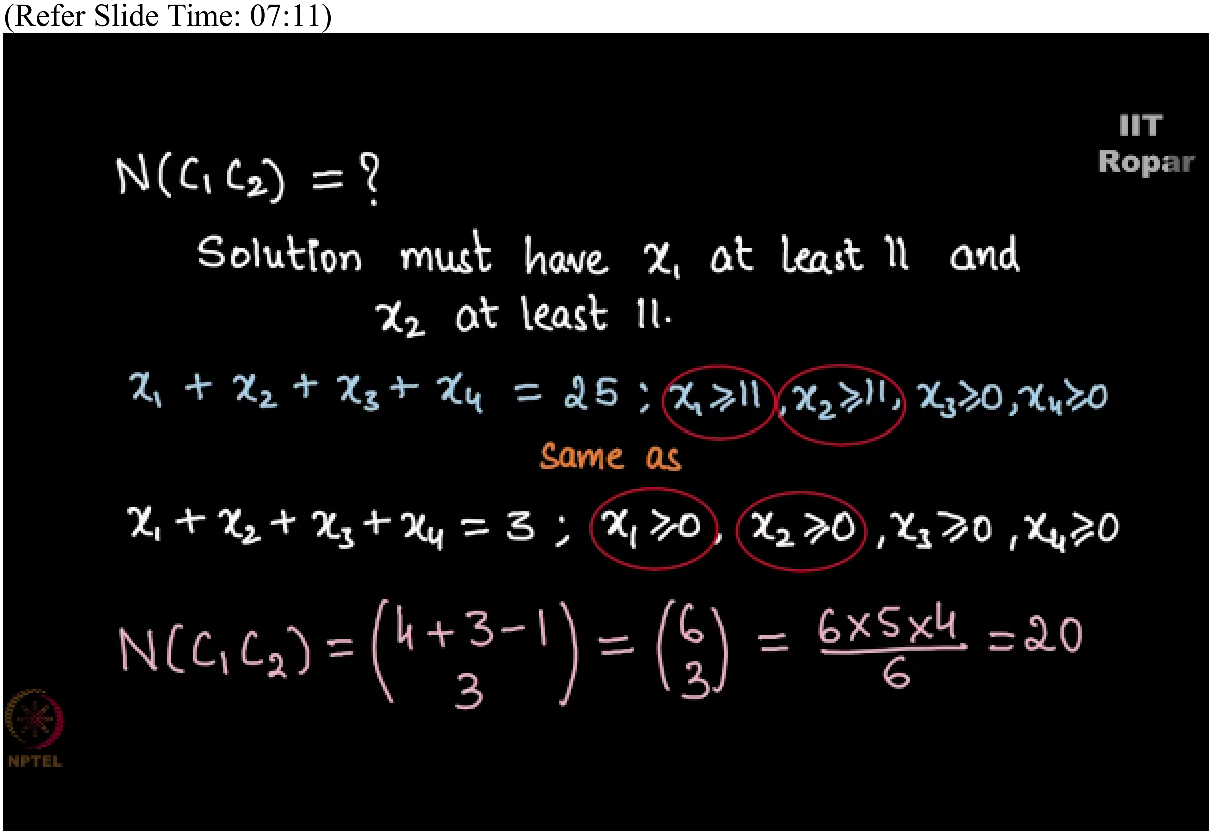

Now N(C2) (C3) will also be the same, it is 6 choose 3, why? Because N(C1) + (C2) or X1 + X2 + X3 + X4 = 25 with X2 and X3 being at least 11, it's the same as X1 + X2 + X3 + X4 = 3, just like the previous one,

