NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 2

Examples on Proper coloring

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Coloring is one of the most interesting topics in graph theory, now we are going to color some graphs using the proper coloring methods,
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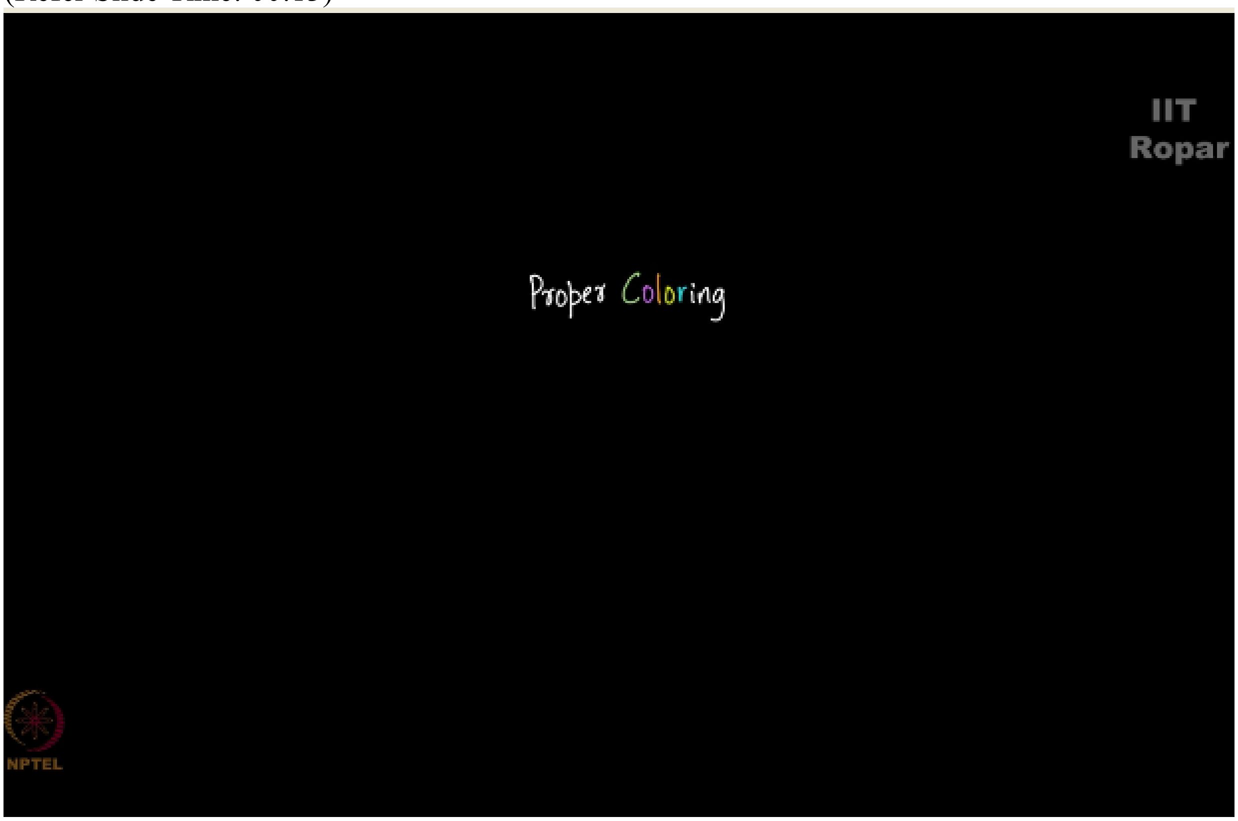

we are not going to assign the same color to 2 adjacent vertices and will be using minimum number of colors, consider the star graph on 5 vertices like this,
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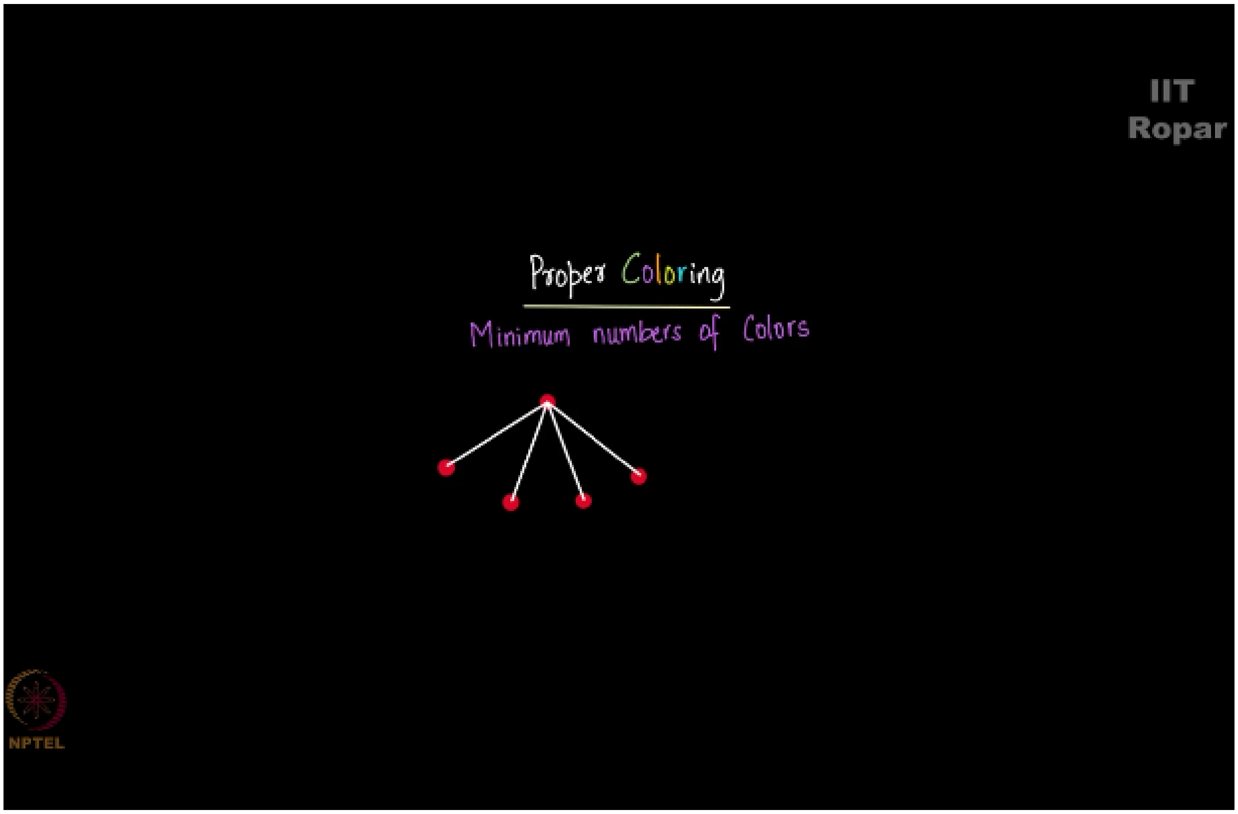

let me label the vertices as 1, 2, 3, 4, 5,

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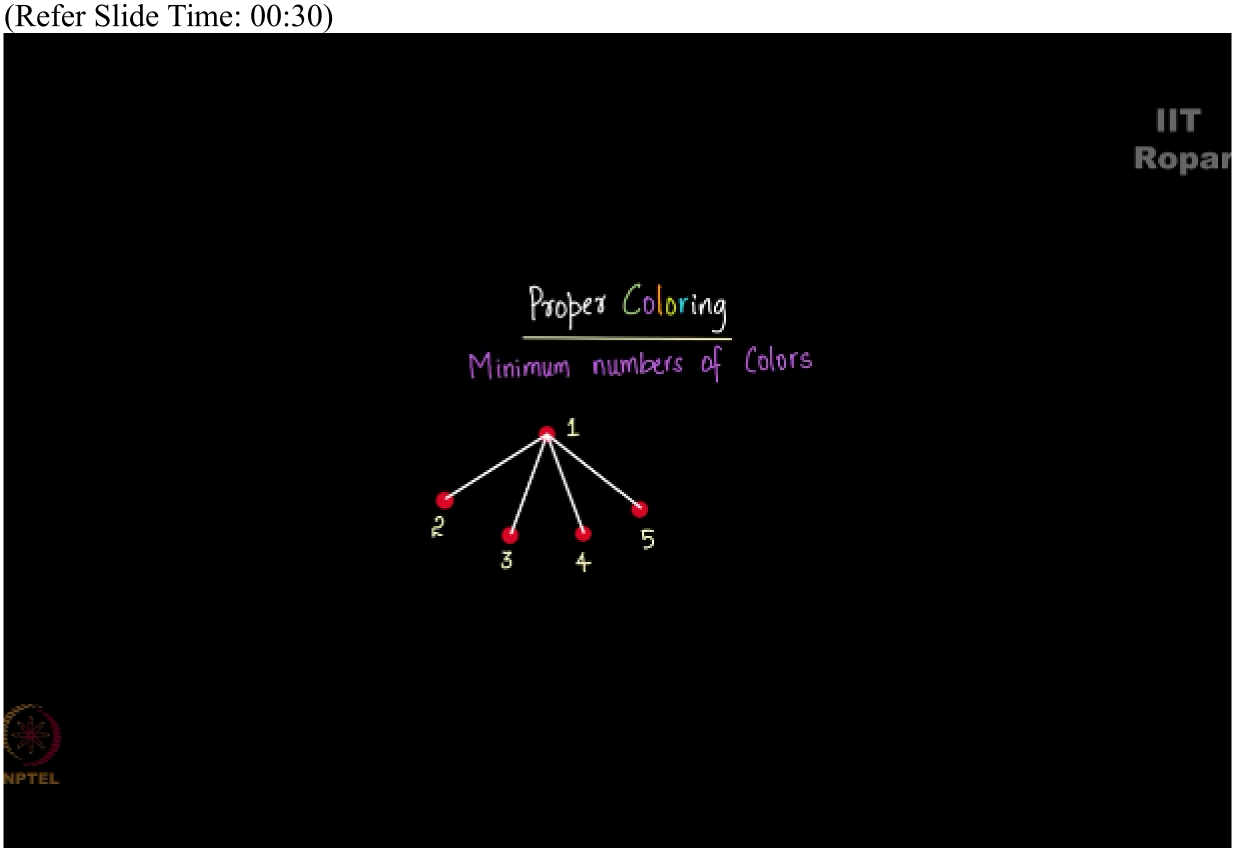

and I'm going to give colors to this graph, I give red color to vertex 1, vertex 2 can get blue, because it can definitely not get red, red is already done, it is adjacent to red, 3 cannot definitely get red, but it can get blue, so blue, blue don't you think the same reason holds true for 4 and 5 as well, think for a minute, it is connected, all these vertices are connected to vertex 1,

