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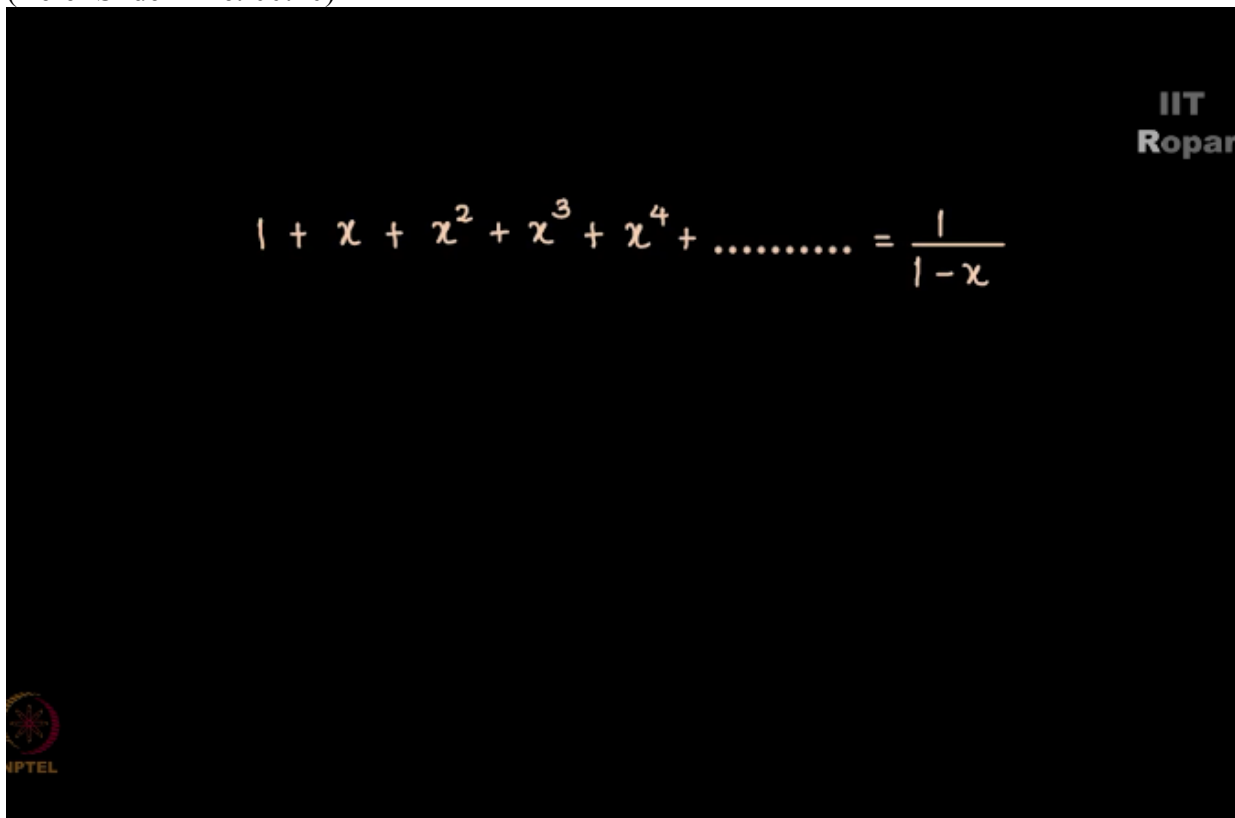
NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 3 &
Generating Functions

Generating function examples - Part 1

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So in the previous video we have seen that $1 + X + X^2 + X^3 + X^4 + \dots$ gives $1/(1-X)$ as the closed form,
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$$1 + x + x^2 + x^3 + x^4 + \dots = \frac{1}{1-x}$$

so the series can be written as $1/(1-X)$, now this is the generating function for the sequence 1, 1, 1, 1 and so on,
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$$1 + x + x^2 + x^3 + x^4 + \dots = \frac{1}{1-x}$$

