NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 2

Complement of a disconnected graph is connected -Solution

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Look at this example,
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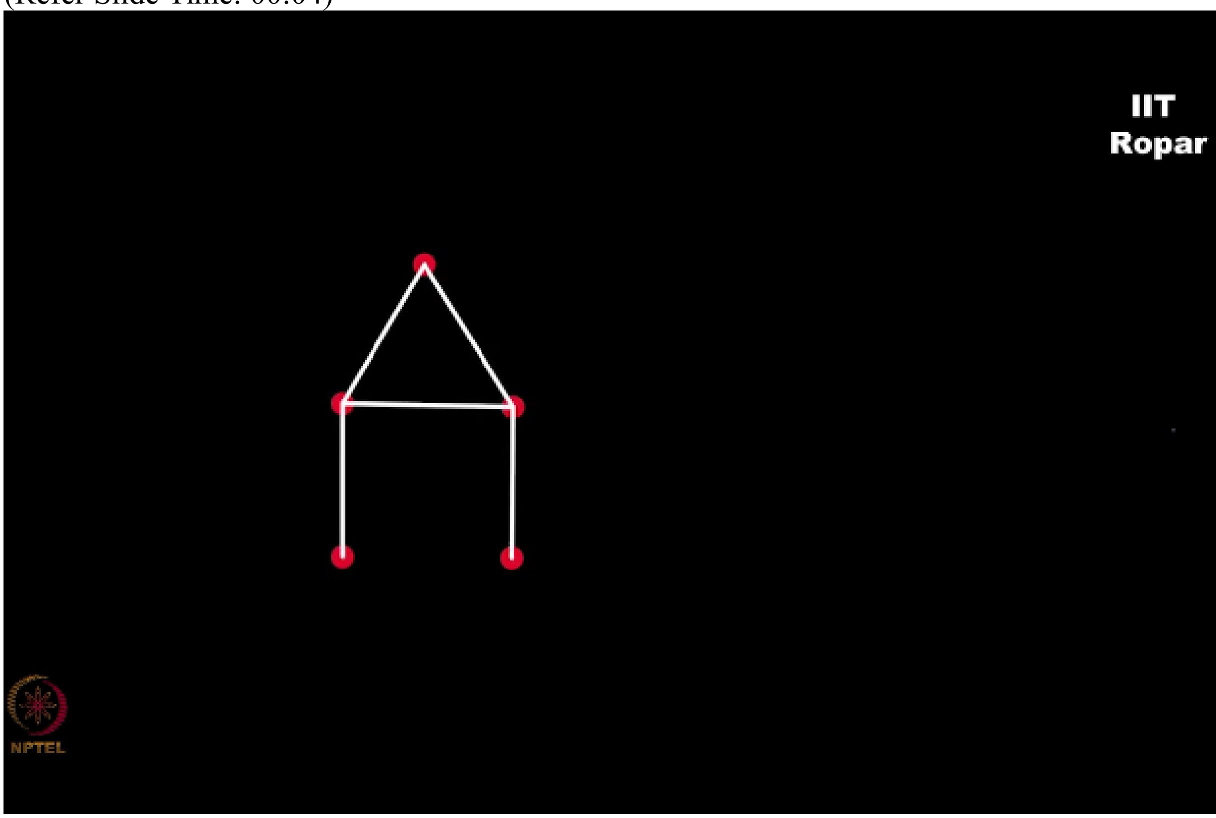

here is a connected graph, take its complement you are getting another connected graph.
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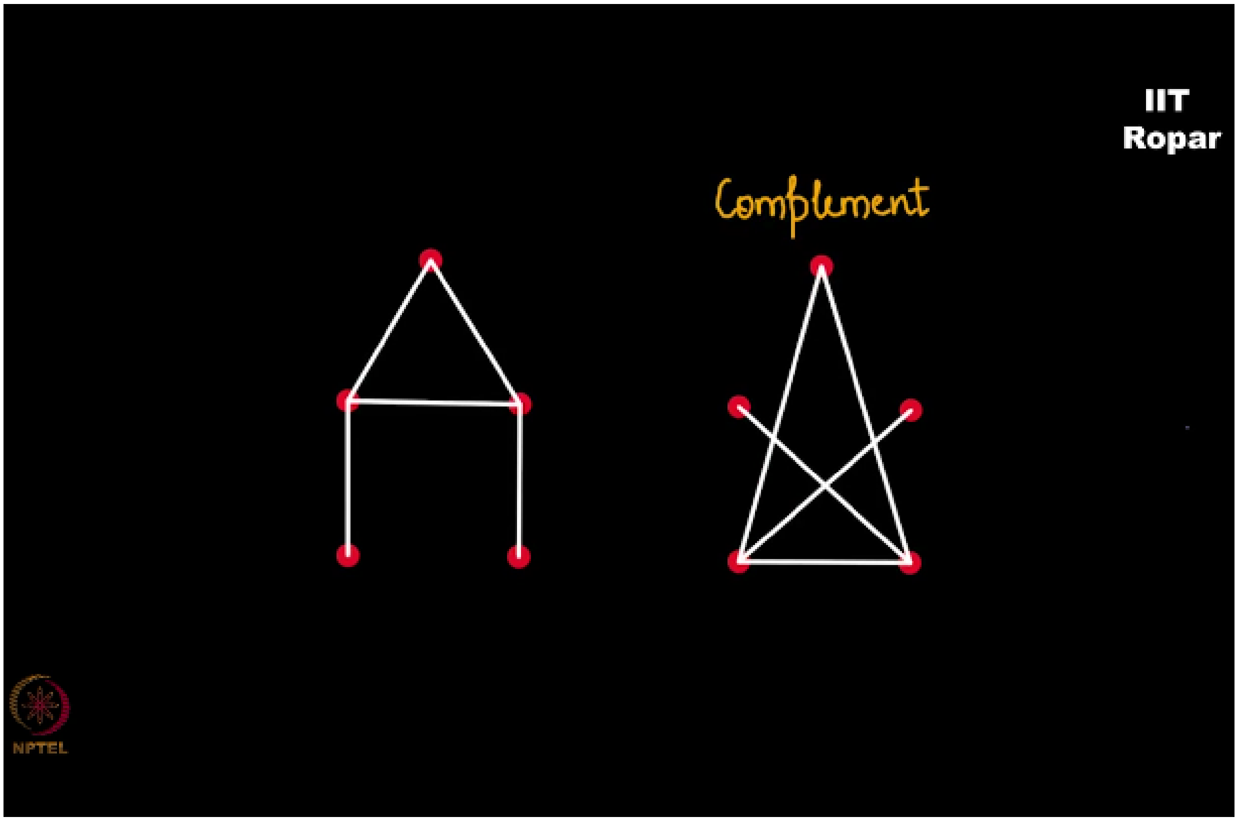

