NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 3 &
Generating Functions

NetworkX - Bipartite graphs

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Here goes another question, in how many ways can 15 identical chocolates be distributed among 4 children, so that each child gets at least 2 chocolates,
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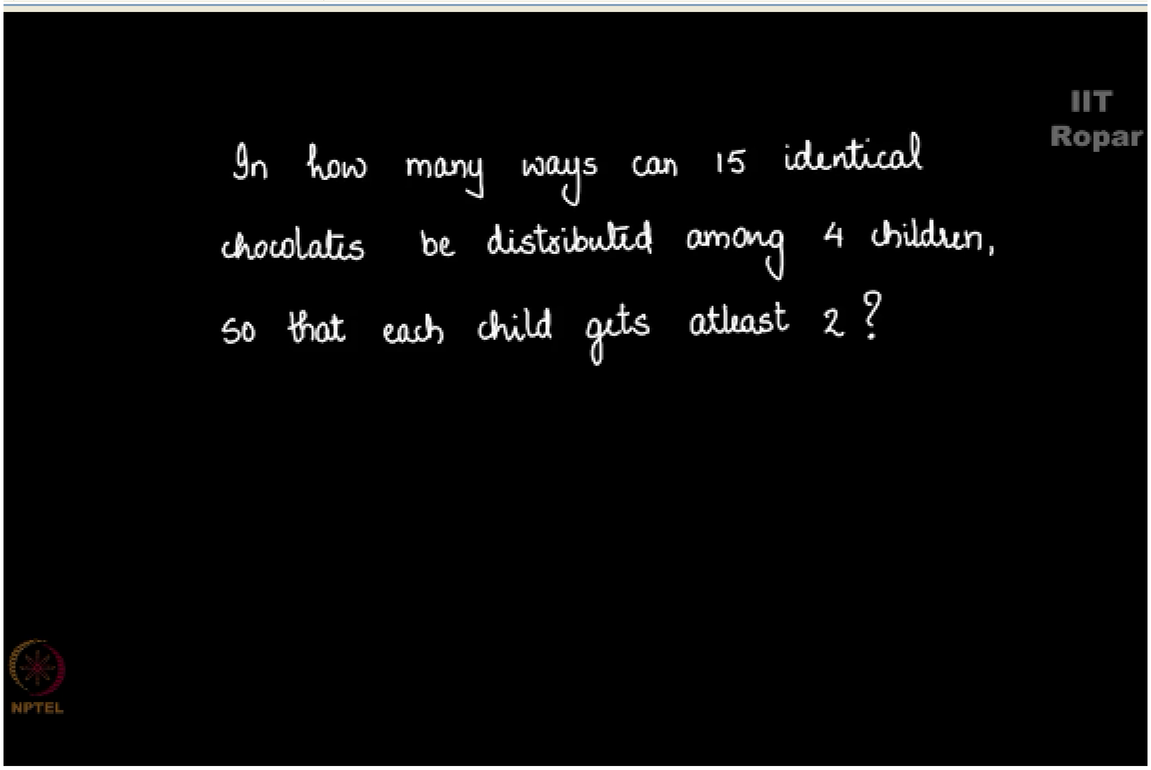

so we have 15 chocolates identical ones and it has to be distributed among 4 children so that each child gets at least 2.

Let us enumerate a few valid possibilities, so 9 2 2 2 this is a valid possibility,
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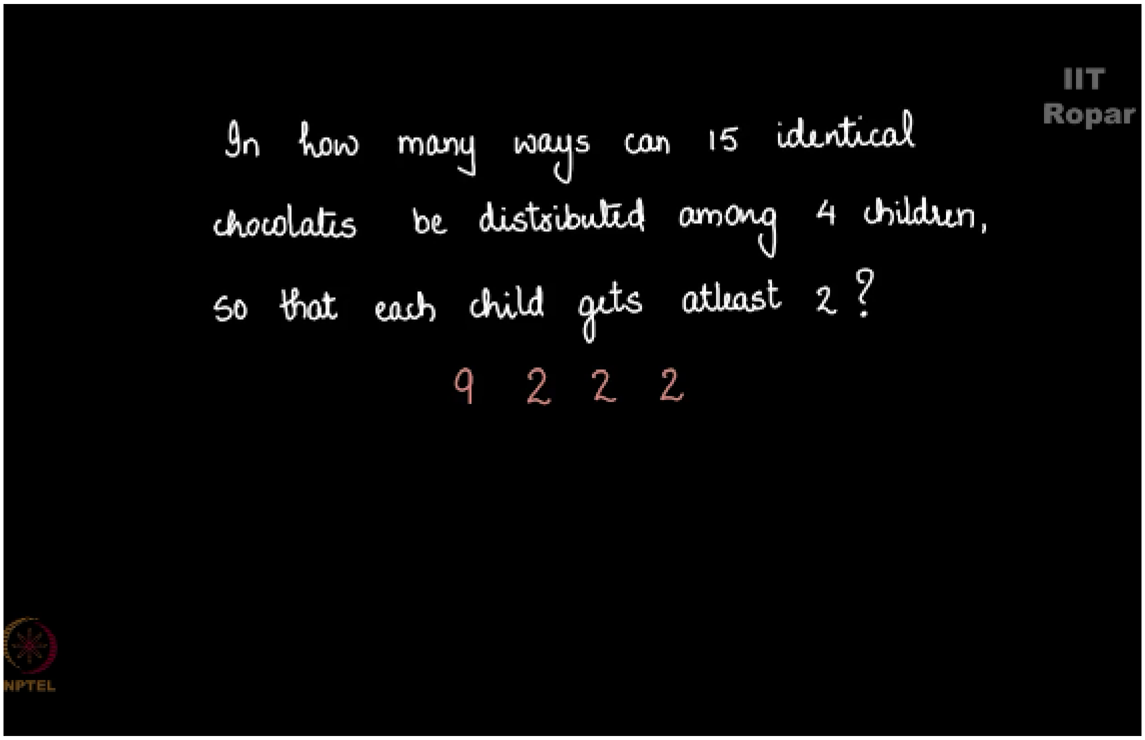