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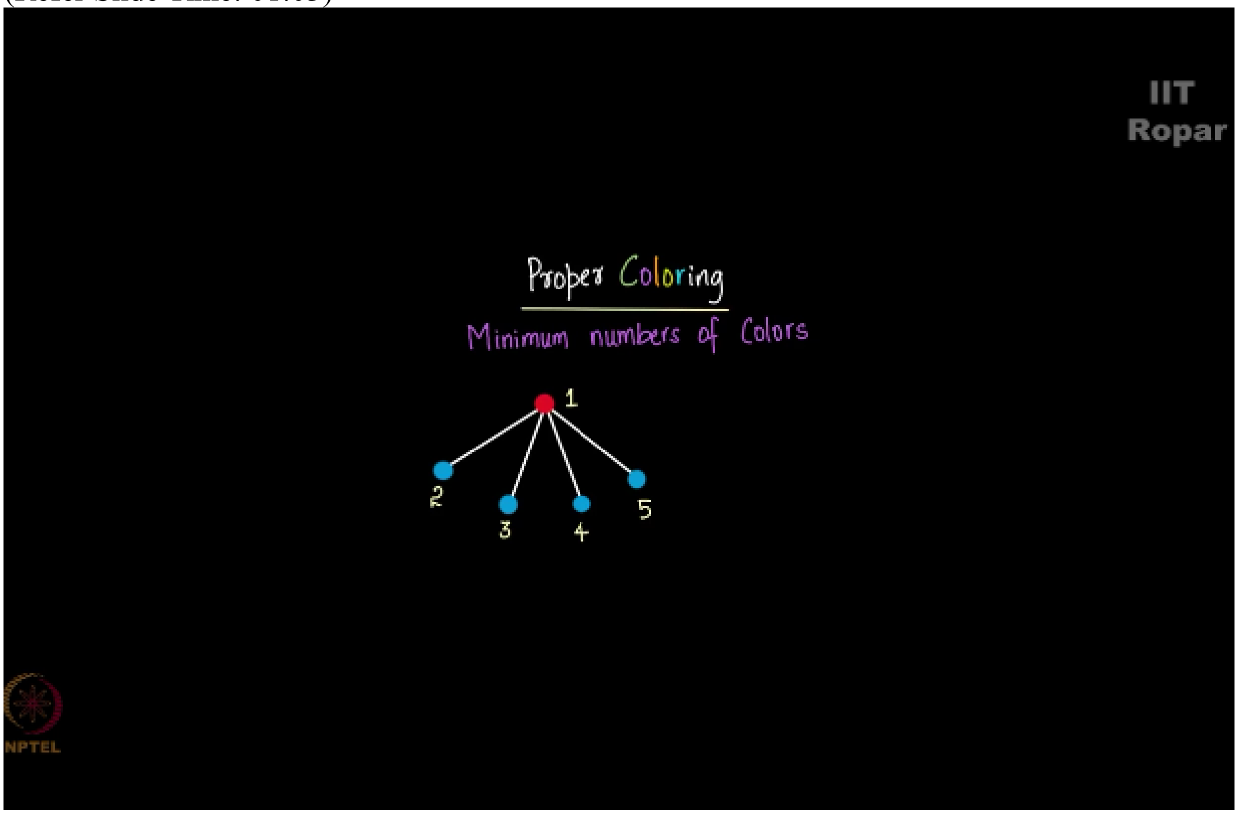

but are not connected to each other, hence all of them can get blue color and 1 can get red, and hence a star graph always is 2 colorable.

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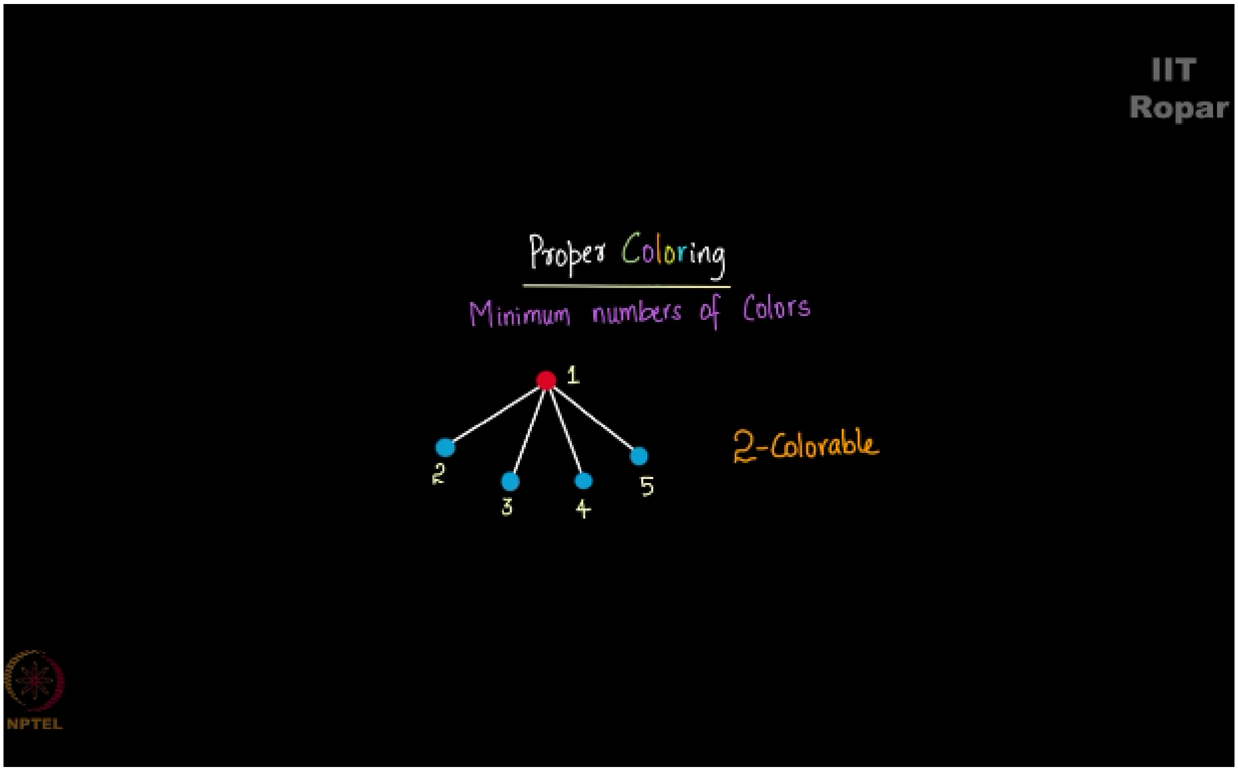

So from now on I’ll be using numbers for coloring and not the actual colors, the formal way or the convention used is coloring the vertices with numbers, consider this graph K5, it’s a complete graph on 5 vertices
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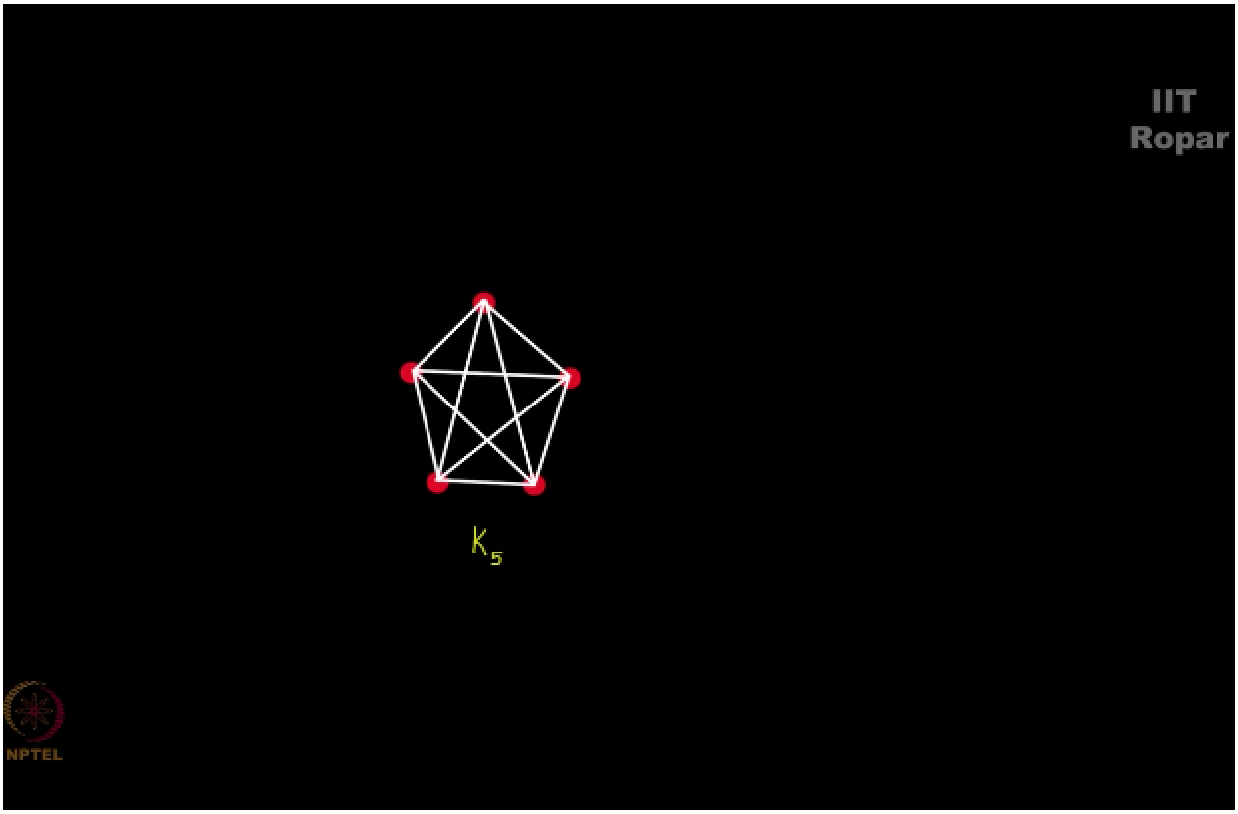

