NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 3 &
Generating Functions

Example 2 - Picking five balls

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Let us look at another example to illustrate the power of polynomials in counting, here goes the question you have 5 blue, 5 red and 5 green balls, right, and these are to be picked in such a way that you make 5 balls out of it, and these 5 balls should have at least 1 blue, 1 red and 1 green ball, as you can see 5 blue balls are indistinguishable,
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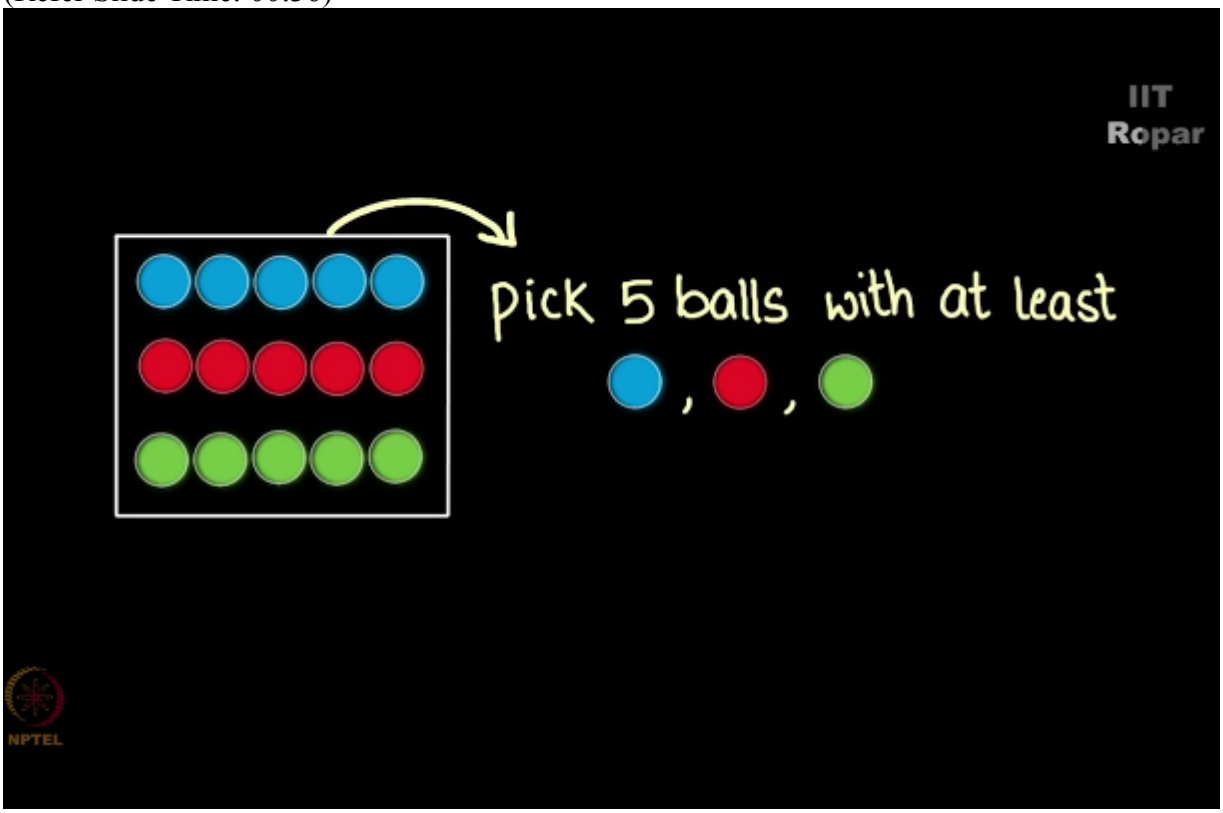

which means all of them are identical, but they are different from the 5 red balls, different from the 5 green balls, you've got to pick 5 balls such that you have a blue, a red, a green, at least but over and beyond it you can pick whatever you want, but the total number of balls you're going