there are 4 children, now 6 4 3 2 this is yet another valid possibility, 15 0 0 0 is this valid? No, because each child must get at least 2, 10 5 0 0 is also not valid because we have 0 0 here,

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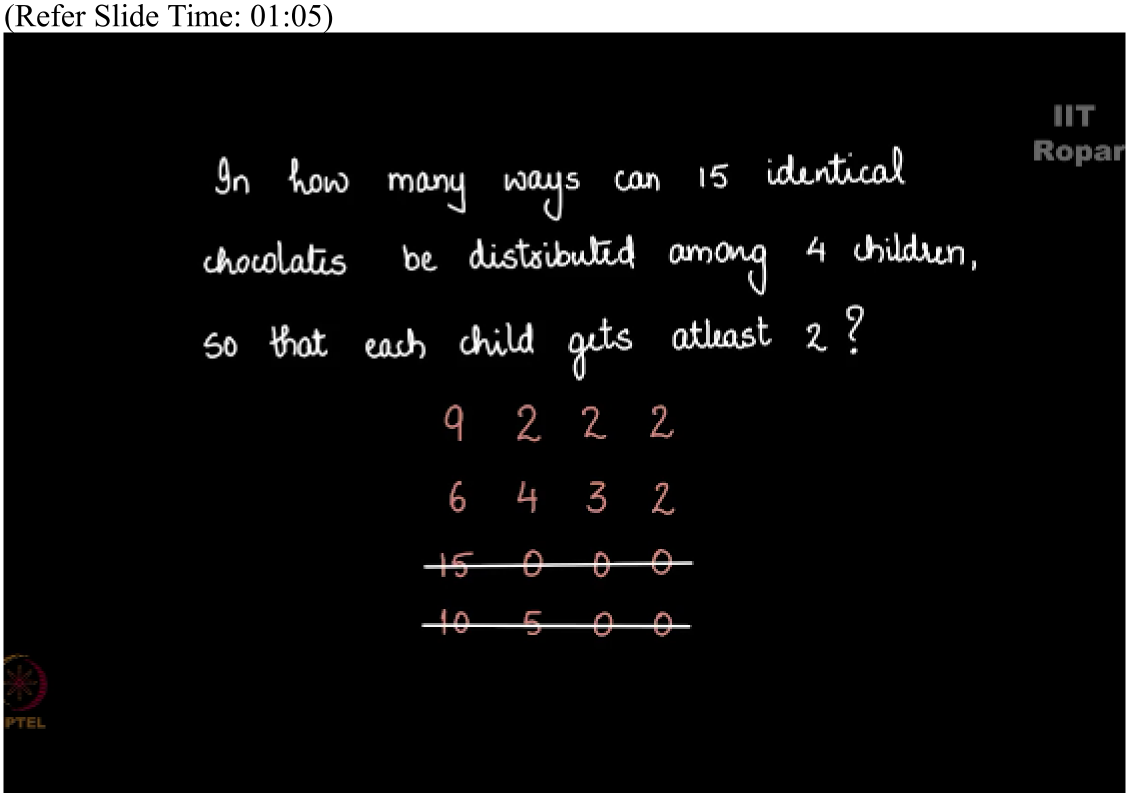

right, and so on so you can enumerate several possibilities, but don't you think it is very cumbersome, here is the importance or significance of generating functions I can represent this difficult problem using just one expression and you will be able to figure out the answer very soon.