I'll color this vertex as 1,

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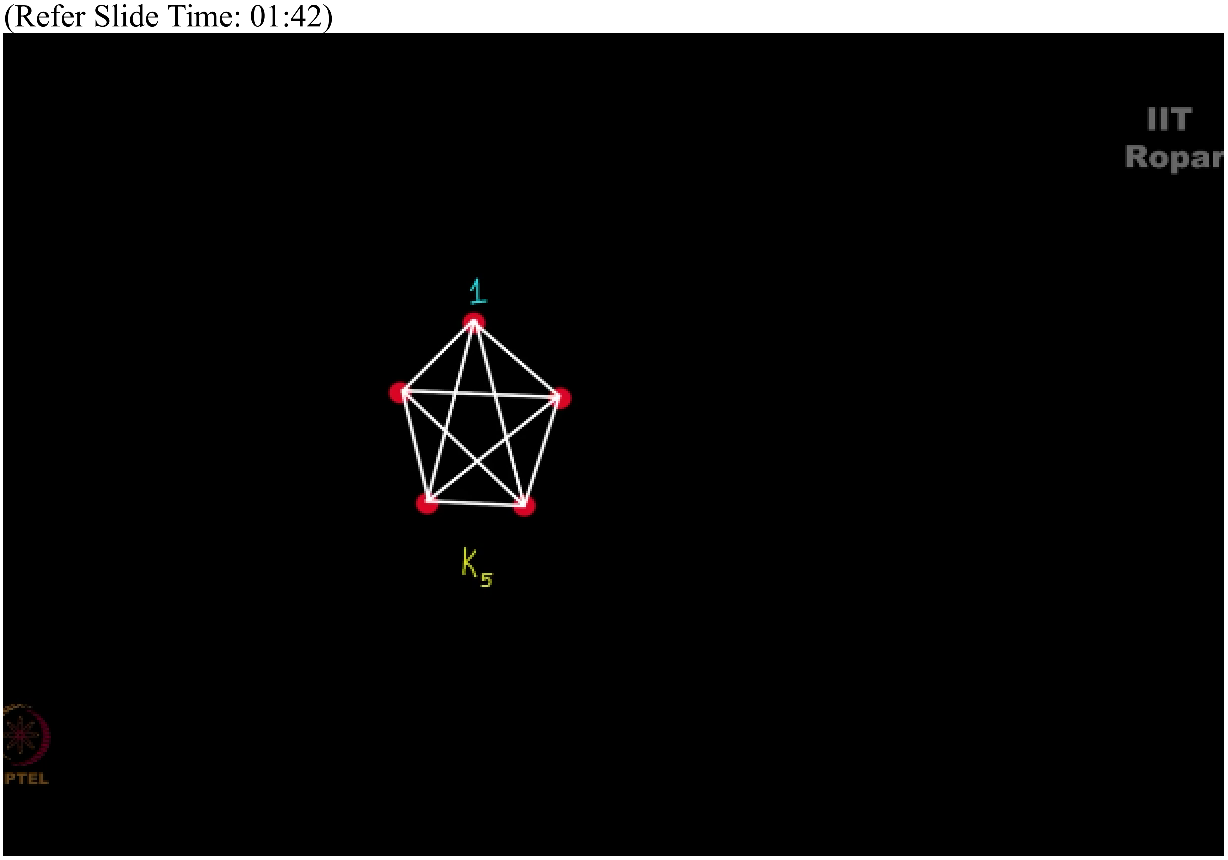

now the adjacent vertices can definitely not get 1, and do you observe that it’s a complete graph and hence all vertices are adjacent to each other, hence we must use 5 colors here, so I’ll randomly give 1, 2, 3, 4, and 5,
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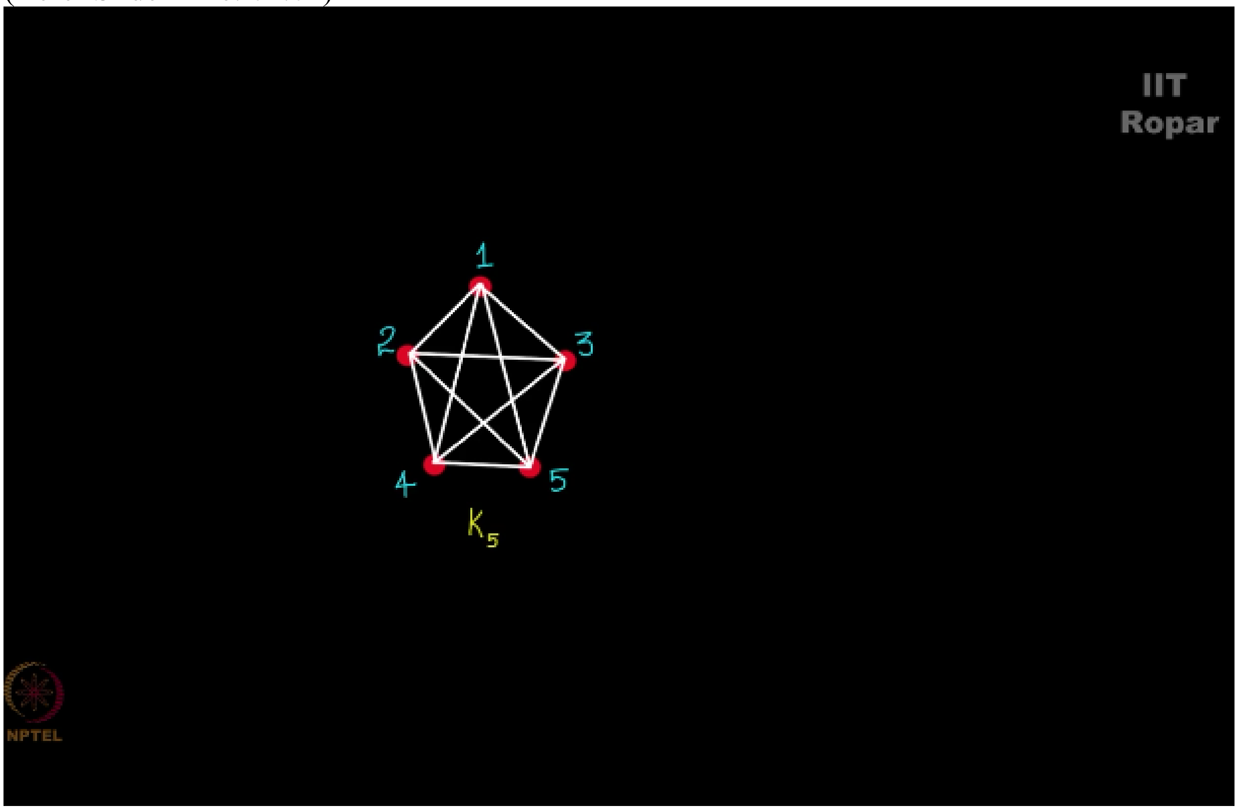

so a complete graph on n vertices always requires n colors to color it properly, why? Because every vertex is adjacent to each other.
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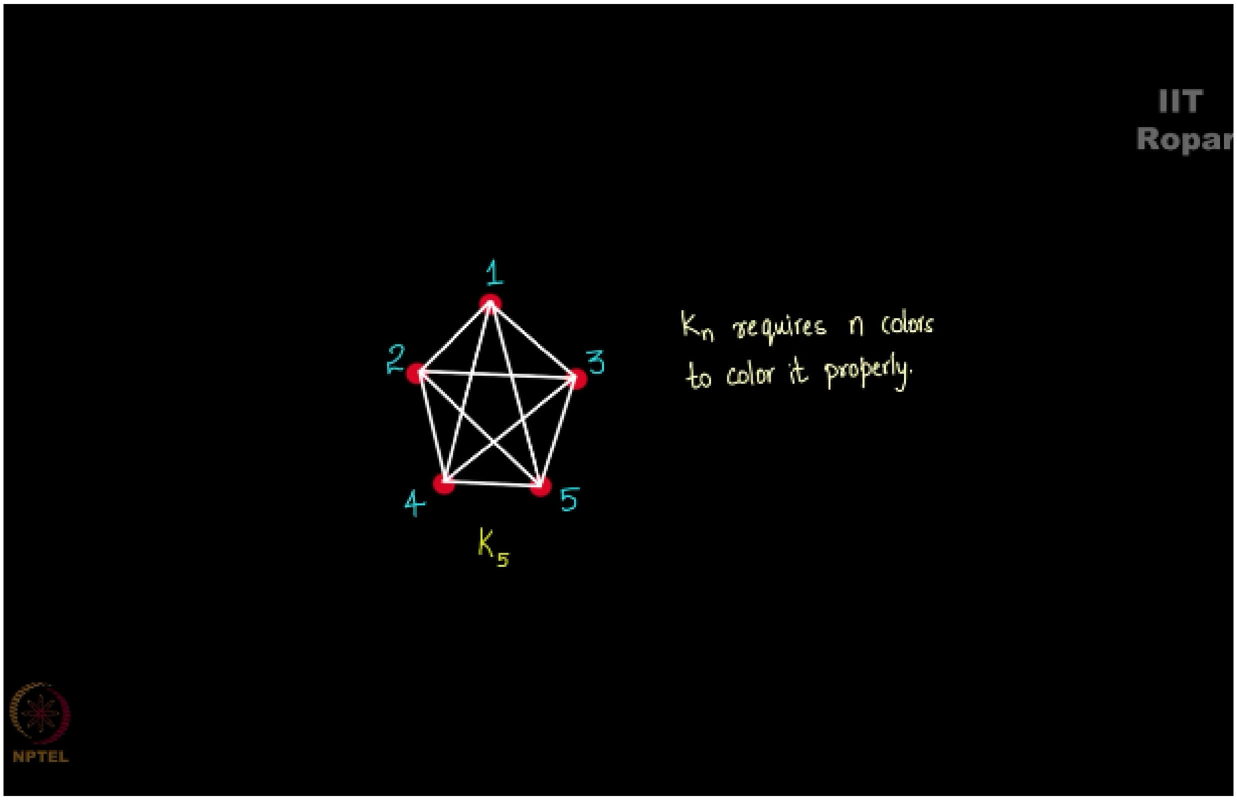

