NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Proof of Inclusion - Exclusion formula
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Let us now look at the general result and its proof, if you can recollect N bar = N – N(C1), (C2), (C3), (C4) up to (C5) is what we solved and so on is what we solved for the 5 node graph problem.

And now let us assume that there are K such conditions C1, C2, up to CK and entities satisfying this conditions are put into the corresponding vowels
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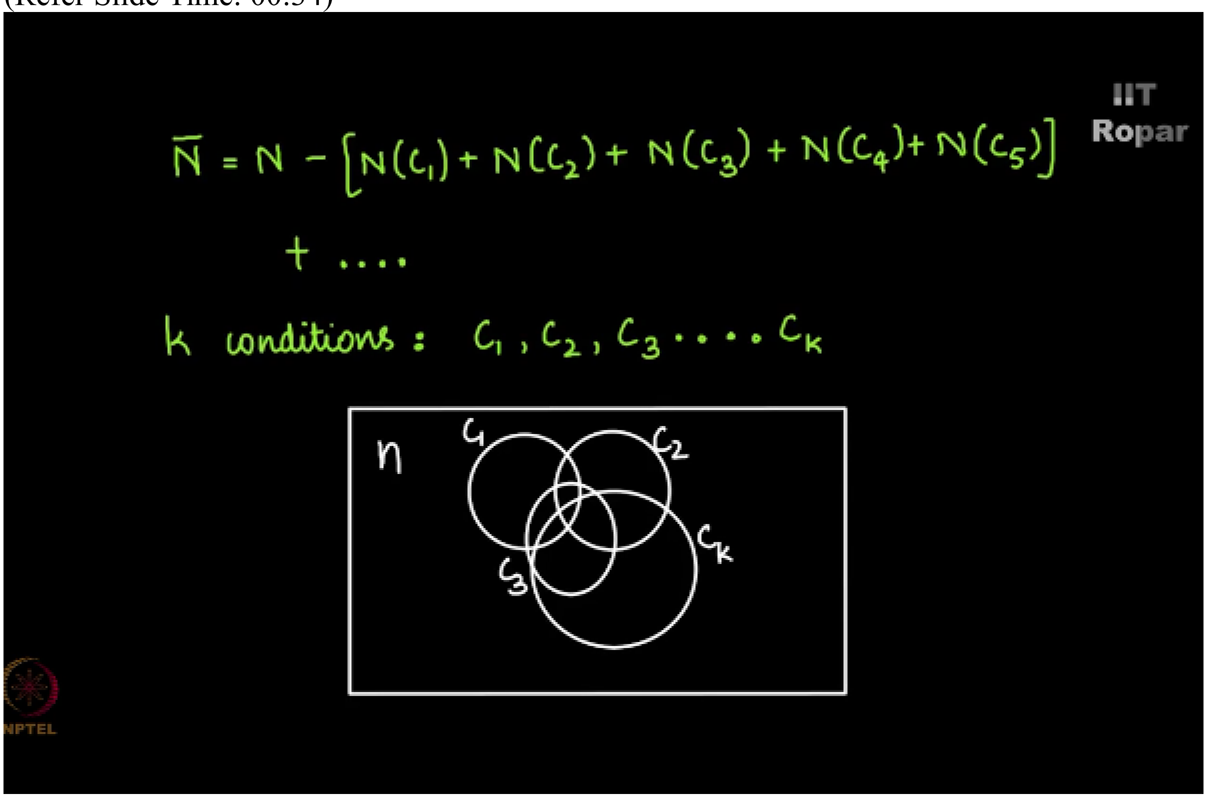

and look at this big rectangle, the total number of elements inside this rectangle is N in number, and what we need to count is total possible entities that are outside all these K number of CI's, C1, C2 up to CK, right, okay, so the formula goes generalizing what we discussed a while before, the formula goes something like this, N bar = N – N(C1) + N(C2) + N(C3) up to N(CK) and then an alternating sign + N(C1, C2) + N(C1, C3) + N(C1, C4) and so on up to the last term of N(CK-1, CK), and then again –N of 3 terms (C1, C2, C3), (C1, C2, C4) and so on, as you can see in the previous term, previous entity there were K choose 2 terms, here there will be K choose 3 terms, and then K choose 4 terms and so on.