Now the possibilities or the number of ways in which one child can get chocolates can be represented as X square + X cube + X to the 4, so on up to X to the 15, why?
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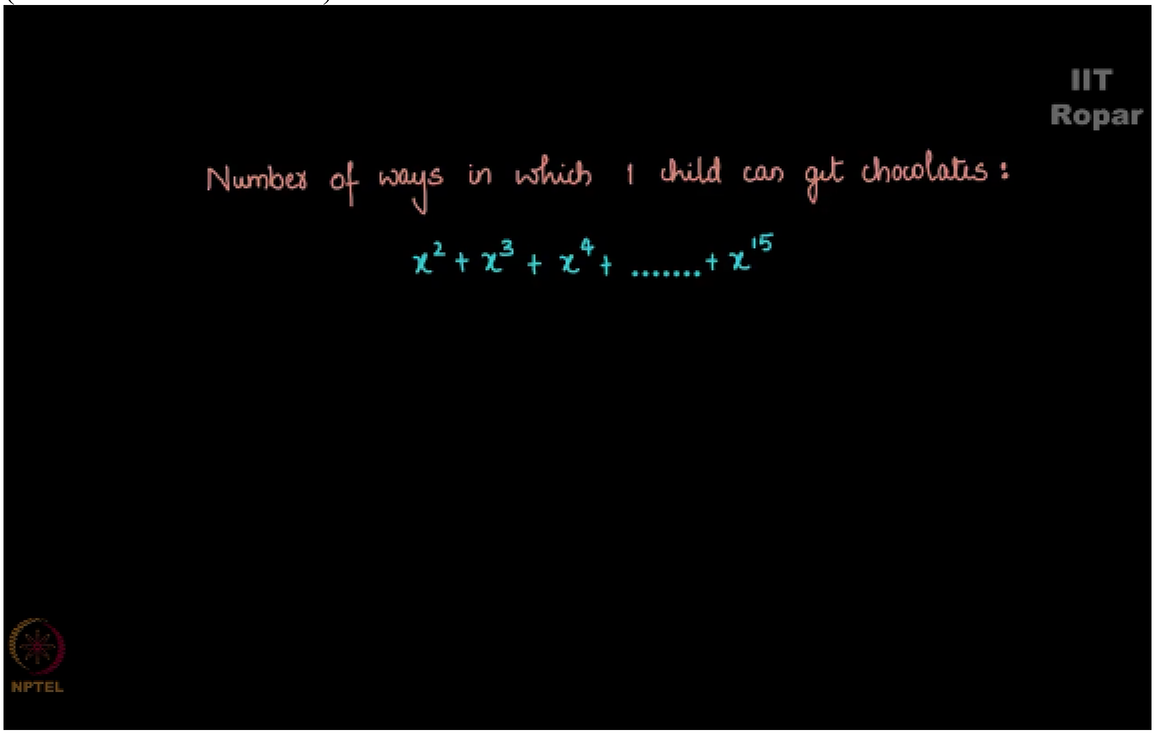

Because the minimality condition is that each child must get at least 2, and hence I started with X square, what is the maximum possibility? I can go up to X to the 15 although a child cannot get 15 chocolates but must get at least 2, I have written X to the 15 here, it doesn't matter do not worry much, now this is for one child, for 4 children it will be X square + X cube + X to the 4 + X to the, up to X to the 15 the whole power 4, because it is the same for all the 4 children,
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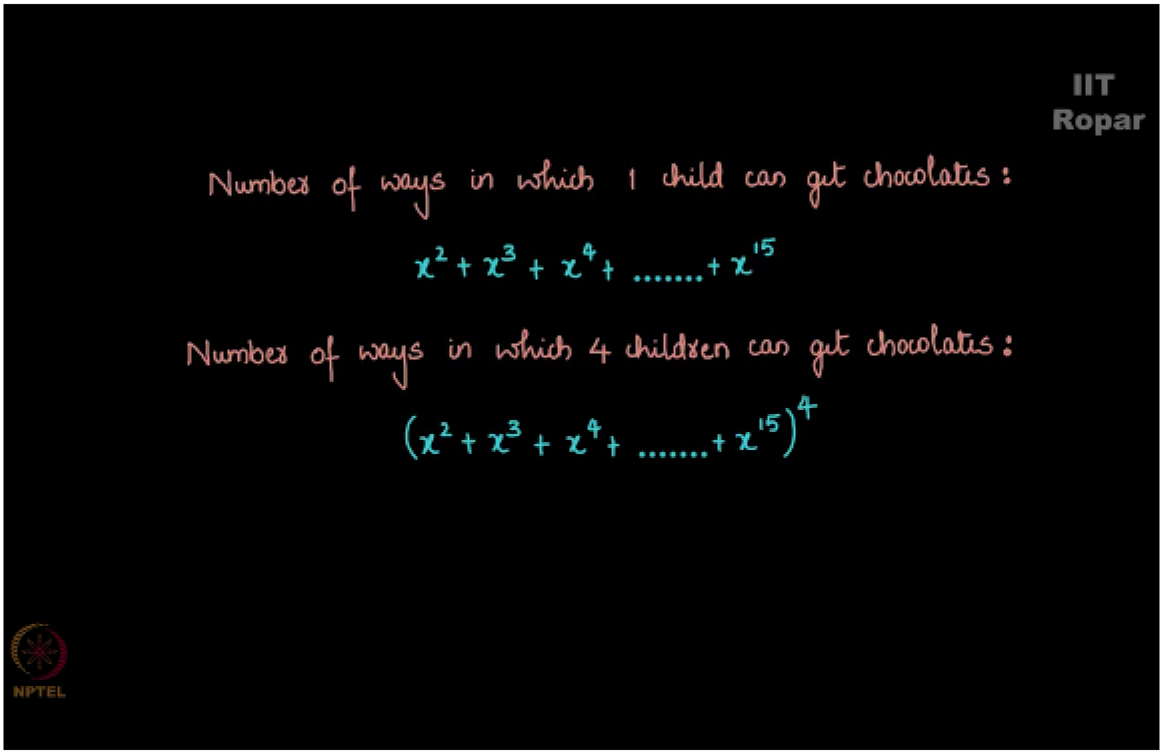