Now coming to the complement of complete graph, no vertex is adjacent to any vertex, and hence don’t you think we can give the same color like this,
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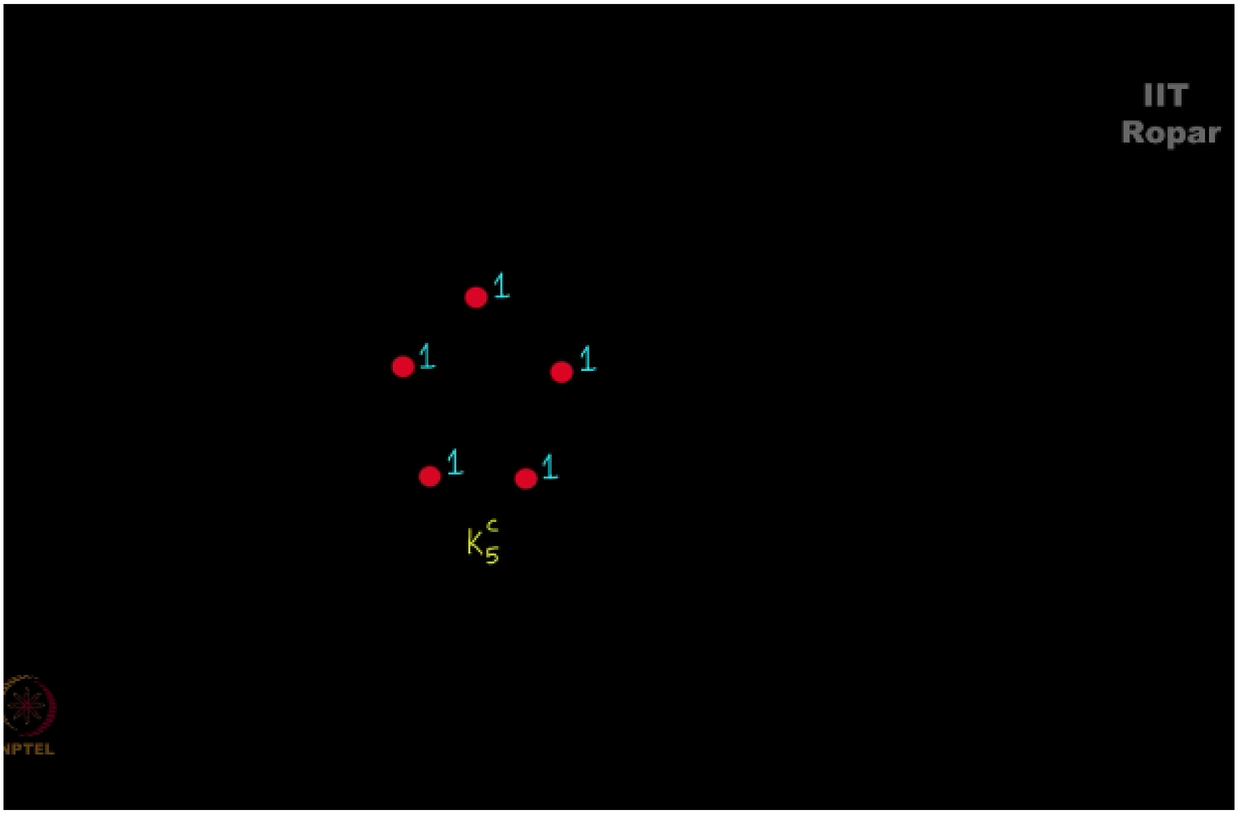

to all the vertices, if you assign 2 colors it is not minimum, right, there are no edges here in between and hence one color is sufficient.

Consider this bipartite graph K2, 2, here there are 2 vertices and 2 vertices like this,
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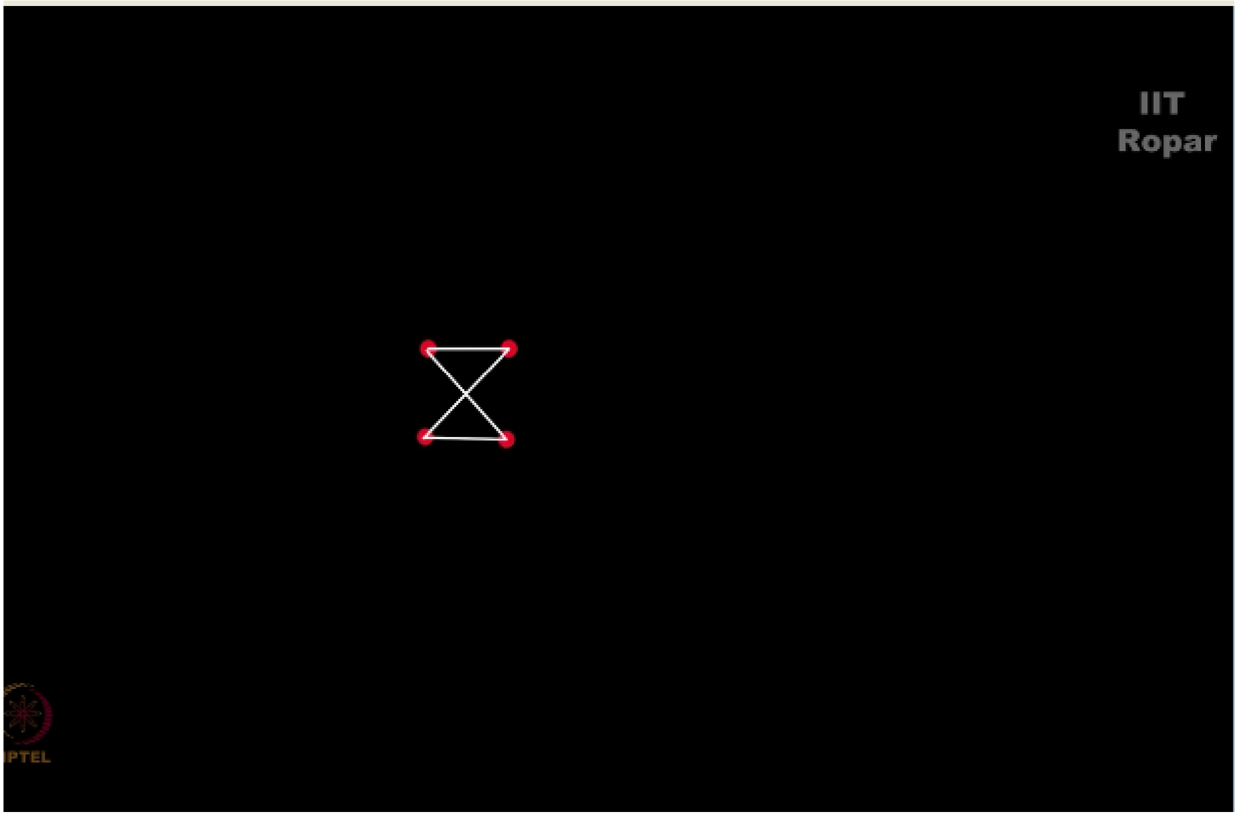

I can assign one here, and then assign one here again,
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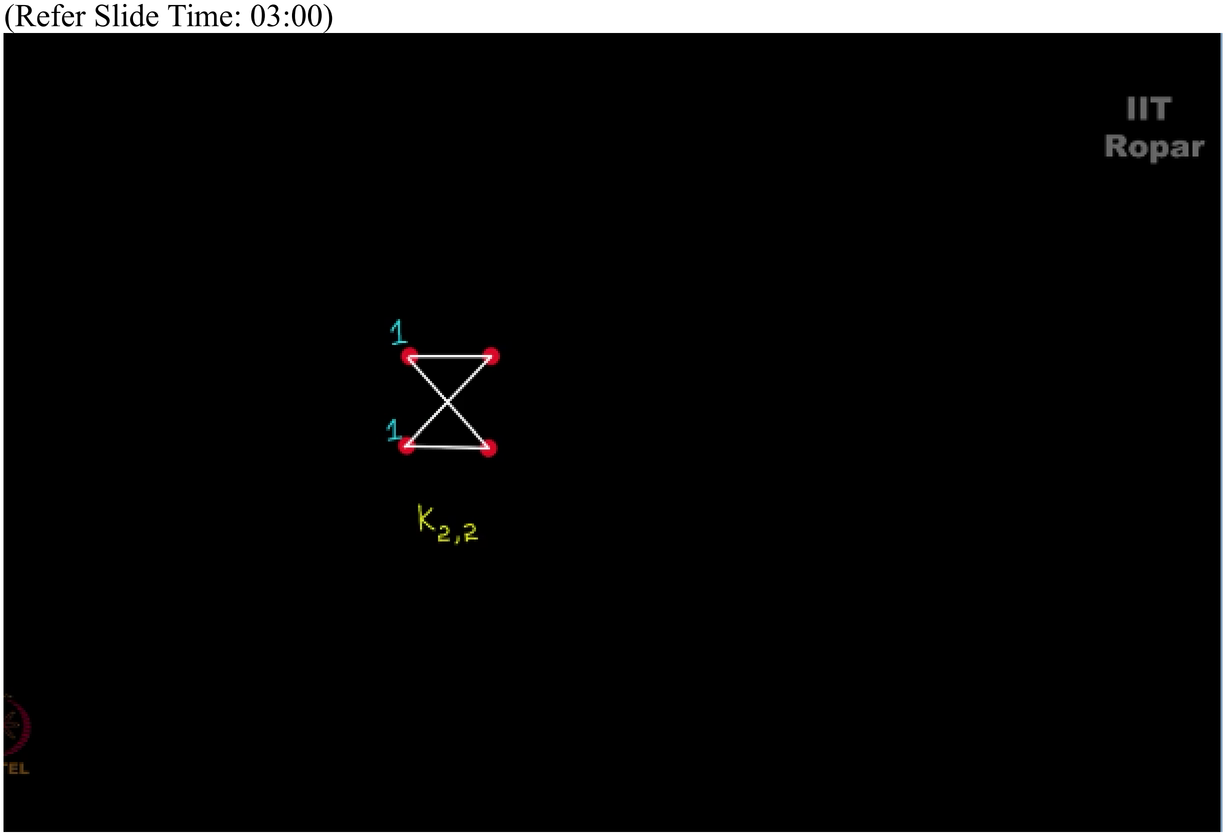

because these 2 vertices in the same partition are not adjacent, the vertices in 1, in the same partition are never adjacent in a bipartite graph and hence all the vertices inside this partition can get the same color.

