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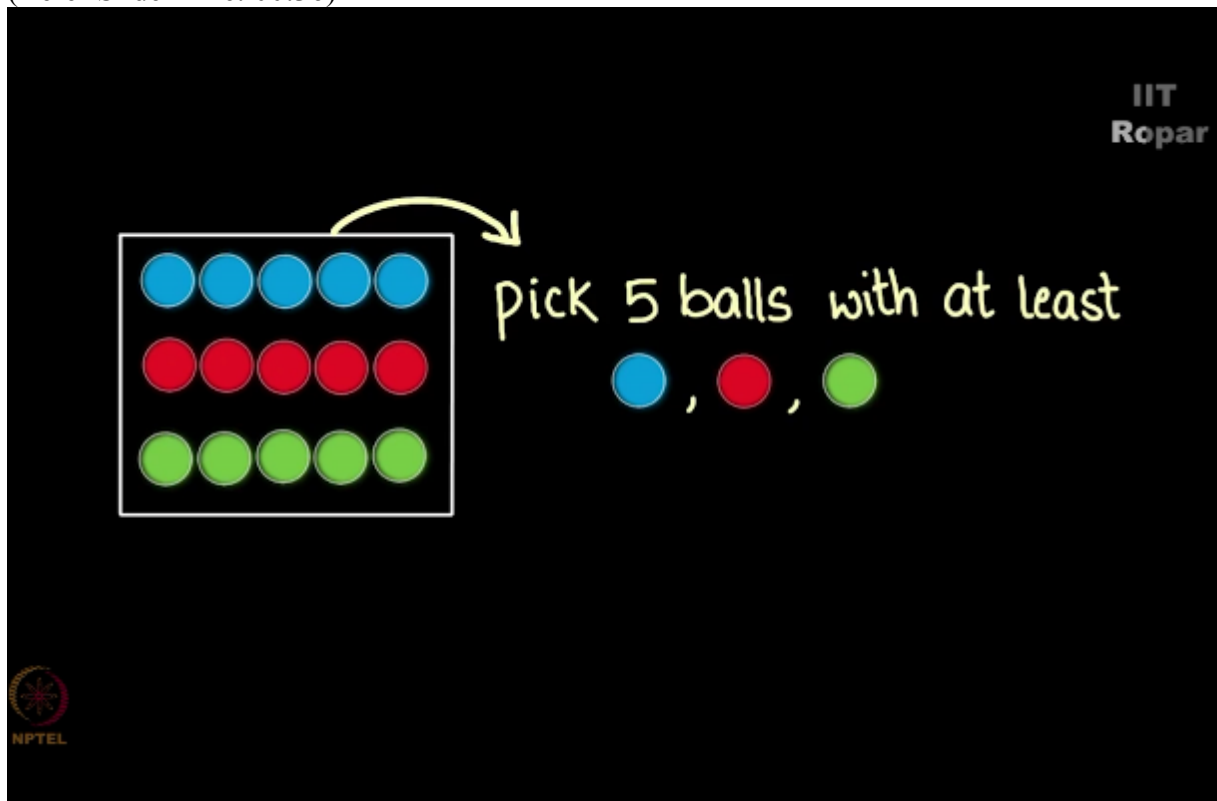
NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 3 &
Generating Functions

Example 2 - Picking five balls

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Let us look at another example to illustrate the power of polynomials in counting, here goes the question you have 5 blue, 5 red and 5 green balls, right, and these are to be picked in such a way that you make 5 balls out of it, and these 5 balls should have at least 1 blue, 1 red and 1 green ball, as you can see 5 blue balls are indistinguishable, (Refer Slide Time: 00:36)



which means all of them are identical, but they are different from the 5 red balls, different from the 5 green balls, you've got to pick 5 balls such that you have a blue, a red, a green, at least but over and beyond it you can pick whatever you want, but the total number of balls you're going