the coefficient of X to the 15 in the product of X square + X cube + X to the 4 so on up to X to the 15 whole power 4 will give us the answer for the question that in how many ways can 15 identical chocolates be distributed among 4 children with the constraint.

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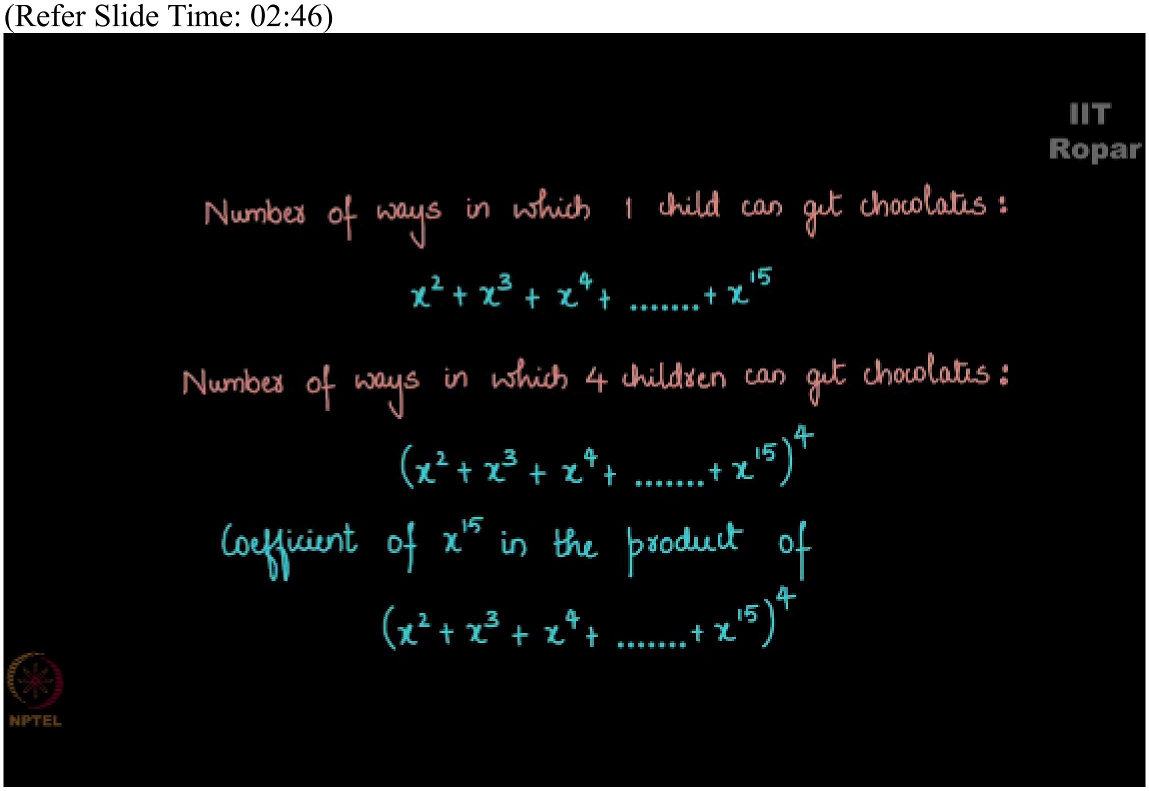