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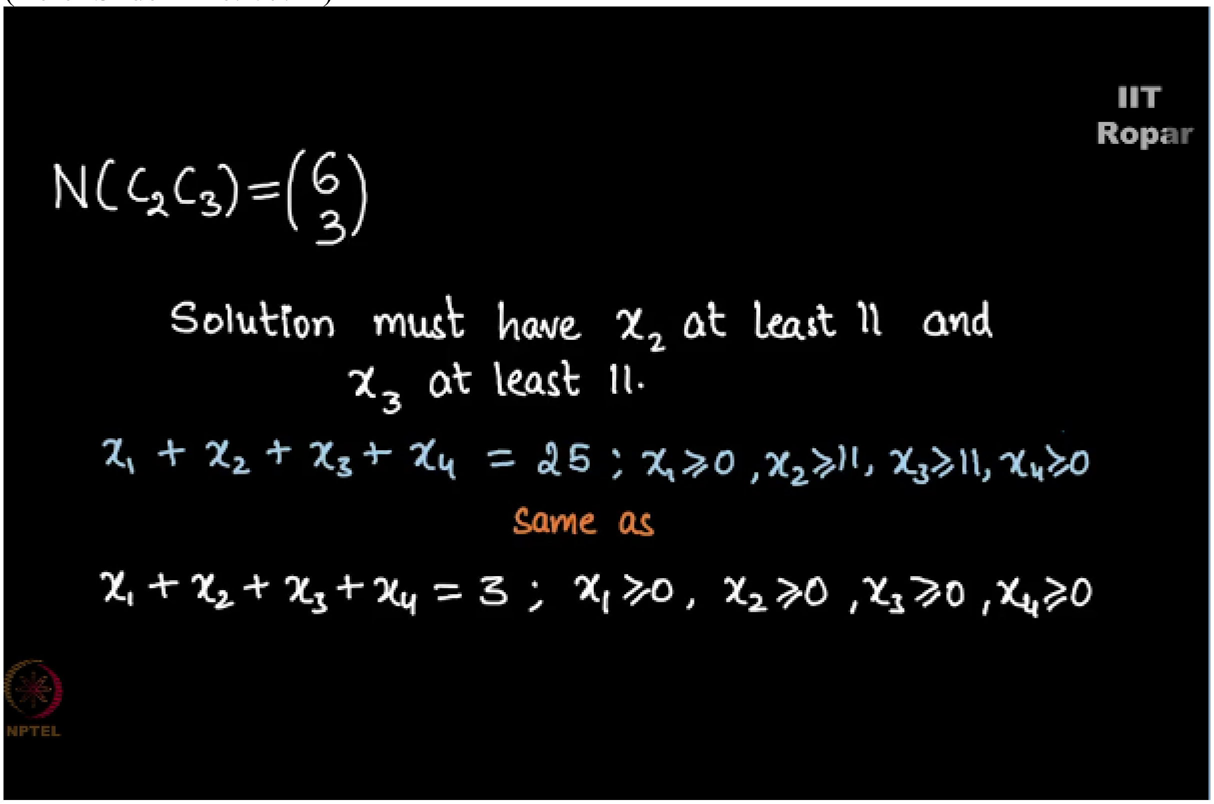

it is the same equation and hence it is 6 choose 3 for all the 6 possibilities, N(C1, C2), N(C2, C3), (C3, C1), (C1, C4), (C2, C4) and (C3, C4), right, so all of these are 6 choose 3.