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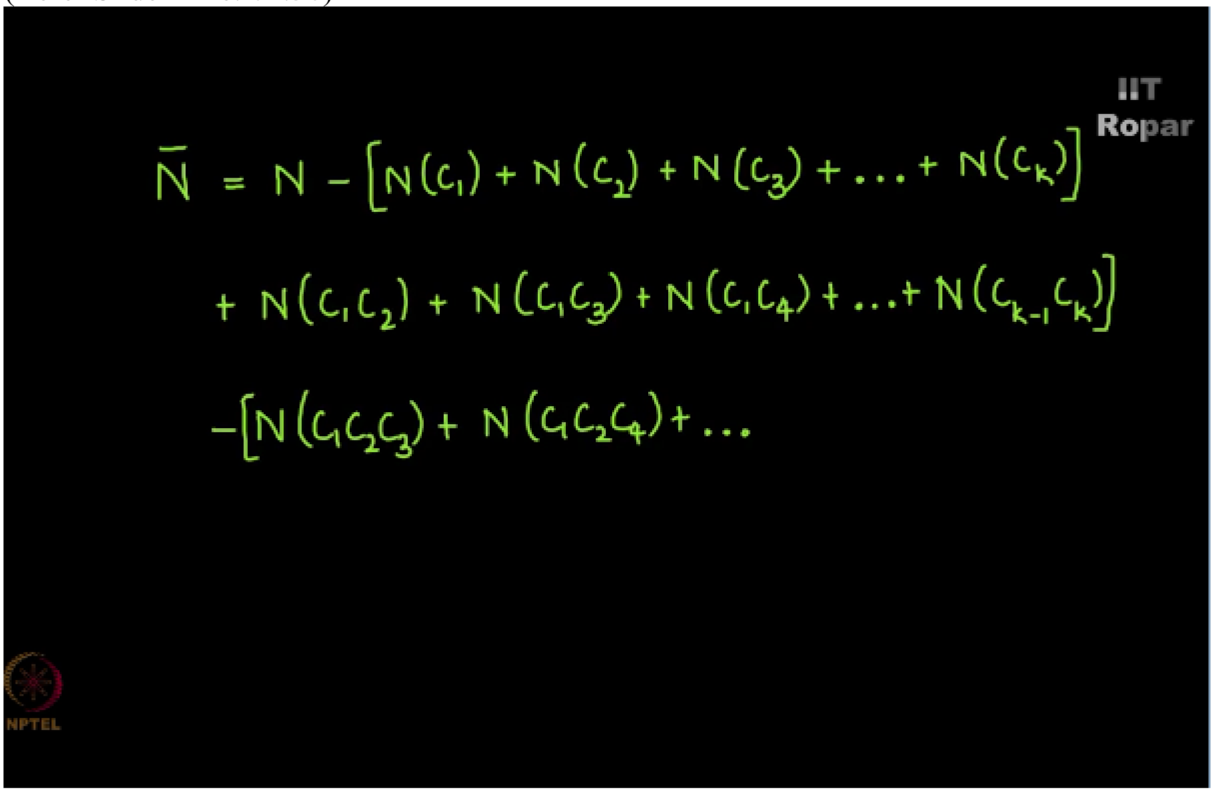

So on the last entity will be N(C1, C2, up to CK), only one term you see because all K conditions will be put here, and the symbol in front of it depends on, you see minus plus, minus plus is alternated, a little observation tells you it's going to be -1 to the power of K, it's a standard trick in mathematics, most of you probably know it, if not observe it deeply, will realize why it is -1 to the power of K.

