NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 2

A result on connectedness

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Let us solve an exercise problem, look at this graph, a graph with the following condition that degree of every single vertex happens to be greater than or equal to n/2, what do we mean by this? By this we mean every single vertex is adjacent to half the vertex set.
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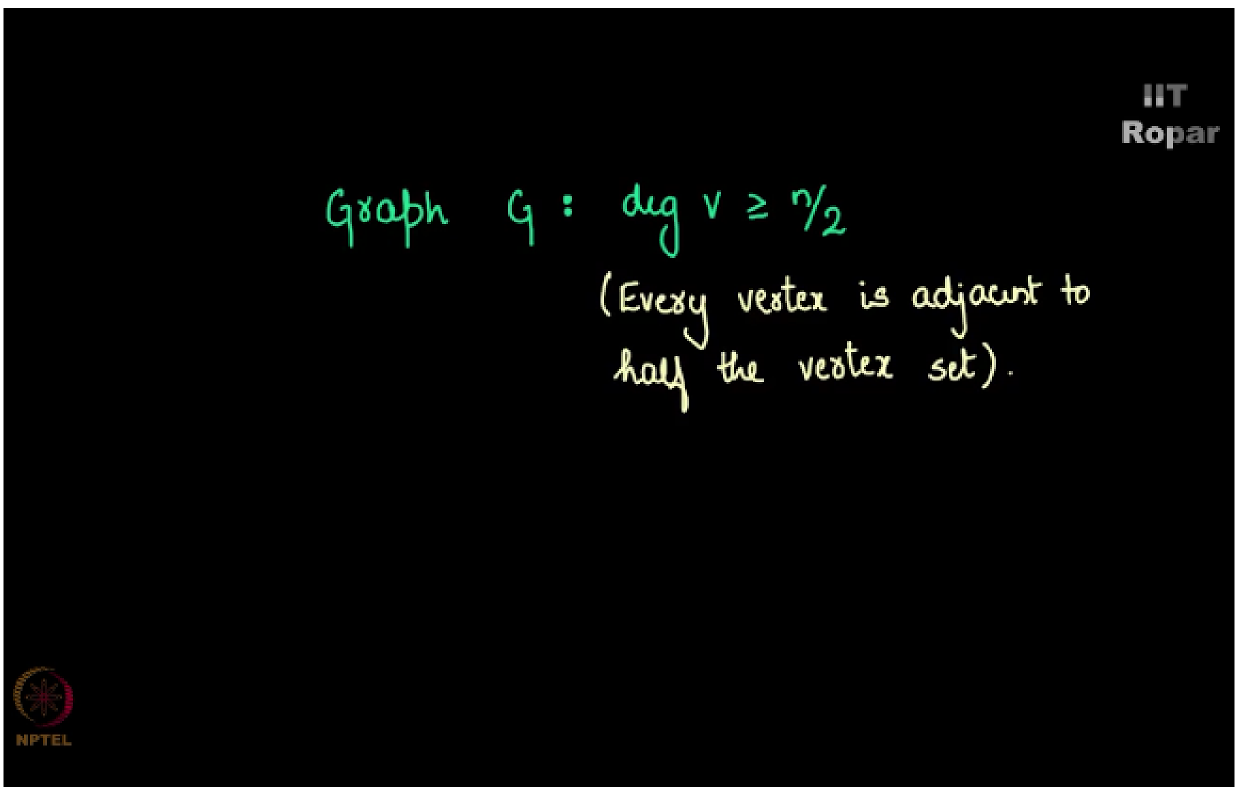

N/2 denotes half of the vertex set, right, okay.

Now what can you say about such a graph, this graph has a very interesting property that it has to be connected, you cannot have the graph disconnected if you have this property that every single person in this network is adjacent to at least half of the entire network, now this is what we mean by emergence of a property.