Now what I'm going to do is I am going to take out X square common here, so what does it become? X square into 1 + X + X square + X cube so on, I'm not going to end it up with a finite term, I am going to leave it as it is, it will not matter much you will observe it very soon,
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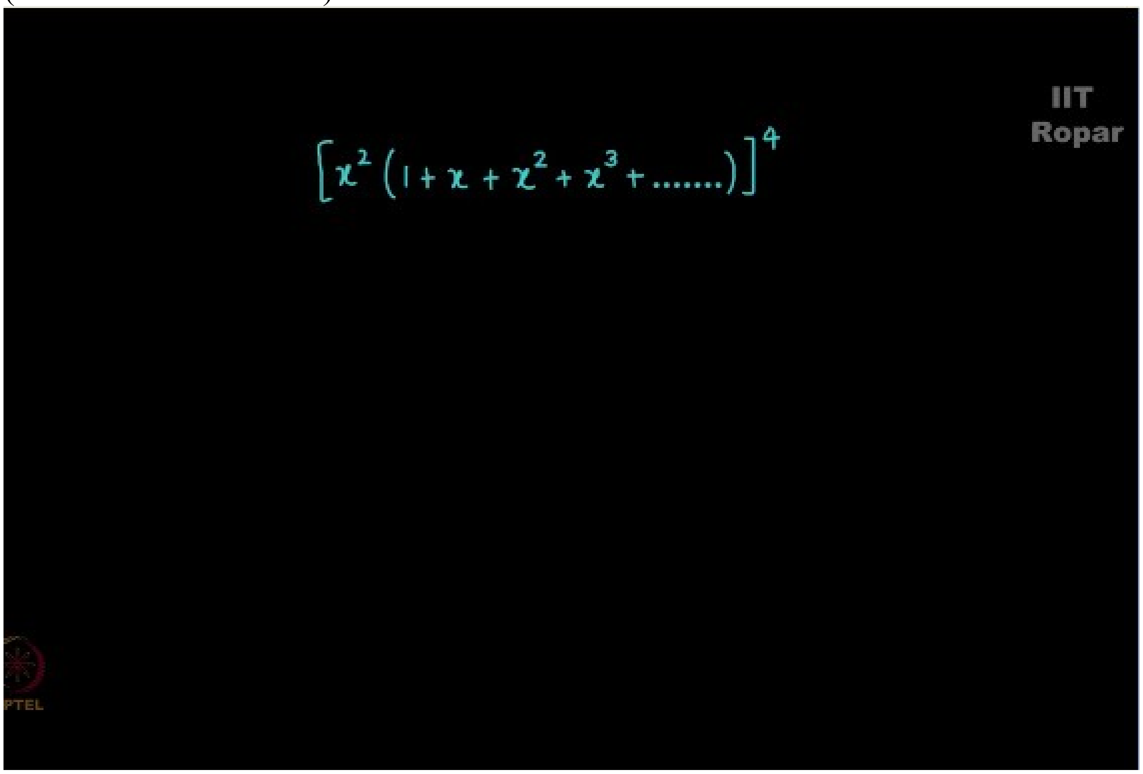

the whole power 4, now X square to the 4 is X to the 8 into 1 + X + X square so on the whole power 4, don't you observe that 1 + X + X square + X cube, so 1 can be written as 1/1-X, we have seen that that 1/1-X is the closed form of 1 + X + X square + X cube and so on,
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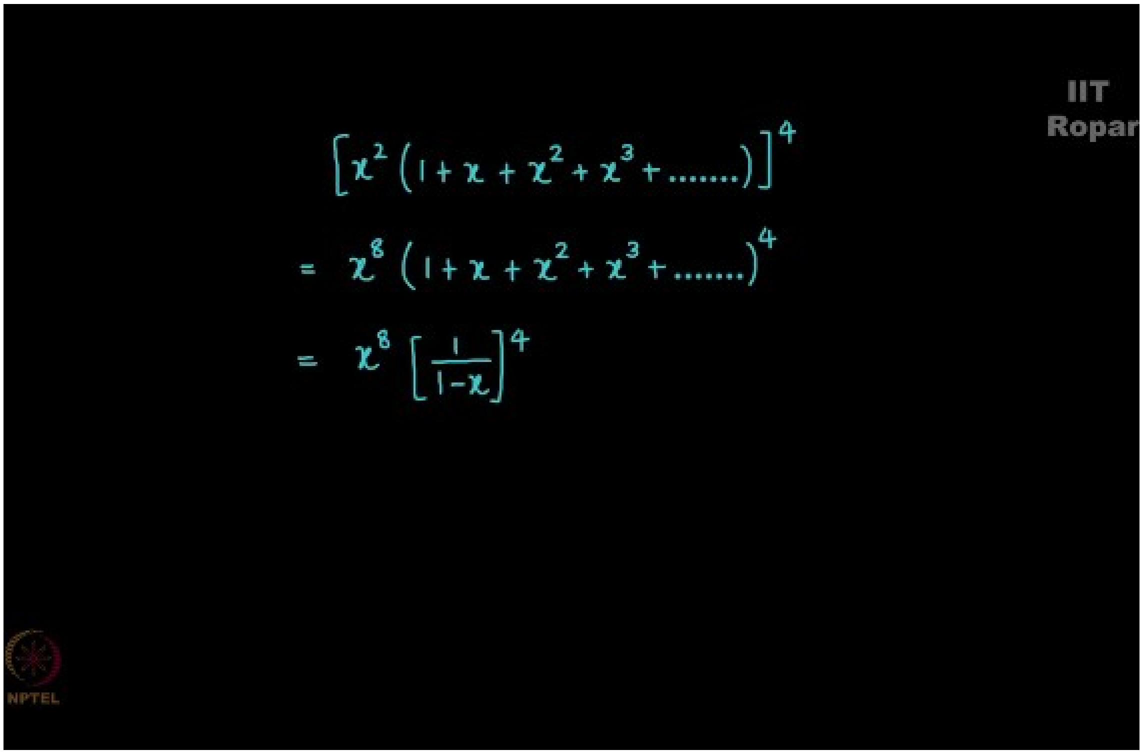