Look at the next example, here is a disconnected graph take its complement,
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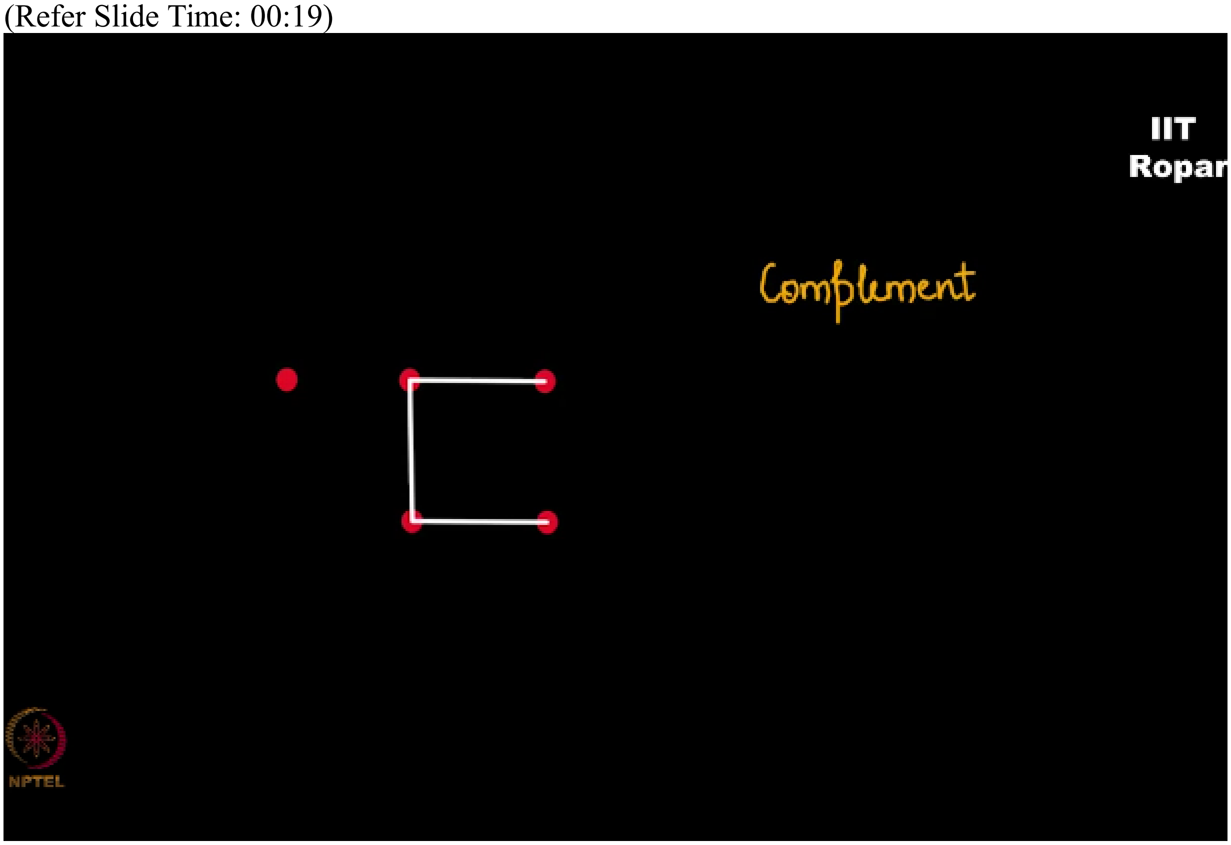