to pick should again be 5, a valid example would be 1 blue, 2 red, and then 2 green, this is a valid pick,
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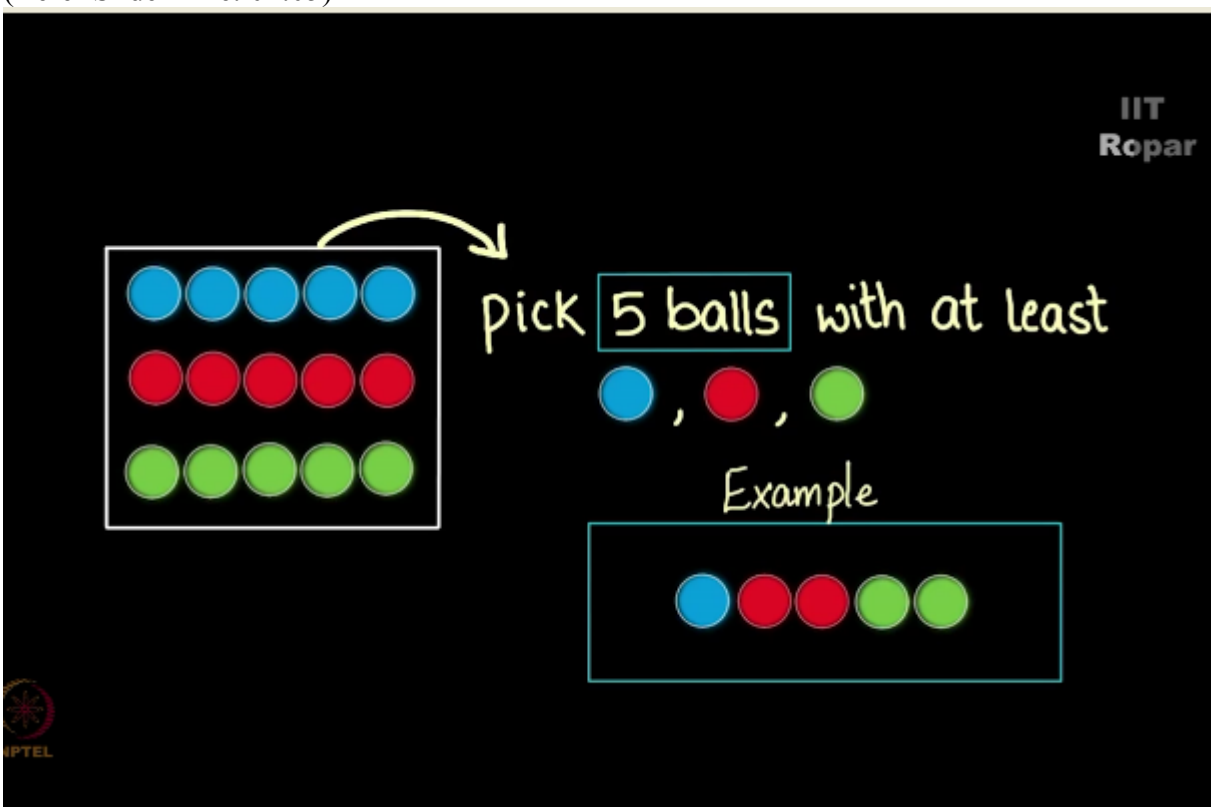