so this can be written as X square the whole power 4 into 1/1-X the whole power 4, clear, if it is not clear please rewind the video, play it several times and you will be able to understand, X square into 4 is X to the 8, right and I'm going to retain 1/1 -X whole power 4 as it is, so I can write it as X to the 8 into 1-X whole to the -4, do you observe that?

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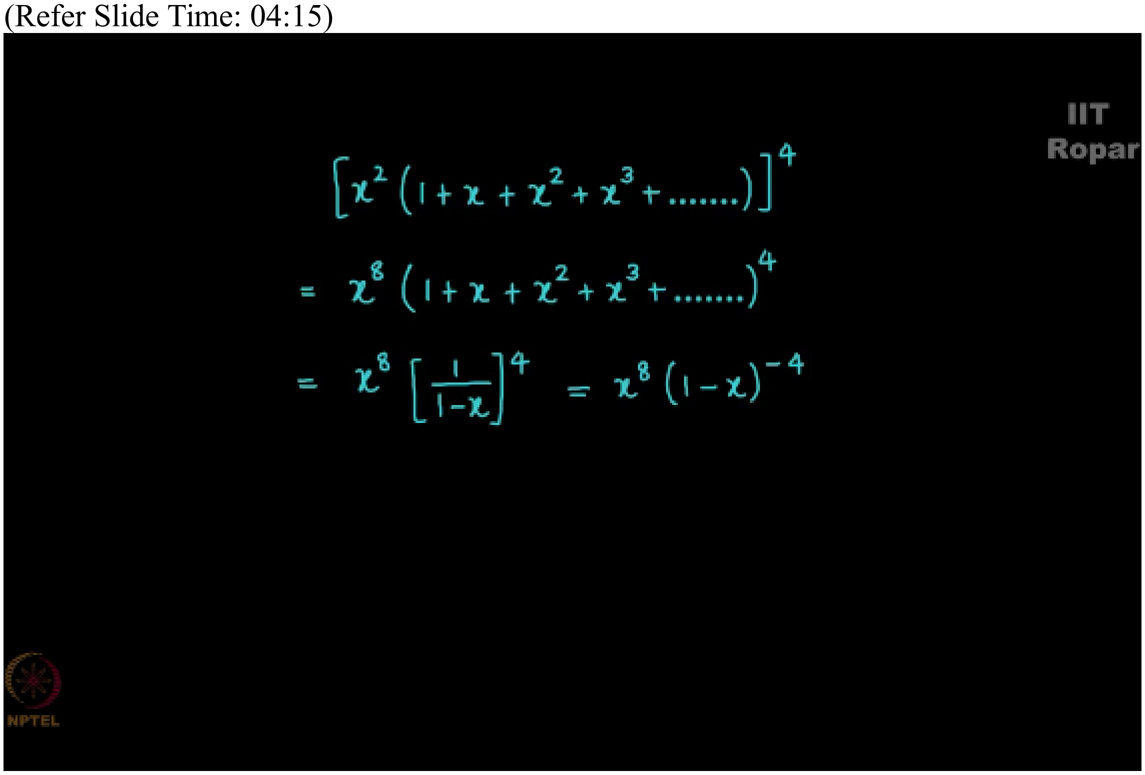