it’s a connected graph,
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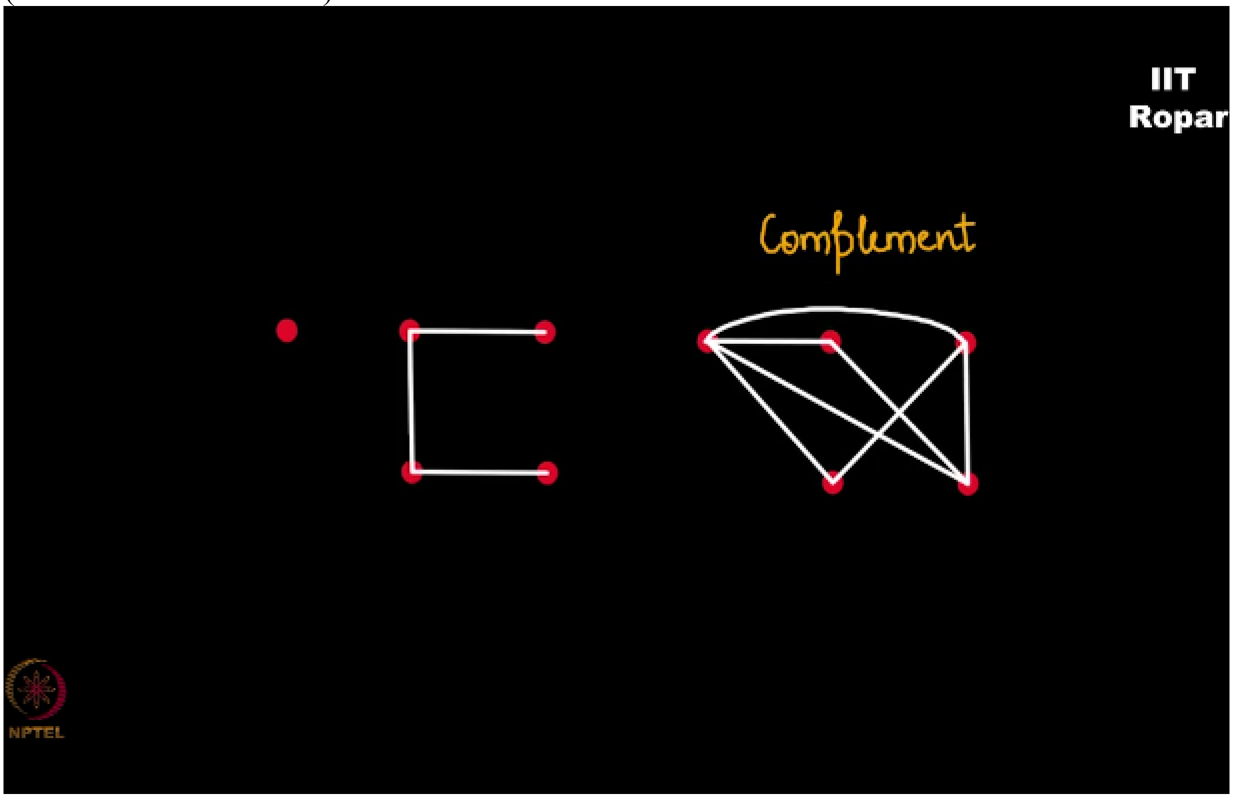

if you are not sure what is complement this is a right time for you to pause and then check what is complement, and how the right side graph is a complement of the left side graph, okay.

When you take the complement of a connected graph, you get a connected graph,
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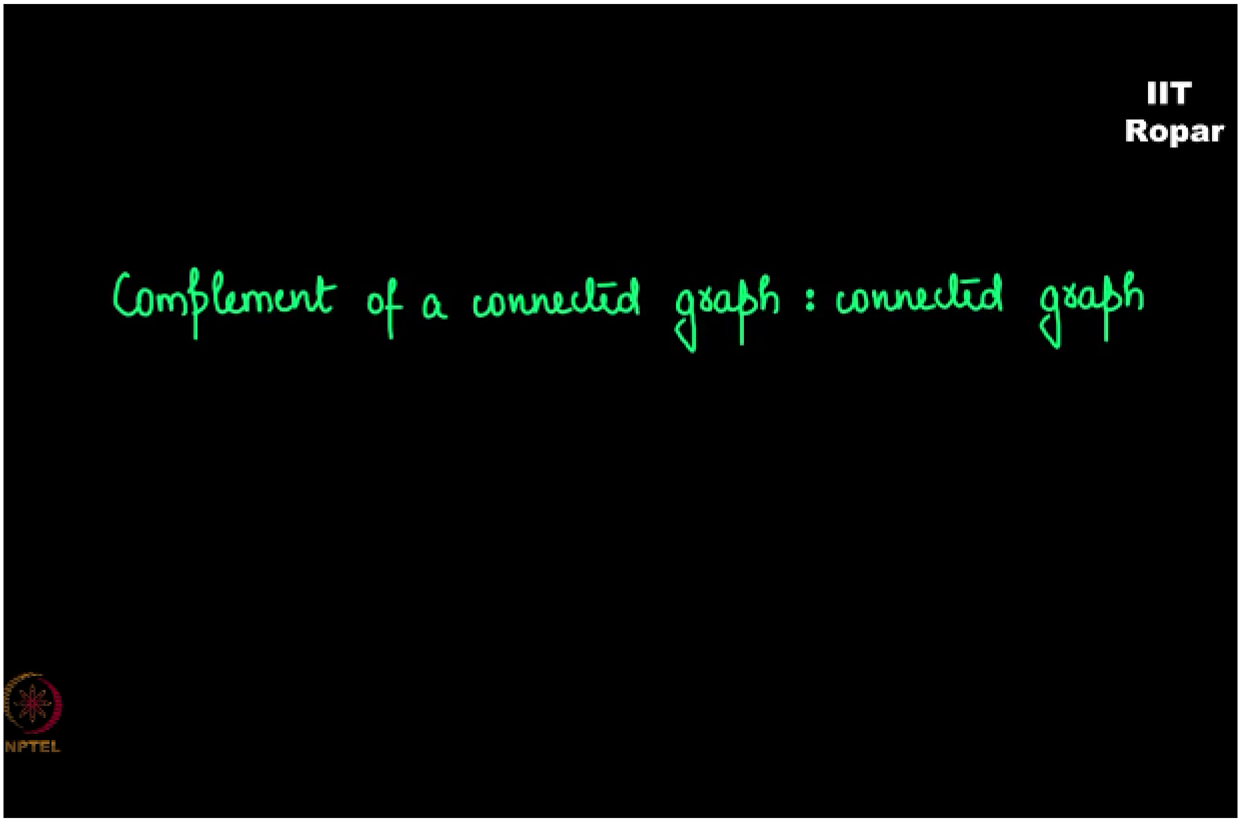