if you explore all possible ways in which you can do it, you will see that there are the following 6 ways,
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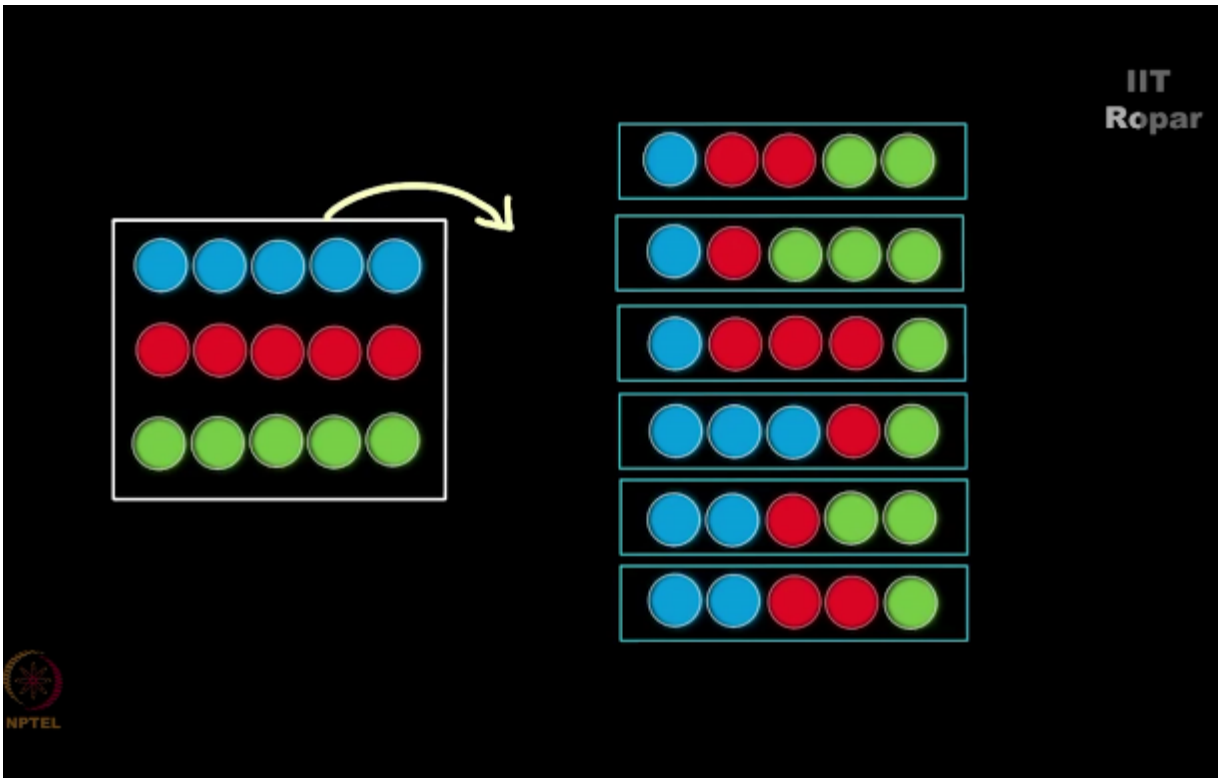

how can we think of a polynomial here, what is polynomials to do here, let me look at this problem in a different light, look at this polynomial X + X square + X cube, write this once more and then once more,