Because it was 1 to the 4 by 1-X whole to the 4, right I just took it to the numerator and hence it becomes 1-X whole to the -4, I have already written X to the 8 outside, I need to find out the coefficient of X to the 15, how do I get X to the 15, X to the 8 is already taken out, what do we have to look for is the coefficient of X to the 7 in the expansion of 1-X whole to the -4,
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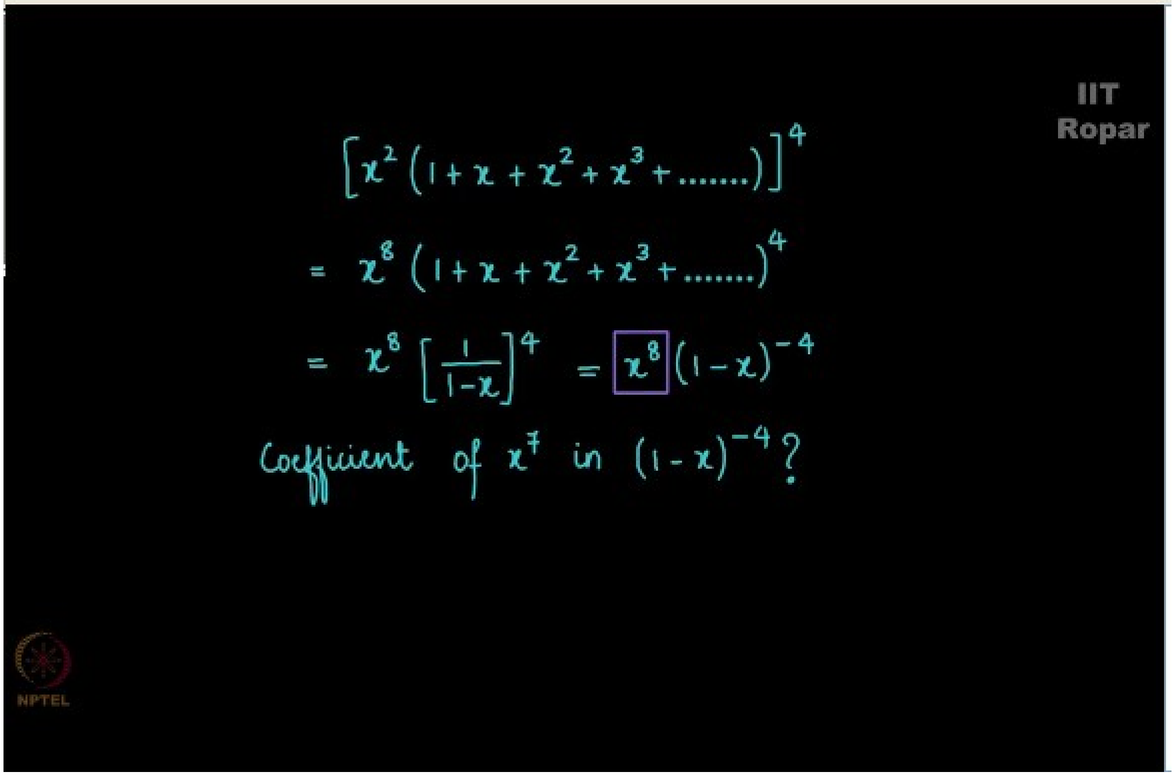

because X to the 8 is taken out, it is locked, the only option available for us is X to the 7, because X to the 8 into X to the 7 will give us X to the 15, do you observe that? Now these are very certain things, you might have to pay attention and just watch the video once or twice, so the coefficient of X to the 7 and 1-X whole to the -4 this is very simple, you people know how to do this applying the formula it gives me summation and choose R, what is N choose R here? It is -4 or rather I'll write it as 4, 4 choose R into -X to the R, right.

Now R is 7 here and hence -4 choose 7 into -1 whole to the 7, this is the coefficient of X to the 7 here,
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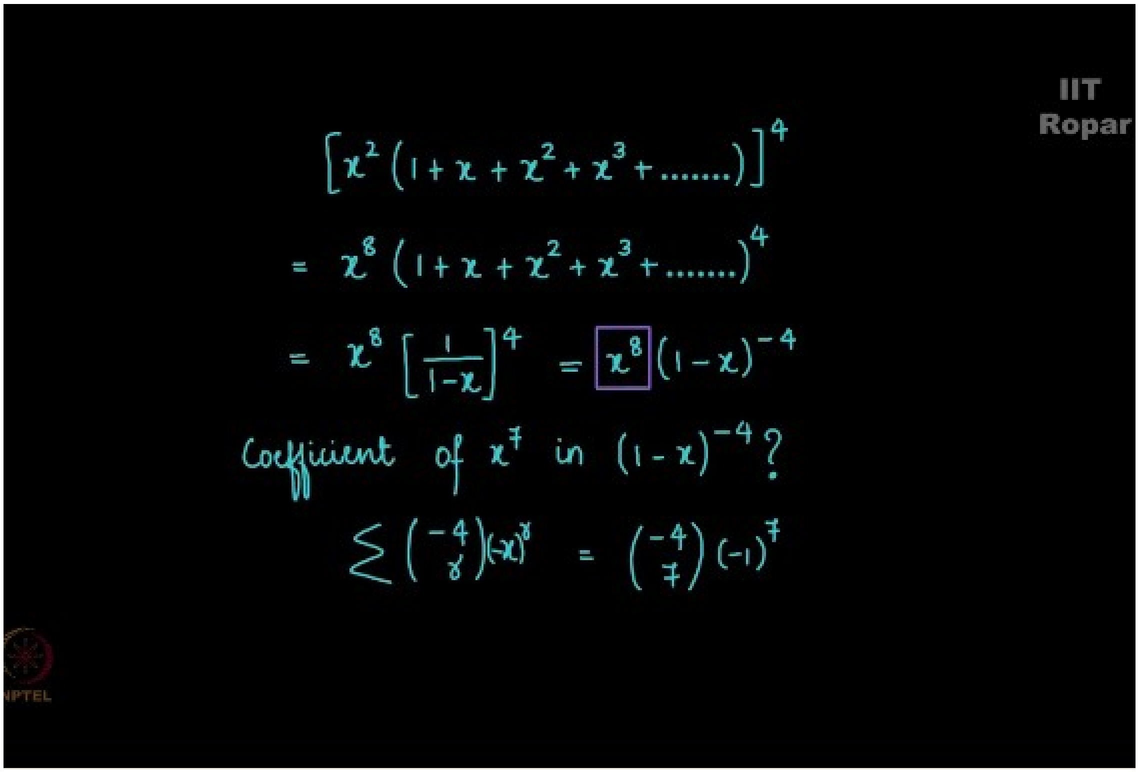