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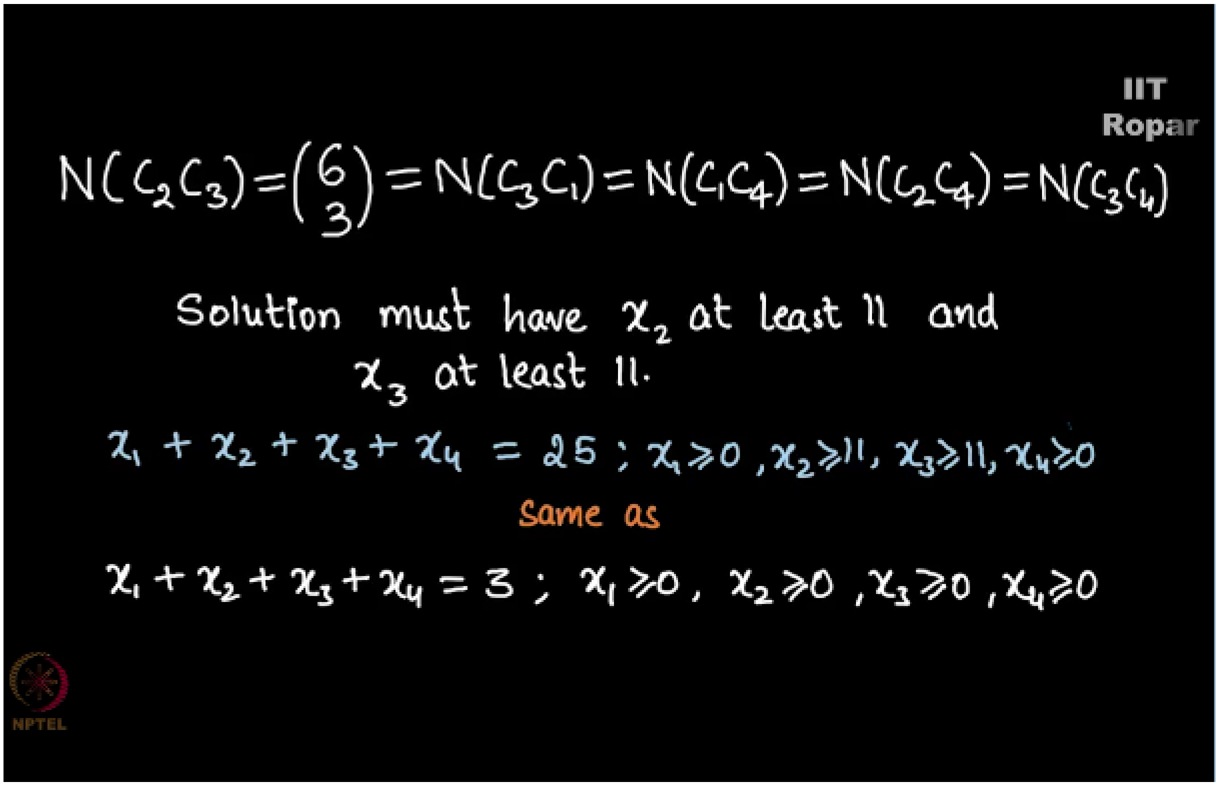
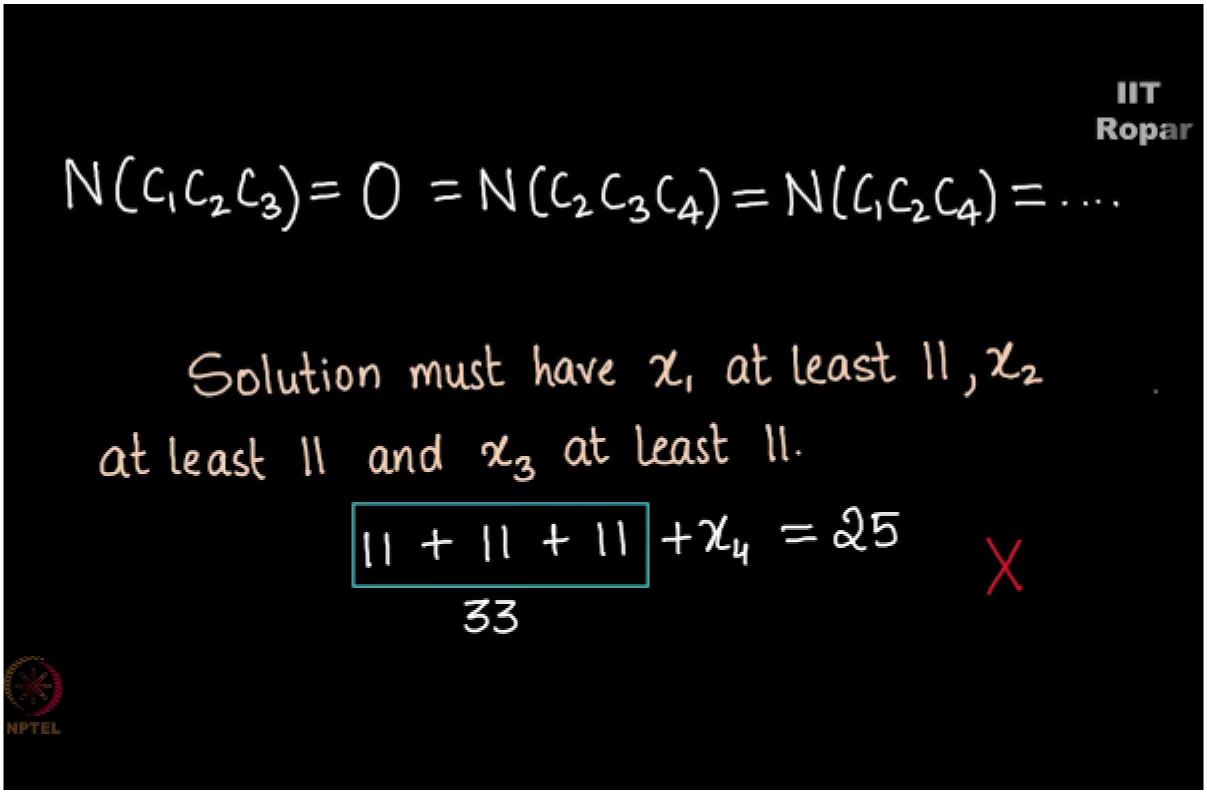


