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NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 3 &
Generating Functions

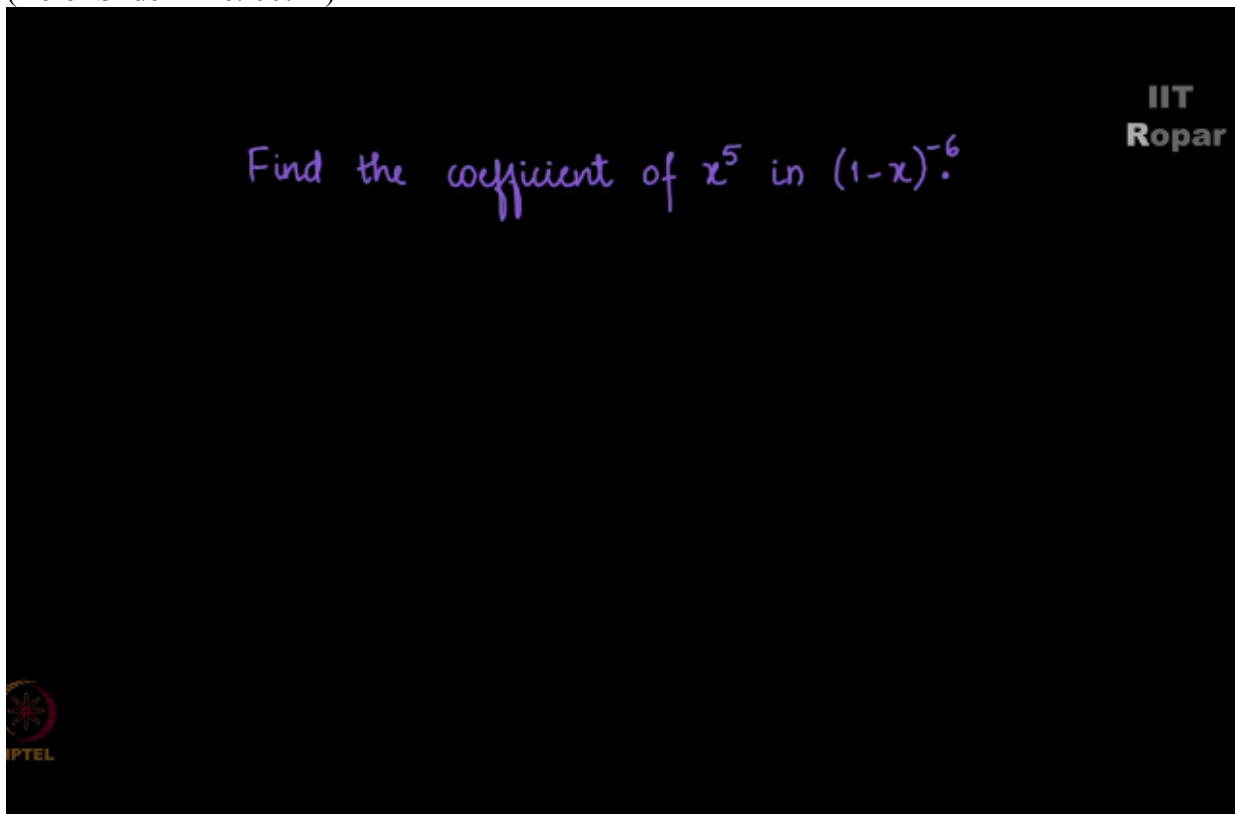
Generating functions - Problem 1

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The professor just taught the expansion of $1 + X$ whole to the $-N$, we are going to see that in action now, the first question goes like this, find the coefficient of X to the 4 in the expansion of $1 - X$ whole to the -6 ,

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well, we have learnt how to expand $1 + X$ whole to the $-N$, but here we are have $1-X$ whole to the $-X$, so how do we go about? Very simple, substitute in place of $-X$ another variable Y , so the question now becomes $1+Y$ whole to the -6 ,