clear because it was -X to the 7, I'll take out -1 common to the power 7 it is, right, once you solve the problem yourself things will be much clearer to you.

Now -4 to the 7 it is nothing but -N choose R, how can it be written as? -1 whole to the R into N+R -1 choose R right, so the answer goes like this -1 to the 7 or -1 to the 7 into 4 + 7 - 1 gives me 10, so it is 10 choose 7 into -1 whole to the 7, so -1 into -1 gives 1, 1 into 10 choose 7, so 10 choose 7 once we apply 10 C7 the formula we see that it is 10 x 9 x 8/3 x 2 which gives me 120 up on calculation, so we see that 120 is the answer, that is it is the coefficient of X to the 15, so in 120 different ways 15 identical chocolates can be distributed among 4 children, so that at least each child gets 2 chocolates.
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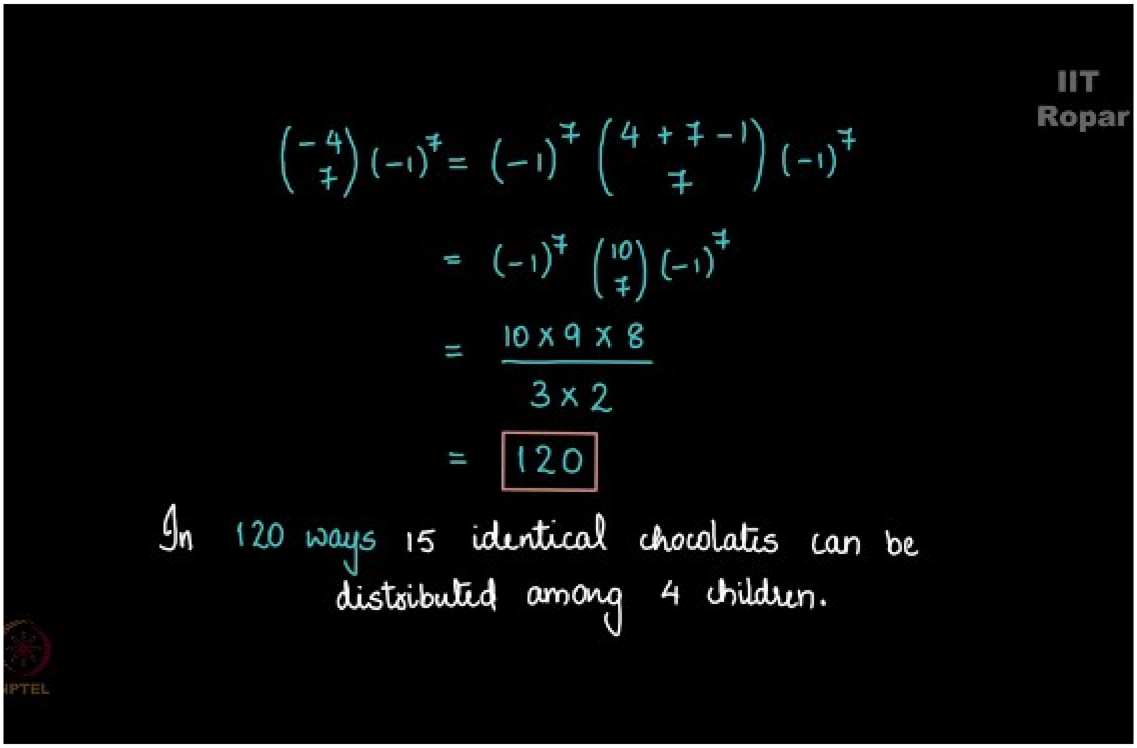

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