Now what would happen to (C1, C2, C3)? The number of possibilities of (C1, C2, C3) will be, you see all of these 3 should happen, that is X1 must be at least 11, X2 must be at least 11, and X3 must be at least 11, well, will this ever happen? Because the moment I give, just 11, well just 11 as the value to X1, X2 and X3 we get 33 as the answer and I'm not even reached X4, and hence N(C1,C2,C3) is 0, the same holds true for (C2, C3,C4) (C1, C2,C4) and all of them, (Refer Slide Time: 08:42)



N(CI, CJ, CK) is 0, I, J, K lying between 1 to 4, now if this is 0 what will happen to C1, C2, C3, C4 it is very obvious that they will be 0 as well, you can try it out yourself, so N(CI, CJ, CK, CL) is 0 for all I, J, K, L lying between 1 to 4,

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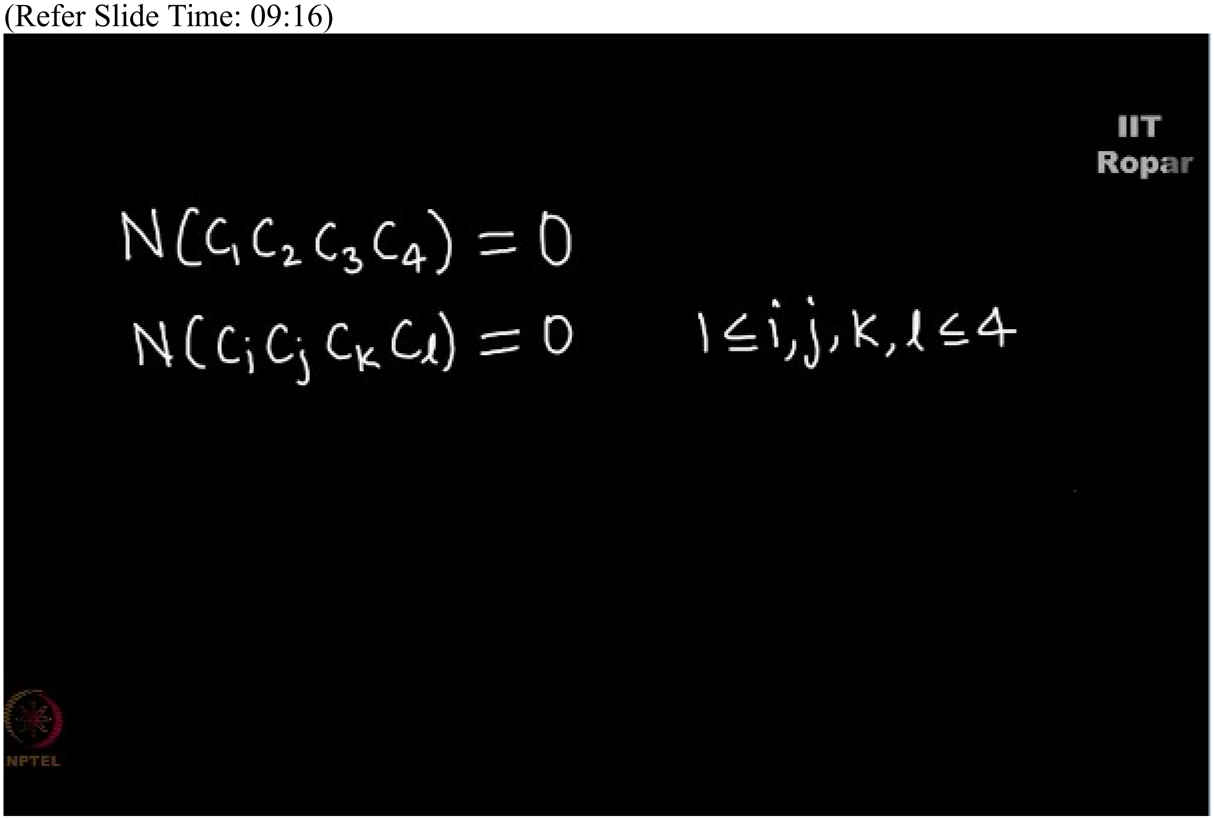

