NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 2

Bipartite graphs - Converse part of the puzzle

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We just now saw that in a bipartite graph you cannot afford to have an odd cycle, odd cycle will disturb the very bipartite property, how about the other way round? What is the other way around? Think about it.

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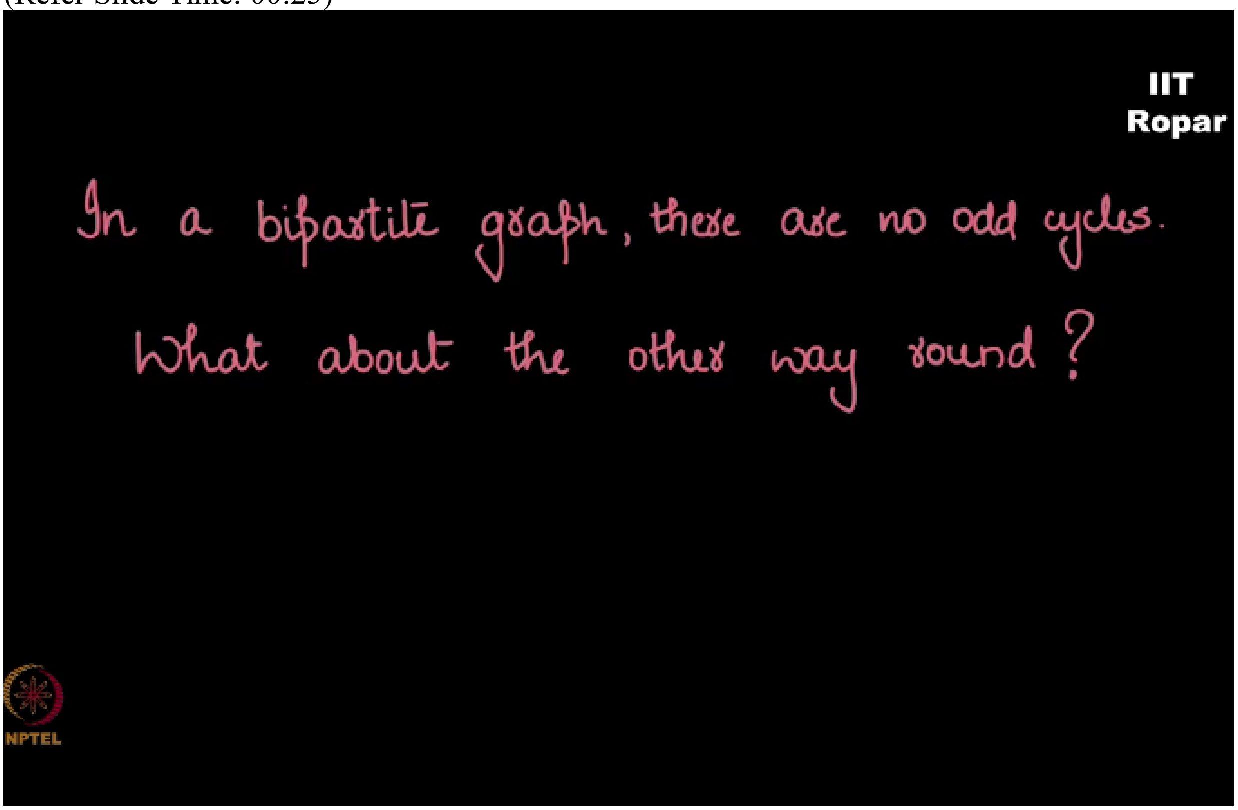

If a bipartite graph is given then I said you cannot have odd cycle, if a graph is given where there are no odd cycle, can you guarantee that it is bipartite, I repeat bipartite graph implies no odd cycles in the graph, no odd cycles in a given graph does it imply that it is bipartite and that you can partition them in to two parts so that all edges are across and not within, let us try looking at it with that example.