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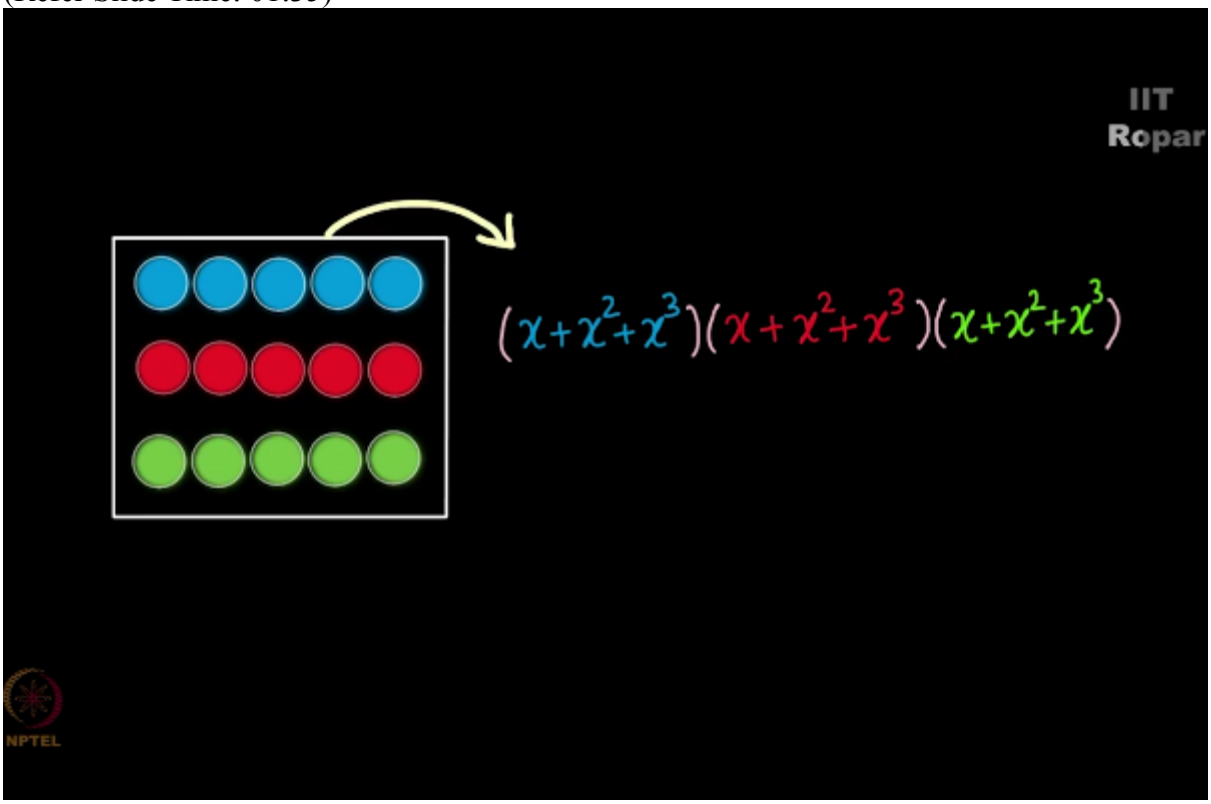

so you have X + X square + X cube written thrice, now whenever you pick 1 blue ball you must at least pick 1 blue ball as you know that's the condition in the puzzle, that is equal to picking X here in the first house, I call this house because on each bracket represents blue, red, and green respectively, these three brackets, right, okay, so picking 1 blue ball corresponds to an X here, picking 2 blue balls corresponds to 2X square here, picking 3 blue balls correspond to picking X cube here,

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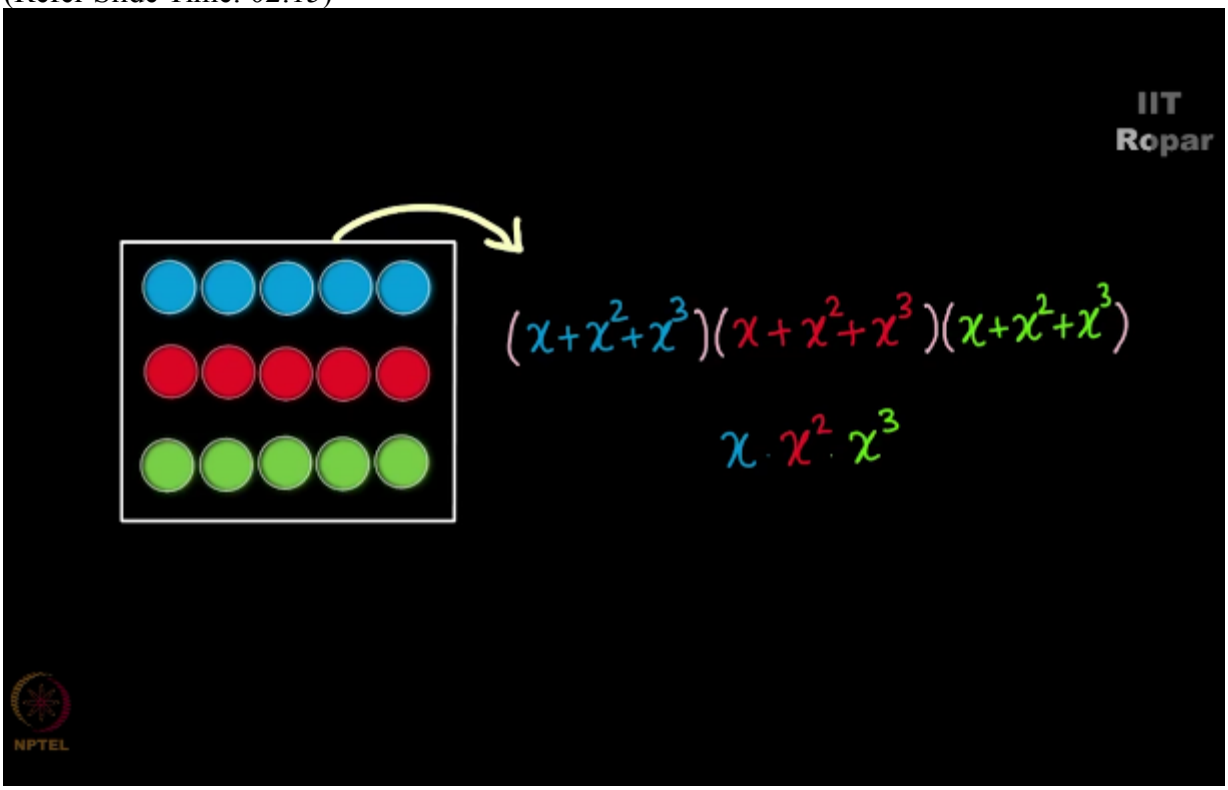