1, 1, 1, 1,



you saw this, now we differentiate it $1/1-X$ and we saw that $X/1-X$ the whole square is the generating function for 0, 1, 2, 3, 4 so on, so this was the sequence, right.
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$$1 + x + x^2 + x^3 + x^4 + \dots = \frac{1}{1-x}$$

1, 1, 1, 1,

$\frac{1}{(1-x)^2}$ generates 0, 1, 2, 3, 4,



Now the professor asked if you can find out the generating function which can give the sequence 0, 1 square, 2 square, 3 square, 4 square, 5 square and so on, I'm going to give you the answer for that if you have to write it, very good, if you haven't you can follow from what I'm going to tell now.

Now we had earlier seen that $X/(1-X)$ the whole square = $X + 2X$ square + $3X$ cube + $4X$ to the 4 + so on, now I'm going to differentiate both the sides, so by simple calculus we can see that the derivative of $X/(1-X)$ the whole square if you apply the formula of differentiating the fractions VU dash – UV dash/ V square will give you,

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Generating function for $1^2, 2^2, 3^2, 4^2, \dots$

$$\frac{d}{dx} \left(\frac{x}{(1-x)^2} \right) = \frac{d}{dx} (x + 2x^2 + 3x^3 + \dots)$$

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if you apply this formula you will see that the derivative of $X/(1-X)$ the whole square is $X+1/(1-X)$ the whole cube, we have differentiate it the left hand side, the right hand side X the derivative is 1, $2X$ squares derivative is $4X$, $3X$ cube derivative is $9X^2$, $9X$ square + $16X$ cube and so on.

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Generating function for $1^2, 2^2, 3^2, 4^2, \dots$

$$\frac{d}{dx} \left(\frac{x}{(1-x)^2} \right) = \frac{d}{dx} (x + 2x^2 + 3x^3 + \dots)$$

$$\frac{x+1}{(1-x)^3} = 1^2 + 4x + 9x^2 + 16x^3 + \dots$$



Now what we have obtained is $1 + 4X + 9X^2 + 16X^3$ and so on, rather I can write it as $1^2 + 2^2 X + 3^2 X^2 + 4^2 X^3 + \dots$
(Refer Slide Time: 02:54)

Generating function for $1^2, 2^2, 3^2, 4^2, \dots$

$$\frac{d}{dx} \left(\frac{x}{(1-x)^2} \right) = \frac{d}{dx} (x + 2x^2 + 3x^3 + \dots)$$

$$\frac{x+1}{(1-x)^3} = 1^2 + 2^2 x + 3^2 x^2 + 4^2 x^3 + \dots$$



so do you see that $X + 1/1-X$ the whole cube it is the generating function for the sequence 1 or 1 square, 2 square, 3 square, 4 square and so on.
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The image shows a handwritten derivation on a black background. At the top right, it says "IIT Ropar". The main text is written in white and pink. It starts with "Generating function for $1^2, 2^2, 3^2, 4^2, \dots$ ". Then it shows the derivative of $\frac{x}{(1-x)^2}$ with respect to x , which is $\frac{d}{dx}(x + 2x^2 + 3x^3 + \dots)$. This is set equal to $\frac{x+1}{(1-x)^3} = 1^2 + 2^2x + 3^2x^2 + 4^2x^3 + \dots$. Finally, it states that $\frac{x+1}{(1-x)^3}$ generates $1^2, 2^2, 3^2, 4^2, \dots$. In the bottom left corner, there is a small NPTEL logo.

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Generating function for $1^2, 2^2, 3^2, 4^2, \dots$

$$\frac{d}{dx} \left(\frac{x}{(1-x)^2} \right) = \frac{d}{dx} (x + 2x^2 + 3x^3 + \dots)$$
$$\frac{x+1}{(1-x)^3} = 1^2 + 2^2x + 3^2x^2 + 4^2x^3 + \dots$$

$\frac{x+1}{(1-x)^3}$ generates $1^2, 2^2, 3^2, 4^2, \dots$

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