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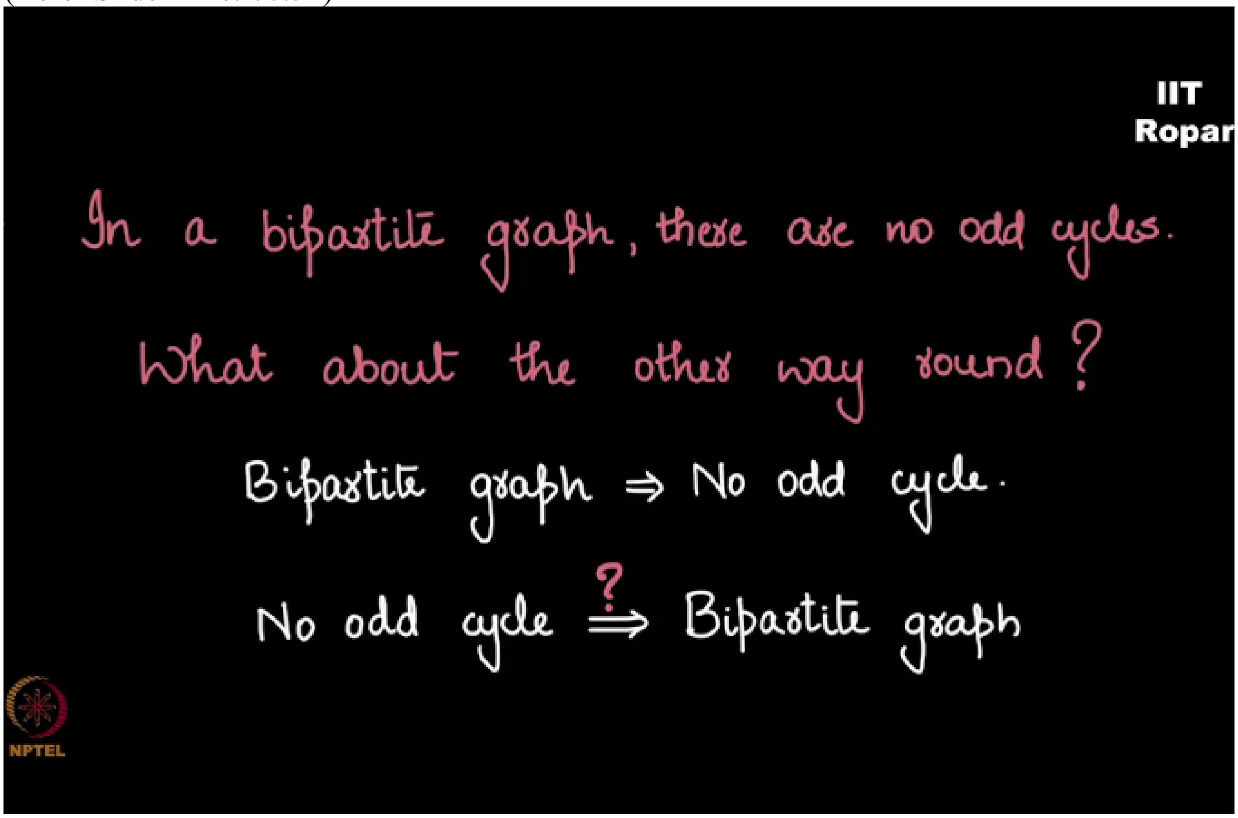