here is an example where complement of a connected graph is actually a disconnected graph,
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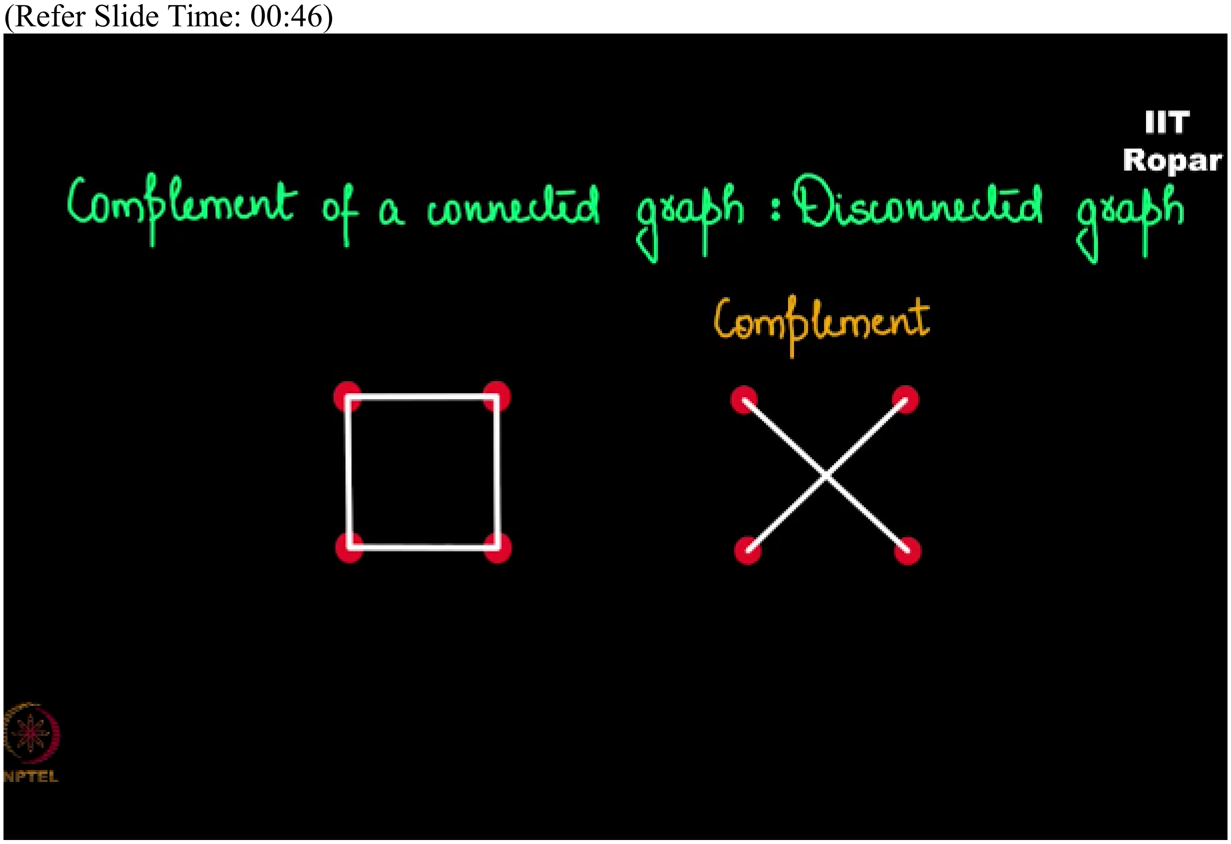

but whenever you take a disconnected graph its complement is always connected and this is the result we are going to prove now,
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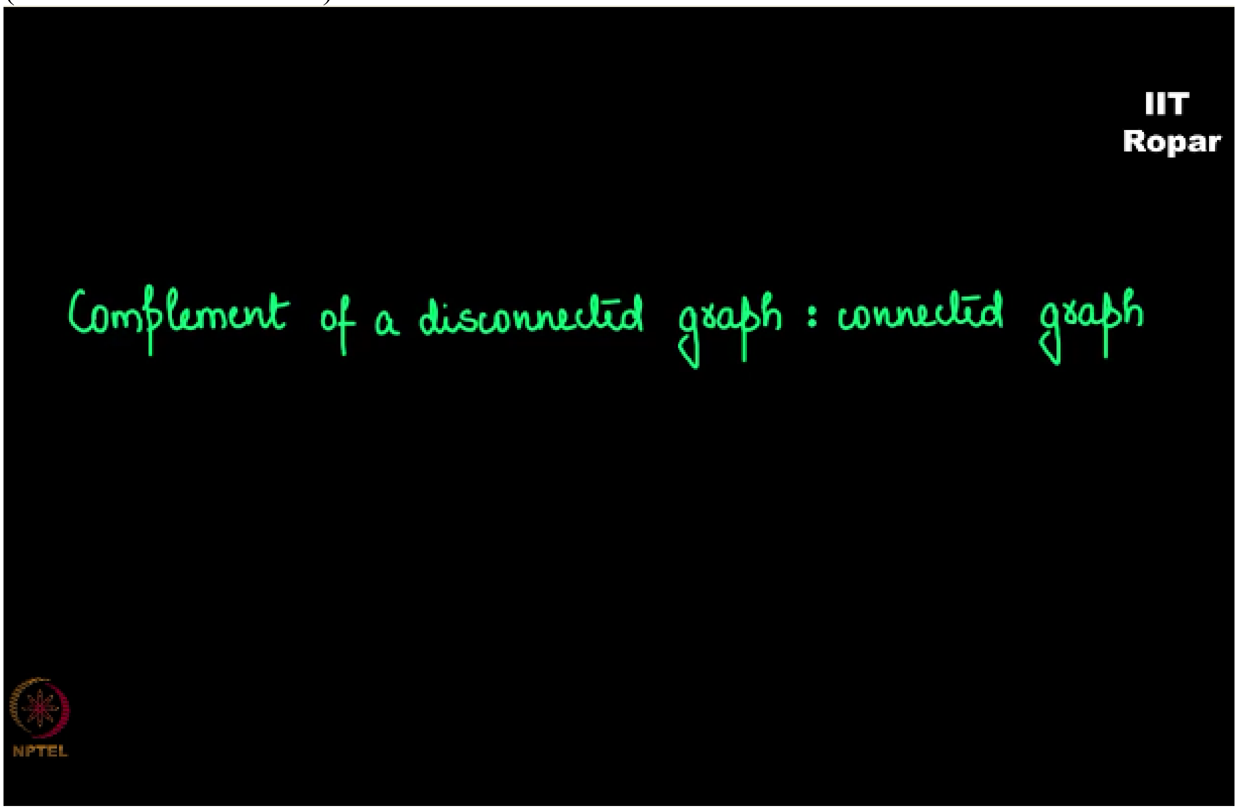