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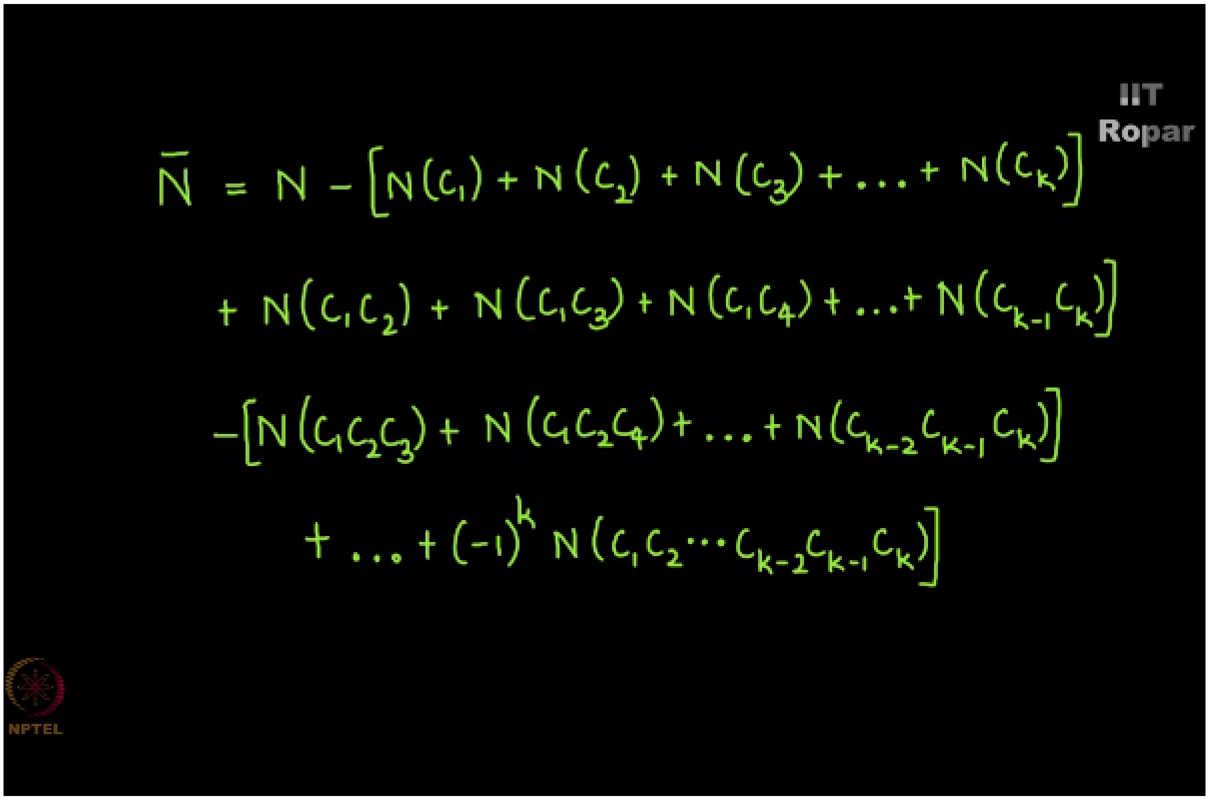

Now how do we show this is true? Here is a very neat trick, what I do is I will observe that every single entity, inside this rectangle contributes equally to the left hand side, as well as to the right hand side, this is a new form of proof, it may take some time for you to even start understanding, be beginning to understand what this means, right, so with time it will get familiar, you will be familiarized, but then you shouldn't be surprised if you don't get it in the first attempt, okay.

So I repeat for every single entity inside the rectangle I'm going to show you that it contributes equally to the left hand side of this equation, and the right hand side of this equation, thus ensuring that this formula indeed is true, okay, how do I do that? Look at this, take an element outside all this CI's, that is counted once in N bar obviously, because N bar is all such elements outside this vowels, it's corrode once in N bar and of course once in N, because it's indeed inside the rectangle, correct, so once in N bar, once in N and you see in N(C1) it is not counted, because it is not inside the C1, in none of these entities is it counted? Please note a zero contribution from the first term in this N(C1) + N(C2) up to N(CK) it is not counted, so 0, the next one is also 0 so on and so forth everything is 0,