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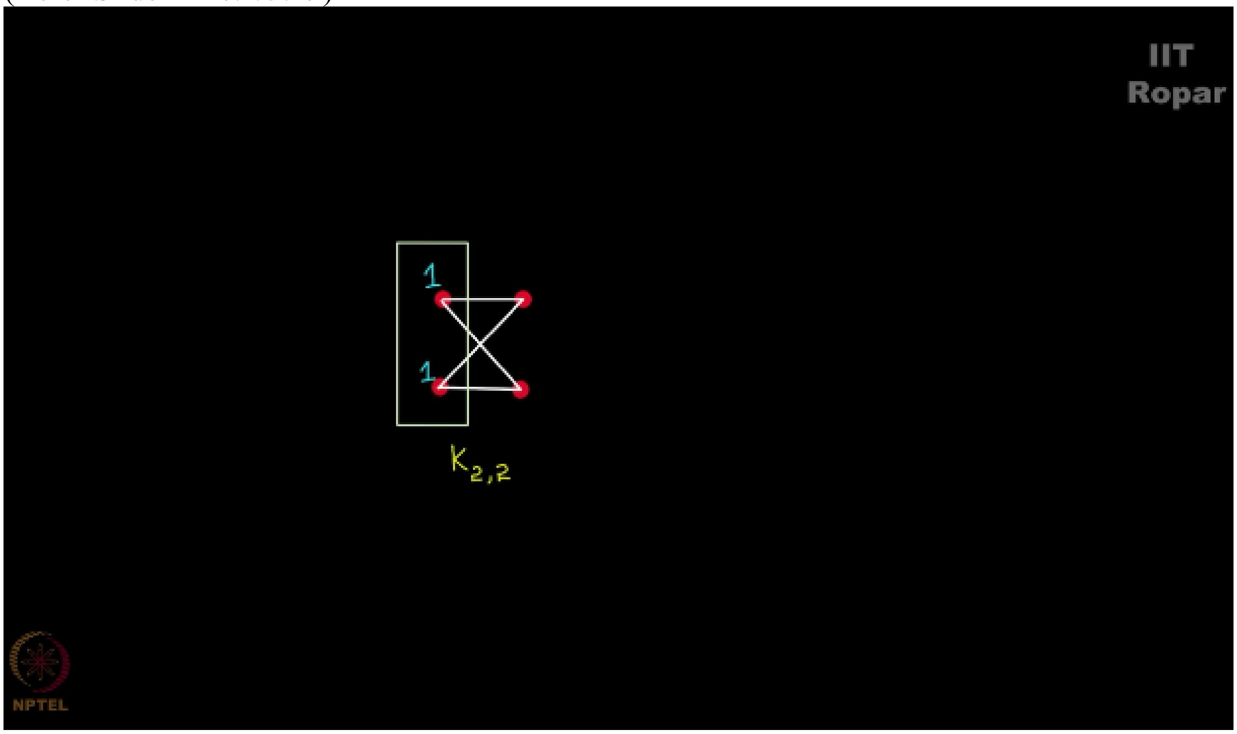

Moving on to this vertex, it cannot get the color 1, because it is adjacent to both of these and hence we can assign color 2 here, and with the same analogy as earlier we can assign color 2 here,

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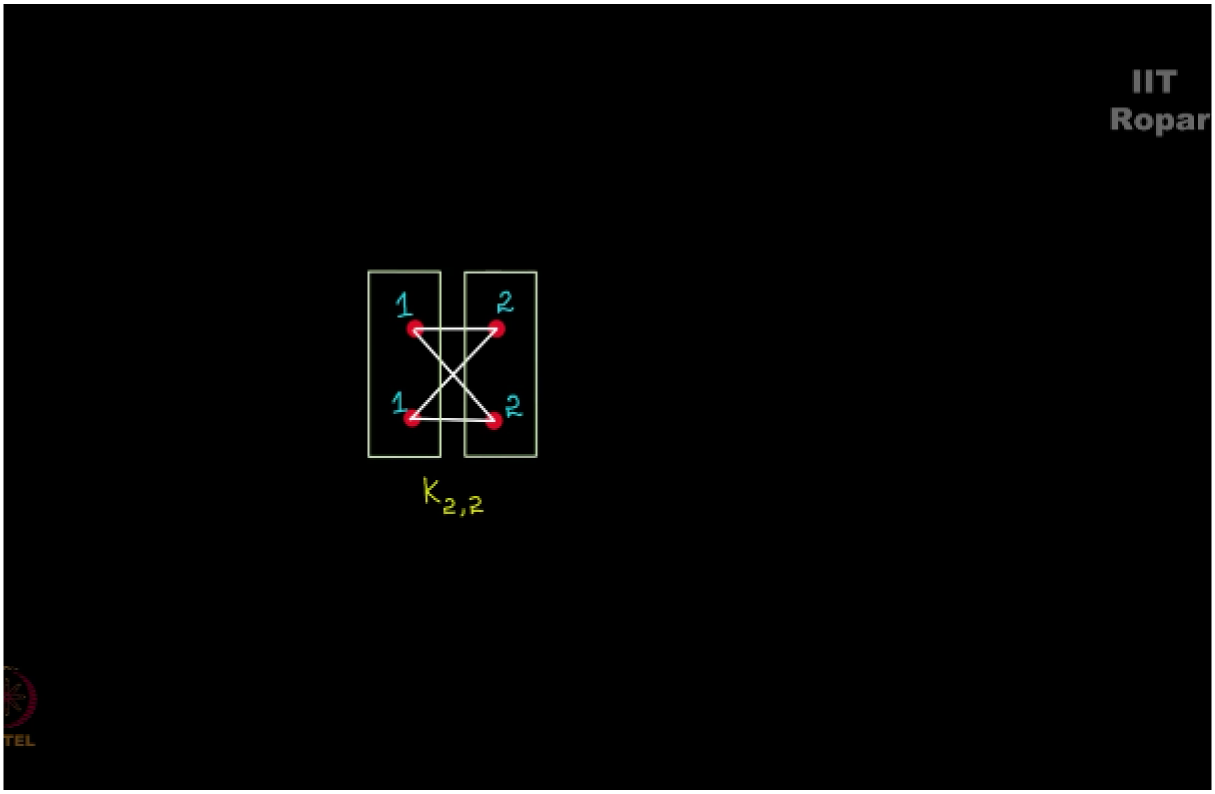

and hence in a bipartite graph 2 colors are sufficient, a bipartite graph as a star graph is always 2 colorable.

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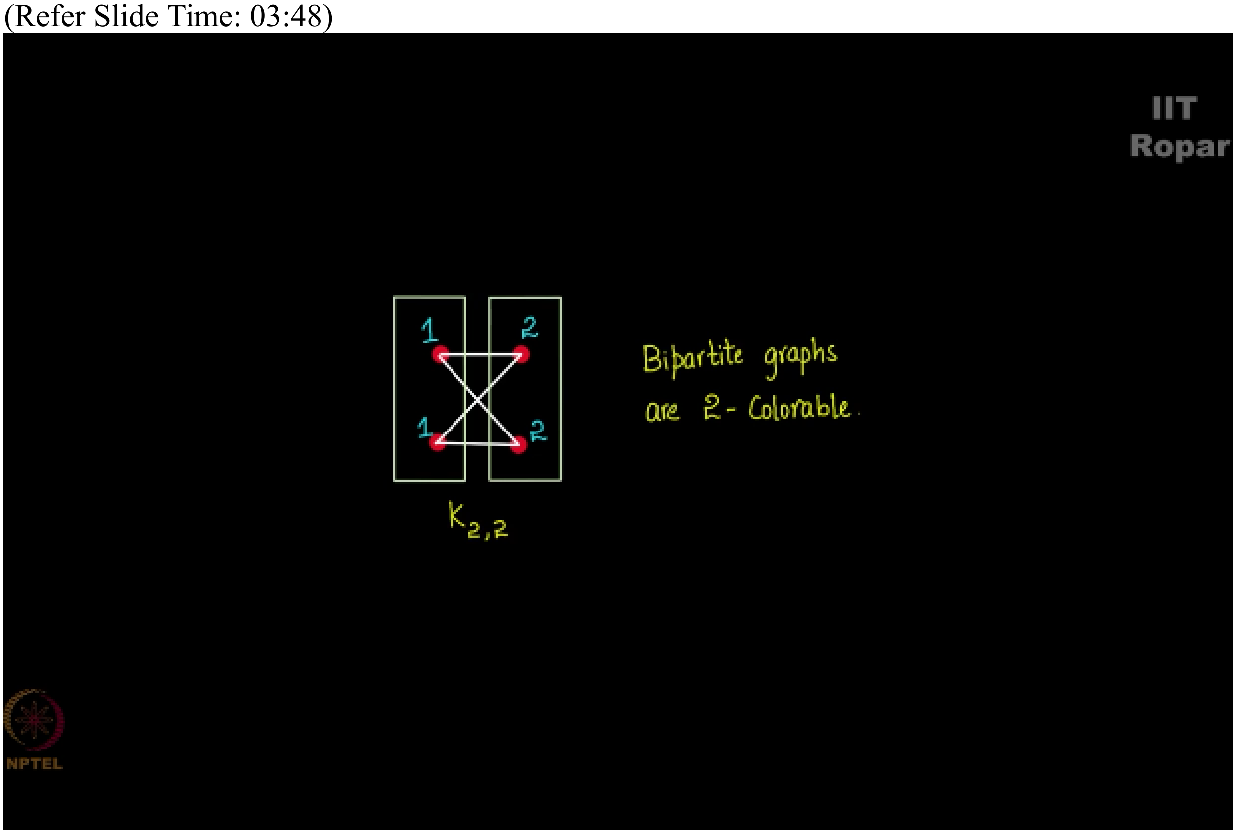