so what is the final answer? We wanted those enumerations or possibilities where XI's lie between 0 to 10, so the answer is N(C1 bar, C2 bar, C3 bar and C4 bar), if N(CI, CJ, CK) and N(CI, CJ, CK, CL) is 0 we need not consider them, right, in the formula, and the final formula they need not consider, so the simplified version goes like this, N(C1 bar, C2 bar, C3 bar, C4 bar) = N – N(C1) + N(C2) + N(C3) + N(C4) which is nothing but S naught, - S1, + S2, S3 and S4 are 0, so I am not considering them, +S2, S2 is what? N(C1, C2), N(C2, C3) and so on, right.

So now N is 28 choose 25, all possibilities that is S naught, S1 happens to be 4 times 17 choose 14,

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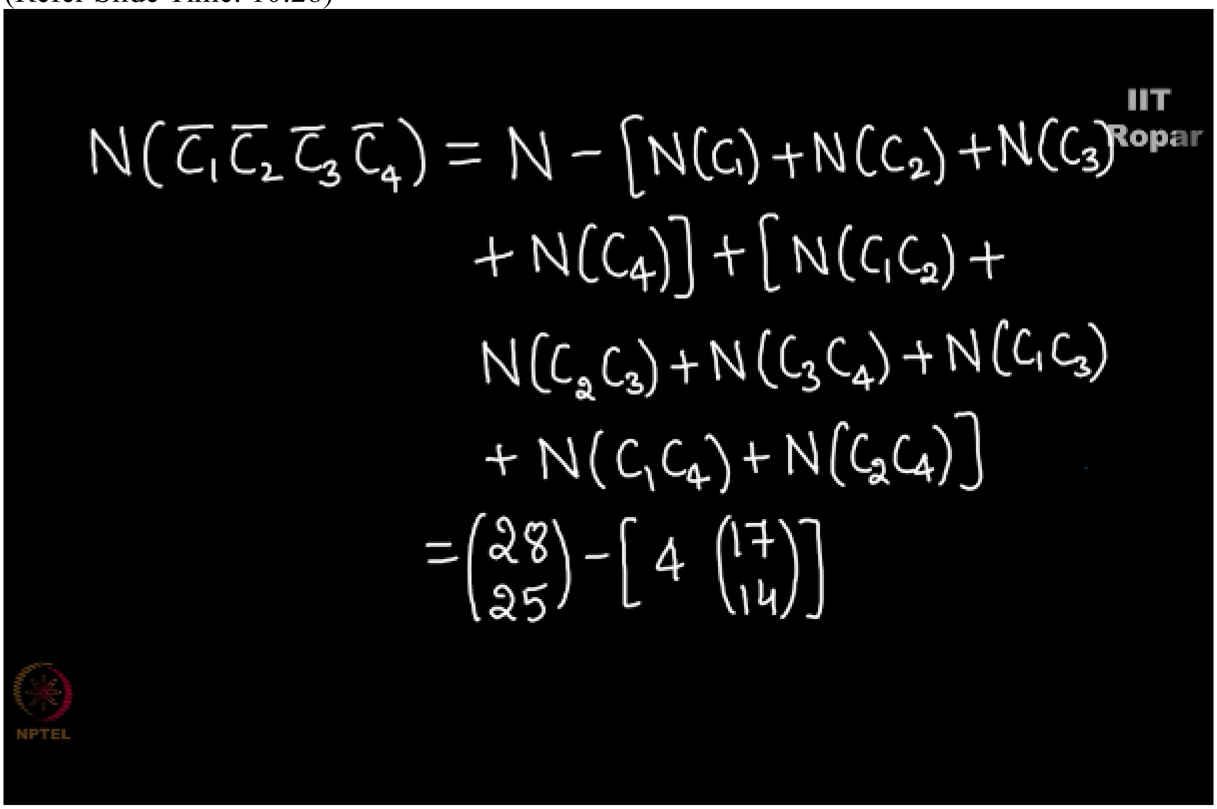