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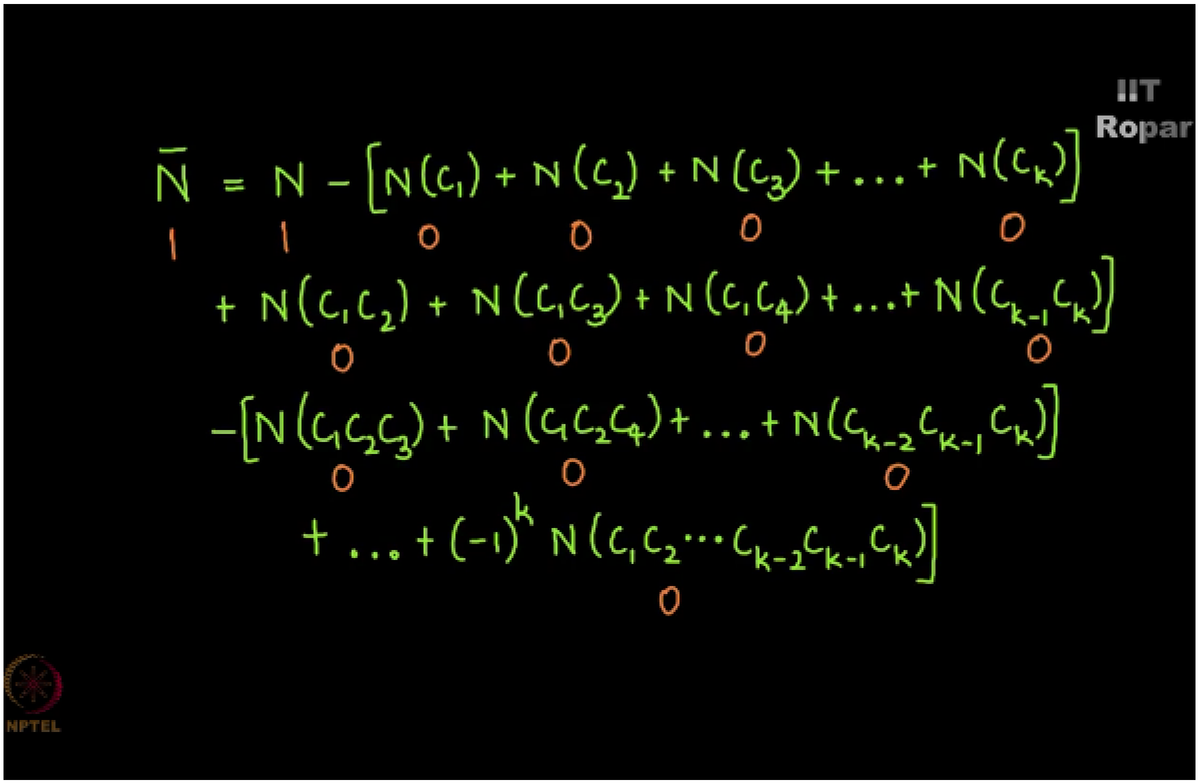

so one on the left hand side and 1 – of 0 on the right side which is one on the right hand side, so an entity outside a vowel, all this vowels which is not in any of the vowels, contributes one on the left hand side and one on the right hand side.