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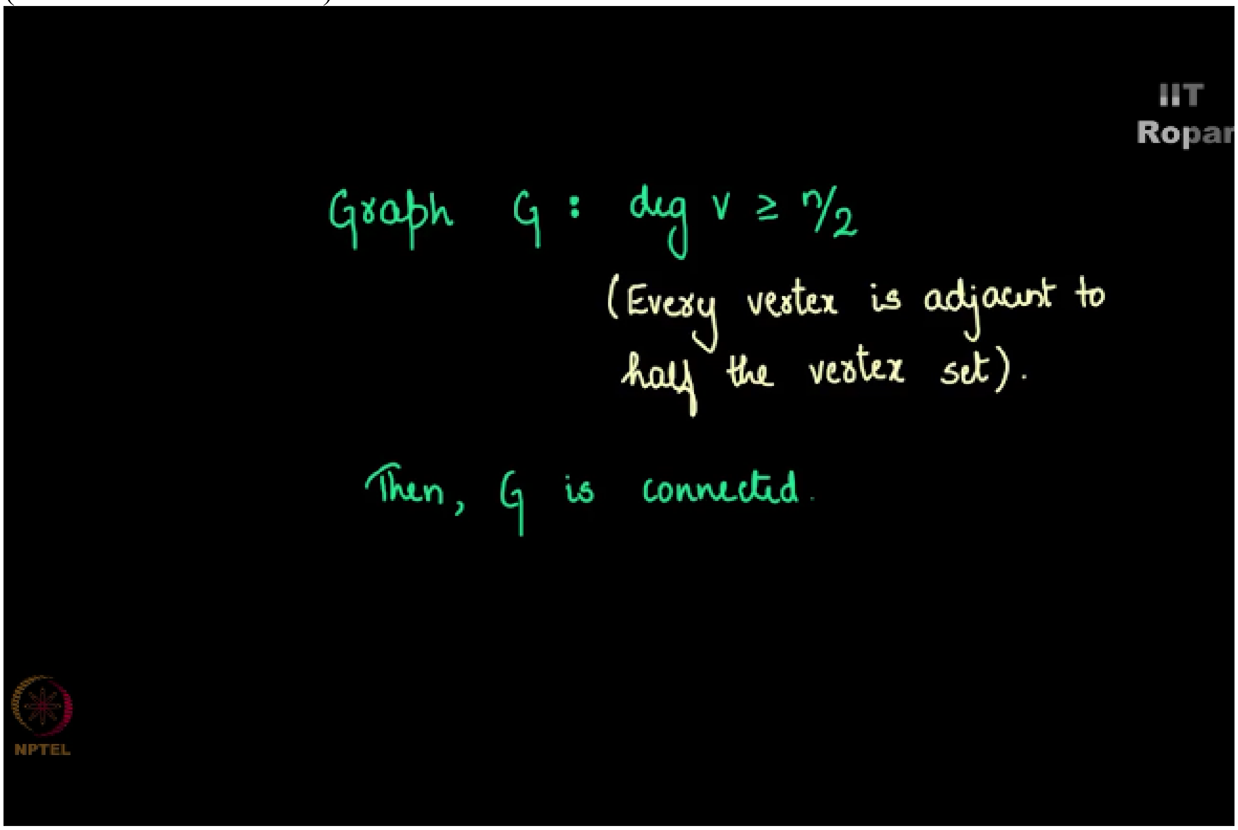

Do you see how this true? This is a right point where you should pause a video and then think why is the graph connected. Now let us see why exactly a graph becomes connected when the degree of every single vertex is greater than or equal to n/2.

Now let us prove this by contradiction, assuming that the graph was disconnected then it will have at least 2 components with different number of vertices, here let say M1 and then M2 and M1 + M2 will be equal to N,