Coming to this cycle C3, do you see that this is nothing but K3, it's a complete graph on 3 vertices, and hence 3 colors are necessary to color it properly, but is it the same case with C4? This was C3, but is it the same answer for C4, let us see, if I give this vertex color 1, I cannot give this vertex color 1, once I go for 2, but this vertex can get the color 1 because it is not adjacent to this one, and hence you see the same reason holds true for this vertex as well as with color 2, and hence I can assign color 2 here,

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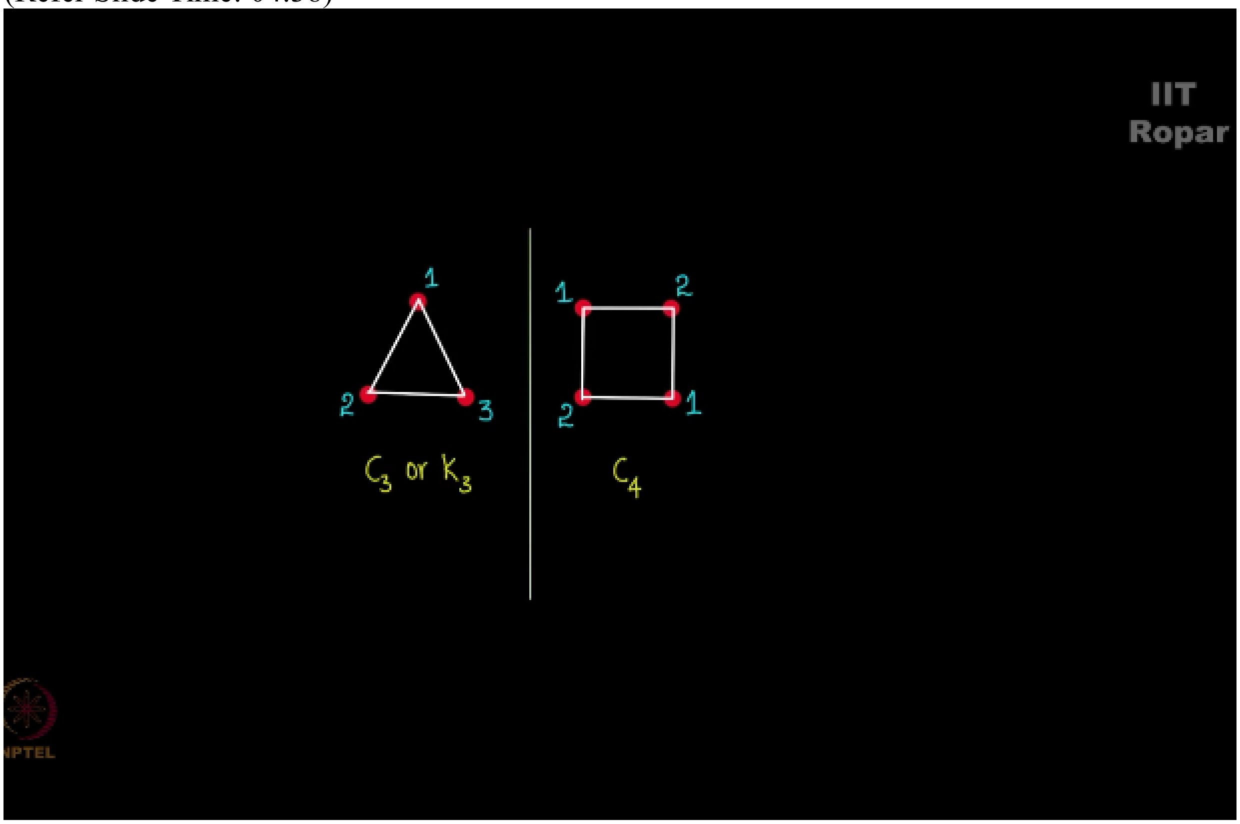

I can go on doing this way if the vertices are even in number, and hence if the number of vertices in CN, N is even then 2 colors are sufficient, but for a cycle with odd number of vertices, 3 colors are required, take a moments break to think about why this is happening, why do we say that in C4 or in CN when N is even, 2 colors are sufficient and then N is odd, 3 colors are required, if you can write down the proof for it, it will be great.

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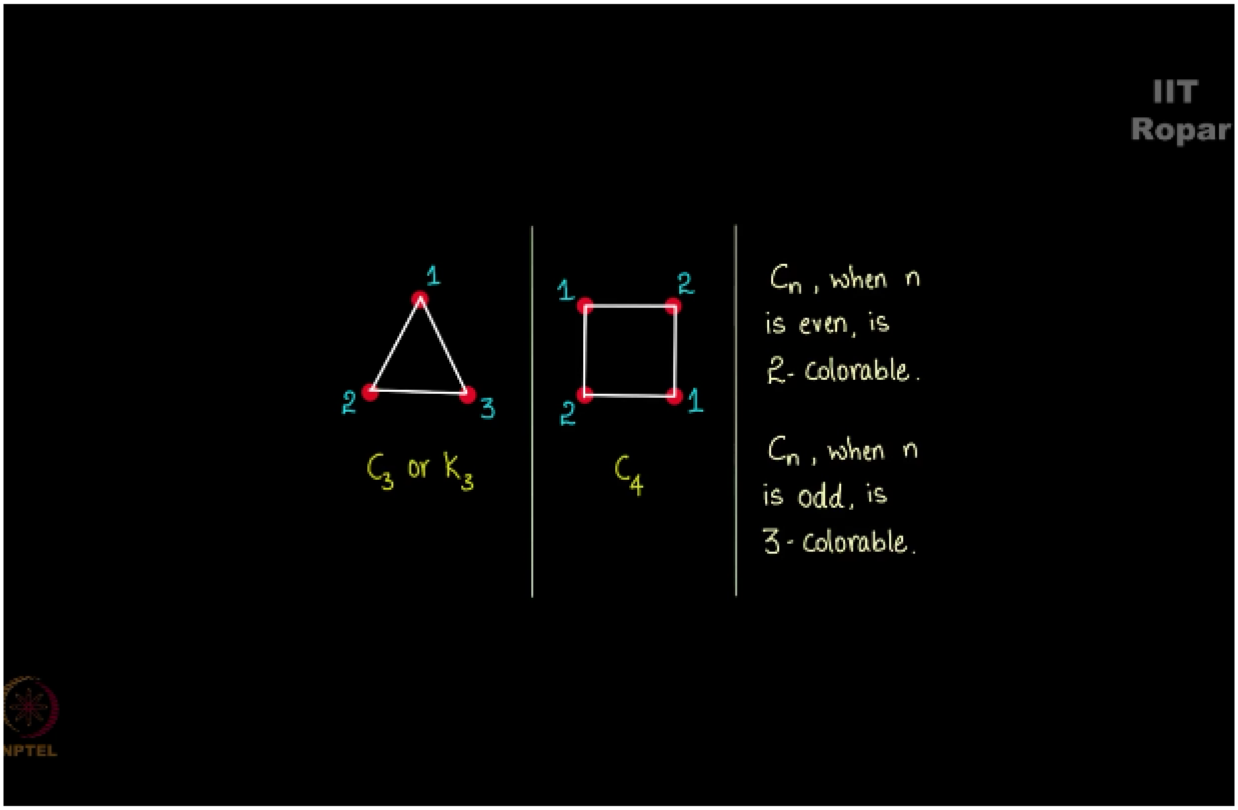

Consider this graph on 6 vertices, I assign this vertex the color 1,
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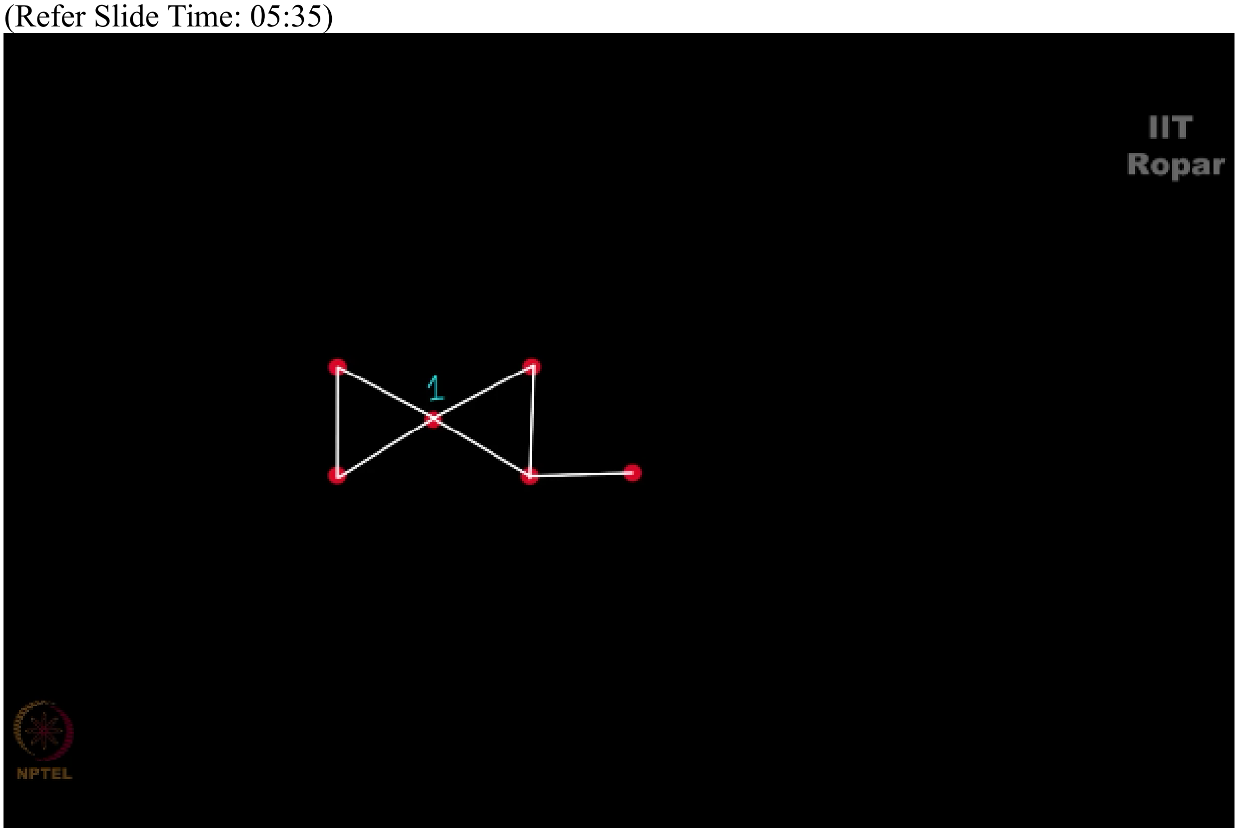