Look at this little graph with 8 vertices, 1, 2, 3, 4, 5, 6, 7, 8,

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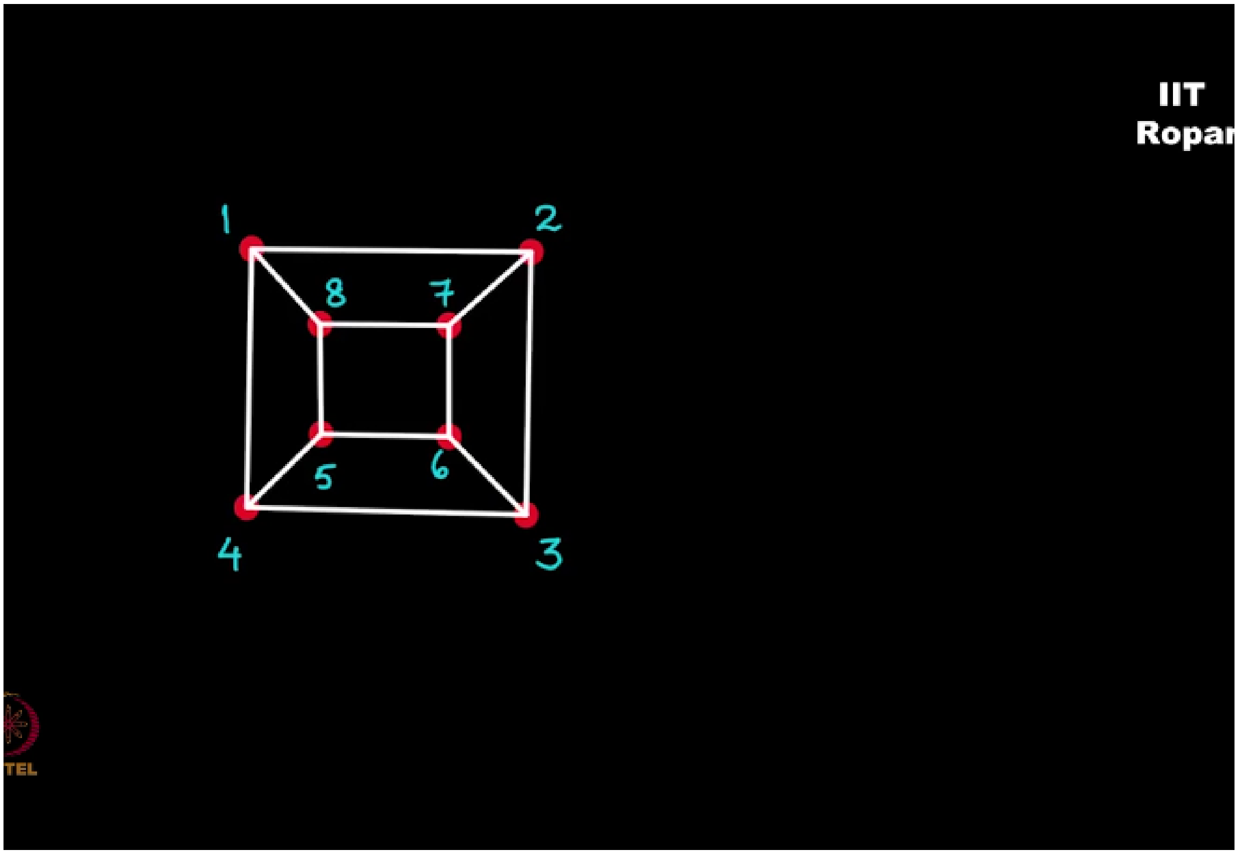
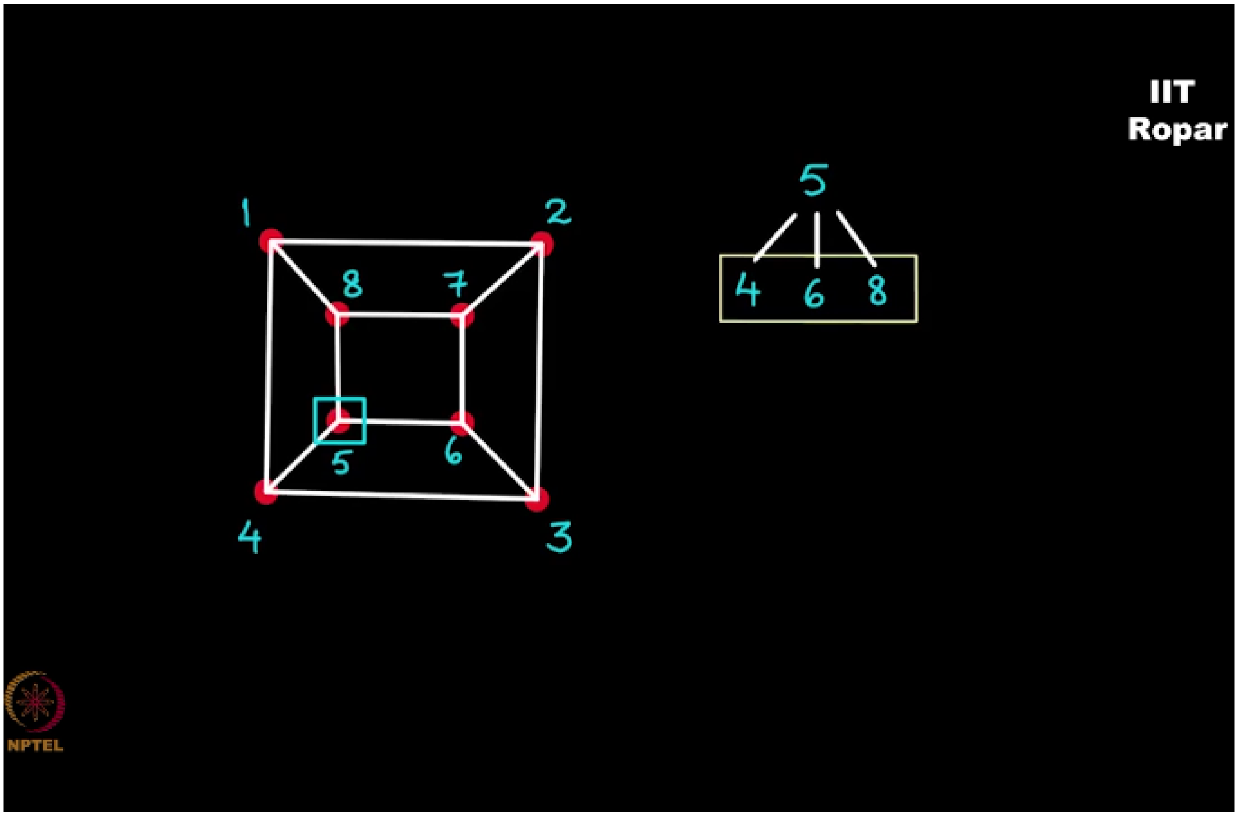