so what's the statement of the theorem? Complement of a connected graph we don't know what can happen with it, it can be connected or disconnected, statement of the theorem is complement of a disconnected graph is always connected, let's see the proof, firstly what do you mean by connected graph? Let's recall, connected graph means given any two vertices there is a path between them, right,
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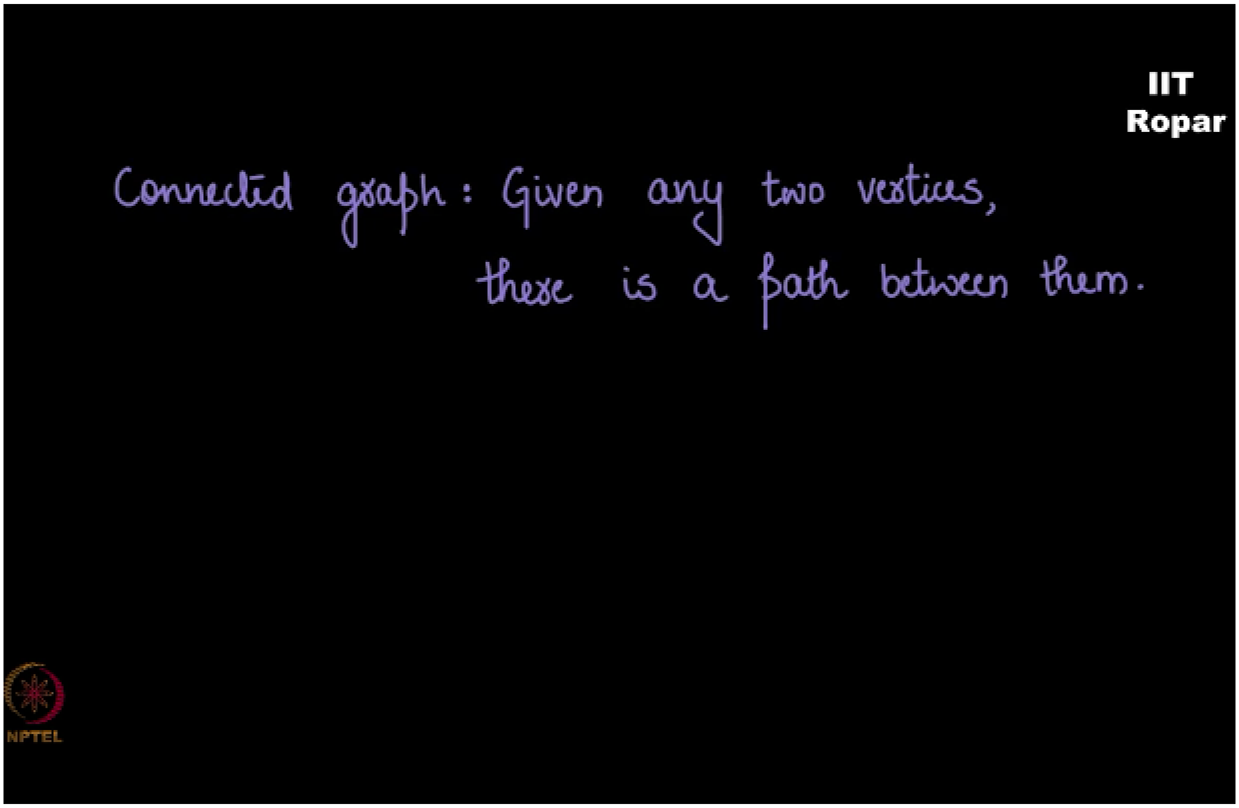

the very word connected in English means that two locations are connected if there is a road from this location to that location, although it can pass through several other locations, but there is a path, so by a connected graph we mean given any two pairs of vertices there is always a path between them, at least one path between them.