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Find the coefficient of x^5 in $(1-x)^{-6}$.

$$-x = y$$



you have to find the coefficient of X to the 4, right, so we'll first expand $1+Y$ whole to the -6 and we will proceed further. So the expansion of this goes like this summation are from 0 to infinity -6 choose R into Y to the R ,
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Find the coefficient of x^5 in $(1-x)^{-6}$.

$$(1+y)^{-6} = \sum_{r=0}^{\infty} \binom{-6}{r} y^r$$

$-x = y$



I hope this step is clear, if not you can refer to the formula which is stated in the previous video, this is nothing but summation R from 0 to infinity -6 choose R into $-X$ whole to the R, in place of Y I have substituted $-X$.

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Find the coefficient of x^5 in $(1-x)^{-6}$.

$$\begin{aligned}
 & -x = y \\
 (1+y)^{-6} &= \sum_{r=0}^{\infty} \binom{-6}{r} y^r = \sum_{r=0}^{\infty} \binom{-6}{r} (-x)^r
 \end{aligned}$$



Now the coefficient of x^5 is you see $-N$ choose R is what? -1 to the R into $N + R - 1$ choose R ,
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Find the coefficient of x^5 in $(1-x)^{-6}$.

$$\begin{aligned}
 & -x = y \\
 (1+y)^{-6} &= \sum_{r=0}^{\infty} \binom{-6}{r} y^r = \sum_{r=0}^{\infty} \binom{-6}{r} (-x)^r \\
 \binom{-n}{r} &= (-1)^r \binom{n+r-1}{r}
 \end{aligned}$$



so -6 choose 5 can be written as -1 to the 5 , R as 5 here and N is 6 , so it becomes -1 to the 5 into $N + R - 1$ is $6 + 5 - 1$ which is 10 , so 10 choose 5 into -1 to the 5 into X to the 5 it is,
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Find the coefficient of x^5 in $(1-x)^{-6}$.

$$(1+y)^{-6} = \sum_{r=0}^{\infty} \binom{-6}{r} y^r = \sum_{r=0}^{\infty} \binom{-6}{r} (-x)^r$$

$$\binom{-n}{r} = (-1)^r \binom{n+r-1}{r}$$

$$\binom{-6}{5} = (-1)^5 \binom{6+5-1}{5} = (-1)^5 \binom{10}{5}$$

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and hence for the coefficient of X to the 5 we consider -1 to the 5 into 10 choose 5 into -1 to the 5 ,
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Coefficient of x^5 is

$$(-1)^5 \binom{10}{5} (-1)^5$$



so -1 and -1 become +1 here, now what remains is 10 choose 5, so the coefficient of X to the 5 is 10 choose 5 on simple calculation we see that it is $10 \times 9 \times 8 \times 7 \times 6$ divided by $5 \times 4 \times 3 \times 2 \times 1$,

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Coefficient of x^5 is

$$\begin{aligned} & (-1)^5 \binom{10}{5} (-1)^5 \\ &= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} \end{aligned}$$



now you must be knowing this formula very well by now, we have studied in week 1 right, so I didn't stated explicitly assuming that you guys know the formula of NCR.

Now on simplification this becomes 18×14 , I've canceled 5 and 10, and 4 and 8, 6 and 6 get canceled here, and so on, so you can check it out yourself,
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Coefficient of x^5 is

$$\begin{aligned} & (-1)^5 \binom{10}{5} (-1)^5 \\ &= \frac{2 \cancel{10} \times 9 \times 8 \times 7 \times 6}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 18 \times 14 \end{aligned}$$



so the answer would be 252, so the coefficient of X to the 4 in 1-X whole to the -6 is 252.
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Coefficient of x^5 is

$$\begin{aligned} & (-1)^5 \binom{10}{5} (-1)^5 \\ &= \frac{2 \cancel{10} \times 9 \times 8 \times 7 \times 6}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 18 \times 14 \\ &= \boxed{252} \end{aligned}$$



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