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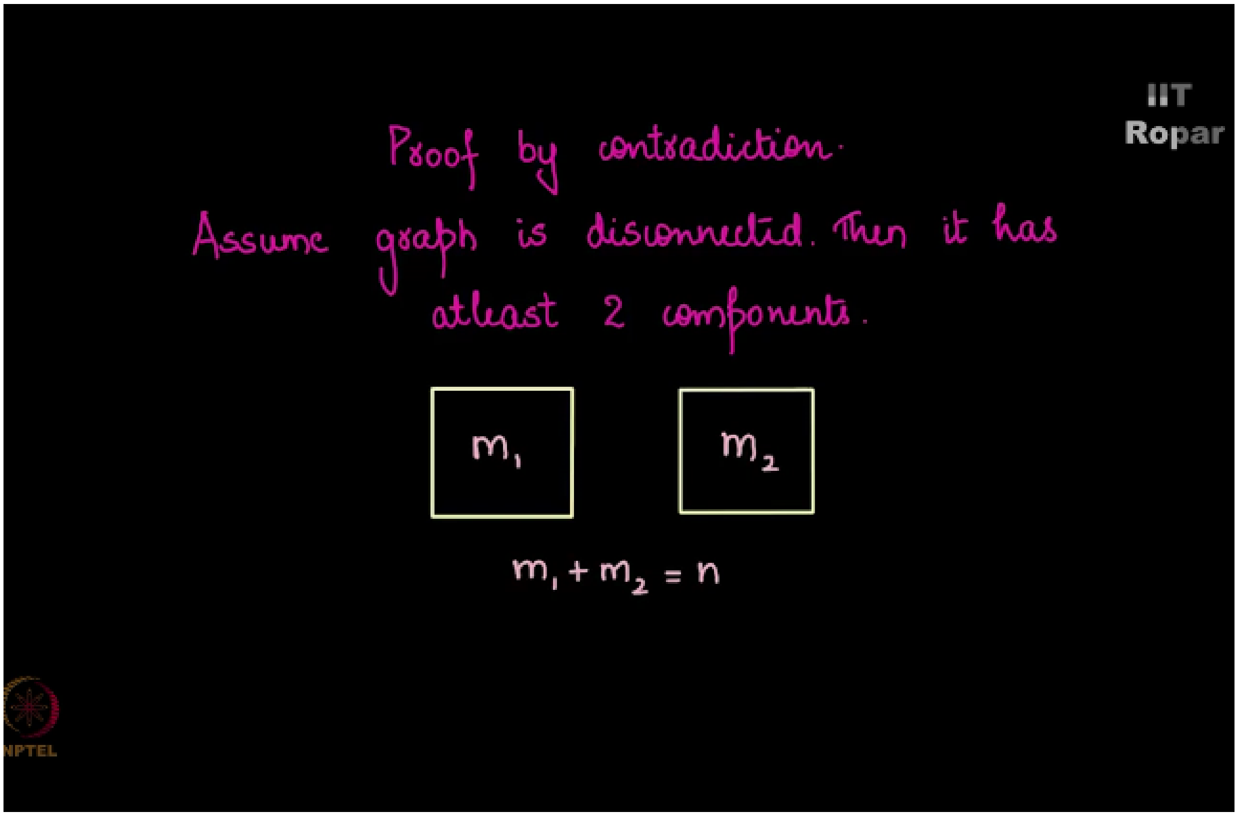

so for simplicity sake I'll take N has 100, then M1 can be 60 and M2 will be then 40, or M1 can be 30, M2 can be 70 something like that, okay.

When M1 is 60, the degree of a vertex here can go up to 59, and M2 is 40 then degree of a vertex here can go up to 39, do you see something, do you note that 39 is not greater than or equal to n/2 which is 50 here, correct,

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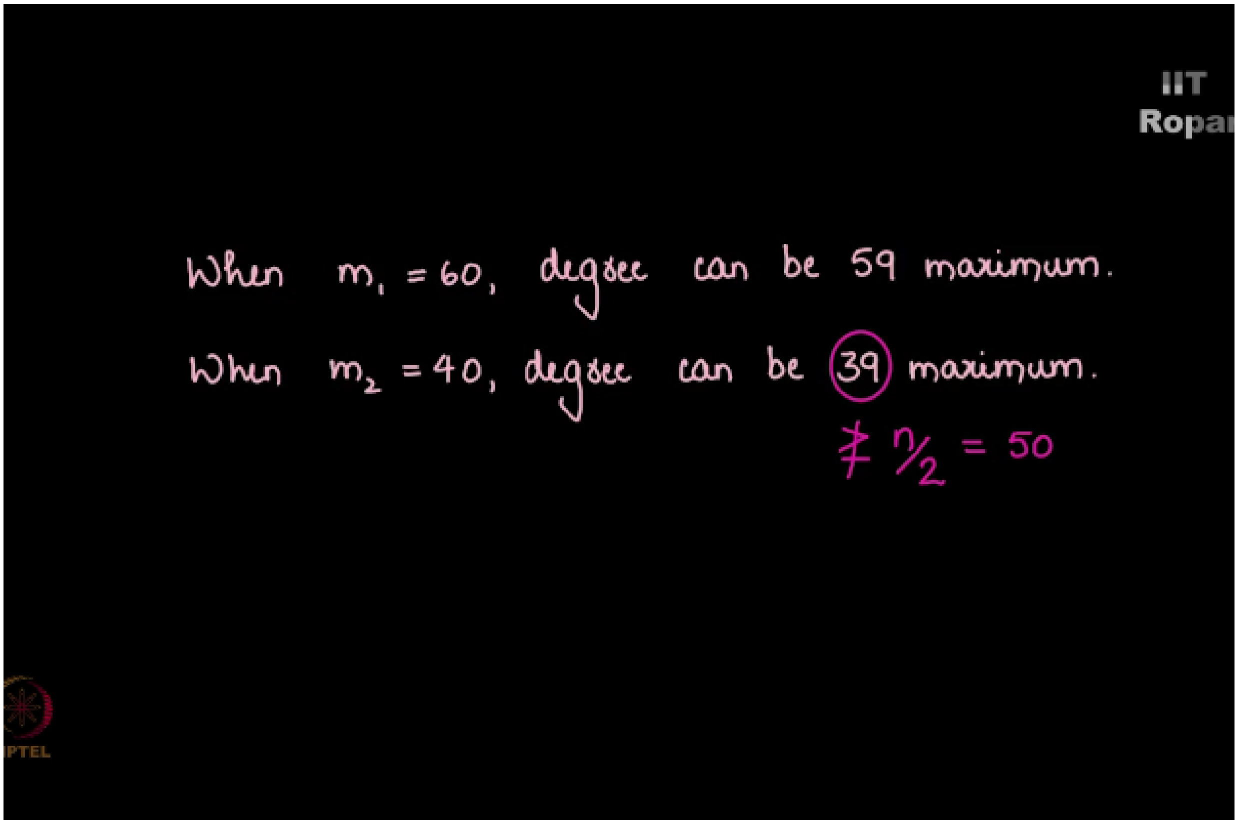