Now what if you take an entity that is in C1, or let's say that is an element that satisfies some random K conditions, okay, C1, C2 up to CK are some C2, C7, C21, C48 so on up to some K number, I'm sorry, let say R number of conditions satisfies, let's assume that, then if it satisfies R number of conditions, and if let's assume it is not outside any of this vowels it's inside, R means R is greater than or equal to 1 is all I'm saying, I'm pretty straight forward, think about it, so which means it doesn't contribute anything to N bar, it contributes a 0 to N bar, and of course it's within N so it contributes 1 to N, and then to N(C1), N(C2) up to N(CK) it satisfies R number of conditions which means it contributes to R such CI's, so here those R, CI's will have a 1, 1, 1, 1, 1, the rest of it will be 0, 0, so all put together R entities here will be 1, so 1 + 1 + 1, R times is R, so this term here will have this R number, okay, it contributes this entity which satisfies R conditions will contribute R here.
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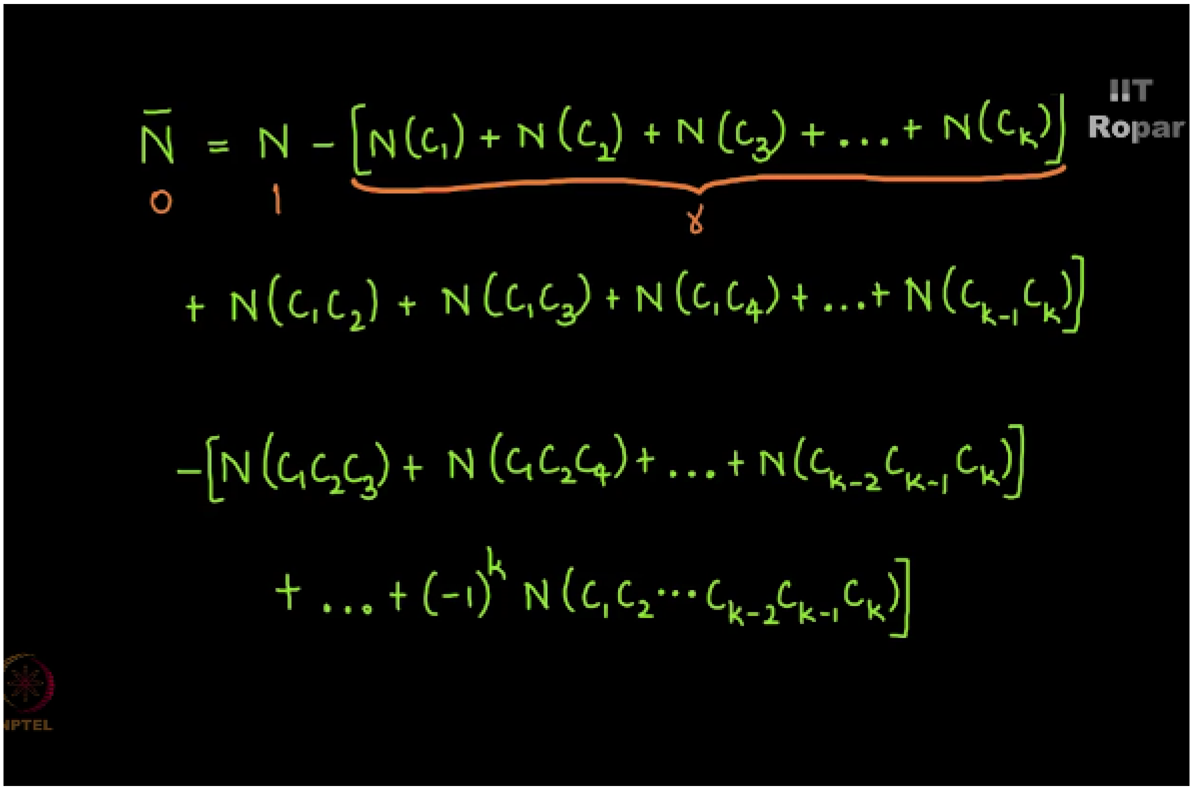

Similarly think about it, when it satisfies R conditions, given any two conditions from this R conditions it will satisfy, so in the second term it will satisfy R choose 2 conditions, and the rest it won't, think about it, again a point of confusion could be here, you may want to spend some time here, and then see why it is R choose 2.

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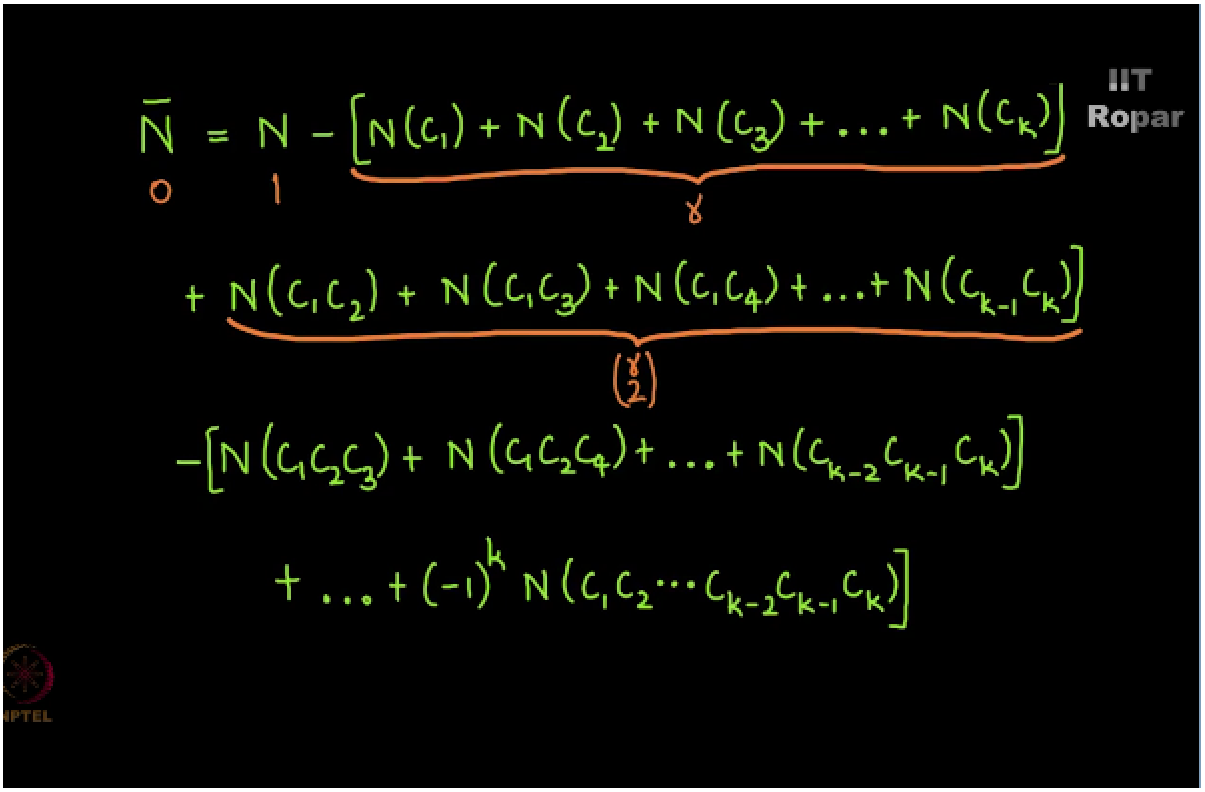