right, a neat looking graph, if you think of it there are no odd cycles here, I can guarantee you that you can also verify it manually, right, okay.

Now look at this, let me take some vertex here, let's say 5, and let me write down all the vertices adjacent to 5 that is those vertices of distance 1, distance 1 means shortest path is through a single edge which is 4, 6, and 8, look there are no edges between 4, 6, and 8, correct, (Refer Slide Time: 01:37)



and hence there cannot be any triangles here, 5 4 6, 5 4 8, 5 6 8, impossible, there cannot be triangles here because there are no triangles in the original graph, right, no odd cycles in the original graph, so 5, 4, 6, 8, at the first level whatever are the vertices I'll enumerate them like this and then I'll go to the next level, level 2, so let me call 5 as level 0, 4 6 8 as level 1, any vertex that is right adjacent to 5 is L1, any vertex that is adjacent to the elements of L1, but not adjacent to 5, because all vertices adjacent to 5 are already listed, correct, so L2 will be 1, 3, 7, and similarly L3, what is L3? Nodes that are 1, 2, 3 units away from 5, shortest path between 5 and that vertex should be 3, there is only one such vertex namely 2.

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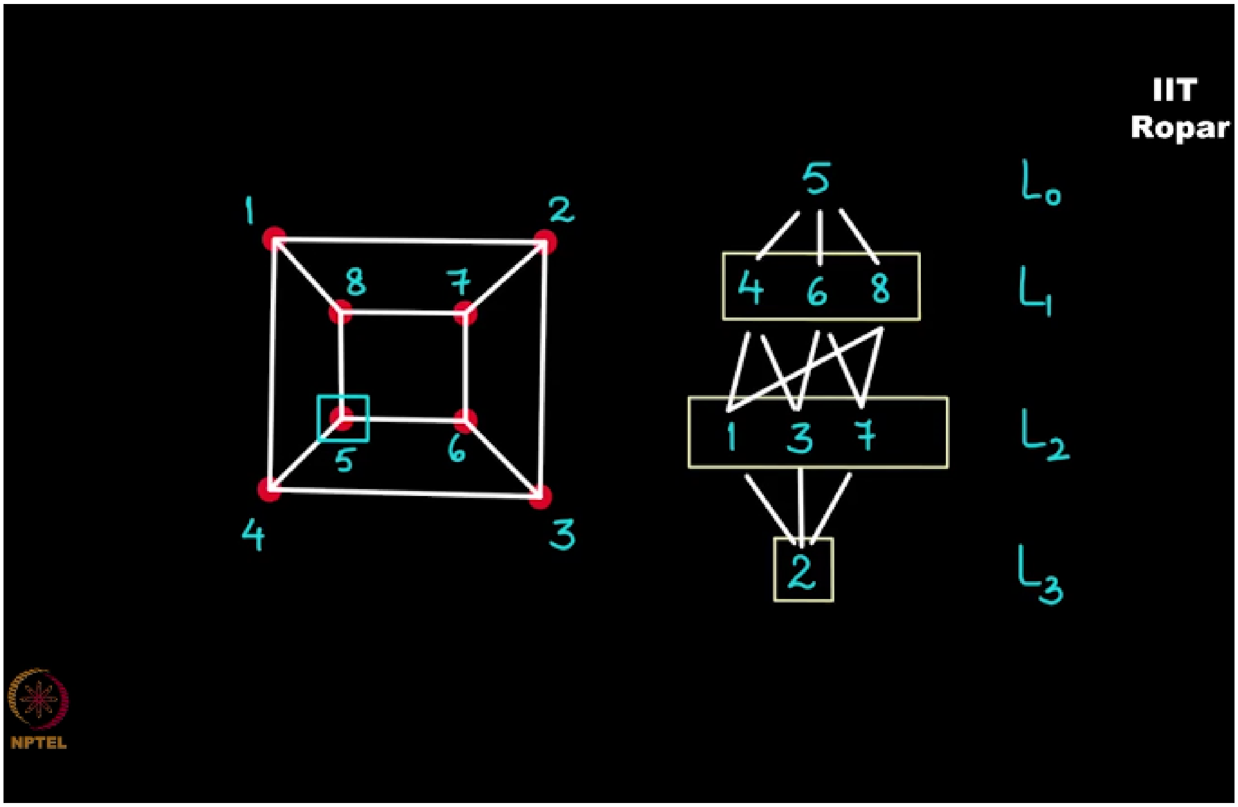