to pick should again be 5, a valid example would be 1 blue, 2 red, and then 2 green, this is a valid pick,
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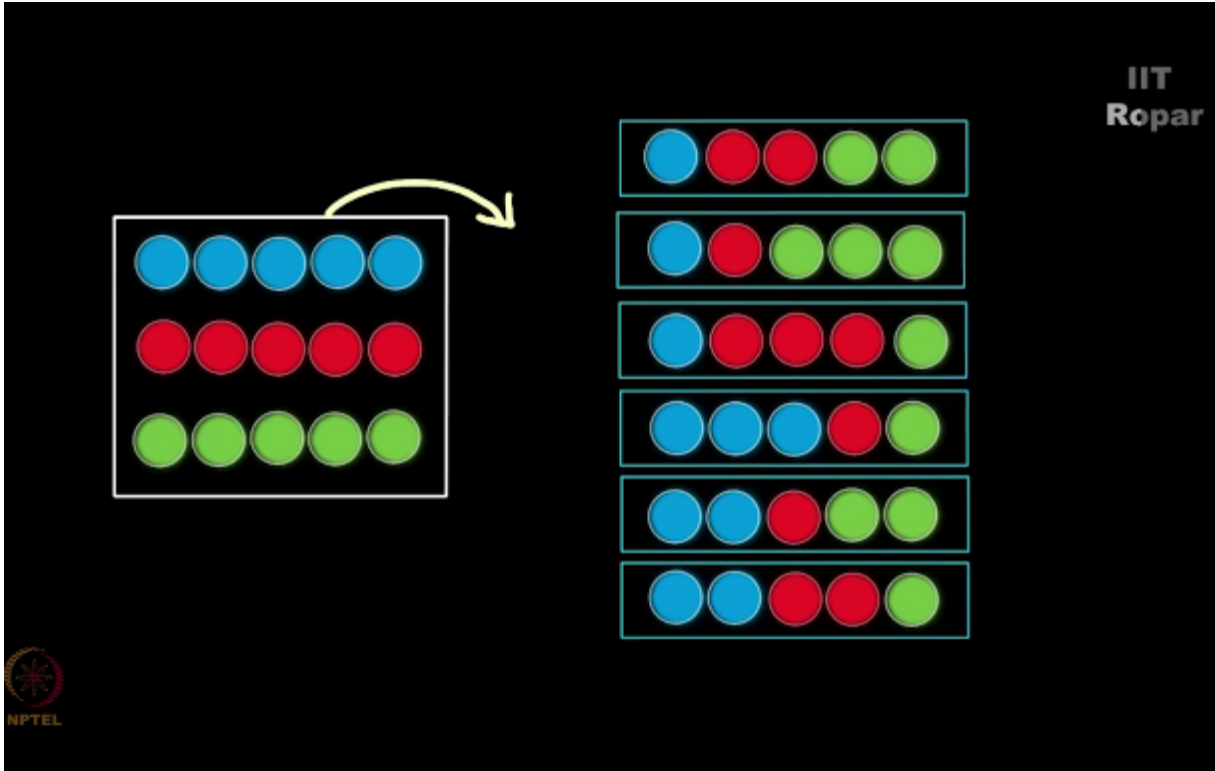
pick 5 balls with at least
●, ●, ●
Example