Now what's a definition of a disconnected graph? As you know that negation of this, what is that? There exists at least a pair of vertices where there is no path from U to V, correct,
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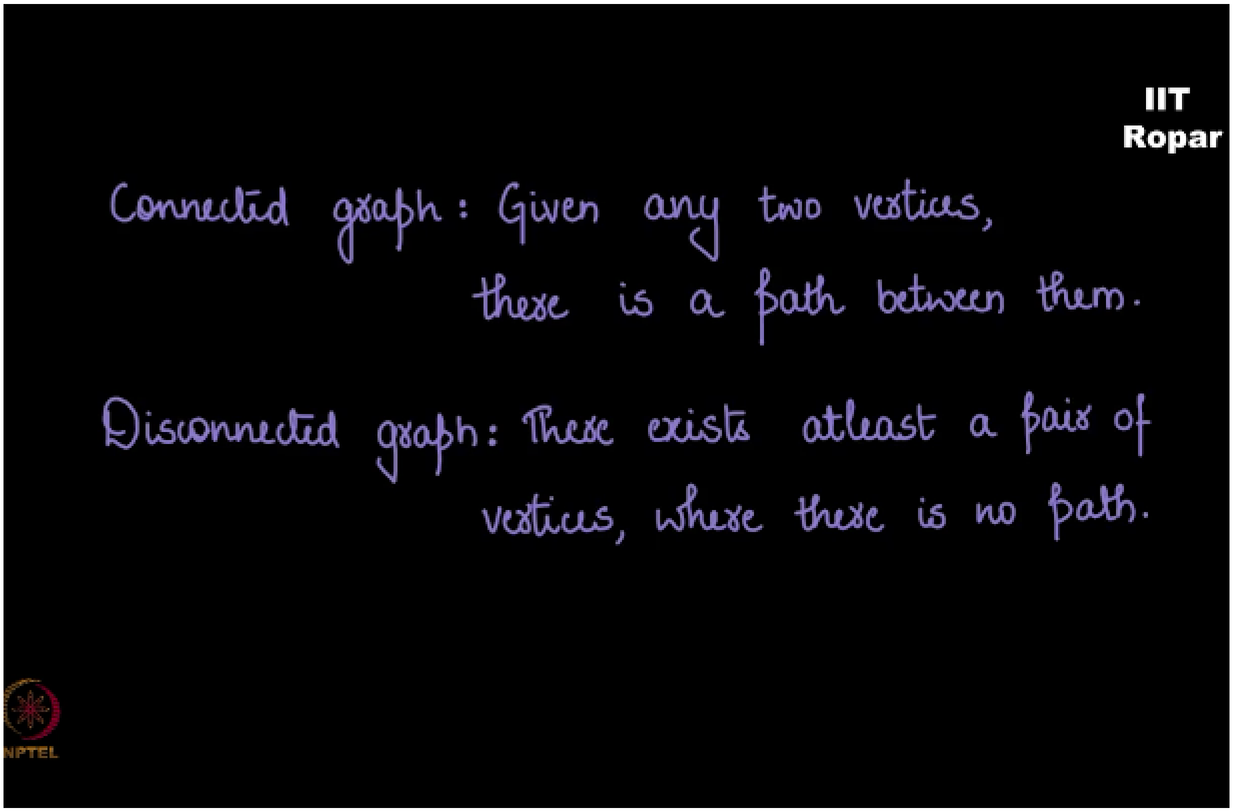

okay, let’s use this definition in showing that the complement of a disconnected graph is always connected, let us look at an example look at this example of a disconnected graph it has 2 components, component 1, component 2 of this disconnected graph,

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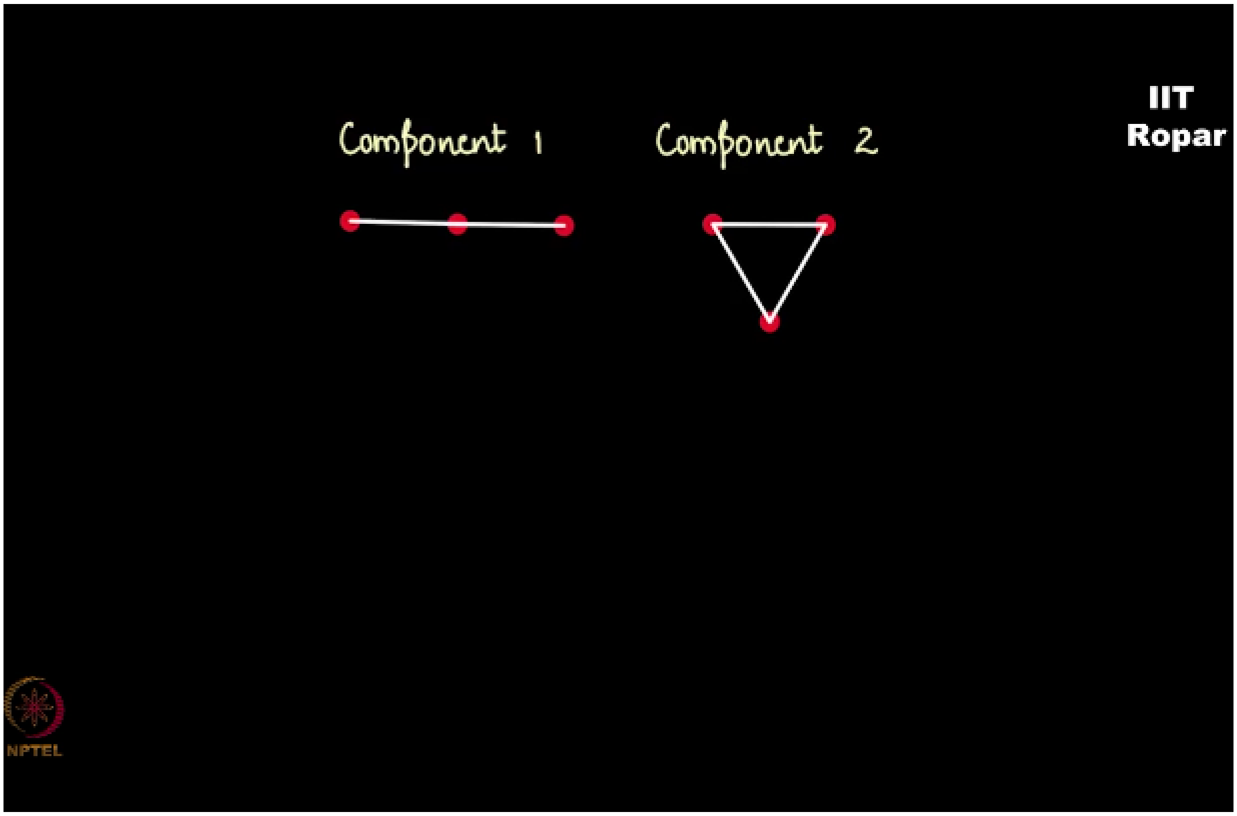