Next it will be minus R choose 3, and next it will be + R choose 4 and so on, and so on up to the point where it actually reaches R conditions, in the Rth step it will be R choose R, after that there will be R+1 conditions and hence it will be 0 everywhere, again a point of warning, think about how it stops at R conditions, I mean R lines here, okay.

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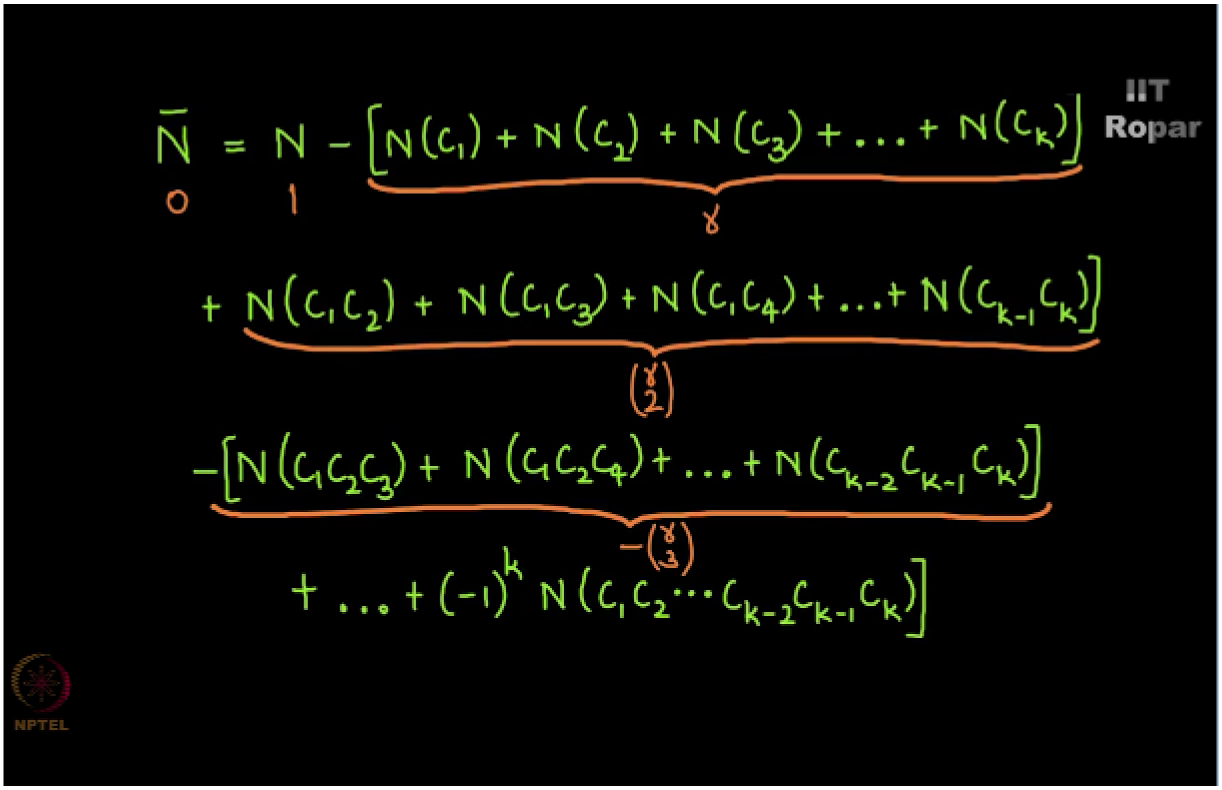