I don't have X to the 4 here, because you never pick 4 blue balls, because if you were to pick 4 blue balls you are supposed to pick all together 5 balls, if you pick 4 blue balls you will not be able to pick 1 red at least and one green at least you see,

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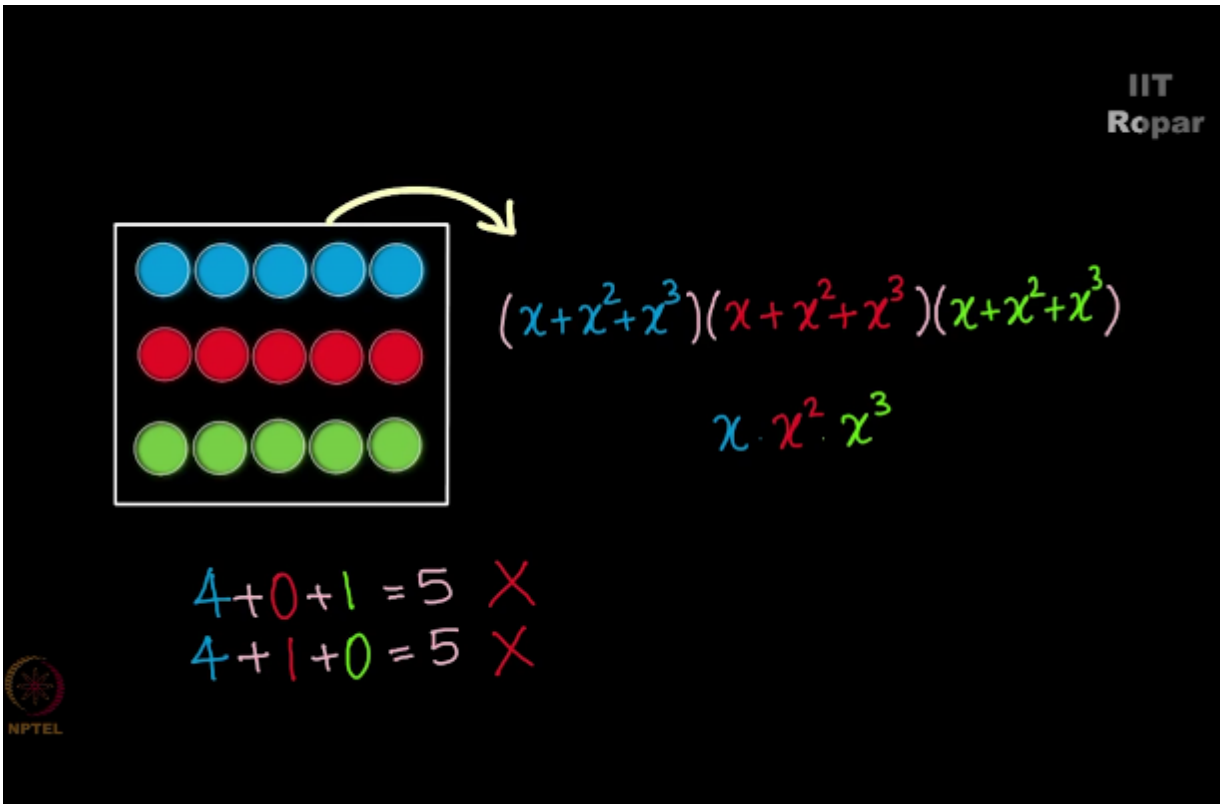

