

NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics  
Graph Theory – 3 &  
Generating Functions

Picking five balls – Another version

By  
Prof. S.R.S Iyengar  
Department of Computer Science  
IIT Ropar

Now let me give a slightly different question, the question is that I'm not going to put any constraint on the same one as earlier, so there are 5 red, 5 blue and 5 green balls, but there is no constraint that you have to pick at least one among them, one among each of them, so you have to pick 5 balls from these given 15 balls, so in how many ways can we do that?

(Refer Slide Time: 00:30)

Q: 5 red, 5 blue, 5 green balls. Pick 5 balls from the 15 balls. No constraints.

IIT Ropar

NPTEL

Well do you see that here we can have 5 0 0, as a valid possibility 5 blue, 0 red, and 0 green, or 0 blue, 5 red, and 5 green, this is also valid, or 0 blue, 0 red and 5 green. Now 4 1 0 is also valid, 4 blue, 1 red and 0 green because there is no constraint that I have to pick at least one of

each ball, so this is also valid, so the next valid possibility will be 4 0 1, and then we have 0 4 1, 0 1 4 and so on,

(Refer Slide Time: 01:21)

The slide features a black background with white and colored text. In the top right corner, the IIT Ropar logo is visible. The main text is a question: "Q: 5 red, 5 blue, 5 green balls. Pick 5 balls from the 15 balls. No constraints." Below the question, seven possible combinations of red, blue, and green balls are listed, each on a new line. The numbers are color-coded: red for red balls, blue for blue balls, and green for green balls. The combinations are: (5, 0, 0), (0, 5, 0), (0, 0, 5), (4, 1, 0), (4, 0, 1), (0, 4, 1), and (0, 1, 4). In the bottom left corner, there is a small circular logo with the text "PTEL" below it.

IIT Ropar

Q: 5 red, 5 blue, 5 green balls. Pick 5 balls from the 15 balls. No constraints.

5	0	0
0	5	0
0	0	5
4	1	0
4	0	1
0	4	1
0	1	4

PTEL

now please note that you can also include or you should also include the possibilities which we had seen earlier along with these extra possibilities.

Now the nice or the challenging step would come here when we have to write the polynomial, well what would the polynomial be? Since there is no constraint the polynomial starts with  $X$  to the 0, unlike the previous one where we started with  $X + X$  square we started the first term was  $X$  because we needed at least one ball, so here the polynomial starts with  $X$  to the 0, so for blue 4 blue balls the polynomial would be  $X$  to the 0 +  $X$  +  $X$  square +  $X$  to the 4 +  $X$  to the 5, what does this mean? I can either pick 0 balls or 1 ball or 2 balls or 3 balls or 4 balls or 5 balls,

(Refer Slide Time: 2:18)

$$(\chi^0 + \chi + \chi^2 + \chi^3 + \chi^4 + \chi^5)$$



now the same holds true for green and red as well,  
(Refer Slide Time: 02:26)

$$(\chi^0 + \chi + \chi^2 + \chi^3 + \chi^4 + \chi^5) \times$$

$$(\chi^0 + \chi + \chi^2 + \chi^3 + \chi^4 + \chi^5) \times$$

$$(\chi^0 + \chi + \chi^2 + \chi^3 + \chi^4 + \chi^5)$$



so now a task for you would be to expand these three polynomials, that is  $1 + X + X^2 + X^3 + X^4 + X^5$ ,  
(Refer Slide Time: 02:39)

$(x^0 + x + x^2 + x^3 + x^4 + x^5)x$   
 $(x^0 + x + x^2 + x^3 + x^4 + x^5)x$   
 $(x^0 + x + x^2 + x^3 + x^4 + x^5)$   
 $(1 + x + x^2 + x^3 + x^4 + x^5)^3$

IIT  
Ropar

NPTEL

you have to multiply this 3 times so  $1 + X + X^2 + X^3 + X^4 + X^5$  rather  $X^3 + X^4 + X^5$  whole cube you have to expand this and check the coefficient of  $X^5$  in the expansion, right,  
(Refer Slide Time: 02:56)

$$(x^0 + x + x^2 + x^3 + x^4 + x^5)x$$

$$(x^0 + x + x^2 + x^3 + x^4 + x^5)x$$

$$(x^0 + x + x^2 + x^3 + x^4 + x^5)$$

$$(1 + x + x^2 + x^3 + x^4 + x^5)^3$$

Coefficient of  $x^5$  ?



now the coefficient of  $X$  to the 5 will give you the answer for the number of ways in which you can pick 5 balls without any constraint this is in the same lines as the previous question I am not going to expand as it would take a lot of time it is left as an exercise for you people to solve but I am going to give the last answer, the final answer for the coefficient of  $X$  to the 5 is 21 in the product of these polynomials, which means there are 21 ways in which you can pick 5 balls from the given 5 blue, 5 red and 5 green balls.

(Refer Slide Time: 03:40)

$$(x^0 + x + x^2 + x^3 + x^4 + x^5) x$$

$$(x^0 + x + x^2 + x^3 + x^4 + x^5) x^2$$

$$(x^0 + x + x^2 + x^3 + x^4 + x^5) x^3$$

$$(1 + x + x^2 + x^3 + x^4 + x^5)^3$$

Coefficient of  $x^5$  ?

21



IIT MADRAS PRODUCTION

Founded by  
Department of Higher Education  
Ministry of Human Resources Development  
Government of India

[www.nptel.iitm.ac.in](http://www.nptel.iitm.ac.in)

Copyrights Reserved