Now look at this, I'll put 5 in the left wing, and then all its adjacent vertices on the right wing, and all the elements of level 2 in the left wing and all the elements of level 3 in the right wing, now my claim is my observation is there cannot be an edge within the wing here,

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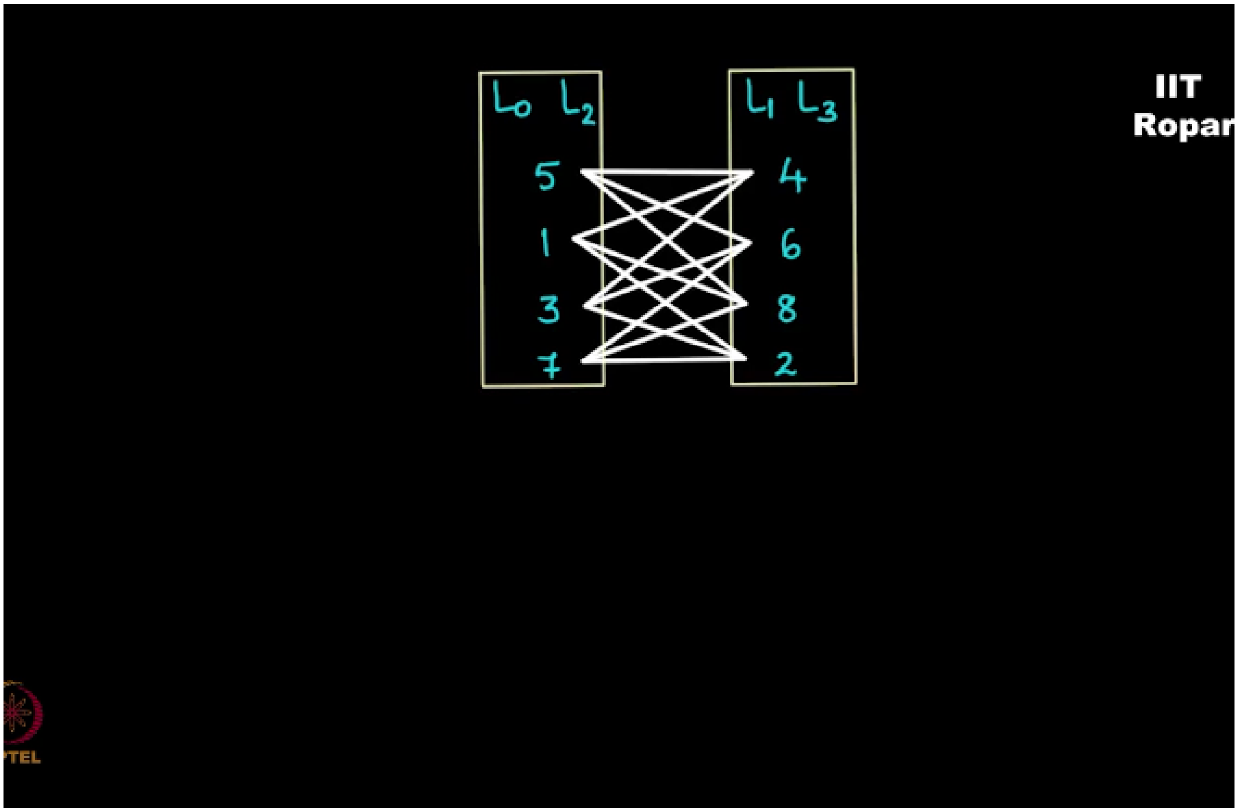