a total of 5 is required and one each of these colors, so the answer to this question is same as the answer to the question of what is the coefficient of X to the 5 in the expansion of X + X square + X cube whole to the 3, do you see the connection?

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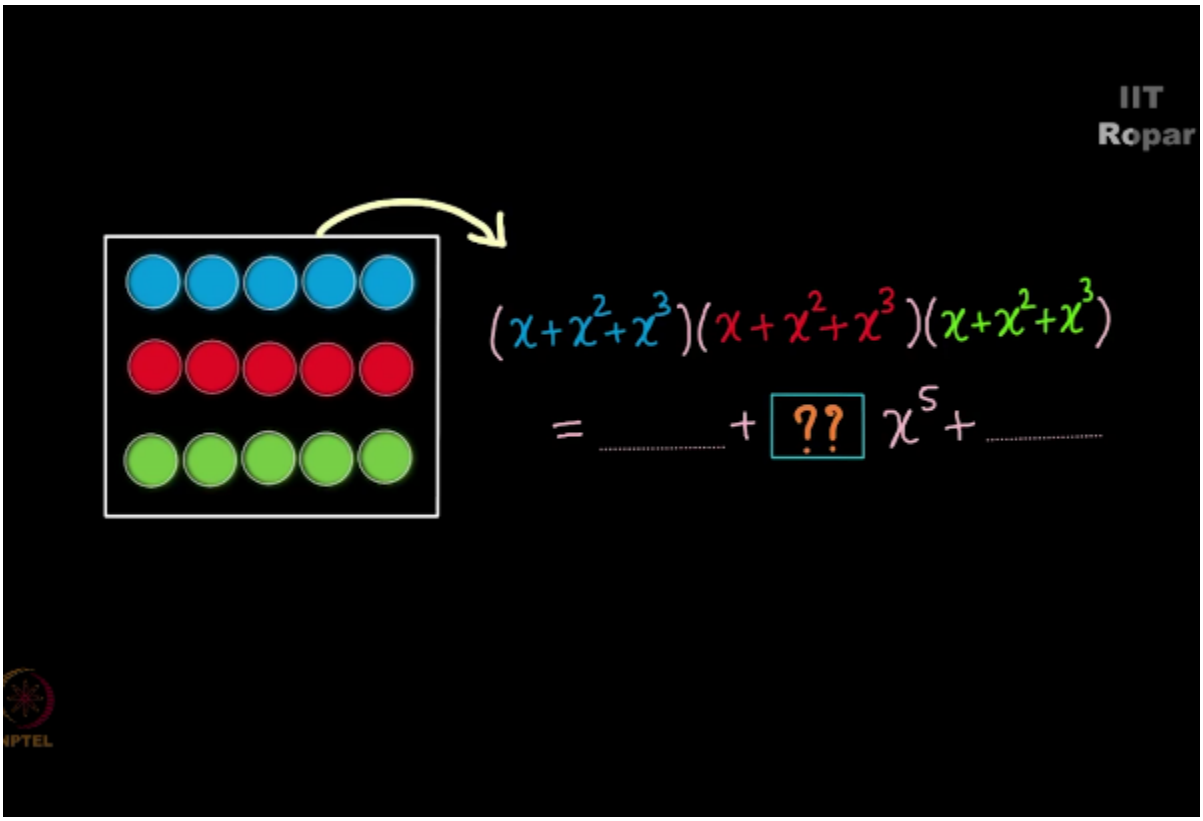

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