and it has 4 adjacent vertices so none of them can get the color 1, right, I'll assign the color 2 here and 3 here, why? You see a C3 here, right, and hence 3 colors are anyway required.
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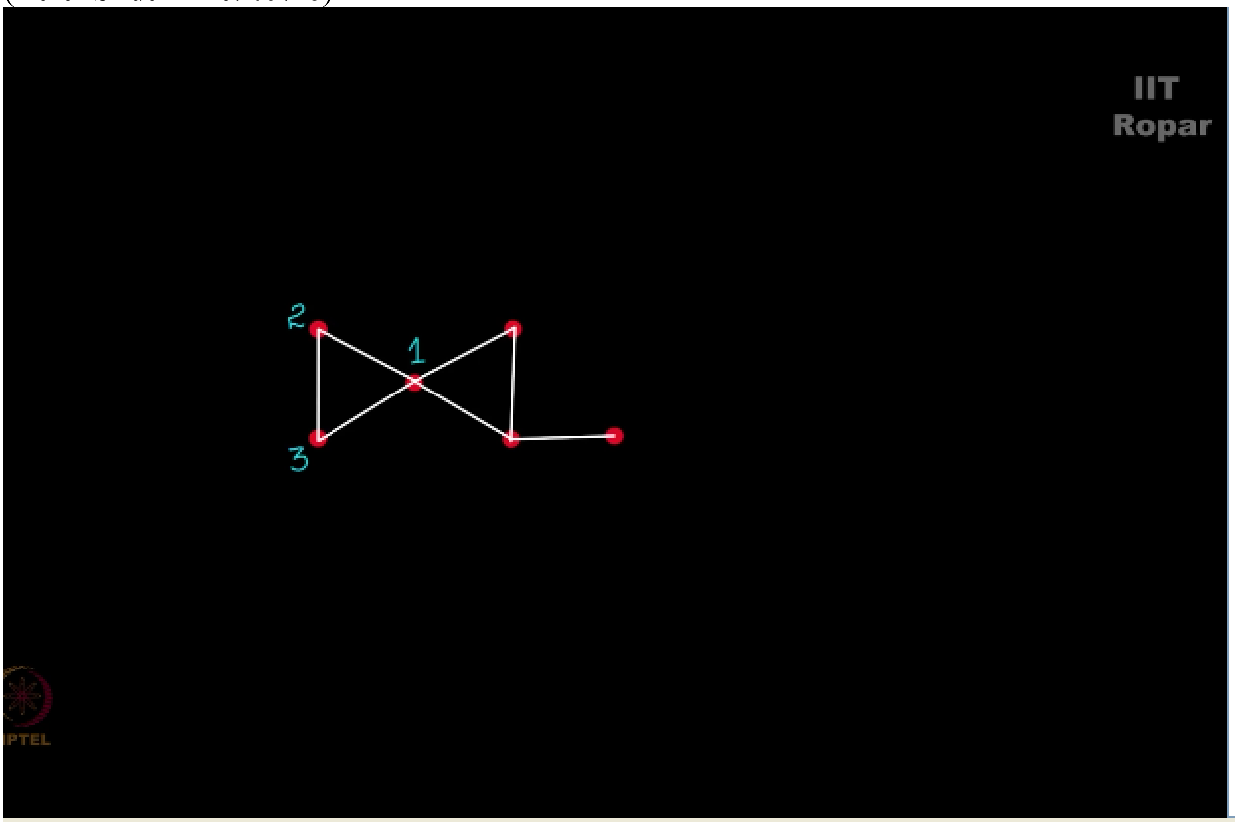

Now for this vertex I can assign the color 2 or 3, since I have already assigned 2 here, I have to assign 3 here, now this vertex is not adjacent to 2 or 1, I can give either 2 or 1, and just like that give 1, hence this graph is 3 colorable.
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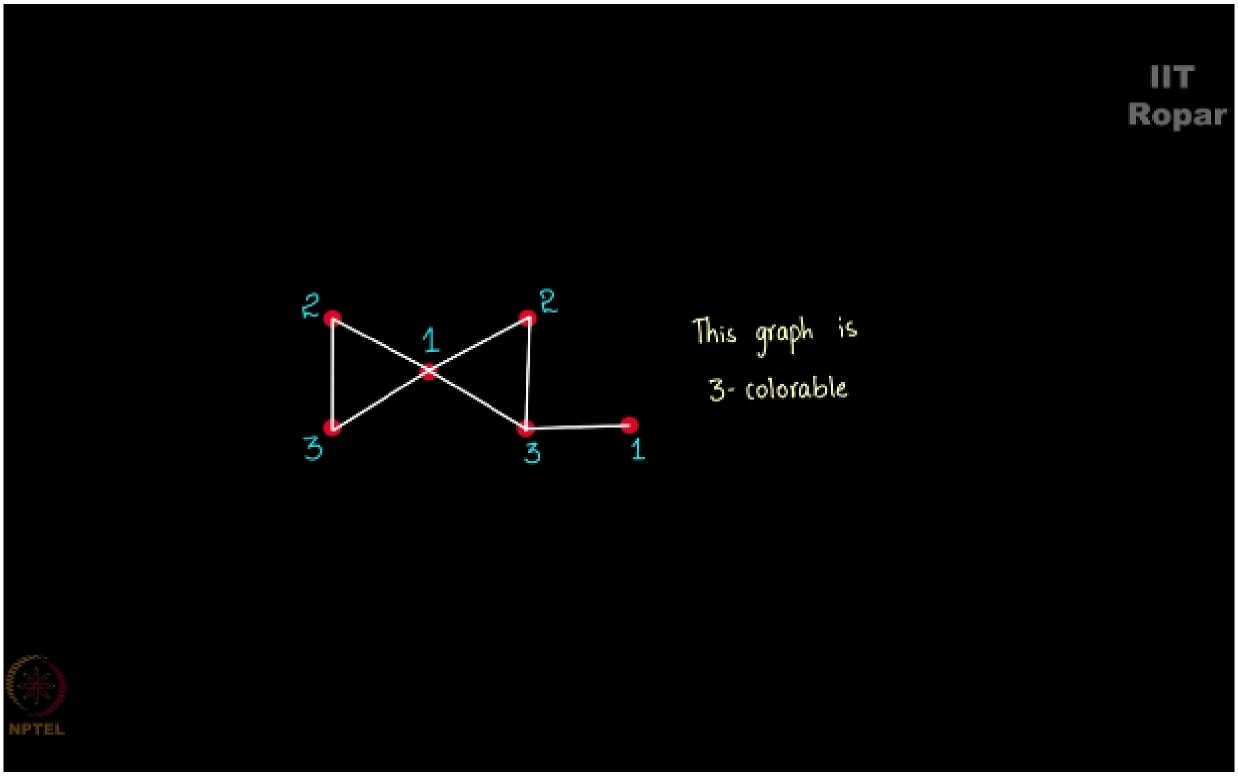

Here goes the Peterson graph, you see I have labeled the vertices from 1 to 10, we see that it has 10 vertices,

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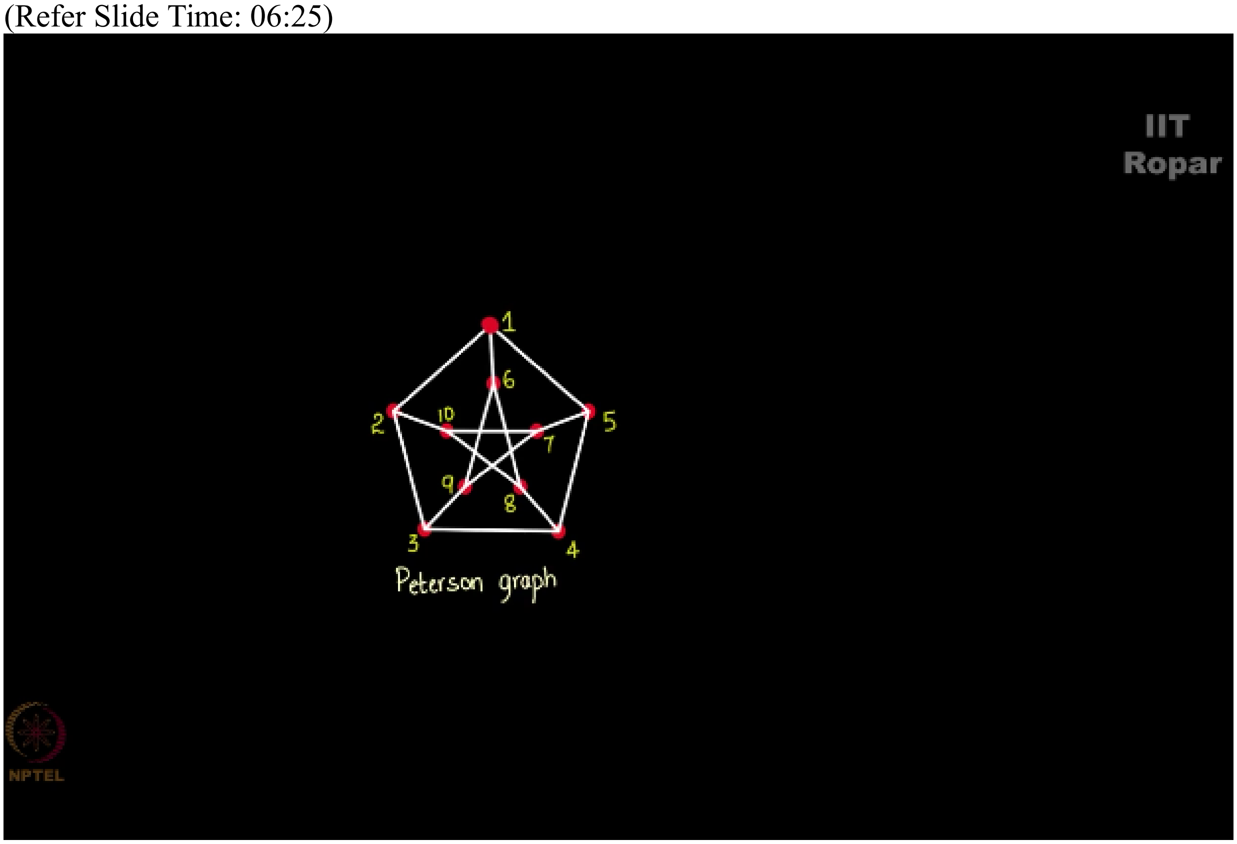