pick any two vertices, let's say one from component 1, one from component 2, there will definitely be an edge here, why? Because they're disconnected and hence there is no edge between a vertex in C1 and a vertex in C2, so this will actually have a simple one edge which means given a vertex in C1 and a vertex in C2 you have an edge which is actually the path as per the definition of connectedness you can find the path from this vertex U, to this vertex V.

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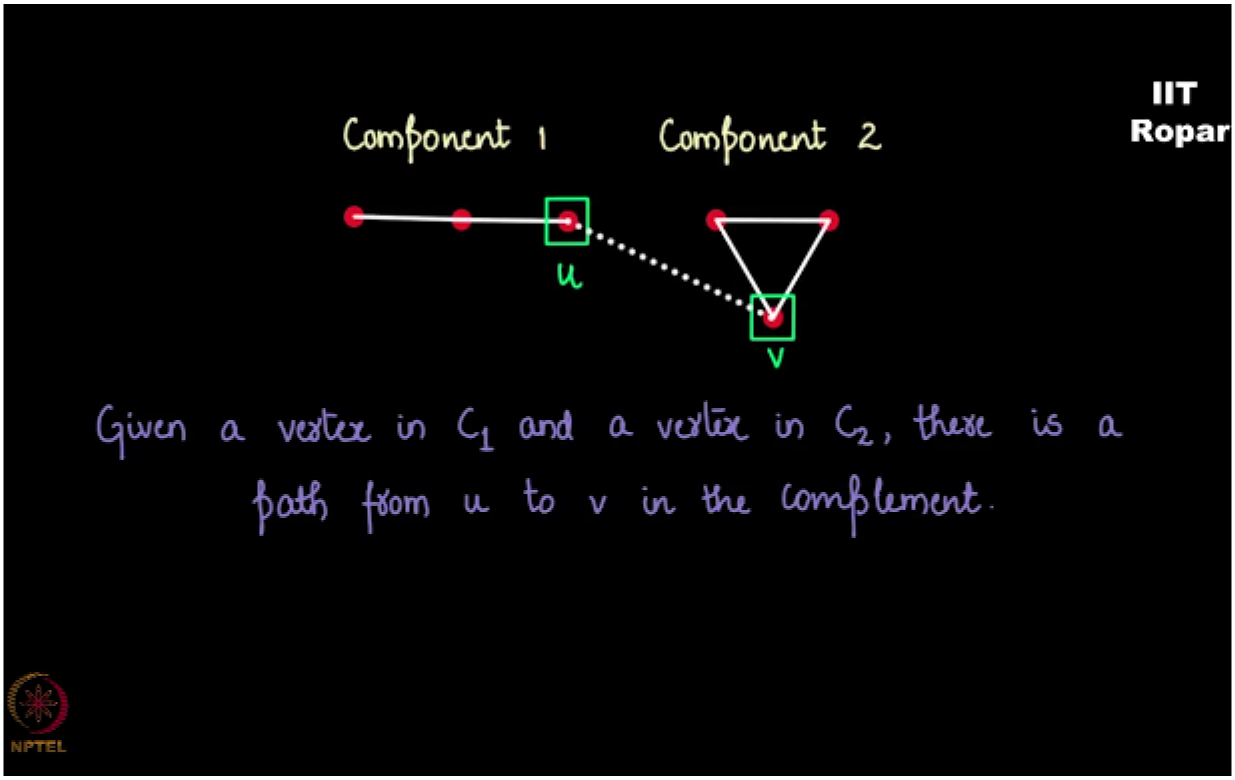

Now if you take two vertices within C1, let’s say X and Y within C1, moving C1 there is a path from X to Y, but in a complement will there be a path? Yes, there will be a path because pick any vertex alpha from C2, there is an edge from X to alpha, and an edge from Y to alpha, so there is a path from X to Y through alpha X, alpha Y.