And now it contributes 0 the left hand side and 1 to N and then – R choose 1 + R choose 2, because R is R choose 1, so R choose 2 – R choose 3 + R choose 4 so on up to -1 to the R, again see how it is -1 to the R here, times R choose R, so –R choose 3, 3 is an odd number so you put a minus, 2 is a even number so you're putting a plus, so you observe that it will be -1 to the R, now if you a stair at this you will realize that this entity is nothing else but 0, so 0 = 0, why is this 0? It is nothing else but 1-1 whole to the power of R

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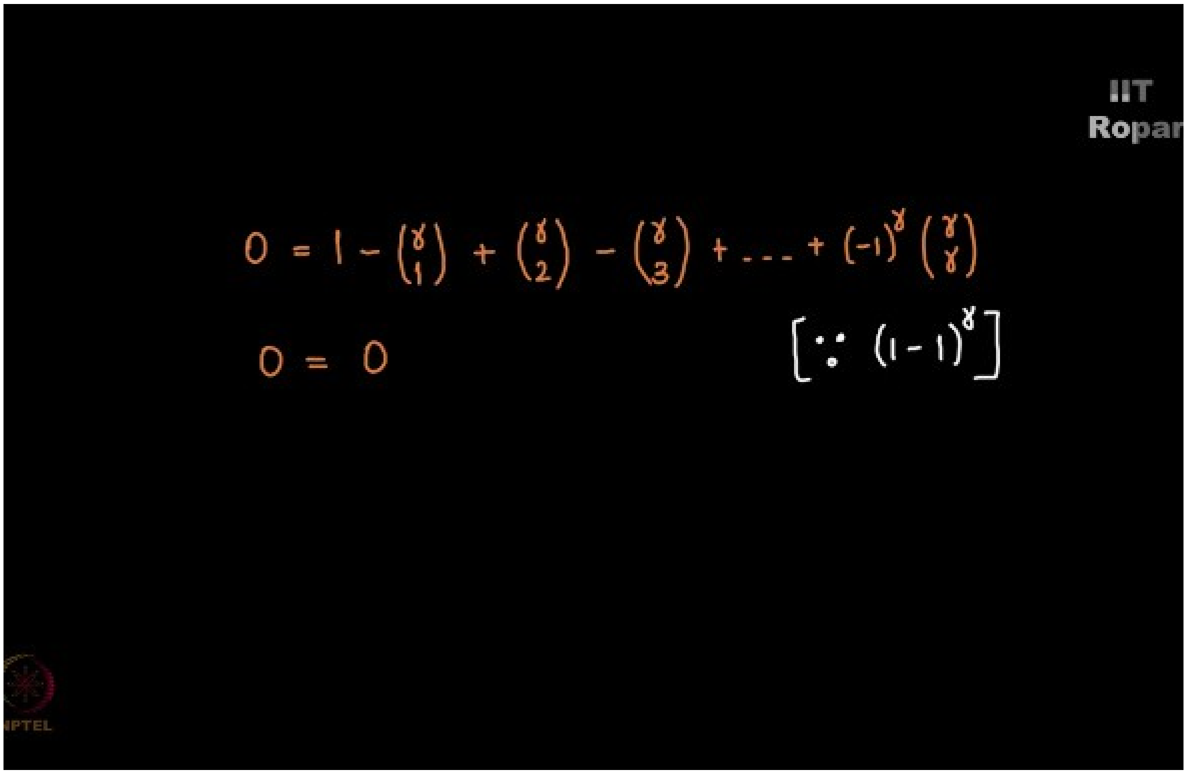

you see, think, so 0 = 0 and hence no matter what element you take inside the rectangle it contributes equally to the left hand side, and to the right hand side and hence the equation is true, now look at the very weird way of proving a formula, there is yet another way of proving it, and that is using induction we'll leave that to you as an exercise.

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