let me now start coloring this graph, I give vertex 1 red color, at vertex 2 I cannot give red, hence I give it blue, coming to vertex number 10, 10 is not adjacent to 1, and hence I give it red, you see I can also give it green, because anyway I'm not using blue, and I can give green, but please remember we always want minimum number of colors, and hence I give it red.

Moving to vertex 7, I cannot give it red because it is adjacent here, and hence but you see 7 is not adjacent to 2, and hence it gets the colored blue. Coming to vertex 6, 6 is not adjacent to 7, but it is adjacent to 1, so I cannot give red, but I can definitely give blue, right, though 10 has red, it is adjacent to 1, so we cannot give the color red to 6.

Now coming to vertex number 5, 5 is adjacent to both 7 and 1 which have red and blue, I cannot give both of these colors, I'm now going to introduce a new color green, do you see, only when we are in the critical condition that we cannot use the colors used so far, do we introduce a new color, now 5 gets the color green, coming to vertex 4, I cannot give it green, but I can either give blue or give red, what I'm going to do is I'll choose one of them, I'll give red.

Now consider vertex number 8, I cannot give it red because either sides that is 10 and 4 it is having red color, but it's not connect, but you see it is connected to 6 also here, 6 has the color blue, so neither can it get blue color nor red color, but it is not connected to 5 and hence I can give green color to it.

Vertex number 9 can get the color, red is not there, it is connected to 6, 6 has blue color and 7 has blue color, definitely we cannot assign blue color to it, but we can assign red color and hence I'm going ahead with red, we can also give green. Now coming to vertex 3, 3 is connected to 2, 9, and 4, 2, 9 and 4 these vertices have blue and red respectively, so what I'm going to do is I'm going to give the color green, because it is not connected to either 8 or 5, so do you see with these colors I have satisfied all the conditions and colored the Peterson graph using just 3 colors, hence Peterson graph is 3 colorable.
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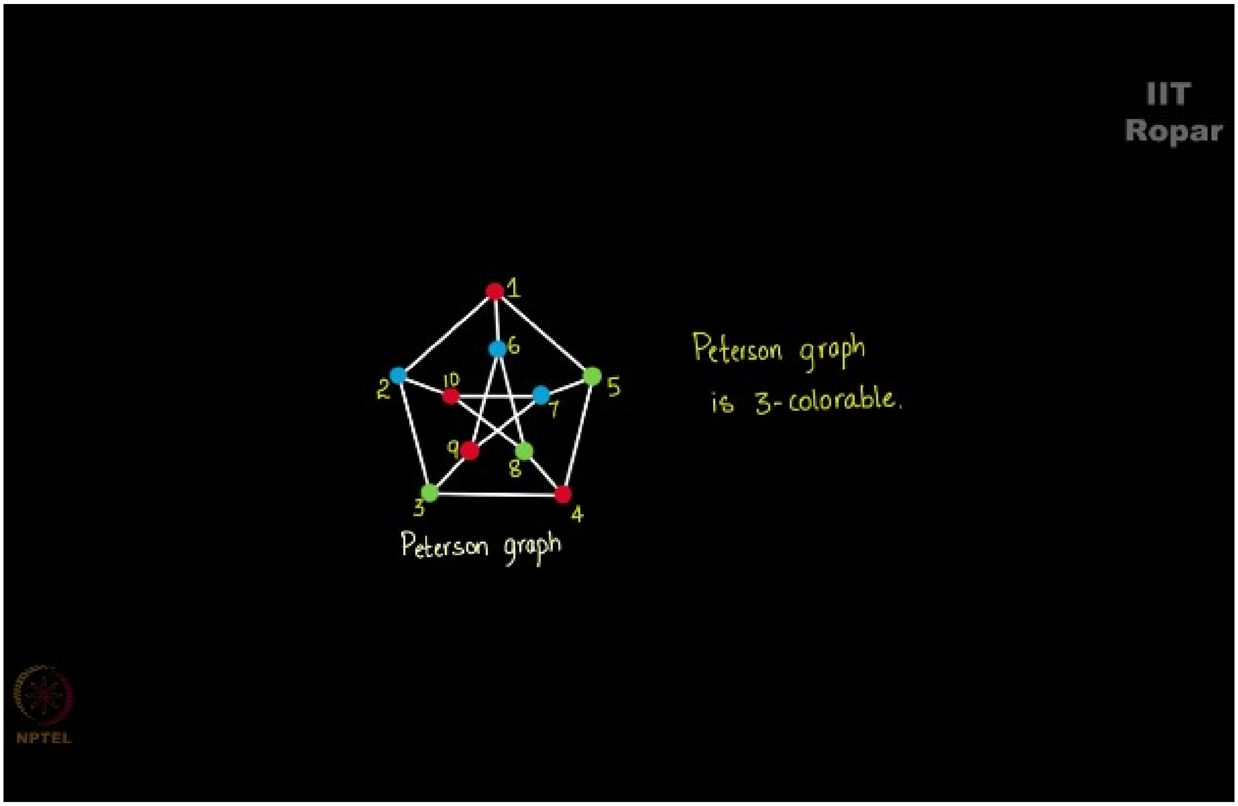

You might have given some other way, you might have given the colors in some other way and obtain another assignment to these 10 vertices, if you have colored the graph using 3 colors, then you are done, the chromatic number is 3 for a Peterson graph, no matter how you assign the colors you must be able to do it with just 3 colors, go ahead and give another assignment.
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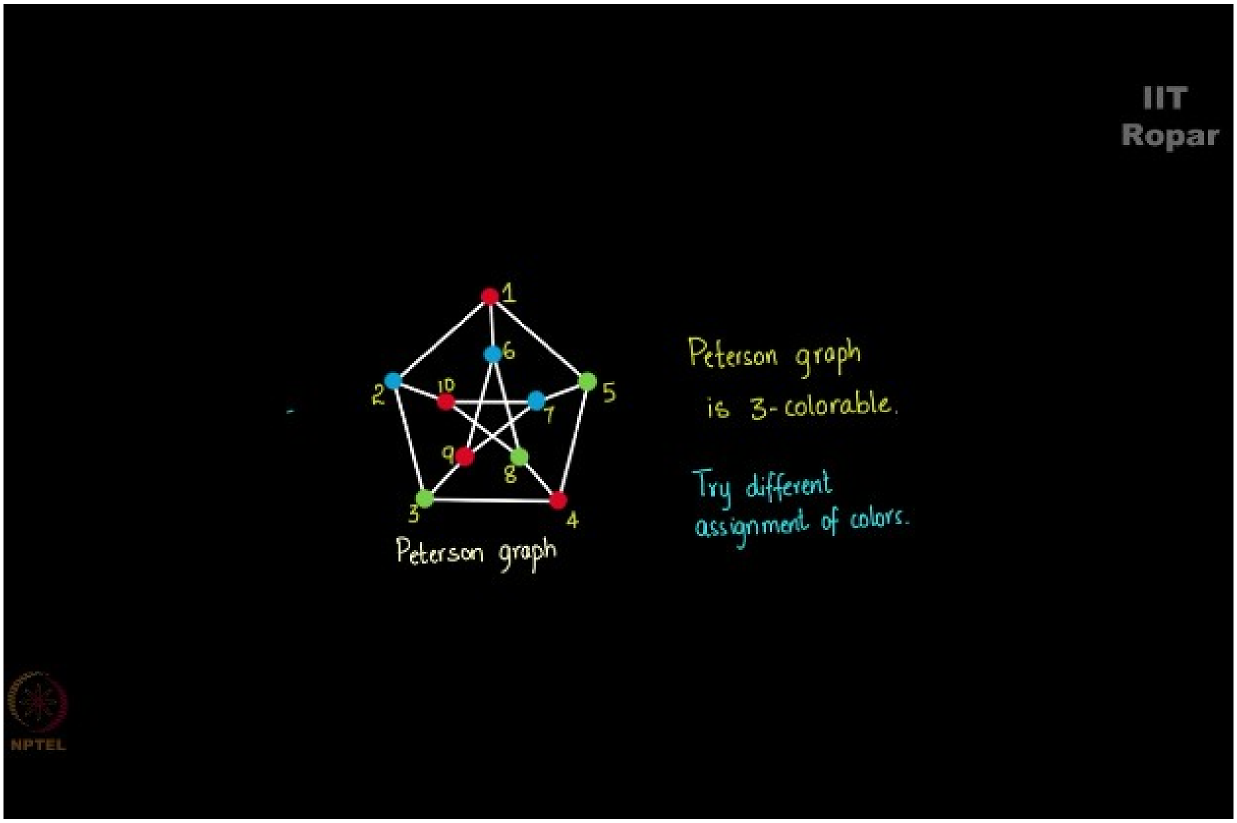

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