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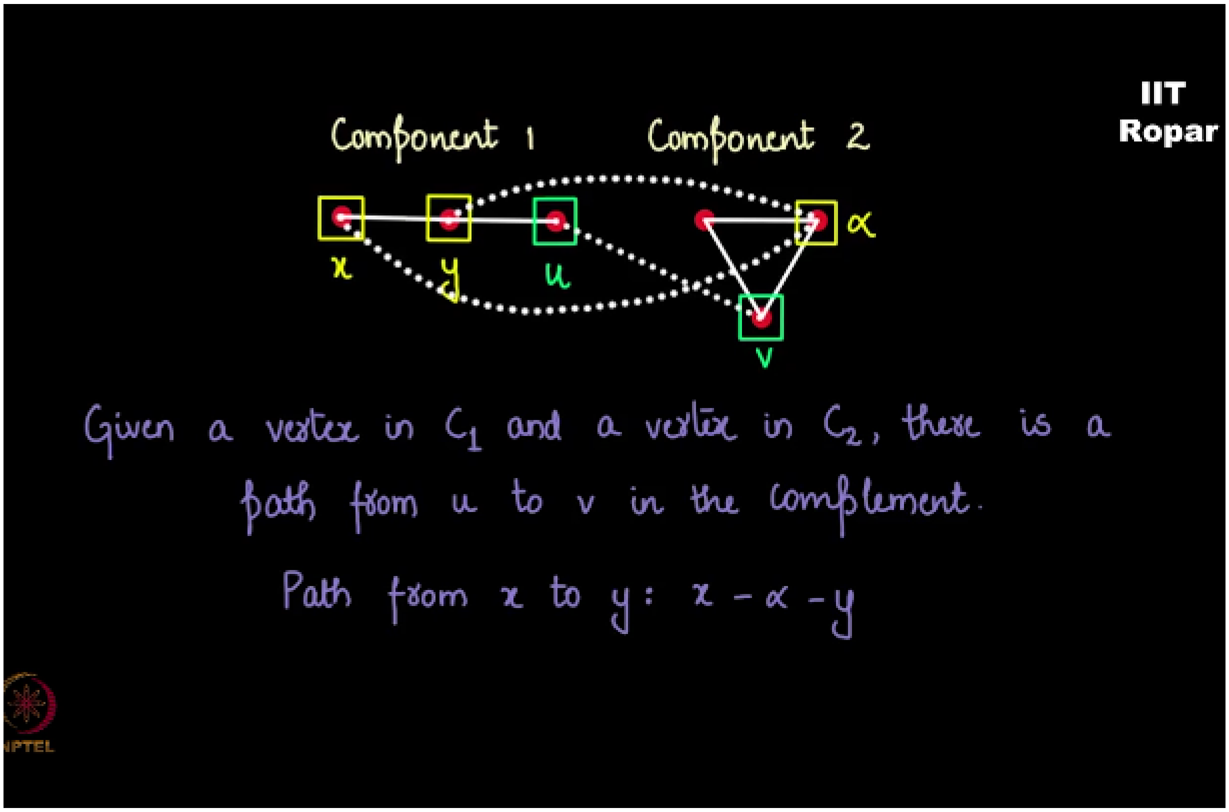

Similarly even in C2 if you take two vertices, there will be some beta in C1 through that you can always go to the other vertex in C2, starting from C2 which means I just showed you that all possible cases of you picking two vertices from this big graph comprising of two components you will always find a path from a vertex to any other vertex, which proves that my complement graph will actually be connected.

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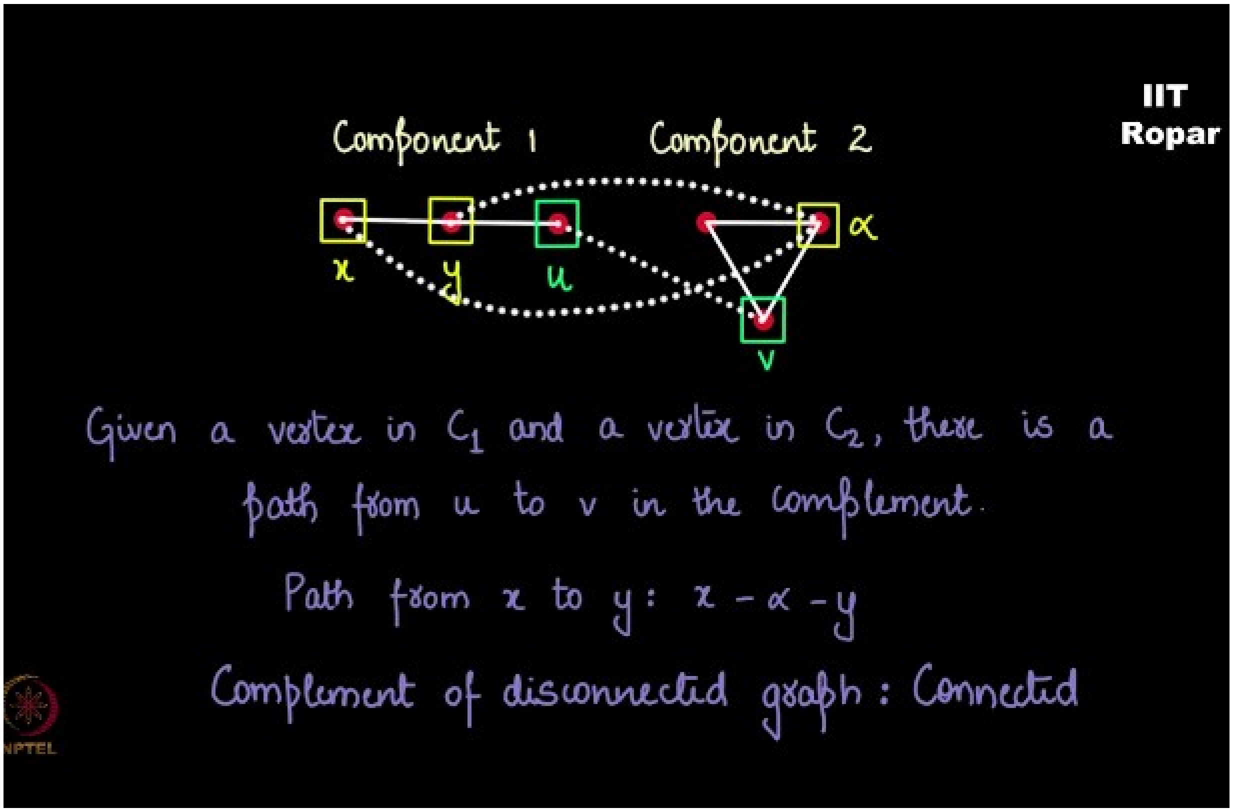

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