wing is basically this box, right, within the box there is no edge because if there was an edge within the box it means either there is an edge at the same level or there is an edge across two or more levels, right, so let's say 8 and 2 can be adjacent here, if it's adjacent then 8 cannot be in level 1 and 2 cannot be in level 3, correct, because the distance from 5 to 2 is 3, revise the notion of distance, if it's not clear to you, so what I'm trying to show here in general, I actually give a general proof that whenever a graph G is given, if you start from some vertex not necessarily 5 here, some vertex and write down the next level and then the next level, and then the next level, if you don't find any edges within each of this levels then it means there are no odd cycles, and you can alternatively put these levels into the box as left and right, and the graph will magically become bipartite,

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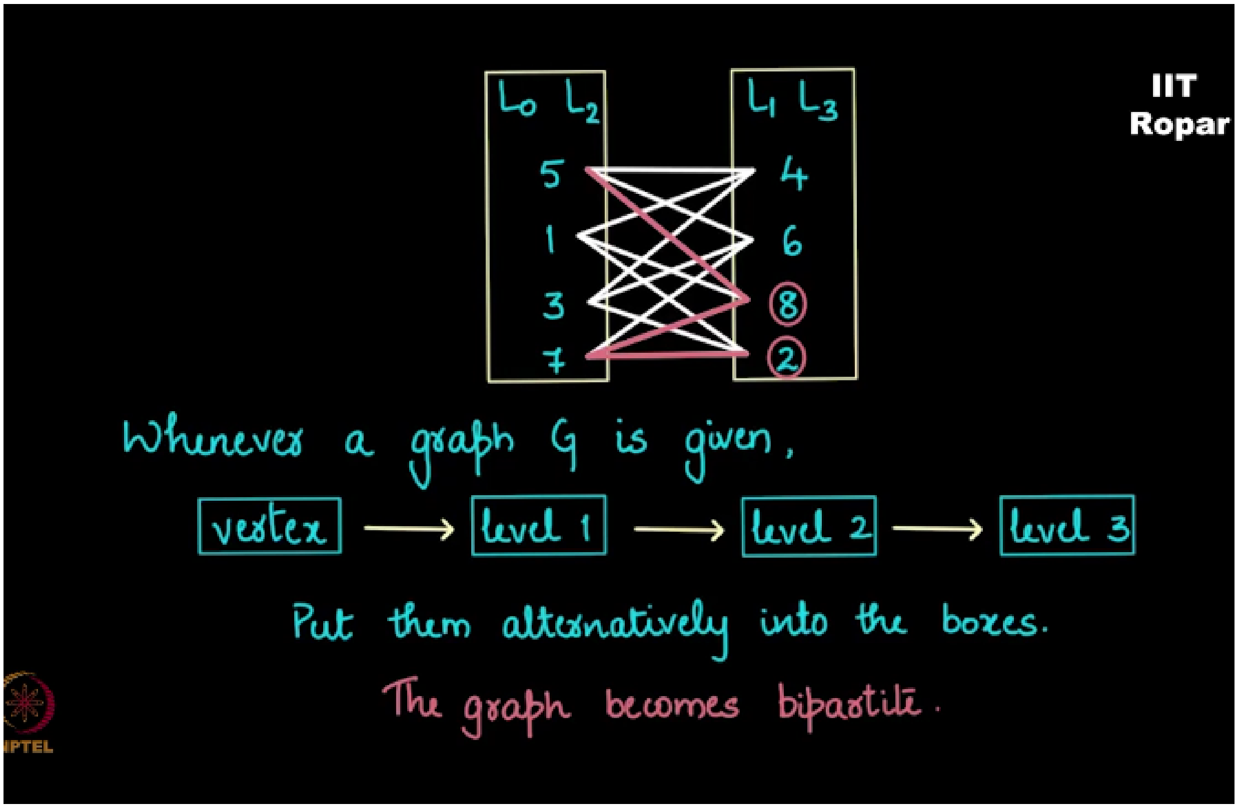

this result is best observed all by yourself than we explaining it, think about it, it will be crystal clear to you.

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