● ● ● ● ●
● ● ● ● ●
● ● ● ● ●

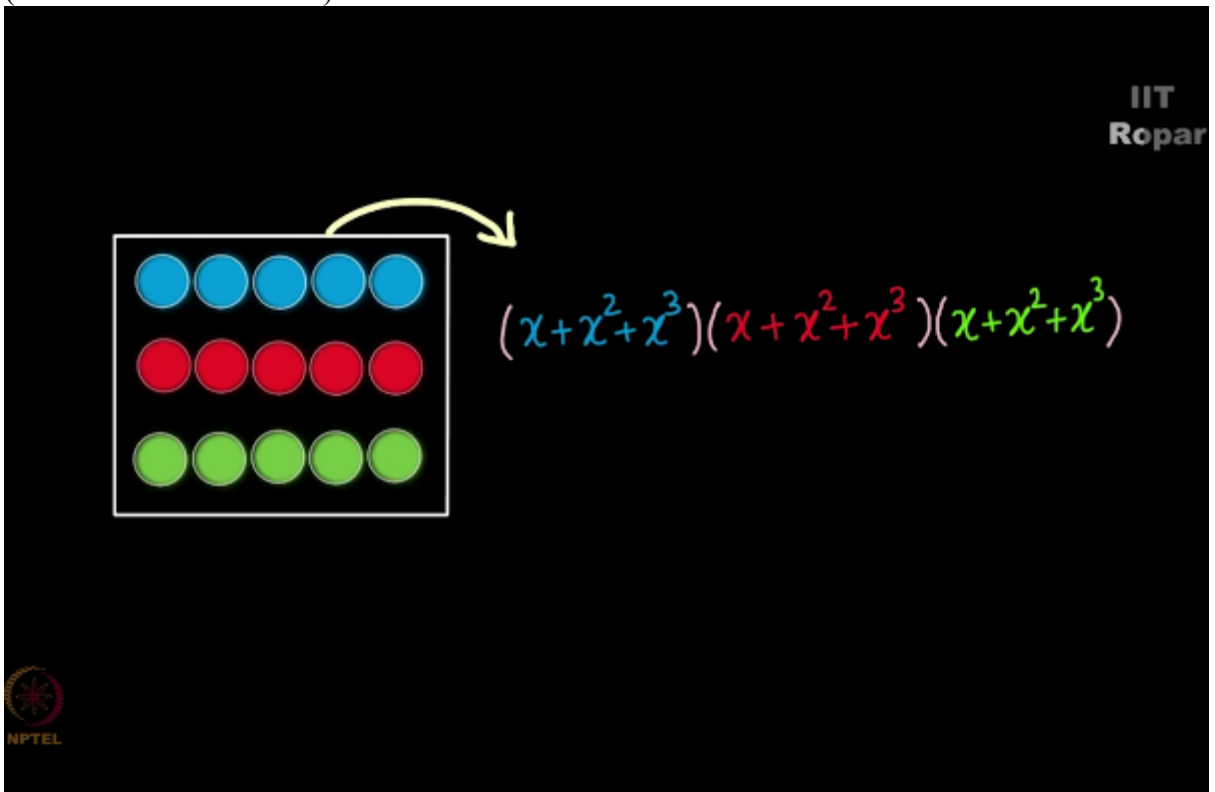
● ● ● ● ●

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if you explore all possible ways in which you can do it, you will see that there are the following 6 ways,
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how can we think of a polynomial here, what is polynomials to do here, let me look at this problem in a different light, look at this polynomial $X + X^2 + X^3$, write this once more and then once more,
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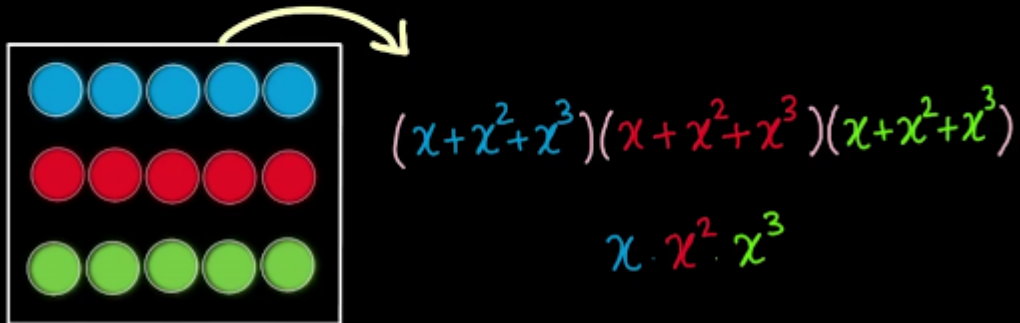
so you have $X + X^2 + X^3$ written thrice, now whenever you pick 1 blue ball you must at least pick 1 blue ball as you know that's the condition in the puzzle, that is equal to picking X here in the first house, I call this house because on each bracket represents blue, red, and green respectively, these three brackets, right, okay, so picking 1 blue ball corresponds to an X here, picking 2 blue balls corresponds to X^2 here, picking 3 blue balls correspond to picking X^3 here,

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The slide features a 3x5 grid of colored balls: the top row has 5 blue balls, the middle row has 5 red balls, and the bottom row has 5 green balls. A yellow arrow points from the top row of balls to the first bracket of the algebraic expression. The expression is $(x + x^2 + x^3)(x + x^2 + x^3)(x + x^2 + x^3)$, with the terms in each bracket colored blue, red, and green respectively. Below this, the terms $x \cdot x^2 \cdot x^3$ are written in blue, red, and green. The slide includes the IIT Ropar logo in the top right and the NPTEL logo in the bottom left.

I don't have X to the 4 here, because you never pick 4 blue balls, because if you were to pick 4 blue balls you are supposed to pick all together 5 balls, if you pick 4 blue balls you will not be able to pick 1 red at least and one green at least you see,

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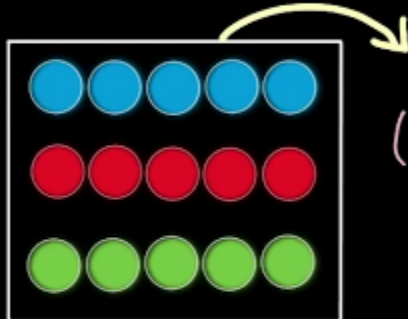


$$4+0+1=5 \quad \times$$

$$4+1+0=5 \quad \times$$

a total of 5 is required and one each of these colors, so the answer to this question is same as the answer to the question of what is the coefficient of X to the 5 in the expansion of $X + X^2 + X^3$ whole to the 3, do you see the connection?

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A 3x5 grid of colored circles (blue, red, green) is shown on the left. An arrow points from the top row of circles to the first polynomial term in the equation on the right.

$$\begin{aligned} & (\chi + \chi^2 + \chi^3)(\chi + \chi^2 + \chi^3)(\chi + \chi^2 + \chi^3) \\ &= \dots + \boxed{??} \chi^5 + \dots \end{aligned}$$



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