so you cannot have a graph satisfying this degree property, that degree of every single node is greater than or equal to n/2, and at the same time the graph is disconnected, this cannot happen, so maybe 60, 40 was a very weird bifurcation of the graph, vertices into two connected components, maybe I'll try 50 and 50, than degree of each vertex here will be 49 at the most, still violating the degree condition being the fact that degree of U is greater than or equal to n/2

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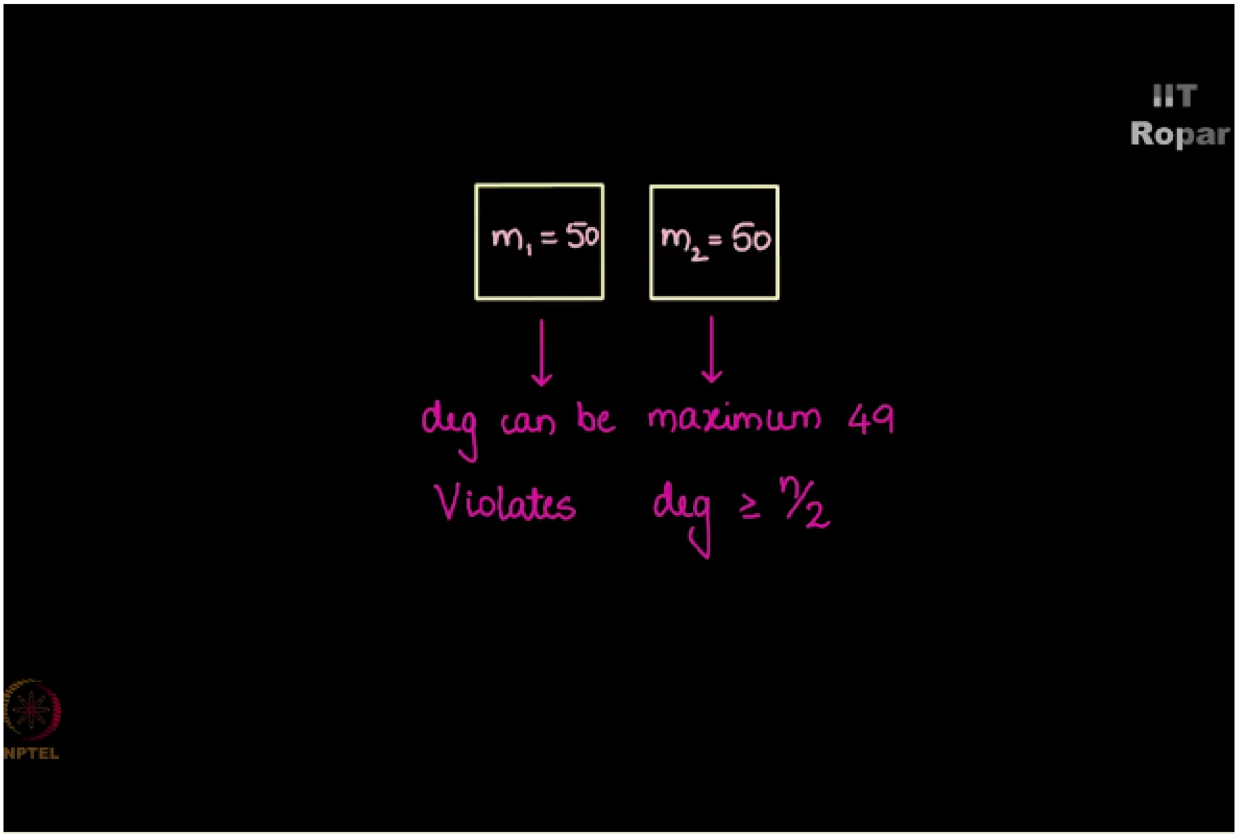