rather I can tell it as 4 choose 1 times 17 choose 14, and S2 happens to be 4 choose 2, or 6 times 6 choose 3, so the final answer goes like this, 28 choose 25 – 4 choose 1 x 17 choose 14 + 4 choose 2 x 6 choose 3.

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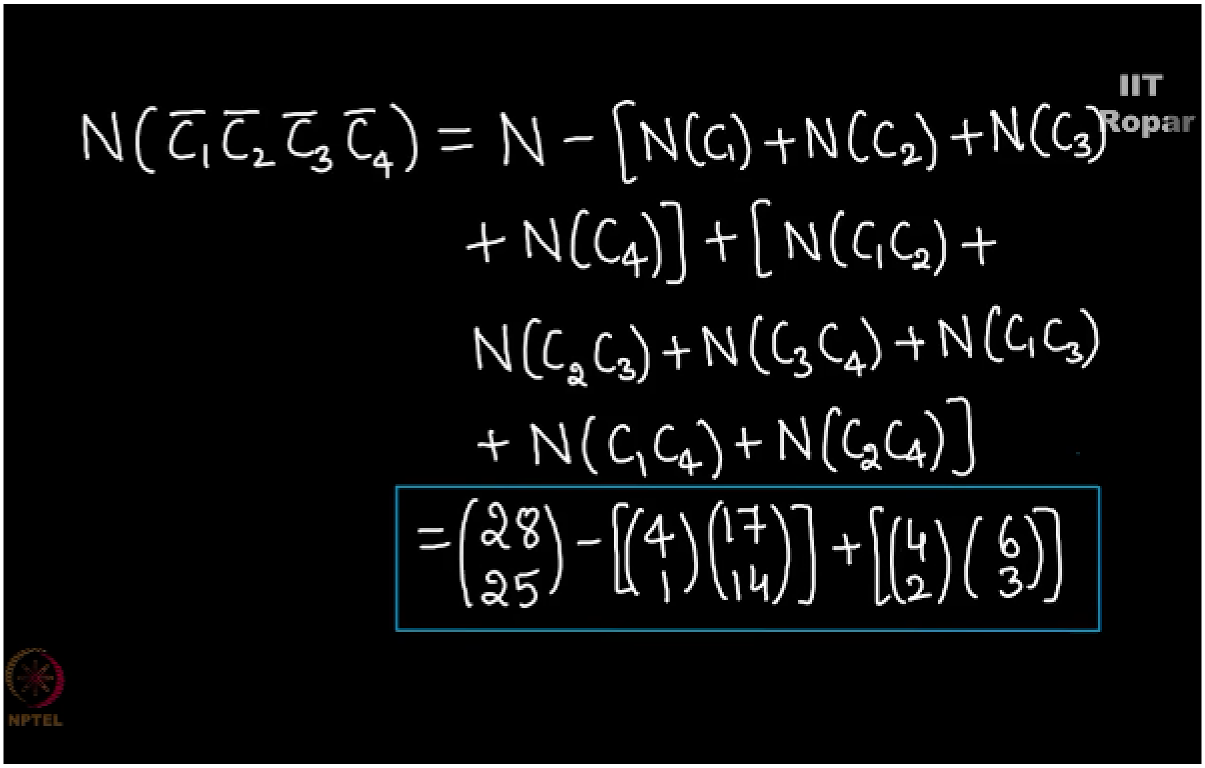

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