and what do we infer from this? We infer that whenever a graph G has the property that the degree of every single node is at least n/2 and the graph has to be connected,

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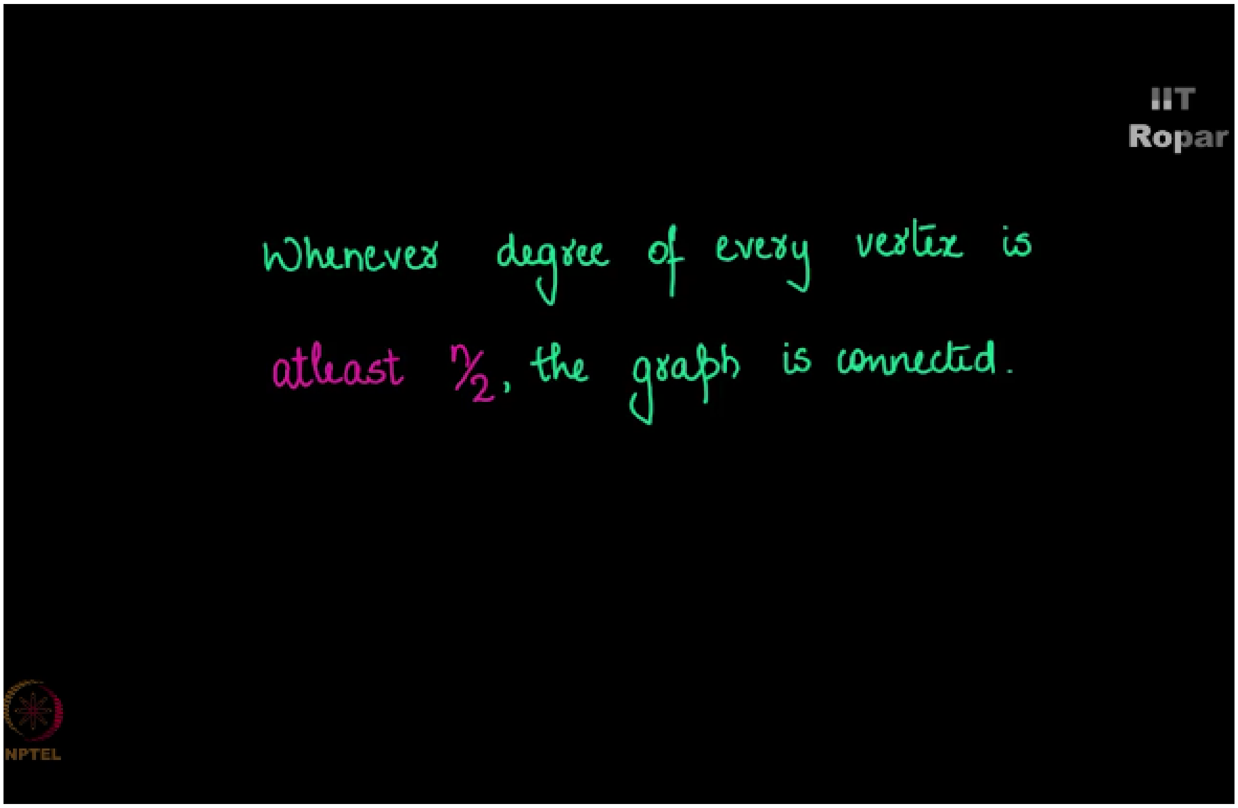

there is no other go, if you try constructing a disconnected graph with this property you will not get one, so here is the statement of the theorem, take a look at it and try to construct the proof all by yourself, and see what exactly is the logic that we have used here.

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